Homework 6

1) Solve by giving a 2-term asymptotic expansion

\[ \varepsilon y'' + \gamma y' - y = 0 \]
\[ y(0) = 0 \]
\[ y(1) = e^2 \]

2) Solve

\[ \frac{d^2 u}{dx^2} = \phi^2 \mu \nu \]
\[ \frac{d^2 \nu}{dx^2} = \phi^2 \mu \nu \]
\[ \mu(0) = 1 \quad \mu'(1) = 0 \]
\[ \nu(0) = 0 \quad \nu'(1) = 0 \]

3) Solve the hanging cable problem to whatever extent you can

\[ \varepsilon \phi'' + k \varepsilon \phi - 2 \varepsilon \phi = 0 \]
\[ \phi(0) = \phi(1) = 0 \]
5) \[ y + y + \varepsilon \left( -y' + \frac{1}{3} y'^3 \right) = 0 \quad \varepsilon \ll 1 \]
\[ y(0) = a \quad y'(0) = 0 \]

has only one periodic solution for some value \( a = a^* \). Find it using Strumian coordinates and give a 3-term expansion.

Hint: Expand \( y \) as a power series of \( \varepsilon \) and compute coefficients to avoid secular terms.

6) Let \( r(x, y) = xe^{\delta(x - \frac{1}{\phi})} \) in the CSTR problem.

Solve for \( \phi \gg 1 \) and for \( \phi \ll 1 \).

7) Let \( x^3 y''' - y = 0 \)

Find a solution of the form
\[ y = x^\mu e^{\delta(x)} \left[ 1 + a_1 x^\alpha + a_2 x^\beta + a_3 x^\gamma \right] \]

Attempt to obtain at least \( a_2 \) and \( a_3 \).

8) Let \( y^{(11)} + x^{-3/2} y' + x^{-2} y = 0 \).

a) \( x = \infty \) is an irregular singular point. Prove it.

b) If you try \( y = e^{\delta(x)} \). Assuming \( s^{(1)} \ll s^{(2)} \), terms out not to be correct. Try other balancing \( s^{(1)} \sim x^{-3/2} \), \( s^{(2)} \sim x^{-1/2} \), or \( s^{(1)} \ll x^{-1/2} \), until you find one that works. Show the solution is of the form
\[ y = x^{s_1} + x^{s_2} + x^{s_3} + \ldots \]