Sequential Location and Price Choices in Two-Dimensional Spatial Competition

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Abstract

This paper examines a sequential entry game in two-dimensional space covering a range of transportation costs for three different distributions defining consumer locations. Firms evaluate their location and pricing choices conditional on consumers' characteristics. As the transportation cost rises, we observe dramatic increases in prices, profits, and a decline in social welfare. In contrast to much of the spatial literature, the first mover does not always appear to have an advantage over subsequent entrants; a later entrant receives on average higher profit for a low transportation cost. As the transportation cost continues to increase, all firms' profits are slowly eroded. Further increases in transportation cost create spatial barriers between firms that transform the market structure from competitive to monopolistic. Firm distance (describing the level of product differentiation) is best characterized by a U-shaped curve with the minimum value realized for intermediate transportation cost.

JEL Classification: L11, L13

Key Words: Spatial competition, strategic location choice, sequential entry and pricing decisions

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[2]The authors would like to thank Dimitrios V. Papavassiliou and Jiandong Ju for valuable comments.

1 Introduction

This paper analyzes a two dimensional spatial model of oligopolistic competition with sequential entry and pricing decisions. We consider the effect of this market structure on firm behavior. We analyze firm-level results on relative prices, location decisions, and profits for three consumer distributions and a variable transportation cost. We also evaluate how market-level characteristics, such as social welfare, Herfindahl index, and product differentiation, are affected by increasing transportation costs.

In the seminal paper on localized spatial competition, Hotelling (1929) considers a framework of simultaneous choices. He evaluates a market structure in which products are differentiated by location and firms compete in prices. He identifies minimum product differentiation as the optimal solution using assumptions such as a linear transportation cost, a one dimensional surface, and a uniform distribution of consumers.

Since then, much of the literature has relaxed these and other assumptions of the Hotelling model in order to assess the robustness of the results. D'Aspremont et al. (1979) employ a quadratic, rather than linear, transportation cost, which produces an equilibrium characterized by maximum product differentiation. Economides (1986a) finds that firms do not realize maximum product differentiation for some specifications of the transportation cost. Eaton and Lipsey (1975) and Economides (1986b) relax the one-dimensional surface assumption by considering spatial competition occurring over a disk. Veendorp and Majeed (1995) investigate a two-dimensional rectangle with a uniform distribution of consumers. Further modifications to the traditional Hotelling model have been made by specifying alternative consumer distributions (see Claycombe (1996) for a bi-modal normal distribution). Nevertheless, in most cases the distribution used is uniform.

Using a sequential paradigm, our paper proposes to relax three assumptions of the Hotelling model. We will assess spatial competition (a) for three consumer distributions (uniform, corner and bi-modal bi-variate distributions) instead of the uniform distribution alone, (b) for a range of transportation costs,[3] and (c) in a two dimensional space allowing non-localized competition, in which every firm competes against the entire market. Our goal is to provide better heuristics for framing real-world problems. Comparison of results derived from generalizing these assumptions to those of the Hotelling framework will offer better insight into the robustness of the original assumptions.

Our methodological framework allows us to examine problems of price competition and firm location in geographic space. Alternatively, we can examine the selection of product characteristics whose desirability varies across the population. In the latter case, a consumer's location in space corresponds to the most preferred combination of characteristics defining an ideal product. In fact, the relative choice of location for a firm can be thought of as indicating intensity of competition across a set of product characteristics that define consumers' preferences in a fashion not necessarily monotonic. In this framework, we can consider horizontal differentiation, such as stylistic differences, across items. For example, we can consider differences across the range of classic versus contemporary styles for some products, shades of colors and sizes for others. Our methodology also helps us increase computational efficiency. We use a nodal network technique. We superimpose an flexible grid over the space and assign consumers to different nodes conditional on the population density.

When considering the spatial framework, our model can help explain business competition at different stages of urban development. In the 1970's urban areas were characterized by a central business district (CBD) surrounded by residential places. In the 1990's the CBD declined and suburban employment centers emerged (Button (2000)). The urban structure is the outcome of the interplay between the intensity of face to face communications and the level of commuting costs. Low commuting costs foster the emergence of a single CBD. Dispersed or polycentric structures may emerge when higher commuting costs prevail. Low levels of the transportation cost can push firms into agglomeration.[4] Our choice of three

[3]Here we assume quadratic transportation costs and vary the coefficient of the transportation cost component.

[4]There is empirical evidence that transportation costs have increased from the early 1980's to late 1990's.

distinct consumer distributions evaluated over a range of the transportation cost can shed light into the intensity of firm competition arising at various stages of urban development.

Our modeling process is similar to Neven (1987), Gotz (2005), and Economides et al. (2004), who employ a first stage sequential location decision. Our second stage is characterized by a sequential pricing decision. Similar to Economides (2004) and Veendorp (1995), we discretize the problem, and solve it using computational methods. We then use backward induction to identify the solution(s) to the spatial oligopolistic competition game.

We find that transportation costs play a critical role in characterizing the extent of spatial dispersion or, alternatively, the nature of product differentiation. Product differentiation increases as rising transportation costs initially transform the perfectly competitive market into a monopolistically competitive one. However, as the transportation cost continues to rise, firm behavior changes. At intermediate levels of the transportation cost, we observe a steep decline in differentiation. As the transportation cost increases even more, firms are driven further apart achieving greater differentiation and approaching a local monopolistic configuration. The essential reason is that firms are not guaranteed the purchases of fringe consumer groups since these groups may decide to drop out of the market due to the increased pressure of higher transportation costs.[5]

In contrast to much of the spatial literature, the first entrant does not appear to have an advantage over other competitors for the entire range of transportation costs. For low transportation costs, the benefit to being a subsequent entrant can be substantial. A subsequent firm with its location choice reduces the first mover's market share by making its product a less costly alternative (in terms of the transportation cost) to the first firm's product. In some sense, this location choice behavior might then be construed as undercutting. Close inspection

Using NHTS/NPTS data, Polzin (2006) show that there has been an increase in daily travel between 1983 and 2001 by 32.8 minutes. Our model indicates that, this increase in commute time can lead to an increase in spatial dispersion of the firms.

[5]A change in the transportation cost has two distinct, but opposing effects in our model. On the one hand, for low levels of transportation costs product differentiation increases placing an upward pressure on both prices and profits. On the other hand, for high levels of transportation cost the number of consumers in the market drop leading to reduced market share and profit.

of our results also reveals that later entrants tend to match or undercut the first entrant's price for low values of the transportation cost. This undercutting behavior produces higher profits on average for later entrants. Although these results might seem counter-intuitive in a sequential setting, Gal-Or (1985) has also identified a sequential game in which undercutting strategies make the second entrant's profits greater than the first mover's. However, as the transportation cost increases in our model, the incidences of price undercutting become less frequent and the importance of entry order diminishes.

The paper is organized as follows: Section 2 discusses the theoretical model, and section 3 analyzes our use of the nodal network. Section 4 outlines the set–up of the simulation, while section 5 provides a general interpretation of the model. Section 6 discusses the simulation results. Section 7 offers some concluding remarks.

