FINANCIAL RISK

CHE 5480
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Scope of Discussion

We will discuss the definition and management of financial risk in any design process or decision making paradigm, like…

- Investment Planning
- Scheduling and more in general, operations planning
- Supply Chain modeling, scheduling and control
- Short term scheduling (including cash flow management)
- Design of process systems
- Product Design

Extensions that are emerging are the treatment of other risks in a multiobjective (?) framework, including for example

- Environmental Risks
- Accident Risks (other than those than can be expressed as financial risk)
Introduction – Understanding Risk

Consider two investment plans, designs, or operational decisions

Profit Histogram

- Investment Plan I - $E[Profit] = 338$
- Investment Plan II - $E[Profit] = 335$

Probability of Loss for Plan I = 12%
Conclusions

- Risk can only be assessed after a plan has been selected but it cannot be managed during the optimization stage (even when stochastic optimization including uncertainty has been performed).

- There is a need to develop new models that allow not only assessing but managing financial risk.

- The decision maker has two simultaneous objectives:
  - Maximize Expected Profit.
  - Minimize Risk Exposure
What does Risk Management mean?

One wants to modify the profit distribution in order to satisfy the preferences of the decision maker.

REDUCE THESE FREQUENCIES

OR... INCREASE THESE FREQUENCIES

OR BOTH!!!!
Characteristics of Two-Stage Stochastic Optimization Models

Philosophy

• Maximize the *Expected Value* of the objective over all possible realizations of uncertain parameters.
• Typically, the objective is *Expected Profit*, usually *Net Present Value*.
• Sometimes the minimization of *Cost* is an alternative objective.

Uncertainty

• Typically, the uncertain parameters are: *market demands, availabilities, prices, process yields, rate of interest, inflation, etc.*
• In Two-Stage Programming, uncertainty is modeled through a finite number of independent *Scenarios*.
• Scenarios are typically formed by *random samples* taken from the probability distributions of the uncertain parameters.
First-Stage Decisions

- Taken before the uncertainty is revealed. They usually correspond to structural decisions (not operational).
- Also called “Here and Now” decisions.
- Represented by “Design” Variables.
- Examples:
  - To build a plant or not. How much capacity should be added, etc.
  - To place an order now.
  - To sign contracts or buy options.
  - To pick a reactor volume, to pick a certain number of trays and size the condenser and the reboiler of a column, etc.
Characteristics of Two-Stage Stochastic Optimization Models

- **Second-Stage Decisions**
  - Taken in order to adapt the plan or design to the uncertain parameters realization.
  - Also called “Recourse” decisions.
  - Represented by “Control” Variables.
  - Example: the operating level; the production slate of a plant.
  - Sometimes first stage decisions can be treated as second stage decisions. In such case the problem is called a multiple stage problem.

- **Shortcomings**
  - The model is unable to perform risk management decisions.
**Two-Stage Stochastic Formulation**

Let us leave it linear because as is it is complex enough.!!

**LINEAR MODEL SP**

\[
\begin{align*}
\text{Max} \quad & \sum_{s} p_s q_s^T y_s - c^T x \\
\text{Recourse} & \quad \text{First-Stage} \\
\text{Function} & \quad \text{Cost} \\
\text{s.t.} & \\
Ax = b & \quad \text{First-Stage Constraints} \\
T_s x + Wy_s = h_s & \quad \text{Second-Stage Constraints} \\
x \geq 0, \quad x \in X & \\
y_s \geq 0 & \quad \text{Second Stage Variables}
\end{align*}
\]

**Technology matrix**

**First stage variables**

**Recourse matrix (Fixed Recourse)**

**Sometimes not fixed (Interest rates in Portfolio Optimization)**

**Complete recourse**: the recourse cost (or profit) for every possible uncertainty realization remains finite, independently of the first-stage decisions (x).

**Relatively complete recourse**: the recourse cost (or profit) is feasible for the set of feasible first-stage decisions. This condition means that for every feasible first-stage decision, there is a way of adapting the plan to the realization of uncertain parameters.

