TEST WEEK 1 ENGR 3723

PROBLEM 1 (10 points)

State the difference between chopping, rounding and truncation error

ANSWER:

-"One method of approximation [of a number] would be to merely omit, or "chop-off", … higher terms" - Chapra and Canale- 5th Ed., Page 73.

-Rounding is when the last digit of the approximation of a number is chosen to be the lowest or the highest approximation of the remainder, depending on whether this remainder is lower or higher than 0.5 $10^{-(n+1)}$, where n is the last significant digit used.

-"Truncation errors are those that result from using an approximation in place of an exact mathematical procedure" - Chapra and Canale- 5th Ed., Page 73

PROBLEM 2 (15 points)

a) Determine the number of terms needed to approximate e^{-x} to 5 significant figures. For x=1.3. Use $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ....$

ANSWER:

Term	***Value of Term***	***Series***	***Significant figures***
$1-x$	-0.3	-0.3	0
$x^2/2!$	0.845	0.545	0
$-x^3/3!$	-0.36616666666	0.17883333333	0
$x^4/4!$	0.11904167	0.2978375	1
$-x^5/5!$	-0.030941083	0.266896417	1
$x^6/6!$	0.006703901	0.273600318	2
$-x^7/7!$	-0.00124501	0.272355308	3
$x^8/8!$	0.000202314	0.272557622	4
$-x^9/9!$	-0.0000292232	0.272528399	4
$x^{10}/10!$	0.00000379901	0.272532198	5
$-x^{11}/11!$	-0.00000379901	0.272531749	5

TEN (10) Terms are needed

Repeat using

$$e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots}.$$

ANSWER:

Term	***Value of Term***	***Series***	***1/Series***	***Significant figures***
$1+x$	2.3	2.3	0.43478261	0
$x^2/2!$	0.845	3.145	0.31796502	0
$x^3/3!$	0.366166666	3.51116666666	0.28480562	1
$x^4/4!$	0.11904167	3.630170833	0.27546913	2
$x^5/5!$	0.030941083	3.661111917	0.27314106	2
$x^6/6!$	0.006703901	3.667815818	0.2726483	3
$x^7/7!$	0.00124501	3.669060828	0.27254931	4
$x^8/8!$	0.000202314	3.669263142	0.27253428	5
$x^9/9!$	0.0000292232	3.669292366	0.27253211	5
$x^{10}/10!$	0.00000379901	3.669296165	0.27253183	6
$x^{11}/11!$	0.00000379901	3.669296614	0.27253180	6

NINE (9) Terms are needed

b) Look how the series changes after you add a term in both cases. Can you explain the difference?

ANSWER:

The first series has alternate signs and therefore it oscillates around the final true value. The second in turn approximates the value from above and therefore converges faster.

PROBLEM 3 (25 points)

Consider

$\frac{dc}{dt} = -kc$, where k is a constant.

a) Obtain the formula for *c* vs. time.

ANSWER:

$$\frac{dc}{c} = -kdt \quad \Rightarrow \quad d\ln c = -kdt \Rightarrow \quad \ln c = -kt + K' \Rightarrow \quad c = Ke^{-kt}$$

But c=c_o at t=0, Then $c = c_o e^{-kt}$

b) Integrate numerically and compare. Use k=1.

ANSWER:

See file attached

PROBLEM 4 (10 points)

Write down the algorithm for bracketing. If you do not remember, reason it out!!! Draw the graphs and establish the logic!!

ANSWER:

1) Take two points x1 and x2
2) Calculate f1 and f2
3) If f1>0, f2>0, then go to 4, otherwise go to 6
4) IF f1>f2, then a) x1=x2 and b) x2=x2+Δx, and c) go to 2; Otherwise go to 5,
5) a) x2=x1 and b) x1=x1-Δx, and c) go to 2
6) If f1<0, f2<0, then go to 7, otherwise go to 9
7) IF f1>f2, then a) x2=x1 and b) x1=x1-Δx, and c) go to 2; Otherwise go to 8,
8) a) x1=x2 and b) x2=x2+Δx, and c) go to 2
9) Calculate x3=(x1+x2)/2
10) Calculate f3
11) If (x3-x1)<epsilon, stop. Solution found!!
12) If f3*f1<0, then a) x2=x3, b) go to 9; Otherwise a)x1=x3, b) go to 9

PROBLEM 5 (20 points)

Consider

$$f(x) = 2\sin(\sqrt{x}) - x$$

Use a fixed point method to obtain the smallest positive root. This will make you think carefully about your starting point. Write down your considerations…

ANSWER:

$$x = g(x) = 2\sin(\sqrt{x})$$

Do not start at x=0. It is actually one root, but we look for a positive one. So pick a positive number.

PROBLEM 6 (20 points)

Consider

$$f(x) = 2\sin(\sqrt{x}) - x$$

a) Use Newton Raphson to obtain the root. Start at x_0=1.

ANSWER:

See attached file.

b) What happens if you start at x_0=0.1.

ANSWER:

See attached file.

c) Is starting at at x_0=0 a good idea? Why?

ANSWER:

First, the derivative at zero does not exist. But, one could actually "trick"the method by being clever. Indded

$$f(x)/f'(x) = \left[2\sin(\sqrt{x}) - x\right] / \left[\cos(\sqrt{x})/\sqrt{x} - 1\right] =$$
$$= \sqrt{x}\left[2\sin(\sqrt{x}) - x\right] / \left[\cos(\sqrt{x}) - \sqrt{x}\right]$$

which renders

$$f(0)/f'(0) = \sqrt{0}\left[2\sin(\sqrt{0}) - 0\right] / \left[\cos(\sqrt{0}) - \sqrt{0}\right] = 0 * [0-0]/[1-0] = 0$$

But even if this problem is "solved", the NR will not move away from x=0 (because it is already a root).