QUIZ #2 ENGR 3723

PROBLEM 1

Write down

a) the first 5 terms of the Taylor expansion for a function of one variable $f(x)$.

ANSWER:

$$f(x_2) = f(x_1) + f'(x_1)(x_2 - x_1) + f''(x_1)\frac{(x_2 - x_1)^2}{2!} + f'''(x_1)\frac{(x_2 - x_1)^3}{3!} + f^{(iv)}(x_1)\frac{(x_2 - x_1)^4}{4!} + \dots\dots$$

b) All terms of the Taylor expansion terms of a function of three variables, $f(x_1, x_2, x_3)$, up to the third derivative.

ANSWER:

$$\begin{aligned}
f(x_{1,i+1}, x_{2,i+1}, x_{3,i+1}) = {} & f(x_{1,i}, x_{2,i}, x_{3i}) + \frac{\partial f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_1}(x_{1,i+1}, -x_{1,i}) \\
& + \frac{\partial f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_2}(x_{2,i+1}, -x_{2,i}) \quad + \frac{\partial f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_3}(x_{3,i+1}, -x_{3,i}) + \\
& + \frac{\partial^2 f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_1^2}\frac{(x_{1,i+1}, -x_{1,i})^2}{2!} \\
& + \frac{\partial^2 f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_2^2}\frac{(x_{2,i+1}, -x_{2,i})^2}{2!} \quad + \frac{\partial^2 f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_3^2}\frac{(x_{3,i+1}, -x_{3,i})^2}{2!} \\
& + \frac{\partial^2 f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_1 \partial x_2}\frac{(x_{1,i+1}, -x_{1,i})\ (x_{2,i+1}, -x_{2,i})}{2!} \\
& + \frac{\partial^2 f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_1 \partial x_3}\frac{(x_{1,i+1}, -x_{1,i})\ (x_{3,i+1}, -x_{3,i})}{2!} \\
& + \frac{\partial^2 f(x_{1,i}, x_{2,i}, x_{3,i})}{\partial x_2 \partial x_3}\frac{(x_{2,i+1}, -x_{2,i})\ (x_{3,i+1}, -x_{3,i})}{2!}
\end{aligned}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_1^3}\frac{(x_{1,i+1},-x_{1,i})^3}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_2^3}\frac{(x_{2,i+1},-x_{2,i})^2}{3!} \quad +\frac{\partial^3 f(x_{1,i},x_{2,i},x_{2i})}{\partial x_3^2}\frac{(x_{1,i+1},-x_{1,i})^3}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_1\,\partial x_2\partial x_3}\frac{(x_{1,i+1},-x_{1,i})(x_{2,i+1},-x_{2,i})\ (x_{3,i+1},-x_{3,i})}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_1\,\partial x_2^2}\frac{(x_{1,i+1},-x_{1,i})(x_{2,i+1},-x_{2,i})^2}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_1\,\partial x_3^2}\frac{(x_{1,i+1},-x_{1,i})(x_{3,i+1},-x_{3,i})^2}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_2\partial x_1^2}\frac{(x_{2,i+1},-x_{2,i})(x_{1,i+1},-x_{1,i})^2}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_2\partial x_3^2}\frac{(x_{2,i+1},-x_{2,i})(x_{3,i+1},-x_{2,i})^2}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_3\partial x_1^2}\frac{(x_{3,i+1},-x_{3,i})(x_{1,i+1},-x_{1,i})^2}{3!}$$

$$+\frac{\partial^3 f(x_{1,i},x_{2,i},x_{3,i})}{\partial x_3\partial x_2^2}\frac{(x_{3,i+1},-x_{3,i})(x_{2,i+1},-x_{2,i})^2}{3!}$$

c) The general expression (using summations) of the Taylor expansion of a function of n variables $f(x_1, x_2, \ldots\ldots, x_n)$.

ANSWER:

$$f(x_{1,i+1},x_{2,i+1},\ldots,x_{n,i+1}) = f(x_{1,i},x_{2,i},\ldots,x_{n,i})$$

$$+\frac{1}{2!}\sum_{i_1=1}^{n}\left[\frac{\partial^2 f(x_{1,i},x_{2,i},\ldots,x_{n,i})}{\partial x_{i_1}^2}(x_{i_1,i+1},-x_{i_1,i})^2\right]$$

$$+\sum_{i_1=1}^{n}\ \ldots\sum_{\substack{i_2=1\\ i_2\neq 1}}^{n}\left[\frac{\partial^2 f(x_{1,i},x_{2,i},\ldots,x_{n,i})}{\partial x_{i_1}\partial x_{i_2}^2}(x_{i_1,i+1},-x_{i_1,i})(x_{i_2,i+1},-x_{i_2,i})\right] \quad +\ldots\ldots$$

<u>PROBLEM 2</u>

Consider

$$f(x) = \frac{1.225}{x}\left[1 - e^{-2*x}\right] - 1 = 0$$

The root is around 1.1. Apply bisection, step by step and show all your work. Start at x_{low} and x_{high}, one below 1 and the other above 1.5.

ANSWER:

xlow	xhigh	xr	flow	fhigh	f(xr)
0.5	1.6	1.05	0.548695	-0.26558	0.023801
1.05	1.6	1.325	0.023801	-0.26558	-0.14079
1.05	1.325	1.1875	0.023801	-0.14079	-0.06437
1.05	1.1875	1.11875	0.023801	-0.06437	-0.02189
1.05	1.11875	1.084375	0.023801	-0.02189	0.000537
1.084375	1.11875	1.101563	0.000537	-0.02189	-0.01078
1.084375	1.101563	1.092969	0.000537	-0.01078	-0.00515
1.084375	1.092969	1.088672	0.000537	-0.00515	-0.00231
1.084375	1.088672	1.086523	0.000537	-0.00231	-0.00089
1.084375	1.086523	1.085449	0.000537	-0.00089	-0.00018
1.084375	1.085449	1.084912	0.000537	-0.00018	0.00018