2 Theoretical Model

A population of consumers is continuously distributed over $\{x \mid x \in R^2, 0 \leq x \leq 1\}$. Consumers are assigned to groups on the basis of their location in space. Consumer group i has an indirect utility function for firm j's product, which is given by

$$U_{ij} = \alpha_i - p_j - \lambda \left(\frac{\sqrt{\sum_{k=1}^{2} (x_{kj} - x_{ki})^2}}{\sqrt{2}} \right)^2 \tag{1}$$

where α_i is the reservation price of consumer group i, p_j is firm j's price, λ represents per unit transportation cost, x_{ki} is the location in the k^{th} dimension for consumer group i, x_{kj} is the location in the k^{th} dimension for firm j and the numerator of the last term represents the Euclidean norm capturing the distance between firm j and consumer group i. This measure is normalized to take values in the interval $[0, 1]$ in order to avoid scale issues as the dimensionality of the problem increases. Indirect utility is then assumed to be a quadratic function of the scaled norm distance. It is further assumed that $\alpha_i = 1 \; \forall \; i$.[6]

[6] The reservation price is set low enough to create the possibility that, for some firm choices, consumers will drop out of the market completely.

Consumer groups can choose from J total firms in the market. Let M be the sub-set of firms that, because of a combination of price and distance, yield the maximum positive utility level to consumer group i. The utility of this consumer group is given by the following function:

$$V_i = \{ \begin{array}{cc} \max(U_{im}) & \text{if} \quad max(U_{im}) \geq 0 \\ 0 & \text{otherwise} \end{array} \tag{2}$$

where $m \in M$. If $V_i = 0$, then group i chooses not to purchase the product. However, if V_i is greater than zero, then group i shops exclusively at one of the firms contained in M. In particular, group i's population, δ_i, is equally split between the h firms contained in M. Therefore, consumer group i's demand for firm j's product is given by

$$q_{ij} = \{ \begin{array}{cc} \frac{\delta_i}{h} & \text{if} \quad V_i > 0 \\ 0 & \text{otherwise} \end{array}$$

Firm j, having chosen a particular location, l_j (with coordinates x_{1j} and x_{2j}), and price, p_j (conditional on the prices, p_{-j}, and locations, l_{-j}, of the other firms), aggregates the consumers' demand for its product over all possible locations, L, to obtain its market demand

$$Q_j(p_j, l_j | p_{-j}, l_{-j}) = \sum_{i=1}^{L} q_{ij} \ . \tag{3}$$

A total of J firms participate in a two-stage sequential game. The first stage is characterized by the location decision while the second stage is defined by the pricing decision. Consumer group locations are fixed and provide, in the first stage, potential locations for the firms. Firms will not reside at the same location, since Bertrand price competition would drive profits down to zero.[7] Firms cannot re-locate once they have selected a location.

A firm cannot discern the residence of individual consumers, so it is unable to engage in spatial price discrimination. Therefore, it will select a single price, p_j, for all consumers from a set, P, which is made up of N discrete prices such that

$$P = \{p_1, p_2, ..., p_N\} = \left\{ \frac{\alpha_i}{N+1}, \frac{2 \cdot \alpha_i}{N+1}, ..., \frac{N \cdot \alpha_i}{N+1} \right\} \ . \tag{4}$$

[7] Economides (2004) considers a similar setting for price competition.

This set P is bounded above by α_i and below by zero (the marginal cost). At the very minimum, this range implies that consumers will always shop at the firm which shares their location. Two firms can charge the same prices.

There are $\frac{L!}{(L-J)!} \cdot N^J$ possible combinations for L locations, N prices, and J firms. Each firm's profit is calculated for every possible outcome. Firm j's profit function is given by [8]

$$\pi_j = p_j \cdot Q_j(p_j, l_j | p_{-j}, l_{-j}). \tag{5}$$

The decision making process for each firm is driven by three hierarchical objectives.[9] First, firms restrict their choice set to those which yield maximum expected profits ($E(\pi_j)$). Given that there are multiple solutions in this restricted set, a firm will narrow down its choice set by further maximizing its expected market share ($E(Q_j)$). Finally, if multiple solutions still exist, then the set is further refined by selecting those choices which maximize expected social welfare. Social welfare is given by

$$S = \sum_{i=1}^{L} V_i \ \delta_i. \tag{6}$$

This process is described in more technical details below.

For stage 1:

$$l_j = argmax_{l_j} \ E(S) \text{ s.t. } l_j \in \left(argmax_{l_j} \ E(Q_j) \text{ s.t. } l_j \in \left(argmax_{l_j} \ E(\pi_j) \text{ s.t. } l_j \in L \right) \right). \tag{7}$$

For stage 2:

$$p_j = argmax_{p_j} \ E(S) \text{ s.t. } p_j \in \left(argmax_{p_j} \ E(Q_j) \text{ s.t. } p_j \in \left(argmax_{p_j} \ E(\pi_j) \text{ s.t. } p_j \in P \right) \right). \tag{8}$$

[8] The maximum attainable profit for a monopolist operating in a market where all consumers ($Q_j = 1$) with reservation price $\alpha_i = 1$ are purchasing the good is given by $\pi_j = \frac{N}{N+1}$.

[9] This decision making process is based on three criteria that have been identified in the literature. Gannon (1973) explores the case in which firms maximize market share rather than profits in spatial competition. Hwang and Mai (1986) look at the difference in location patterns between the profit maximizing and welfare maximizing outcomes. Bade (2005) investigates a Cournot game with multiple objectives.

3 Nodal Locations

Individuals in the continuous consumer distribution (Ω) are assigned to various groups in order to make the problem more tractable.[10] This transformation is typically achieved by superimposing a grid over the consumer distribution. Areas defined between consecutive grid lines are known as grid cells. Integrating the consumer distribution contained in each grid cell yields the fraction of the population that resides within that particular area. Each grid cell can have a different population when the grid is formed by cells of equal area.

Insert Figure 1

The use of uniform grid cell sizes with varying population, however, imposes a substantial computational burden. Low population density grid cells would probably not be selected by a firm, yet they would produce a high computational cost.

In order to increase the computational efficiency of the simulations, we use a grid transformation so that the population in each transformed grid cell is equal. This transformation is done by first dividing the first dimension into equal population weight zones, and then subdividing these zones into equal population weight grid cells. We define the population weight of a Zone l to be:

$$Z_l = \frac{\int_0^1 \int_{x_1^S(l)}^{x_1^N(l)} \Omega(x_1, x_2) dx_1 dx_2}{\int_0^1 \int_0^1 \Omega(x_1, x_2) dx_1 dx_2}. \tag{9}$$

Then we define the population weight of grid cell l to be:

$$\omega_l = \frac{\int_{x_2^W(l)}^{x_2^E(l)} \int_{x_1^S(l)}^{x_1^N(l)} \Omega(x_1, x_2) dx_1 dx_2}{\int_0^1 \int_0^1 \Omega(x_1, x_2) dx_1 dx_2} \tag{10}$$

where $x_2^E(l)$, $x_2^W(l)$, $x_1^S(l)$, $x_1^N(l)$ are the East, West, South, and North boundaries of grid cell l, respectively. The minimum possible value for the West and South boundaries is zero, whereas the maximum possible value for the East and North boundaries is one.

