

Name: _____

Closed book- closed notesPROBLEM 1

Write the expression of the Jacobian in terms of the gradient vectors.

ANSWER:

$$J(x_i) = \begin{bmatrix} [\nabla f_1(x_i)]^T \\ [\nabla f_2(x_i)]^T \end{bmatrix} = \begin{bmatrix} \left[\begin{array}{c} \frac{\partial f_1(x_i)}{\partial x_1} \\ x_1 \\ \frac{\partial f_1(x_i)}{\partial x_2} \\ x_2 \end{array} \right]^T \\ \left[\begin{array}{c} \frac{\partial f_2(x_i)}{\partial x_1} \\ x_1 \\ \frac{\partial f_2(x_i)}{\partial x_2} \\ x_2 \end{array} \right]^T \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1(x_i)}{\partial x_1} & \frac{\partial f_1(x_i)}{\partial x_2} \\ x_1 & x_2 \\ \frac{\partial f_2(x_i)}{\partial x_1} & \frac{\partial f_2(x_i)}{\partial x_2} \\ x_1 & x_2 \end{bmatrix}$$

PROBLEM 2

Consider

$$5x^2 - y^2 = 2$$

$$5x - 2y^2 = -6$$

- a) Obtain the roots analytically

ANSWER:

$$5x - 2y^2 = -6 \Rightarrow y^2 = \frac{5x+6}{2} \Rightarrow y^2 = 2.5x + 3$$

$$\Rightarrow 5x^2 - y^2 - 2 = 5x^2 - 2.5x - 3 - 2 = 5x^2 - 2.5x - 5 = 0$$

$$\Rightarrow x^2 - x/2 - 1 = 0 \Rightarrow x = 1/4 \pm \sqrt{1/16 + 1}$$
$$\Rightarrow y = \sqrt{2.5x + 3}$$

See attached excel file

- b) Use ONE STEP of NR $\left[x_{i+1} = x_i - J^{-1}(x_i) f(x_i) \right]$ to solve for x and y. Use $x=0$ and $y=1$ as starting points. Do not invert the Jacobian, just leave it indicated

as $J^{-1}(x_i) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and write down the final expression for the next point.

ANSWER:

See attached excel file