CAPACITY EXPANSION
INVESTMENT PLANNING MODEL

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Capacity Expansion Planning

Deterministic Planning
Process Planning Under Uncertainty

**GIVEN:**
- Process Network
  - Set of Processes
  - Set of Chemicals
- Forecasted Data
  - Demands & Availabilities
  - Costs & Prices
  - Capital Budget

**DETERMINE:**
- Network Expansions
  - Timing
  - Sizing
  - Location
- Production Levels

**OBJECTIVES:**
- Maximize Net Present Value
Capacity Investment Planning

**Design Variables:** to be decided before the uncertainty reveals

\[ x = \{ Y_{it}, E_{it}, Q_{it} \} \]

- \( Y_{it} \): Decision of building process \( i \) in period \( t \)
- \( E_{it} \): Capacity expansion of process \( i \) in period \( t \)
- \( Q_{it} \): Total capacity of process \( i \) in period \( t \)

**Control Variables:** selected after uncertain parameters become known.
Assume them known for the time being!!!!

\[ y_s = \{ S_{jlt}, P_{jlt}, W_{it} \} \]

- \( S_{jlt} \): Sales of product \( j \) in market \( l \) at time \( t \) and scenario \( s \)
- \( P_{jlt} \): Purchase of raw mat. \( j \) in market \( l \) at time \( t \) and scenario \( s \)
- \( W_{it} \): Operating level of of process \( i \) in period \( t \) and scenario \( s \)
Example

Project Staged in 3 Time Periods of 2, 2.5, 3.5 years

- Chemical 5
  - Process 1
  - Chemical 1
  - Process 2
  - Chemical 6
  - Chemical 2

- Chemical 3
  - Process 4
  - Process 5
  - Chemical 7
  - Chemical 8
  - Chemical 4
Solution

Period 1
2 years

Chemical 1
5.27 kton/yr

Process 1
10.23 kton/yr

Chemical 5
5.27 kton/yr

Chemical 7
19.60 kton/yr

Process 3
22.73 kton/yr

Chemical 3
19.60 kton/yr
Solution

Period 2
2.5 years

Chemical 1
4.71 kton/yr

Process 1
10.23 kton/yr

Chemical 5
4.71 kton/yr

Chemical 3
41.75 kton/yr

Process 3
22.73 kton/yr

Process 4
22.73 kton/yr

Chemical 7
20.87 kton/yr

Process 5
22.73 kton/yr

Chemical 8
20.87 kton/yr
**Period 3**

3.5 years

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**Solution**

Chemical 1
44.44 kton/yr

Process 1
80.77 kton/yr

Chemical 5
14.95 kton/yr

Process 2
80.77 kton/yr

Chemical 6
29.49 kton/yr

Process 3
22.73 kton/yr

Chemical 7
21.88 kton/yr

Process 4
22.73 kton/yr

Chemical 3
43.77 kton/yr

Process 5
22.73 ton/yr

Chemical 4
21.88 kton/yr

Process 2
29.49 kton/yr

Chemical 8
21.88 kton/yr
MODEL

SETS

I : Processes i = 1, ..., NP
J : Raw materials and Products, j = 1, ..., NC
T: Time periods. T = 1, ..., NT
L: Markets, l = 1, ..., NM

Yit: An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)
Eit: Expansion of capacity of process i in period t.

Qit: Capacity of process i in period t.

Wit: Utilized capacity of process i in period t.

Pjlt : Amount of raw material/intermediate product j consumed from market l in period t
Sjlt : Amount of intermediate product/product j sold in market l in period t

ηij : Amount of raw material/intermediate product j used by process i
μij : Amount of product/intermediate product j consumed by process i
γjlt : Sale price of product/intermediate product j in market l in period t
Γ jlt : Cost of product/intermediate product j in market l in period t

δit : Operating cost of process i in period t

αit : Variable cost of expansion for process i in period t
βit : Fixed cost of expansion for process i in period t

Lt : Discount factor for period t

Eit,Lit, Uit : Lower and upper bounds on a process expansion in period t

aLjlt, aUjlt : Lower and upper bounds on availability of raw material j in market l in period t

dLjlt, dUjlt : Lower and upper bounds on demand of product j in market l in period t

CI, : Maximum capital available in period t

NEXP, t : maximum number of expansions in period t
MODEL

OBJECTIVE FUNCTION

\[
\text{Max} \; \text{NPV} = \sum_{t=1}^{NT} L_t \left( \sum_{l=1}^{NM} \sum_{j=1}^{NC} (\gamma_{jlt} S_{jlt} - \Gamma_{jlt} P_{jlt}) - \sum_{i=1}^{NP} \delta_{it} W_{it} \right) - \sum_{i=1}^{NP} \sum_{t=1}^{NT} (\alpha_{it} E_{it} + \beta_{it} Y_{it})
\]

\[\begin{align*}
Y_{it}: & \text{An expansion of process } I \text{ in period } t \text{ takes place } (Y_{it}=1), \text{does not take place } (Y_{it}=0) \\
E_{it}: & \text{Expansion of capacity of process } i \text{ in period } t \\
W_{it}: & \text{Utilized capacity of process } i \text{ in period } t \\
P_{jlt}: & \text{Amount of raw material/interm. product } j \text{ consumed from market } l \text{ in period } t \\
S_{jlt}: & \text{Amount of intermediate product/product } j \text{ sold in market } l \text{ in period } t
\end{align*}\]

\[\begin{align*}
\gamma_{jlt}: & \text{Sale price of product/intermediate product } j \text{ in market } l \text{ in period } t \\
\Gamma_{jlt}: & \text{Cost of product/intermediate product } j \text{ in market } l \text{ in period } t \\
\delta_{it}: & \text{Operating cost of process } i \text{ in period } t \\
\alpha_{it}: & \text{Variable cost of expansion for process } i \text{ in period } t \\
\beta_{it}: & \text{Fixed cost of expansion for process } i \text{ in period } t \\
L_t: & \text{Discount factor for period } t
\end{align*}\]
MODEL