[10] A consumer distribution integrated from negative infinity to positive infinity does not have to equal one. (Therefore, this distribution is not necessarily the same as a probability density function.)

Our goal is to generate a grid cell structure with identical population weights so that ω_l=ω for all grid cells. For example, if the number of possible firm locations is L, then we need $\omega = \frac{1}{L}$. Such a nodal structure would not only allow for a more efficient use of computational resources, but also for clustering of nodes at high population areas and dispersion of nodes at low population areas.

In order to achieve this goal of generating cells with identical population weights, we proceed as follows:

1. We vary the North boundary of the inner integral of the Zone equation (10) so that each Z_l has $\frac{1}{\sqrt{L}}$ of the population, where

$$\frac{1}{\sqrt{L}} = \frac{\int_0^1 \int_{x_1^S(l)}^{x_1^N(l)} \Omega(x_1, x_2) dx_1 dx_2}{\int_0^1 \int_0^1 \Omega(x_1, x_2) dx_1 dx_2}.$$

2. Having divided the first dimension into equal population weight zones, we then proceed to identify grid cell boundaries in the second dimension using equation (11). By varying the East boundary, each grid cell is forced to possess a population weight of $\frac{1}{L}$ where:

$$\frac{1}{L} = \frac{\int_{x_2^W(l)}^{x_2^E(l)} \int_{x_1^S(l)}^{x_1^N(l)} \Omega(x_1, x_2) dx_1 dx_2}{\int_0^1 \int_0^1 \Omega(x_1, x_2) dx_1 dx_2}.$$

An example might help clarify this process: Suppose we have 16 nodes. We move along the first dimension dividing it up into 4 zones. The following equation corresponds to the Z_1 calculation:

$$\frac{1}{4} = \frac{\int_0^1 \int_{x_1^S(1)}^{x_1^N(1)} \Omega(x_1, x_2) dx_1 dx_2}{\int_0^1 \int_0^1 \Omega(x_1, x_2) dx_1 dx_2}$$

where $x_1^S(1) = 0$. The North boundary, $x_1^N(l)$, is varied until the right hand side yields 25%.

The North boundary value for Z_1 then becomes the South boundary value for Z_2. This same process is repeated for the remaining three zones (see figure 2).

Insert Figure 2

Once we have identified the Zone boundaries, we then proceed to identify the grid cell boundaries. For the first grid cell, we operate in Z_1 and $x_2^W(1)$=0. We vary $x_2^E(1)$ until the area contained in that grid cell contains $\frac{1}{16}^{th}$ of the population, i.e.,

$$\frac{1}{16} = \frac{\int_{x_2^W(1)}^{x_2^E(1)} \int_{x_1^S(1)}^{x_1^N(1)} \Omega(x_1, x_2) dx_1 dx_2}{\int_0^1 \int_0^1 \Omega(x_1, x_2) dx_1 dx_2}.$$

Once $x_2^E(1)$ has been identified, then it becomes west boundary ($x_2^W(2)$) for the second grid cell (see figure 3). This process is repeated until the four grid cells contained in Z_1 have been identified.

Insert Figure 3

This process is then repeated for the three remaining zones. Having identified all grid cell boundaries the nodal location is assumed to be the spatial center of each grid cell, as shown in figure 4.

Insert Figure 4

4 Simulation Set-up

Since the simulation is based on more generalized assumptions than traditional models, inferences gathered from these results may provide better heuristics to explain the real world phenomena. Innovations such as two-dimensional space and various consumer distributions paint a more realistic picture of the market in which firms and consumers interact.

However, using a relaxed set of assumptions does create additional difficulties. Simulation runs produce Nash equilibrium outcomes that are not unique. In order to summarize and present the findings, results are averaged across the set of potential outcomes. The average values are then graphed against the transportation cost coefficient. Thus, inferences drawn from the graphs are based on an aggregation of the information from different outcomes.[11] In addition, obtaining these results is computationally intensive.

[11] The results are representative of the actual outcomes. Unfortunately, it impossible to present a set of graph for each equilibruim outcome due to their large number.

In our analysis, we evaluate the outcomes of these games by the profit, price, market share, social welfare, Herfindahl index[12], and the degree of product differentiation. Our measure of product differentiation, PD, is based on the mean distance among firms, $\bar{D}$, which is given by

$$\bar{D} \; = \frac{1}{0.5 \cdot (J^2 - \sum_{g=1}^{J} g)} \sum_{j=1}^{J-1} \sum_{g=j+1}^{J} \left(\sqrt{\sum_{k=1}^{2} (x_{kj} - x_{kg})^2} \right) \tag{11}$$

where j and g are both indexing firms.

When $\bar{D}$ increases, we observe a rise in product differentiation as firms locate further apart from each other. Therefore, high $\bar{D}$ values indicate that products in the market are different while low values indicate that they are similar. The maximum distance, $\bar{D}_{\max}$, and minimum distance, $\bar{D}_{\min}$, are identified from the set of all combinations of nodal locations for J firms.[13] Using the mean distance associated with the equilibrium locations, $\bar{D}_E$, we construct an index for product differentiation (PD) defined as

$$PD = \frac{\bar{D}_E - \bar{D}_{\min}}{\bar{D}_{\max} - \bar{D}_{\min}} \; . \tag{12}$$

These games are conditioned on the following input parameters: number of firms (J), number of prices (N), number of locations (L), transportation cost (λ) and consumer distribution (Ω). A consumer distribution, Ω, is used to identify the appropriate locations for our nodal network. There are three distributions for which the simulation is run: Ω_1 is uniform, Ω_2 has mass at its corners, and Ω_3 is a bi-variate, bi-modal distribution.

These distributions and their corresponding nodal arrangements are shown in figure 5. An analytical description of these distributions is provided in the appendix. As an example, Ω_3 has means (0.25, 0.25) and (0.85, 0.85) and standard deviation of 0.15 for all variables. It can

[12]The definition of the Herfindahl index is presented in appendix A.

[13]The minimum distance excludes the possibility that all firms will locate at the same node which can never occur in equilibrium.

be seen in figure 5 that this parameterization of Ω_3 yields nodal locations that are not spaced in a uniform manner.

Insert Figure 5

We define a discrete set of prices with N=4, 9, 19, 29. The range between $[0, \alpha_i]$ is divided up into 0.2 increments for N=4, 0.1 increments for N=9, 0.05 increments for N=19, and $0.0\overline{3}$ increments for N=29. As N increases, we approach the traditional assumption of price being a continuous variable.[14]

5 General Interpretation of the Model

This model can be viewed as a spatial model or alternatively as a product characteristic model. While the interpretation of the results is straightforward in the spatial model, the product characteristic model requires additional abstractions.

In the product characteristic model, a product is defined by its underlying characteristic(s). Each dimension represents a range of values that a given characteristic can take (for example, Color: red, orange, green, etc ...). In our two-dimensional model, there are two characteristics that define the product (for example, Color and Size). Consumers' preferences are defined over the product characteristics and are represented in the form of a distribution. Consumers are then separated into groups on the basis of their proclivity for a product with a particular set of characteristics.

Firms take into account consumer group preferences when evaluating what particular characteristics their products should possess. A firm's product is defined by its location in the underlying characteristic space. The mean distance between firms represents the extent that product differentiation is occurring in the market.

