

QUIZ #6

ENGR 3723

PROBLEM 1 (85 points)

Consider

| x_i | y_i |
|-------|-------|
| 0 | 2.1 |
| 1 | 7.7 |
| 2 | 13.6 |
| 3 | 27.2 |
| 4 | 40.9 |

Obtain the

A) (25 points) linear regression ($y = a_0 + a_1x$)

ANSWER:

$$S_r = \sum_{\forall i} (y_i - a_0 - a_1x_i)^2$$

$$\frac{\partial S_r}{\partial a_0} = -2 \sum_{\forall i} (y_i - a_0 - a_1x_i) = 0$$

$$\frac{\partial S_r}{\partial a_1} = -2 \sum_{\forall i} (y_i - a_0 - a_1x_i)x_i = 0$$

Then

$$\sum_{\forall i} y_i = na_0 + a_1 \sum_{\forall i} x_i$$

$$\sum_{\forall i} y_i x_i = a_0 \sum_{\forall i} x_i + a_1 \sum_{\forall i} x_i^2$$

Solve for a_0 and a_1

B) (60 points) second degree polynomial regression

ANSWER:

$$S_r = \sum_{\forall i} (y_i - a_0 - a_1 x_i - a_2 x_i^2)^2$$

$$\frac{\partial S_r}{\partial a_0} = -2 \sum_{\forall i} (y_i - a_0 - a_1 x_i - a_2 x_i^2) = 0$$

$$\frac{\partial S_r}{\partial a_1} = -2 \sum_{\forall i} (y_i - a_0 - a_1 x_i - a_2 x_i^2) x_i = 0$$

$$\frac{\partial S_r}{\partial a_2} = -2 \sum_{\forall i} (y_i - a_0 - a_1 x_i - a_2 x_i^2) x_i^2 = 0$$

Then

$$\sum_{\forall i} y_i = n a_0 + a_1 \sum_{\forall i} x_i + a_2 \sum_{\forall i} x_i^2$$

$$\sum_{\forall i} y_i x_i = a_0 \sum_{\forall i} x_i + a_1 \sum_{\forall i} x_i^2 + a_2 \sum_{\forall i} x_i^3$$

$$\sum_{\forall i} y_i x_i^2 = a_0 \sum_{\forall i} x_i^2 + a_1 \sum_{\forall i} x_i^3 + a_2 \sum_{\forall i} x_i^4$$

Solve for a_0 , a_1 and a_2

PROBLEM 2 (15 points)

Derive the expression for a and b if the proposed formula for regression is $y = a e^{bx}$

ANSWER:

$$\ln y_i = \ln a + b x_i$$

then call $z_i = \ln y_i$

and get $z_i = a' + b x_i$, for which one can use the same formulas as in problem 1a-