QUIZ #1 ENGR 3723

PROBLEM 1

Newton's law of cooling says that the temperature of a body (T)changes at a rate that is proportional to the difference between its temperature and the surrounding medium (usually the ambient) (T_a), that is:

$$\frac{dT}{dt} = -k(T - T_a)$$

where k is the heat transfer coefficient.

a) Obtain the formula for temperature vs. time

b) Integrate numerically and compare, for the temperature of a cup of coffee that is initially at 68^{o}C. (use k=0.017/min. You choose the value of T_a). What is the largest error incurred by the approximate formula?

ANSWER:

$$\frac{dT}{dt} = -k(T - T_a) \quad \Rightarrow \frac{dT}{(T - T_a)} = -kdt \quad \Rightarrow d\ln(T - T_a) = -kdt$$

$$\Rightarrow \quad \ln(T - T_a) = K - kt$$

But T=T_0 at t=0. *Then* $\ln(T_0 - T_a) = K$. Therefore:

$$\ln(T - T_a) = \ln(T_0 - T_a) - kt \quad \Rightarrow \quad \ln\frac{(T - T_a)}{(T_0 - T_a)} = -kt \quad \Rightarrow \quad \frac{(T - T_a)}{(T_0 - T_a)} = e^{-kt}$$

$$\Rightarrow \quad (T - T_a) = (T_0 - T_a)e^{-kt} \qquad \Rightarrow \quad T = T_a + (T_0 - T_a)e^{-kt}$$

Therefore: $$T = T_a + (T_0 - T_a)e^{-kt}$$

Numerical integration : $\frac{dT}{dt} \cong \frac{(T_{i+1} - T_i)}{\Delta t}$. Therefore

$\frac{(T_{i+1} - T_i)}{\Delta t} \cong -k(T_i - T_a) \quad \Rightarrow \quad T_{i+1} \cong T_i - k(T_i - T_a)\Delta t$ See attached Excel file for the implementation

PROBLEM 2

Determine the number of terms needed to approximate $\cos x$ to 8 significant figures. For x=0.3π. Use

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - ....$$

ANSWER:

$$\frac{x^2}{2} = \frac{(0.3\pi)^2}{2} = 0.4444132198 \qquad \cos x = 1 - \frac{x^2}{2} = 0.555867802$$

$$\frac{x^4}{4!} = \frac{(0.3\pi)^4}{24} = 0.032875568 \qquad \cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} = 0.588743370$$

$$\frac{x^6}{6!} = \frac{(0.3\pi)^6}{720} = 0.000973407 \qquad \cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} = 0.587769964$$

Looks like the second significant figure will not change anymore

$$\frac{x^8}{8!} = \frac{(0.3\pi)^8}{40320} = 0.00001544 \qquad \cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} = 0.587785404$$

Looks like the fourth significant figure will not change anymore

$$\frac{x^{10}}{10!} = \frac{(0.3\pi)^{10}}{3628800} = 0.000000152387 \quad \cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} = 0.587785251$$

Looks like the sixth significant figure will not change anymore

$$\frac{x^{12}}{12!} = \frac{(0.3\pi)^{12}}{479001600} = 0.00000000102545$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} = 0.587785251$$

Looks like the eighth significant figure will not change anymore

THE ANSWER IS 7 terms (this includes the zero order term : 1)