[14]The simulation was conducted for 16 nodes. Simulation runs with fewer nodes produced results that were similar, yet noisier. Due to the sequential nature of our model, the computational intensity can be very high in some cases. One of the final 16 node simulation runs, for example, included a combination of 29 prices, 3 firms and 31 values of lambda and, thus, required the evaluation of 2.54 billion records.

6 Discussion of Results

Results of simulation runs for a sequential game in locations and prices and for all distribution types are shown in figures 6 and 7. The bi-modal, bi-variate distribution has been parameterized with means (0.45, 0.45) and (0.65, 0.65) and a standard deviation of 0.65 for all variables.

While underlying location choices change with each distribution, we can nevertheless observe similar patterns of behavior across the three distribution types. This is especially true for uniform and corner distributions. These two distributions have very similar location patterns and choices, which yield very similar results.

For all distributions, low λ values generate markets that exhibit perfectly competitive characteristics. Low prices, low profits, and high social welfare are observed. As one increases λ from zero (the competitive outcome), spatial barriers between firms are created. Firms, seeking to maximize profits, compete based on pricing for fringe consumers groups. Firms, in some cases, will eliminate fringe consumer groups from their market demand in order to maximize their profits by raising their price. At extremely high values of λ, large spatial barriers exist. These spatial barriers induce firms to exhibit more monopolistic behavior (which is observed in the convergence of market demand and profits in the case of uniform and corner distributions. See figures 6a-b and d-e.).

Figures 6 and 7 display the simulation results. Firm-level information on profits, demand and prices is shown in figure 6. Market-level information on the index for product differentiation, social welfare and Herfindahl index is graphed in figure 7. Both firm- and market-level information is plotted as a function of λ. In order to facilitate the discussion of these results, the range of λ is divided into three intervals covering low, intermediate, and high values. Firms exhibit different patterns of behavior in each of these intervals. These results are discussed below.

Insert Figure 6

Insert Figure 7

6.1 Low Values of λ (0 to 1.5)

At very low values of λ, one observes competitive behavior taking place. Prices are extremely low (apparently approaching the zero marginal cost), and, as a result, profits for all firms are extremely low (see figures 6a-c and g-i). All consumer groups benefit from both low prices and low transportation cost. This is indicated by a large value for social welfare in figures 7d-f.

Over this range of λ, the framework yields some interesting insights into the Herfindahl index. Typically, as the market becomes more concentrated, we would expect firms to increase the markup over marginal cost. This price increase would decrease social welfare. Therefore, we would expect to see an inverse relationship between the Herfindahl index and social welfare.

In the spatial competition setting, however, one must also take into account the transportation cost and location of the firm. A firm in a spatial setting can charge a price similar to other firms, yet has a much larger market share due to its location choice. This large discrepancy in market share would produce a large Herfindahl index, while at the same time increase social welfare by reducing the individual's transportation cost due to the firm's location choice. Therefore, in a spatial competition framework, we can observe a positive link between social welfare and the Herfindahl index as shown in figures 7d-i.

As λ increases, prices and profits begin to increase dramatically. Increasing the transportation cost creates spatial barriers around firms insulating them to some extent from the behavior of other firms. This insulation allows firms to increase prices. These barriers are further reinforced by firms locating further apart from each other (see figures 7a-c). Thus, firms are taking advantage of the higher transportation cost in the form of location and price choices in order to realize higher profits. Each consumer group's utility is decreased by the increases in transportation cost and price. This is reflected as a significant decline in social welfare.

Firm specific results for all three distributions indicate that the first entrant observes the smallest increase in profits. It appears the first mover is at a disadvantage over this range of λ. However, this result is not unique to our work. Gal-Or (1985) showed that, when two identical players move sequentially, the player that moves first earns lower profit than the player that moves second, if the reaction functions of the players are upward sloping. Upward sloping reaction functions, for example, refer to markets in which there is price competition and followers are copying or undercutting the leader's choice.

The market demand faced by firms has quite a wide spread over this value of λ as shown in figures 6d-f. Typically, the last firm faces the largest market demand while the first faces the smallest. Firms' market shares are taken away by the strategic behavior of subsequent entrants.

6.2 Intermediate Values of λ (1.5 to 9)

After the initial increase in both prices and profits, further increases in λ place a downward pressure on both measures. The downward pressure in prices comes from the enticement of fringe consumer groups that are deciding whether to purchase the good from a particular firm. If fringe groups decide not to purchase from the firm, they can either purchase it from another firm (whose product generates a higher utility) or not purchase it at all (if all products generate negative utility).

At some point, however, the firm's profit maximizing choice is to drop the fringe consumer group and charge a higher price to the remaining consumer groups. This phenomenon is indicated by a positive jump in the price function (for λ approximately 8 in the case of the uniform distribution, and λ approximately 6 in the case of the corner distribution) and by a corresponding decline in market demand as fringe groups are dropped (see figures 6d-f and g-i).

The benefit of being the last mover is quickly eroded as most firms seem to have similar prices, profits and market shares. All firms in the market are observing small decline in prices, relatively stable market shares, and therefore decline in profit as λ increases. The Herfindahl index is relatively stable over this range taking on a value of about 0.34 for all distributions.

6.3 Large Values of λ (9 to 13)

At higher levels of transportation cost, we observe a convergence of price, market demand, and profits for each firm as they become insulated local monopolies. At higher levels of λ, only about 75% of the population is purchasing the good for the uniform and corner distribution. The remaining 25% does not purchase the good because it generates negative utility.

Each firm recognizes this and decides to locate far enough from its competitors by increasing product differentiation. This allows each firm to become a local monopoly. Inter-firm competition over fringe groups is less fierce since a quarter of the consumer population is not selecting to purchase the good from anyone. This decline in the size of the market produces a drop in social welfare.

6.4 Product Differentiation - Firm Location

Choices regarding the location of firms are relatively sensitive to the level of transportation cost presiding in the market (see figures 7a-c). The relationship between the level of transportation cost and the degree of dispersion is relatively robust to the various consumer distributions reviewed by this paper. In a perfectly competitive market (where $\lambda=0$), consumer preferences are unaffected by the location decision and therefore it is irrelevant to the firm's profit maximizing choice. At low values of λ, the location pattern of firms is dispersed, as shown by the index of product differentiation. Firms locate farther away in order to reduce competition and raise prices. However, as λ increases further, consumers experience a greater disutility from traveling, which means that certain consumer groups may stop purchasing the goods altogether. The firms' profit maximizing response is to minimize the distance to all consumer groups in order to retain those consumer groups who would otherwise drop out.

Firms accomplish this by moving to the market interior which implies that firm locations are closer together in (product) space. This is characterized by a decline in product differentiation. As λ continues to rise, however, the profit-maximizing firms decide to drop these fringe consumer groups and establish a local monopoly market through increased dispersion in the (product) space. The frequency of different equilibrium outcomes characterized by different degree of dispersion vary across the distributions as one can see in figures 8a-c. The uniform distribution supports a number of outcomes with both extreme product differentiation and relative homogeneity.

