**PROBLEM 1**

Write down

a) the first 5 terms of the Taylor expansion for a function of one variable $f(x)$.

**ANSWER:**

$$f(x_2) = f(x_1) + f'(x_1)(x_2 - x_1) + \frac{f''(x_1)}{2!}(x_2 - x_1)^2 + \frac{f'''(x_1)}{3!}(x_2 - x_1)^3$$

$$+ \frac{f^{(iv)}(x_1)}{4!}(x_2 - x_1)^4 + \ldots$$

b) All terms of the Taylor expansion terms of a function of three variables, $f(x_i, x_2, x_3)$, up to the third derivative.

**ANSWER:**

$$f(x_{i+1}, x_{2,j+1}, x_{3,k+1}) = f(x_{i,j}, x_{2,j}, x_{3,k}) + \frac{\partial f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_i}(x_{i+1,j} - x_{i,j})$$

$$+ \frac{\partial f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_2}(x_{2,j+1} - x_{2,j}) + \frac{\partial f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_3}(x_{3,j+1} - x_{3,j})$$

$$+ \frac{\partial^2 f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_i^2}(x_{i+1,j} - x_{i,j})^2$$

$$+ \frac{\partial^2 f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_2^2}(x_{2,j+1} - x_{2,j})^2$$

$$+ \frac{\partial^2 f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_3^2}(x_{3,j+1} - x_{3,j})^2$$

$$+ \frac{\partial^2 f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_i \partial x_2}(x_{i+1,j} - x_{i,j})(x_{2,j+1} - x_{2,j})$$

$$+ \frac{\partial^2 f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_i \partial x_3}(x_{i+1,j} - x_{i,j})(x_{3,j+1} - x_{3,j})$$

$$+ \frac{\partial^2 f(x_{i,j}, x_{2,j}, x_{3,k})}{\partial x_2 \partial x_3}(x_{2,j+1} - x_{2,j})(x_{3,j+1} - x_{3,j})$$
c) The general expression (using summations) of the Taylor expansion of a function of \( n \) variables \( f(x_1, x_2, \ldots, x_n) \).

\[ f(x_{1,i+1}, x_{2,i+1}, \ldots, x_{n,i+1}) = f(x_1, x_2, \ldots, x_n) \]
\[ + \frac{1}{2!} \sum_{i=1}^{n} \left[ \frac{\partial^2 f(x_1, x_2, \ldots, x_n)}{\partial x_i^2} (x_{i,i+1}, -x_{i,i})^2 \right] \]
\[ + \sum_{i_1=1}^{n-1} \ldots \sum_{i_2=1}^{n-1} \sum_{i_3=1}^{n-1} \left[ \frac{\partial^2 f(x_1, x_2, \ldots, x_n)}{\partial x_{i_1} \partial x_{i_2}^2} (x_{i_1,i+1}, x_{i_2,i+1}, -x_{i_1,i}) (x_{i_2,i+1}, -x_{i_1,i}) \right] + \ldots \]
PROBLEM 2

Consider

\[ f(x) = \frac{1.225}{x} \left[ 1 - e^{-2x} \right] - 1 = 0 \]

The root is around 1.1. Apply bisection, step by step and show all your work. Start at \( x_{low} \) and \( x_{high} \), one below 1 and the other above 1.5.

<table>
<thead>
<tr>
<th>xlow</th>
<th>xhigh</th>
<th>xr</th>
<th>flow</th>
<th>fh high</th>
<th>f(xr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.6</td>
<td>1.05</td>
<td>0.548695</td>
<td>0.26558</td>
<td>0.023801</td>
</tr>
<tr>
<td>1.05</td>
<td>1.6</td>
<td>1.325</td>
<td>0.023801</td>
<td>0.26558</td>
<td>-0.14079</td>
</tr>
<tr>
<td>1.05</td>
<td>1.325</td>
<td>1.1875</td>
<td>0.023801</td>
<td>0.14079</td>
<td>-0.06437</td>
</tr>
<tr>
<td>1.05</td>
<td>1.1875</td>
<td>1.11875</td>
<td>0.023801</td>
<td>0.06437</td>
<td>-0.02189</td>
</tr>
<tr>
<td>1.05</td>
<td>1.11875</td>
<td>1.084375</td>
<td>0.023801</td>
<td>0.02189</td>
<td>0.000537</td>
</tr>
<tr>
<td>1.084375</td>
<td>1.11875</td>
<td>1.101563</td>
<td>0.000537</td>
<td>0.02189</td>
<td>-0.01078</td>
</tr>
<tr>
<td>1.084375</td>
<td>1.101563</td>
<td>1.092969</td>
<td>0.000537</td>
<td>0.01078</td>
<td>-0.00515</td>
</tr>
<tr>
<td>1.084375</td>
<td>1.092969</td>
<td>1.088672</td>
<td>0.000537</td>
<td>0.00515</td>
<td>-0.00231</td>
</tr>
<tr>
<td>1.084375</td>
<td>1.088672</td>
<td>1.086523</td>
<td>0.000537</td>
<td>0.00231</td>
<td>-0.00089</td>
</tr>
<tr>
<td>1.084375</td>
<td>1.086523</td>
<td>1.085449</td>
<td>0.000537</td>
<td>0.00089</td>
<td>-0.00018</td>
</tr>
<tr>
<td>1.084375</td>
<td>1.085449</td>
<td>1.084912</td>
<td>0.000537</td>
<td>0.00018</td>
<td>0.00018</td>
</tr>
</tbody>
</table>