We also have found that one can sacrifice efficiency for certain scenarios to improve risk management. We do not know how to call this yet.
Robust Optimization Using Variance (Mulvey et al., 1995)

Maximize $E[\text{Profit}] - \rho \cdot V[\text{Profit}]$

Underlying Assumption: Risk is monotonic with variability
Robust Optimization Using Variance

Drawbacks

• Variance is a symmetric risk measure: profits both above and below the target level are penalized equally. We only want to penalize profits below the target.

• Introduces non-linearities in the model, which results in serious computational difficulties, specially in large-scale problems.

• The model may render solutions that are stochastically dominated by others. This is known in the literature as not showing Pareto-Optimality. In other words there is a better solution \((y_s, x^*)\) than the one obtained \((y_s^*, x^*)\).
Previous Approaches to Risk Management

- Robust Optimization using Upper Partial Mean (Ahmed and Sahinidis, 1998)

Maximize $E[\text{Profit}] - \rho \cdot UPM$

Underlying Assumption: Risk is monotonic with lower variability
Robust Optimization using the UPM

Advantages

• Linear measure

Disadvantages

• The UPM may misleadingly favor non-optimal second-stage decisions.
• Consequently, financial risk is not managed properly and solutions with higher risk than the one obtained using the traditional two-stage formulation may be obtained.
• The model loses its scenario-decomposable structure and stochastic decomposition methods can no longer be used to solve it.
Robust Optimization using the UPM

Objective Function: Maximize $E[\text{Profit}] - \rho \cdot \text{UPM}$

$$\text{UPM} = \sum_{s \in S} p_s \Delta_s$$

$$\Delta_s = \max \left\{ 0 ; \sum_{k \in S} p_k \text{Profit}_k - \text{Profit}_s \right\}$$

<table>
<thead>
<tr>
<th>$\rho = 3$</th>
<th>$\text{Profit}_s$</th>
<th>$\Delta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case I</td>
<td>Case II</td>
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<tr>
<td>S1</td>
<td>150</td>
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<td>S2</td>
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<td>75</td>
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<tr>
<td>S4</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$E[\text{Profit}]$</td>
<td>100.00</td>
<td>81.25</td>
</tr>
<tr>
<td>UPM</td>
<td>18.75</td>
<td>9.38</td>
</tr>
<tr>
<td>Objective</td>
<td>43.75</td>
<td>53.13</td>
</tr>
</tbody>
</table>

Downside scenarios are the same, but the UPM is affected by the change in expected profit due to a different upside distribution. As a result a wrong choice is made.
Both technologies are able to produce two products with different production cost and at different yield per unit of installed capacity.
OTHER APPROACHES


- **Multiobjective Approach**: Considers Downside Risk, ENPV and Process Life Cycle as alternative Objectives.
- **Multiperiod Decision process** modeled as a Markov decision process with recourse.
- The problem is sometimes amenable to be reformulated as a sequence of single-period sub-problems, each being a two-stage stochastic program with recourse. These can often be solved backwards in time to obtain Pareto Optimal solutions.

This paper proposes a new design paradigm of which risk is just one component. We will revisit this issue later in the talk.
OTHER APPROACHES

Risk Premium (Applequist, Pekny and Reklaitis, 2000)

- Observation: Rate of return varies linearly with variability. The dependance is called Risk Premium.
- They suggest to benchmark new investments against the historical risk premium by using a two objective (risk premium and profit) problem.
- The technique relies on using variance as a measure of variability.
Conclusions

• The minimization of Variance penalizes both sides of the mean.
• The Robust Optimization Approach using Variance or UPM is not suitable for risk management.
• The Risk Premium Approach (Applequist et al.) has the same problems as the penalization of variance.

