TEST WEEK 2 ENGR 3723

Name:______________________________

<u>Closed book- closed notes</u>

<u>PROBLEM 1 (10 points)</u>

a) Obtain the gradient vectors of the following functions:

$$f_1(x,y,z) = 5\ln x - y^2 + zx$$
$$f_2(x,y,z) = 5x - 2y^2 + 6zyx - 2$$
$$f_3(x,y,z) = e^{-3xz} + \sin xy - 33x$$

ANSWER:

$$\nabla f_1(x,y,z) = \begin{bmatrix} 5/x + z \\ -2y \\ x \end{bmatrix}; \quad \nabla f_2(x,y,z) = \begin{bmatrix} 5 + 6zy \\ -4y + 6zx \\ 6yx \end{bmatrix}$$

$$\nabla f_3(x,y,z) = \begin{bmatrix} -3ze^{-3xz} + y\cos(xy) - 33 \\ x\cos(xy) \\ -3xe^{-3xz} \end{bmatrix}$$

b) Obtain the Jacobian of the above system of three equations.

ANSWER:

$$J = \begin{bmatrix} \nabla f_1^T(x,y,z) \\ \nabla f_2^T(x,y,z) \\ \nabla f_2^T(x,y,z) \end{bmatrix} = \begin{bmatrix} 5/x + z & -2y & x \\ 5+6zy & -4y+6zx & 6yx \\ -3ze^{-3xz} + y\cos(xy) - 33 & x\cos(xy) & -3xe^{-3xz} \end{bmatrix}$$

PROBLEM 2 (25 points)

Consider

$$\frac{1}{2}\ln x - \ln y \ = \frac{1}{2}$$
$$5x - 15y^2 = -6$$

a) Obtain the roots analytically

ANSWER:

$$5x - 15y^2 = -6 \quad \Rightarrow \quad x = (-6+15y^2)/5$$

$$\Rightarrow \quad \frac{1}{2}\ln\left[(-6+15y^2)/5\right] - \ln y \ = \frac{1}{2}$$

$$\Rightarrow \quad \ln\left[(-6+15y^2)/5\right] - 2\ln y \ = 1$$

$$\Rightarrow \quad \ln\left[(-6+15y^2)/5y^2\right] = 1 \quad \Rightarrow \quad (-6+15y^2)/5y^2 = e$$

$$\Rightarrow \quad (-6+15y^2) = 5y^2 e \quad \Rightarrow \quad y = \sqrt{\frac{6}{15-5e}} = 2.063874$$

$$\Rightarrow \quad x = (-6+15*(2.063874)^2)/5 = 11.57873$$

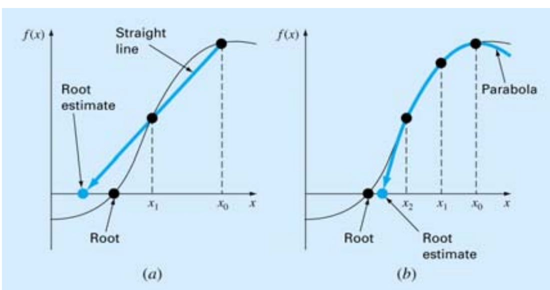
b) Use ONE STEP of NR to solve for x and y. Pick convenient starting points (x between 5 and 10 and y between 1 and 5)

$$J^{-1}(x_i) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \begin{matrix} a = J_{22}/\det(J), b = -J_{21}/\det(J) \\ c = -J_{12}/\det(J), d = J_{11}/\det(J) \end{matrix}$$

ANSWER:
See attached excel file

PROBLEM 3 (10 points)

Describe Mueller's method to obtain the roots of a polynomial. Use words. The picture should help.



ANSWER:
The method consists of

a) taking three points and obtaining the coefficients of a parabola that passes exactly through these three points
b) Equate the expresson of this parabola to zero to find the point where the parabola intersects the x axis
c) Replace the point with highest value of the polynomial (f value) by the new point and start over.

Once you found one root, what do you do to obtain the next root?

ANSWER:
You divide the original polynomial by (x-x^*) where x^* is the root found and start all over.

PROBLEM 4 (20 points)

Consider

$$x - 4x^2 + 3x^4 = -1$$

Find the roots using Mueller's method. Pick three arbitrary points between -1.5 and 0.1 to start

Hints:

If

$h_o = x_1 - x_o \qquad h_1 = x_2 - x_1$

$\delta_o = \dfrac{f(x_1) - f(x_o)}{x_1 - x_o} \qquad \delta_1 = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$(h_o + h_1)b - (h_o + h_1)^2 a = h_o\delta_o + h_1\delta_1$

$h_1 b - h_1^2 a = h_1\delta_1$

$a = \dfrac{\delta_1 - \delta_o}{h_1 + h_o} \qquad b = ah_1 + \delta_1 \qquad c = f(x_2)$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

ANSWER:

See attached excel file

PROBLEM 5 (20 points)

a) Obtain the inverse matrix using LU decomposition (15 points)

$$Ax = b \qquad A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & -9 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

b) Once you have the inverse obtain x (5 points)

ANSWER:
See attached excel file

PROBLEM 6 (15 points)
Apply one iteration of Gauss-Seidel using (0,0,0) as starting point. Show all your steps top to bottom.

$$Ax = b \qquad A = \begin{bmatrix} -5 & 1 & 2 \\ 3 & 5 & 1 \\ 6 & 6 & -9 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

ANSWER:
See attached excel file