

QUIZ #8

ENGR 3723

PROBLEM 1

Consider $f(x) = 3 * \cos(3x - 2)$

A) (10 points) Find the maximum analytically

ANSWER:

$$f'(x) = -9 * \sin(3x - 2) = 0 \quad \Rightarrow 3x - 2 = 0 \quad \Rightarrow x = 2/3$$

$$f''(x) = -27 * \cos(3x - 2) \quad \Rightarrow f''(x) = -27 * \cos(0) = -27$$
$$\Rightarrow f''(x) < 0 \quad \Rightarrow \text{Maximum}$$

B) (35 points) Find the maximum using Newton Raphson. Start from x=0.

ANSWER:

Actually, if one starts from x=0, one converges to a minimum

$$x_{i+1} = x_i - f'(x_i) / f''(x_i)$$

| xi | f '(xi) | f "(xi) |
|----------|----------|----------|
| 0 | 8.183677 | 11.23596 |
| -0.72835 | -7.7773 | 13.5876 |
| -0.15596 | 5.61493 | 21.10102 |
| -0.42206 | -1.11844 | 26.7907 |
| -0.38031 | 0.005839 | 26.99999 |
| -0.38053 | -8.2E-10 | 27 |
| -0.38053 | 1.1E-15 | 27 |
| -0.38053 | 1.1E-15 | 27 |

If one starts from x=1, one converges to a maximum.

| xi | f '(xi) | f "(xi) |
|----------|----------|----------|
| 1 | -7.57324 | -14.5882 |
| 0.480864 | 4.760893 | -22.913 |
| 0.688645 | -0.593 | -26.9413 |
| 0.666635 | 0.000861 | -27 |
| 0.666667 | -2.6E-12 | -27 |
| 0.666667 | 0 | -27 |
| 0.666667 | 0 | -27 |
| 0.666667 | 0 | -27 |

C) (55 points) Find the maximum using Golden Search. Use 0 and 1 as starting points.

ANSWER:

$$d = (\sqrt{5} - 1) / 2, \quad x_1 = x_{\text{low}} + d, \quad x_2 = x_u - d$$

Rules

If $f(x_1) > f(x_2)$ $x_l = x_2, x_u = \text{same}$

else $x_u = x_1, x_l = \text{same}$

| x_l | x_u | d | x_2 | x_1 | $f(x_2)$ | $f(x_1)$ |
|----------|----------|----------|----------|----------|-----------|----------|
| 0 | 1 | 0.618034 | 0.381966 | 0.618034 | 1.9706876 | 2.968127 |
| 0.381966 | 1 | 0.381966 | 0.618034 | 0.763932 | 2.9681272 | 2.873186 |
| 0.381966 | 0.763932 | 0.236068 | 0.527864 | 0.618034 | 2.7436433 | 2.968127 |
| 0.527864 | 0.763932 | 0.145898 | 0.618034 | 0.673762 | 2.9681272 | 2.99932 |
| 0.618034 | 0.763932 | 0.09017 | 0.673762 | 0.708204 | 2.9993204 | 2.976738 |
| 0.618034 | 0.708204 | 0.055728 | 0.652476 | 0.673762 | 2.9972818 | 2.99932 |
| 0.652476 | 0.708204 | 0.034442 | 0.673762 | 0.686918 | 2.9993204 | 2.994465 |
| 0.652476 | 0.686918 | 0.021286 | 0.665631 | 0.673762 | 2.9999855 | 2.99932 |
| 0.652476 | 0.673762 | 0.013156 | 0.660606 | 0.665631 | 2.9995042 | 2.999986 |
| 0.660606 | 0.673762 | 0.008131 | 0.665631 | 0.668737 | 2.9999855 | 2.999942 |
| 0.660606 | 0.668737 | 0.005025 | 0.663712 | 0.665631 | 2.9998822 | 2.999986 |
| 0.663712 | 0.668737 | 0.003106 | 0.665631 | 0.666818 | 2.9999855 | 3 |
| 0.665631 | 0.668737 | 0.001919 | 0.666818 | 0.667551 | 2.9999997 | 2.999989 |
| 0.665631 | 0.667551 | 0.001186 | 0.666365 | 0.666818 | 2.9999988 | 3 |
| 0.666365 | 0.667551 | 0.000733 | 0.666818 | 0.667098 | 2.9999997 | 2.999997 |
| 0.666365 | 0.667098 | 0.000453 | 0.666645 | 0.666818 | 3 | 3 |
| 0.666365 | 0.666818 | 0.00028 | 0.666538 | 0.666645 | 2.9999998 | 3 |
| 0.666538 | 0.666818 | 0.000173 | 0.666645 | 0.666711 | 3 | 3 |
| 0.666538 | 0.666711 | 0.000107 | 0.666604 | 0.666645 | 2.9999999 | 3 |
| 0.666604 | 0.666711 | 6.61E-05 | 0.666645 | 0.66667 | 3 | 3 |
| 0.666645 | 0.666711 | 4.09E-05 | 0.66667 | 0.666685 | 3 | 3 |
| 0.666645 | 0.666685 | 2.53E-05 | 0.66666 | 0.66667 | 3 | 3 |
| 0.66666 | 0.666685 | 1.56E-05 | 0.66667 | 0.666676 | 3 | 3 |
| 0.66666 | 0.666676 | 9.64E-06 | 0.666666 | 0.66667 | 3 | 3 |