THUS,

• Risk should be properly defined and directly incorporated in the models to manage it.
• The multiobjective Markov decision process (Applequist et al, 2000) is very closely related to ours and can be considered complementary. In fact (Westerberg dixit) it can be extended to match ours in the definition of risk and its multilevel parametrization.
**Probabilistic Definition of Risk**

Financial Risk = Probability that a plan or design does not meet a certain profit target

Scenarios are independent events

For each scenario the profit is either greater/equal or smaller than the target

\( z_s \) is a new *binary* variable

Formal Definition of Financial Risk

\[
\text{Risk}(x, \Omega) = P(Profit < \Omega)
\]

\[
\Downarrow
\]

\[
\text{Risk}(x, \Omega) = \sum_s p_s \, P(Profit_s \leq \Omega)
\]

\[
\Downarrow
\]

\[
P(Profit_s < \Omega) = \begin{cases} 
1 & \text{If } Profit_s < \Omega \\
0 & \text{else}
\end{cases}
\]

\[
\Downarrow
\]

\[
P(Profit_s > \Omega) = z_s
\]

\[
\Downarrow
\]

\[
\text{Risk}(x, \Omega) = \sum_s p_s z_s
\]
Profit PDF $f(\xi)$

$\Omega$

$x$ fixed

Area $= \text{Risk} (x, \Omega)$
Our intention is to modify the shape and location of this curve according to the attitude towards risk of the decision maker.
Risk Preferences and Risk Curves

Risk-Averse Investor's Choice
\[ E[\text{Profit}] = 0.4 \]

Risk-Taker Investor's Choice
\[ E[\text{Profit}] = 1.0 \]
A plan or design with Maximum $E[Profit]$ (i.e. optimal in Model SP) sets a theoretical limit for financial risk: it is impossible to find a feasible plan/design having a risk curve entirely beneath this curve.
Max $E[\text{Profit}] = \sum_s p_s q_s y_s - c^T x$

$\text{Min Risk}[\Omega_1] = \sum_s p_s z_{s1}$

$\vdots$

$\text{Min Risk}[\Omega_4] = \sum_s p_s z_{s4}$

s.t.

$Ax = b$

$T_s x + W y_s = h_s$

$x \geq 0 \quad x \in X$

$y_s \geq 0$

$q_s^T y_s - c^T x \geq \Omega - U_s z_s$

$q_s^T y_s - c^T x \leq \Omega + U_s (1 - z_s)$

$z_s \in (0, 1)$

Multiple Objectives:

- At each profit we want minimize the associated risk
- We also want to maximize the expected profit
Parametric Representations of the Multi-Objective Model – Restricted Risk

**Restricted Risk MODEL**

\[
\begin{align*}
\text{Max} & \quad \sum_s p_s q_s y_s - c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad T_s x + W y_s = h_s \\
& \quad x \geq 0 \quad x \in X \\
& \quad y_s \geq 0 \\
& \quad \sum_s p_s z_{si} \leq \varepsilon_i \quad \text{Forces Risk to be lower than a specified level} \\
& \quad q_s^T y_s - c^T x \geq \Omega_i - U_s z_{si} \\
& \quad q_s^T y_s - c^T x \leq \Omega_i + U_s (1 - z_{si}) \\
& \quad z_s \in (0,1)
\end{align*}
\]
Parametric Representations of the Multi-Objective Model – Penalty for Risk

Risk Penalty MODEL

\[
\begin{aligned}
\text{Max} & \quad \sum_s p_s q_s^T y_s - c^T x - \sum_i \rho_i \sum_s p_s z_{si} \\
\text{s.t.} & \quad Ax = b \\
& \quad T_s x + W y_s = h_s \\
& \quad x \geq 0 \quad x \in X \\
& \quad y_s \geq 0 \\
& \quad q_s^T y_s - c^T x \geq \Omega_i - U_s z_{si} \\
& \quad q_s^T y_s - c^T x \leq \Omega_i + U_s (1 - z_{si}) \\
& \quad z_s \in (0,1)
\end{aligned}
\]

Penalty Term

Risk Management Constraints

STRATEGY

Define several profit Targets and penalty weights to solve the model using a multi-parametric approach
**Advantages**

- Risk can be effectively managed according to the decision maker’s criteria.
- The models can adapt to risk-averse or risk-taker decision makers, and their risk preferences are easily matched using the risk curves.
- A full spectrum of solutions is obtained. These solutions always have optimal second-stage decisions.
- Model Risk Penalty conserves all the properties of the standard two-stage stochastic formulation.

**Disadvantages**

- The use of binary variables is required, which increases the computational time to get a solution. This is a major limitation for large-scale problems.
Computational Issues

- The most efficient methods to solve stochastic optimization problems reported in the literature exploit the decomposable structure of the model.

