

QUIZ #7

ENGR 3723

Name: _____

Closed book- closed notes

PROBLEM 1

Consider

x_i	y_i
0	2.1
1	7.7
2	13.6
3	27.2

A) (10 points) What degree is the polynomial that interpolates these data?

ANSWER:

4 points 3rd degree polynomial

B) (50 points) Eliminate one point and obtain the corresponding interpolating polynomial using Newton's method.

ANSWER: We use

x_i	$y_i = f(x_i)$
0	2.1
1	7.7
2	13.6

$$\begin{aligned}x = x_0 & \quad b_0 = f(x_0) \\x = x_1 & \quad b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \\x = x_2 & \quad b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}\end{aligned}$$

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

C) (40 points) Propose a polynomial as follows:

$$p(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n.$$

Use three points to substitute directly: $p(x_i) = a_0x_i^n + a_1x_i^{n-1} + \dots + a_{n-1}x_i + a_n$. Set up a linear system to obtain the coefficients a_i directly.

ANSWER: We use the same points as in

$$p(x) = a_0x^2 + a_1x + a_2$$

$$y_0 = 2.1 = a_0x_0^2 + a_1x_0 + a_2 = a_2$$

$$y_1 = 7.7 = a_0x_1^2 + a_1x_1 + a_2 = a_0 + a_1 + a_2 = a_0 + a_1 + 2.1$$

$$y_2 = 13.6 = a_0x_2^2 + a_1x_2 + a_2 = a_04 + a_12 + a_2 = 4a_0 + 2a_1 + 2.1$$

This system can be solved for a_0 and a_1