LIMITS ON EXPANSION

\[ Y_{it} E_{it}^L \leq E_{it} \leq Y_{it} E_{it}^U \quad i = 1, \ldots, NP \quad t = 1, \ldots, NT \]

TOTAL CAPACITY IN EACH PERIOD

\[ Q_{it} = Q_{i(t-1)} + E_{it} \quad i = 1, \ldots, NP \quad t = 1, \ldots, NT \]

LIMIT ON THE NUMBER OF EXPANSIONS

\[ \sum_{t=1}^{NT} Y_{it} \leq NEX_P \quad i = 1, \ldots, NP \]

LIMIT ON THE CAPITAL INVESTMENT

\[ \sum_{i=1}^{NP} (\alpha_{it} E_{it} + \beta_{it} Y_{it}) \leq CI_t \quad t = 1, \ldots, NT \]

\[ Y_{it}: \text{An expansion of process } I \text{ in period } t \text{ takes place (} Y_{it}=1\text{), does not take place (} Y_{it}=0) \]
\[ E_{it}: \text{Expansion of capacity of process } i \text{ in period } t. \]
\[ Q_{it}: \text{Capacity of process } i \text{ in period } t. \]

**I**: Processes \( i = 1, \ldots, NP \)

**J**: Raw mat./Products, \( j = 1, \ldots, NC \)

**T**: Time periods, \( T = 1, \ldots, NT \)

**L**: Markets, \( l = 1, \ldots, NM \)

**NEXP**: maximum number of expansions in period \( t \)

\[ \alpha_{it}: \text{Variable cost of expansion for process } i \text{ in period } t \]
\[ \beta_{it}: \text{Fixed cost of expansion for process } i \text{ in period } t \]

\( E_{it}^L, E_{it}^U \): Lower and upper bounds on a process expansion in period \( t \).
MODEL

UTILIZED CAPACITY IS LOWER THAN TOTAL CAPACITY

\[ W_{it} \leq Q_{it}, \quad i=1,\ldots,\text{NP}, \quad t=1,\ldots,\text{NT} \]

MATERIAL BALANCE

\[ \sum_{l=1}^{NM} P_{jlt} + \sum_{i=1}^{NP} \eta_{ij} W_{it} \leq \sum_{l=1}^{NM} S_{jlt} + \sum_{i=1}^{NP} \mu_{ij} W_{it}, \quad i=1,\ldots,\text{NP}, \quad t=1,\ldots,\text{NT} \]

BOUNDS

\[ a_{jlt}^L \leq P_{jlt} \leq a_{jlt}^U, \quad d_{jlt}^L \leq S_{jlt} \leq d_{jlt}^U, \quad j=1,\ldots,\text{NC}, t=1,\ldots,\text{NT}, l=1,\ldots,\text{NM} \]

NONNEGATIVITY

\[ E_{it}, Q_{it}, W_{it}, P_{jlt}, S_{jlt} \geq 0, \quad \forall i, j, l, t \]

INTEGER VARIABLES

\[ Y_{it} \in \{0,1\}, \quad i=1,\ldots,\text{NP}, \quad t=1,\ldots,\text{NT} \]

\[ Y_{it}: \text{An expansion of process } I \text{ in period } t \text{ takes place (} Y_{it}=1), \text{ does not take place (} Y_{it}=0) \]

\[ E_{it}: \text{Expansion of capacity of process } i \text{ in period } t. \]

\[ Q_{it}: \text{Capacity of process } i \text{ in period } t. \]

\[ W_{it}: \text{Utilized capacity of process } i \text{ in period } t. \]

\[ P_{jlt}: \text{Amount of raw material/intermediate product } j \text{ consumed from market } l \text{ in period } t \]

\[ S_{jlt}: \text{Amount of intermediate product/product } j \text{ sold in market } l \text{ in period } t \]

1: Processes \( i=1,\ldots,\text{NP} \)

J: Raw mat./Products, \( j=1,\ldots,\text{NC} \)

T: Time periods, \( T=1,\ldots,\text{NT} \)

L: Markets, \( l=1,\ldots,\text{NM} \)

\[ a_{jlt}^L, a_{jlt}^U: \text{Lower and upper bounds on availability of raw material } j \text{ in market } l \text{ in period } t \]

\[ d_{jlt}^L, d_{jlt}^U: \text{Lower and upper bounds on demand of product } j \text{ in market } l \text{ in period } t \]
MATERIAL BALANCE

\[ \sum_{l=1}^{NM} P_{jlt} + \sum_{i=1}^{NP} \eta_{ij} W_{it} \leq \sum_{l=1}^{NM} S_{jlt} + \sum_{i=1}^{NP} \mu_{ij} W_{it} \quad i=1, \ldots, NP \quad t=1, \ldots, NT \]

Reference Component is C

\[ \sum_{l=1}^{NM} P_{Blt} + \eta_{kA} W_{kt} = \sum_{l=1}^{NM} S_{Clt} + \mu_{iD} W_{it} \]

"Stoichiometric" Coefficients

\[ \eta_{kA}, \mu_{iD} \]