- This property means that each scenario defines an independent second-stage problem that can be solved separately from the other scenarios once the first-stage variables are fixed.

- The Risk Penalty Model is decomposable whereas Model Restricted Risk is not. Thus, the first one is model is preferable.

- Even using decomposition methods, the presence of binary variables in both models constitutes a major computational limitation to solve large-scale problems.

- It would be more convenient to measure risk indirectly such that binary variables in the second stage are avoided.
Downside Risk

Downside Risk (Eppen et al, 1989) =

Expected Value of the Positive Profit Deviation from the target

Positive Profit Deviation from Target $\Omega$

The Positive Profit Deviation is also defined for each scenario

Formal definition of Downside Risk

$$DRisk(x, \Omega) = E[\delta(x, \Omega)]$$

$$\delta(x, \Omega) = \begin{cases} 
\Omega - Profit(x) & \text{If } Profit(x) < \Omega \\
0 & \text{Otherwise}
\end{cases}$$

$$\delta_s = \begin{cases} 
\Omega - Profit_s & \text{If } Profit_s < \Omega \\
0 & \text{Otherwise}
\end{cases}$$

$$DRisk(x, \Omega) = \sum_s p_s \delta_s$$
Downside Risk Interpretation

$DRisk(x, \Omega) = \mathbb{E}[\delta(x, \Omega)]$

$DRisk(x, \Omega) = \int_{-\infty}^{\Omega} (\Omega - \xi) f(\xi) d\xi$

Profit PDF $f(\xi)$
Downside Risk & Probabilistic Risk

\[ \text{Risk} (x, \xi) \]

\[ \text{x fixed} \]

\[ \Omega \]

\[ \text{Area} = DRisk (x, \Omega) \]

\[ DRisk (x, \Omega) = \int_{-\infty}^{\Omega} Risk (x, \xi) d\xi \]
Two-Stage Model using Downside Risk

MODEL DRisk

Max \( \mu \left( \sum_s p_s q_s^T y_s - c^T x \right) - \sum_s p_s \delta_s \)

s.t.

\( Ax = b \)
\( T_s x + Wy_s = h_s \)
\( x \geq 0 \quad x \in X \)
\( y_s \geq 0 \)
\( \delta_s \geq \Omega - (q_s^T y_s - c^T x) \)
\( \delta_s \geq 0 \)

Advantages

• Same as models using Risk
• Does not require the use of binary variables
• Potential benefits from the use of decomposition methods

Strategy

Solve the model using different profit targets to get a full spectrum of solutions. Use the risk curves to select the solution that better suits the decision maker’s preference.
Two-Stage Model using Downside Risk

**Warning:** The same risk may imply different Downside Risks.

Immediate Consequence:

Minimizing downside risk does not guarantee minimizing risk.
# Commercial Software

- **Riskoptimizer (Palisades) and CrystalBall (Decisioneering)**
  - Use excel models
  - Allow uncertainty in a form of distribution
  - Perform Montecarlo Simulations or use genetic algorithms to optimize (Maximize ENPV, Minimize Variance, etc.)

- **Financial Software. Large variety**
  - Some use the concept of downside risk
  - In most of these software, Risk is mentioned but not manipulated directly
Process Planning Under Uncertainty

GIVEN:
- Process Network
  - Set of Processes
  - Set of Chemicals
- Forecasted Data
  - Demands & Availabilities
  - Costs & Prices
  - Capital Budget

DETERMINE:
- Network Expansions
  - Timing
  - Sizing
  - Location
- Production Levels

OBJECTIVES:
- Maximize Expected Net Present Value
- Minimize Financial Risk
Process Planning Under Uncertainty

**Design Variables:** to be decided before the uncertainty reveals

\[ X = \{ Y_{it}, E_{it}, Q_{it} \} \]

- \( Y_{it} \): Decision of building process \( i \) in period \( t \)
- \( E_{it} \): Capacity expansion of process \( i \) in period \( t \)
- \( Q_{it} \): Total capacity of process \( i \) in period \( t \)

**Control Variables:** selected after the uncertain parameters become known

\[ Y_s = \{ S_{jlt}, P_{jlt}, W_{its} \} \]