6.5 Comparison to the literature

A significant strand of literature focuses on the issue of product differentiation. De Palma et al. (1985) reformulate the Hotelling problem to show that the principle of minimum differentiation is restored when products and consumers are sufficiently heterogeneous. The principle is shown to hold when the degree of heterogeneity is sufficiently large. This amounts to adding a second, non-spatial dimension which arises from differences in tastes. In addition, firms cannot predict with certainty the decision of a particular consumer and they endow him with a probabilistic choice rule. The paper focuses on equilibria leading to agglomeration. There are no analytical results on the existence and nature of other equilibria. Figures 8a-c are histograms presenting the frequency of Nash equilibrium outcomes with different degrees of product differentiation. They reveal a few instances of agglomeration only for medium levels of the transportation cost. Those outcomes appear with higher frequency in the uniform distribution and are absent in the case of the corner distribution.

Economides (1984) considers product differentiation in a one dimensional two stage game where firms first decide on their location simultaneously and then choose prices for given locations. Consumers have single peaked utility functions in the space of characteristics and are distributed uniformly in the product space. Assuming, as we do here, that the reservation price is not too high so that there are customers at the edges of the market who cannot buy the differentiated products, the marginal relocation tendencies are opposite to the ones proved by Hotelling in the model where the reservation price was infinite. Firms are driven far apart and towards a local monopolistic configuration. We observe the same outcome in our model when the transportation cost is large enough to prevent consumers from purchasing.

Veendorp and Majeed (1995) relax the one-dimensional assumption of the Hotelling model. Their model investigates a simultaneous game in which two firms compete for a consumer population that is uniformly distributed over a rectangular surface and has high reservation prices. In contrast, our paper considers a sequential game characterized by three firms competing in a market where consumers are distributed in three different ways over a square surface and possess relatively low reservation prices.[15] Their model explains why firms differentiate their products in one dimension and not in another. They claim two forces at work. Due to the quadratic transportation cost firms are expected to aim for maximum product differentiation. At the same time making the difference minimal in one dimension they make price competition less severe. For most values of λ our model leads to some product differentiation in both dimensions.[16]

Other literature focuses on pricing decisions and their relationship to profits. In contrast to our solution, Lane (1980) uses a uniform population distribution and finds that firms locating first obtain, in general, higher profit. He considers a sequential model of a decision to locate in a one-dimensional product space and a simultaneous subsequent pricing decision. As Neven (1987) also claims, Lane is constructing the price equilibrium in a way that by assumption rules out all undercutting strategies, which by implication rules out Gal-Or's (1985) result.

Neven (1987) considers a more complete investigation of the strategy space. He considers a Hotelling model with a uniform distribution of consumers and derives a unique equilibrium in prices in a simultaneous choice game. He employs computational techniques to characterize

[15]Those reservation prices are low enough so that for high levels of the transportation cost some consumers decide to drop out of the market.

[16]We considered product differentiation along each dimension separately. There is no tendency for minimum product differentiation along one dimension alone. These results are available by the authors upon request.

the equilibria of the sequential location game. This location pattern involves a rather even distribution of firms over the product range. Economides et al. (2004) derive equilibrium locations, prices and profits in a similar framework. They also consider the level of consumer welfare and market concentration. They find that a range exists for which the profit of the second entrant is lower than that of the third entrant only in the model where entry deterrence is an issue. They also report a discontinuity in the location of firms in the sense that for certain parameter values the first entrant leaves the center location and switches position with the second firm.

7 Conclusions

This examination of a sequential entry game with a two dimensional product differentiation has made two important contributions to the existing literature. The first is a new methodology for analyzing spatial problems, which can be applied to a wide variety of economic problems. The second contribution is an examination of a specific spatial oligopoly problem.

This analysis generalizes three key assumptions of the traditional spatial problem and has provided some intriguing insights as the result of these generalizations. Using this new methodology, we were able to relax the assumption of a uniform distribution and test some alternative distributions. In addition, we were able to extend the model from one to two dimensional space using a quadratic transportation cost.

It is evident from our results that the level of the transportation cost is extremely important in the spatial problem. At low values of λ, higher profits can be made by later entrants while as λ increases convergence in profits is observed with the first entrant having a slight edge. Another interesting result is the shift in location in the (product) space with λ. The index of product differentiation, measuring relative distance, dips over intermediate ranges of λ. Finally, this paper reveals specific changes in firm behavior as we move from a perfectly competitive market to a monopolistic one.

There is an obvious extension within the immediate purview of this paper. An interesting, but involved, area for future research is to examine the concept of entry deterrence. Exploring this extension could provide additional insights into firm behavior operating in a two dimensional spatial setting.

More broadly, the methodology employed by this paper could be extended into other arenas where the spatial aspect of any phenomenon overlaps the economic realm. There are several disciplines within economics which are concerned with space. Examples of these spaces include a regional geographic area (regional science), an international area (international trade), product characteristics (industrial organization), or social space (Akerlof (1997)). All these different facets of economics could be modeled using the nodal network procedure.

8 References

[1] Akerlof, G. : Social distance and social decisions. Econometrica **65**, 1005-1027 (1997).

[2] Bade, S.: Nash equilibrium in games with incomplete preferences. Economic Theory **26**, 309-332 (2005).

[3] Button, K.: Where did the new urban economics Go after 25 years? In "Spatial Economic Science. New Frontiers in Theory and Methodology", edited by A. Reggiani Springer-Verlag, 2000.

[4] Claycombe, R.: Mill pricing and spatial price discrimination: monopoly performance and location with spatial retail markets. Journal of Regional Science **36**, 111-127 (1996).

[5] d'Aspremont, C., Gabszewicz, J., Thisse, J.: On Hotelling's 'Stability in Competition'. Econometrica **47**, 1145-1150 (1979).

[6] de Palma, A., Ginsburg V., Papageorgiou Y., Thisse, J.: The principle of minimum differentiation holds under sufficient heterogenity. Econometrica **53**, 767-782 (1985).

[7] Eaton, B., Lipsey, R.: The principle of minimum differentiation reconsidered: some new developments in the theory of spatial competition. Review of Economic Studies **42**, 27-49 (1975).

[8] Economides, N.: The principle of minimum differentiation revisited. European Economic Review **24**, 345-368 (1984).

[9] Economides, N.: Minimal and maximal product differentiation in Hotelling's duopoly. Economics Letters **21**, 67-71 (1986).

[10] Economides, N.: Nash equilibrium in duopoly with products defined by two characteristics. RAND Journal of Economics **17**, 431-439. (1986).

[11] Economides, N., Howell, J., Meza, S.: Does it pay to be first? sequential locational choice and foreclosure. Working Paper (2004).

[12] Gannon, C.: Optimization of market share in spatial competition. Southern Economic Journal **40**, 66-79 (1973).

[13] Gal-Or, E.: First mover and second mover advantages. International Economic Review **26**, 649-653 (1985).

[14] Gotz, G.: Endogenous sequential entry in a spatial model revisited. International Journal of Industrial Organization **23**, 249-261 (2005).

[15] Hwang, H. and Mai, C.: Welfare-maximizing location versus profit-maximizing location. Annals of Regional Science **20**, 54-64 (1986).

[16] Hotelling H.: Stability in competition. The Economic Journal **39**, 41-57 (1929).

[17] Lane W.: Product differentiation in a market with endogenous sequential entry. Bell Journal of Economics **11**, 237-260 (1980).