- \( S_{jlt} \): Sales of product \( j \) in market \( l \) at time \( t \) and scenario \( s \)
- \( P_{jlt} \): Purchase of raw mat. \( j \) in market \( l \) at time \( t \) and scenario \( s \)
- \( W_{its} \): Operating level of process \( i \) in period \( t \) and scenario \( s \)
Example

- Uncertain Parameters: Demands, Availabilities, Sales Price, Purchase Price
- Total of 400 Scenarios
- Project Staged in 3 Time Periods of 2, 2.5, 3.5 years

![Chemical Flow Diagram]

Chemical 1 ➔ Process 1 ➔ Process 2 ➔ Chemical 6
Chemical 2
Chemical 5

Chemical 3 ➔ Process 4 ➔ Process 5 ➔ Chemical 8
Chemical 7
Chemical 4
Period 3
3.5 years

Example – Solution with Max ENPV

Chemical 1
44.44 kton/yr

Process 1
80.77 kton/yr

Chemical 5
14.95 kton/yr

Chemical 2
29.49 kton/yr
21.88 kton/yr

Chemical 7

Process 3
22.73 kton/yr

Process 2
29.49 kton/yr
80.77 kton/yr

Process 4
22.73 kton/yr

Chemical 6
29.49 kton/yr

Chemical 8
21.88 kton/yr

Process 5
22.73 kton/yr

Chemical 3
43.77 kton/yr
Example – Solution with Min DRisk(Ω=900)

Period 3
3.5 years

Same final structure, different production capacities.
Example – Solution with Max ENPV

\[ E[\text{NPV}] = 1140 \text{ M}\$ \]
Example – Risk Management Solutions

NPV PDF $f(\xi)$

- $\Omega = 900$
- $\Omega = 1100$

PP

NPV (\(\xi\), M$)
Process Planning with Inventory

PROBLEM DESCRIPTION:

MODEL:  ▶ The mass balance is modified such that now a certain level of inventory for raw materials and products is allowed  
▶ A storage cost is included in the objective

OBJECTIVES:  ▶ Maximize Expected Net Present Value  
▶ Minimize Financial Risk
Period 3
3.5 years

Example with Inventory – SP Solution

Chemical 1

- 6.80 kton
- 1.94 kton/yr

Process 1

- 43.14 kton/yr
- 2.09 kton/yr
- 76.81 kton/yr

Chemical 2

- 13.61 kton/yr
- 30.44 kton/yr
- 3.86 kton

Process 2

- 76.81 kton/yr
- 1.03 kton/yr

Chemical 5

- 31.47 kton/yr

Chemical 6

- 1.10 kton/yr

Chemical 7

- 44.13 kton/yr
- 0.04 kton/yr
- 11.64 kton

Process 3

- 22.12 kton/yr
- 26.45 kton/yr

Chemical 8

- 25.41 kton/yr

Process 4

- 25.41 kton/yr
- 26.77 kton/yr

Chemical 4

- 11.91 kton
- 3.40 kton/yr

Process 5

- 26.77 kton/yr

Chemical 3

- 4.77 kton/yr
- 31.09 kton/yr

- 26.77 kton/yr

- 76.81 kton/yr
- 76.81 kton/yr

- 43.14 kton/yr

- 45.65 kton/yr

- 45.65 kton/yr

- 4.77 kton/yr
Example with Inventory 
Solution with Min DRisk ($\Omega=900$)

Period 3
3.5 years

Chemical 1
  7.48 kton/yr
  2.39 kton/yr
  1.07 kton
  0.30 kton/yr

Process 1
  11.23 kton/yr
  0.26 kton/yr
  0.90 kton

Chemical 5
  5.39 kton/yr

Process 2
  11.23 kton/yr
  5.39 kton/yr

Chemical 6
  5.39 kton/yr

Chemical 2

Process 3
  22.15 kton/yr
  23.38 kton/yr
  3.37 kton

Chemical 7
  22.04 kton/yr
  0.96 kton

Process 4
  23.38 kton/yr
  22.85 kton/yr

Chemical 4
  0.51 kton

Process 5
  23.38 kton/yr
  0.15 kton

Chemical 8
  4.11 kton

23.00 kton/yr
1.17 kton/yr

Chemical 3
  43.72 kton/yr
  4.05 kton
  1.16 kton/yr

Period 1
2 years

Period 2
2.5 years

Period 3
3.5 years
Example with Inventory - Solutions

\[ \text{E}[\text{NPV}] = 1140 \text{ M$} \]

\[ \text{E}[\text{NPV}] = 1237 \text{ M$} \]