[18] Neven D.: Endogenous sequential entry in a spatial model. International Journal of Industrial Organization **5**, 419-434 (1987).

[19] Polzin S.E.: The case of moderate growth in vehicle miles of travel: A critical juncture in U.S. Travel behavior trends. Center for Urban Transportation Research, University of South Florida, April 2006.

[20] Veendorp, E., Majeed A.: Differentiation in a two-dimensional market. Regional Science and Urban Economics **25**, 75-83 (1995).

9 Appendix A - Equations

Distributions

The underlying distributions were chosen for their characteristic shapes. Uniform was selected because of its frequent use in the literature. The corner distribution with its mass on the corners might, perhaps, be more indicative of suburban growth and urban decay. The bi-modal, bi-variate case allows us to explore how two nearby population centers could impact the market.

Corner Distribution:

$$\Omega(x_1, x_2) = x_1^2 + x_2^2 + 1 - 3 \cdot x_1 + 3 \cdot x_1^2 - x_1^3 + 1 - 3 \cdot x_2 + 3 \cdot x_2^2 - x_2^3 \tag{13}$$

Bi-modal, Bi-variate Distribution:

$$\begin{aligned} \Omega((x_{11}, x_{21}), (x_{12}, x_{22})) \quad = \quad & \frac{1}{2 \cdot \pi \cdot \sigma^2} \cdot \exp\{-\frac{1}{2} \cdot \left(\left(\frac{x_{11} - \overline{x}_{11}}{\sigma} \right)^2 + \left(\frac{x_{21} - \overline{x}_{21}}{\sigma} \right)^2 \right)\} + \\ & \frac{1}{2 \cdot \pi \cdot \sigma^2} \cdot \exp\{-\frac{1}{2} \cdot \left(\left(\frac{x_{12} - \overline{x}_{12}}{\sigma} \right)^2 + \left(\frac{x_{22} - \overline{x}_{22}}{\sigma} \right)^2 \right)\} \end{aligned} \tag{14}$$

where σ is the same for all variables.

Descriptive Equations

The Herfindahl index is typically employed to measure the concentration of market shares for the largest firms. Markets with larger concentration ratios exhibit more imperfectly competitive characteristics.

Herfindahl Index:

$$H = \sum_{j=1}^{J} \left(\frac{Q_j(p_j, l_j | p_{-j}, l_{-j})}{\sum_{i=1}^{J} Q_i(p_i, l_i | p_{-i}, l_{-i})} \right)^2 \tag{15}$$

APPENDIX B - FIGURES

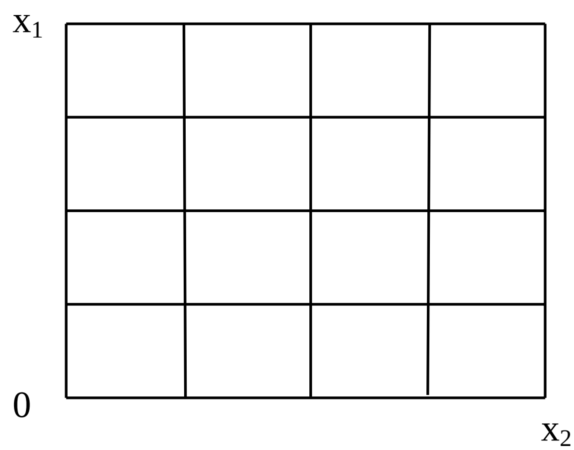

Figure 1. Two-dimensional uniform grid covering the space available for firm location.

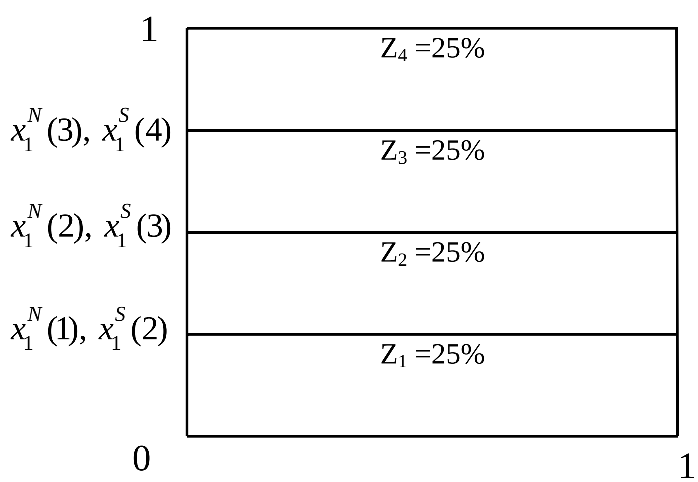

Figure 2. Division of the space available for firm location into four zones of equal population. (A notation example: $x_1^N(3)$ is the North edge of Z_3 which is also the south edge of Z_4).

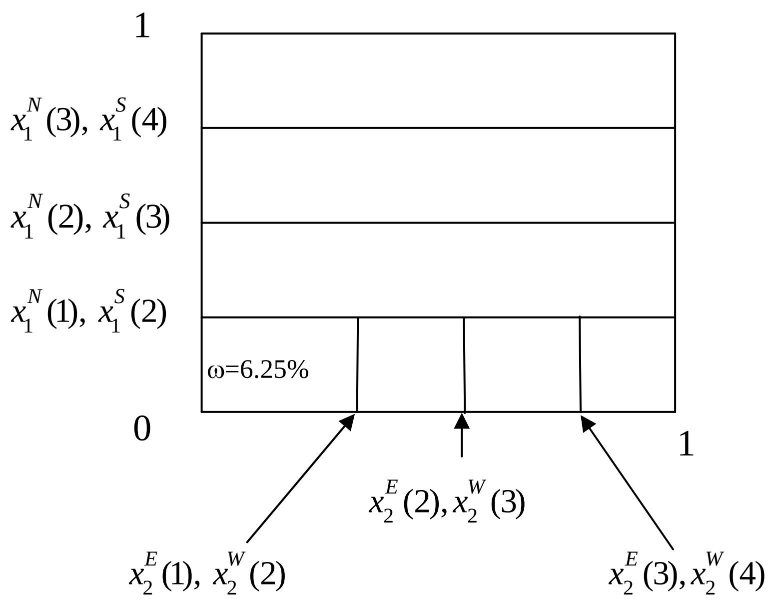

Figure 3. Division of a zone into grid cells of equal population for an example of 16 grid cells.

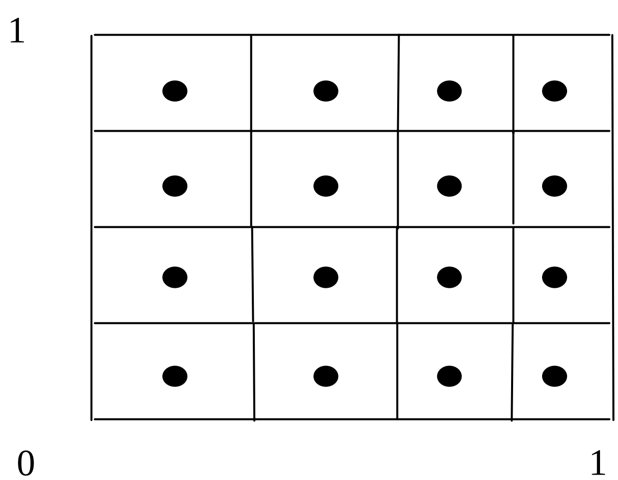

Figure 4. Nodal locations as the centers of each grid cell in a 16-cell example.

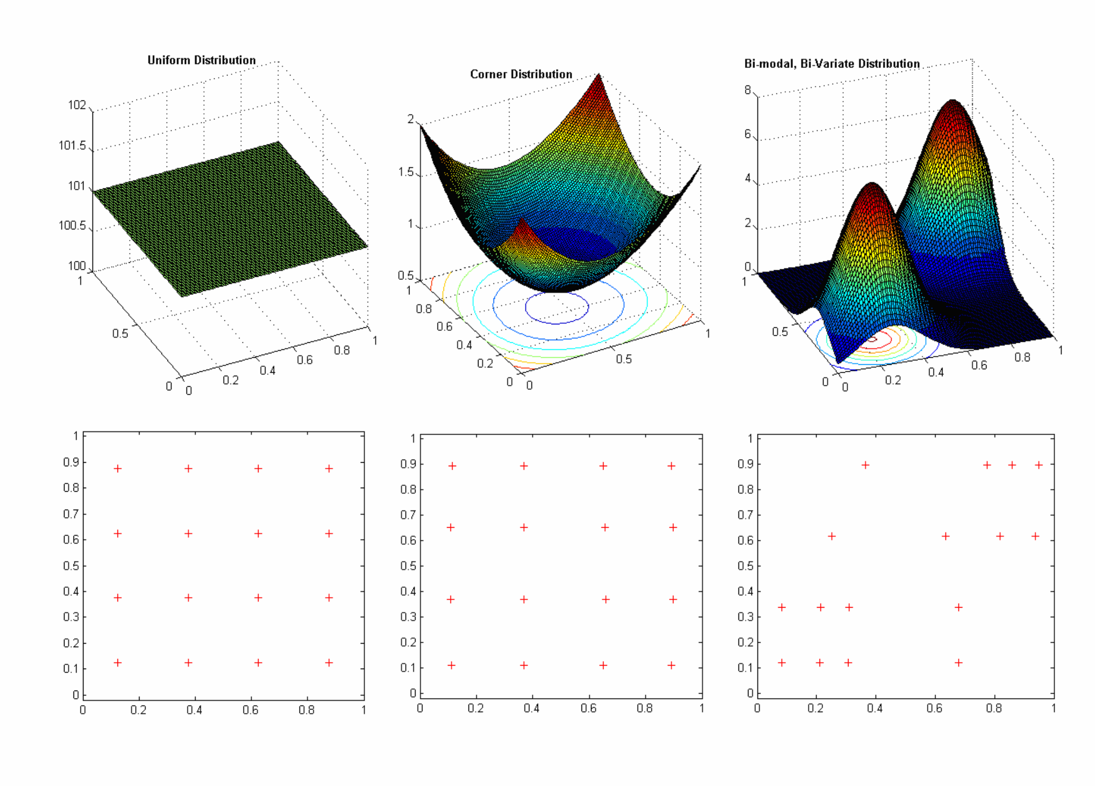

Figure 5. Consumer distributions and node locations for the three distributions employed.

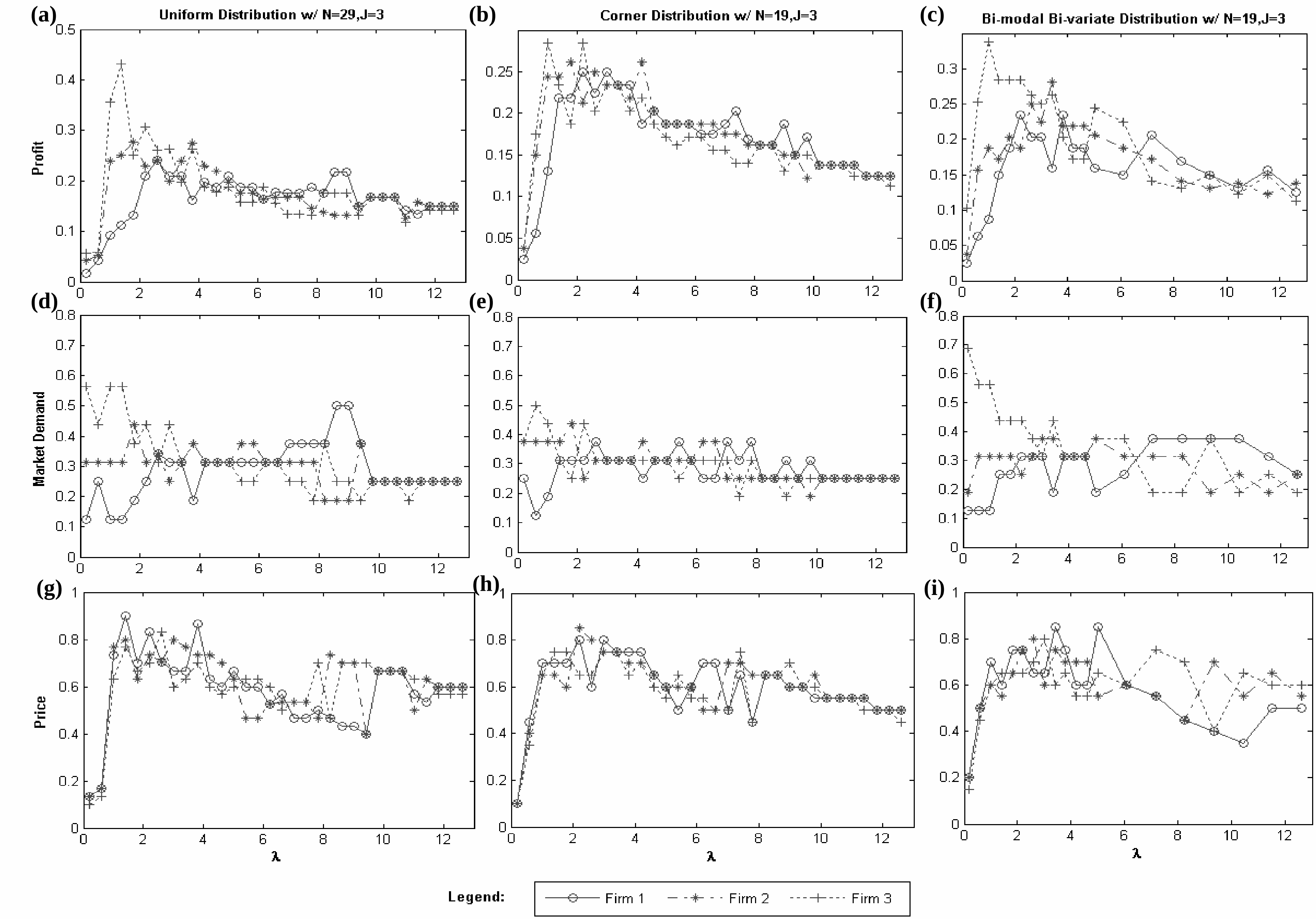

Figure 6. Profit, market demand, and price as a function of λ.

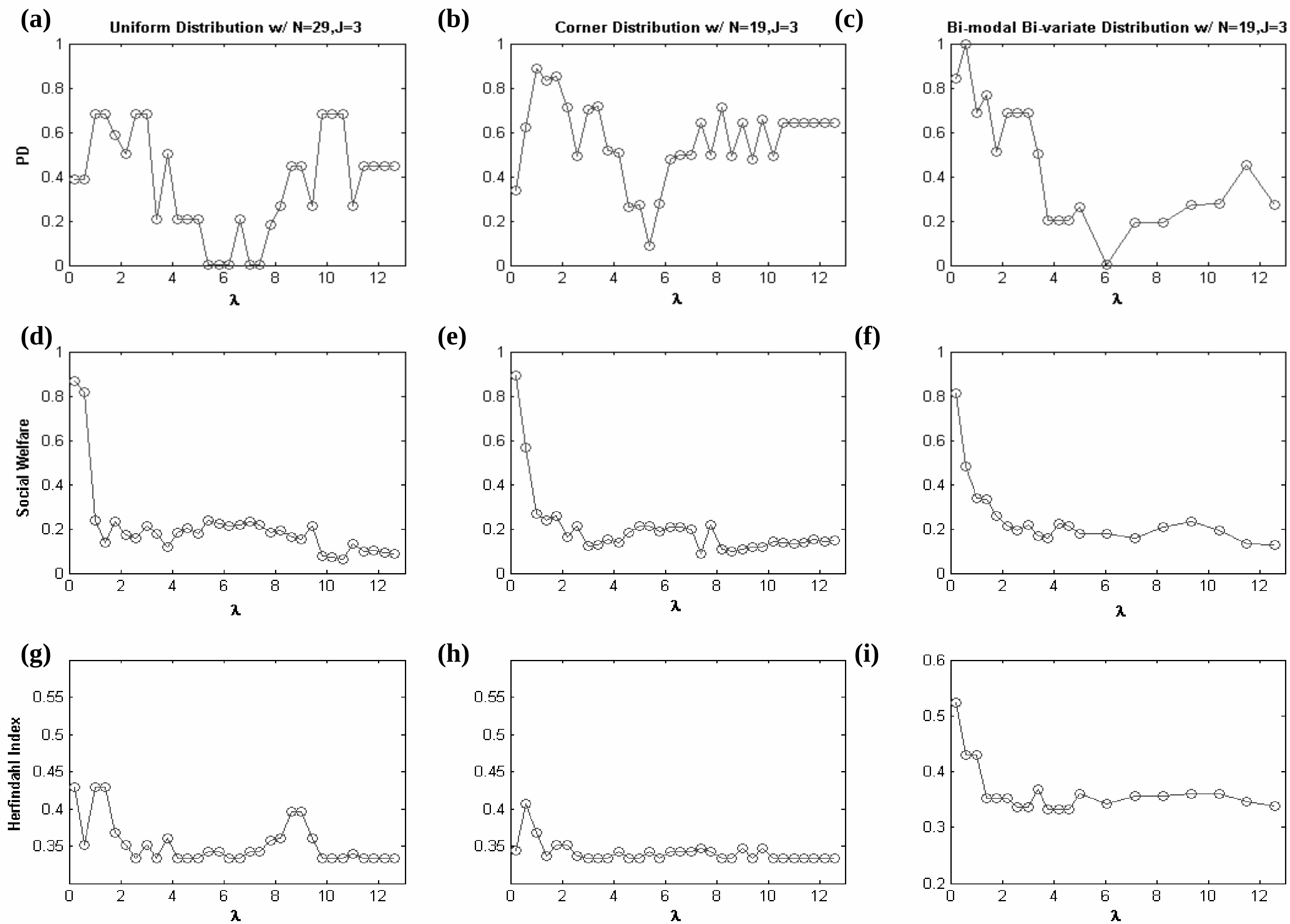

Figure 7. Location information, social welfare, and Herfindahl index as a function of λ.

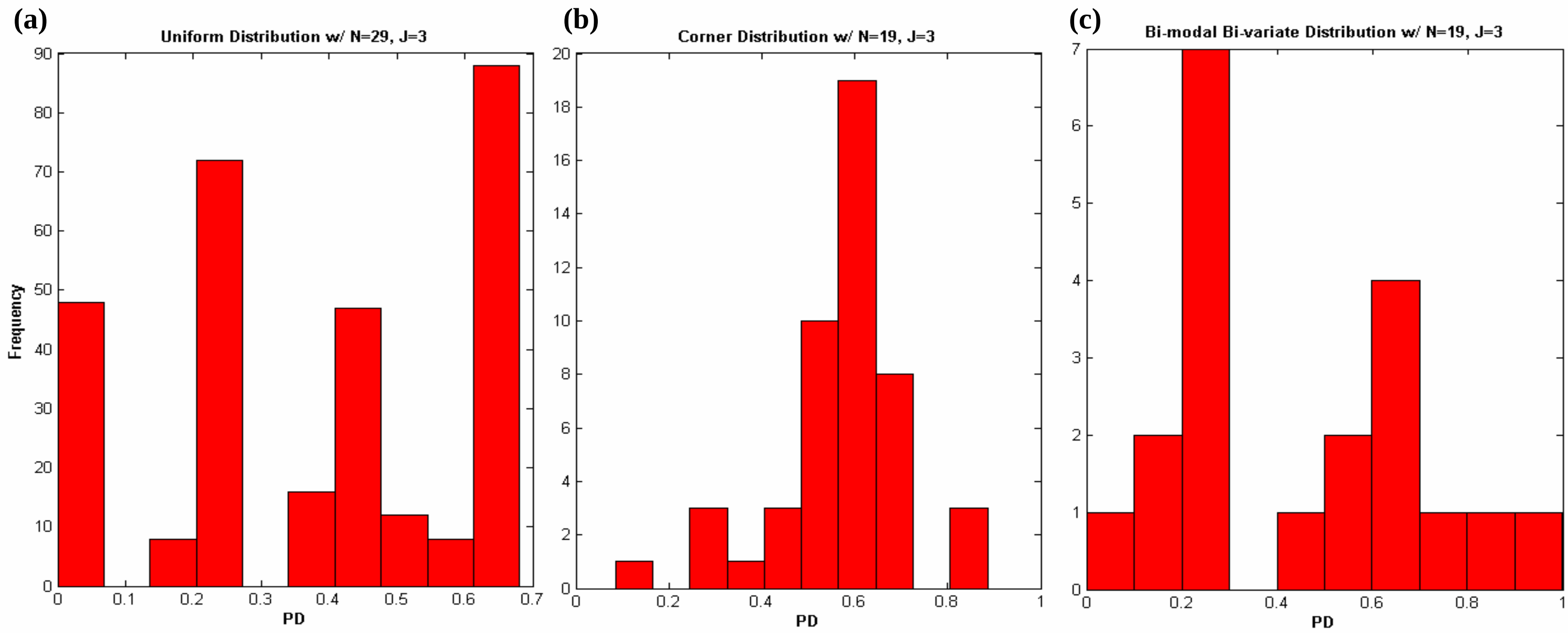

Figure 8. Frequency of product differentiation in two dimensional space for Nash equilibrium outcomes.