With Inventory

\[ \Omega = 900 \]
\[ \text{ENPV} = 980 \]

\[ \Omega = 1400 \]
\[ \text{ENPV} = 1184 \]

Without Inventory

\[ \text{ENPV} = 1140 \]
\[ \text{ENPV} = 1237 \]
**Downside Expected Profit**

**Definition:**

\[ DENPV(x, p, \Omega) = \int_{-\infty}^{\Omega} \xi f(x, \xi) \, d\xi = \Omega \cdot Risk(x, \Omega) - DRisk(x, \Omega) \]

**Up to 50% of risk (confidence?) the lower ENPV solution has higher profit expectations.**
**Definition:** VaR is given by the difference between the mean value of the profit and the profit value corresponding to the $p$-quantile.

\[
VaR(x, p_\Omega) = E[Profit(x)] - Risk^{-1}(x, \Omega)
\]

**VaR=** $z_p \sigma$ for symmetric distributions (Portfolio optimization)
Sampling Average Approximation Method:

- Solve $M$ times the problem using only $N$ scenarios.
- If multiple solutions are obtained, use the first stage variables to solve the problem with a large number of scenarios $N' >> N$ to determine the optimum.

Generalized Benders Decomposition Algorithm

- First Stage variables are complicating variables.
- This leaves a primal over second stage variables, which is decomposable.
Example

Generate a model to:

- **Evaluate** a large network of natural gas supplier-to-market transportation alternatives

- **Identify** the most profitable alternative(s)

- **Manage** financial risk
Network of Alternatives

Suppliers
- Australia
- Indonesia
- Iran
- Kazakhstan
- Malaysia
- Qatar
- Russia

Transportation Methods
- Pipeline
- LNG
- CNG
- GTL
- Ammonia
- Methanol

Markets
- China
- India
- Japan
- S. Korea
- Thailand
- United States
Network of Alternatives
**Results**

**Risk Management (Downside Risk):**

- **Malaysia**
  - GTL
  - ENPV: 4.570
  - DR@ 4: 0.157
  - DR@ 3.5: 0.058

- **Thailand**
  - Indo-GTL
  - ENPV: 4.633
  - DR@ 4: 0.190
  - DR@ 3.5: 0.086
**Value at Risk (VaR):**

VaR is the expected loss for a certain confidence level usually set at 5%

\[ \text{VaR} = \text{ENPV} - \text{NPV} \at p\text{-quantile} \]

Opportunity Value (OV) or Upper Potential (UP):

\[ \text{OV} = \text{NPV} \at (1-p)\text{-quantile} - \text{ENPV} \]
Point measure for the upside

\[
\text{E}(\text{Profit}) = 3.4 \\
\text{VaR} = 1.75 \\
\text{OV} = 3.075
\]
Results

Value at Risk (VaR) and Opportunity Value (OV):

Reduction in VaR: 18.1%  
Reduction in OV: 18.9%
Results

Risk /Upside Potential Loss Ratio

Risk /Upside Potential Loss Ratio: 2.2
Risk /Upside Potential Loss Ratio

Risk /Upside Potential Loss Ratio \[= \frac{O\_Area}{R\_Area} = \frac{\int_{-\infty}^{\psi^-} dNPV}{\int_{\psi^+}^{+\infty} dNPV}\]

where:

\[\psi^+ = \begin{cases} \psi & \text{if } \psi \geq 0 \\ 0 & \text{otherwise} \end{cases}\]

\[\psi^- = \begin{cases} -\psi & \text{if } \psi < 0 \\ 0 & \text{otherwise} \end{cases}\]

\[\psi = \text{Risk}_{NGC}(NPV) - \text{Risk}_{NGC-DR}(NPV)\]
**Upper and Lower Bound Risk Curve**

**Upper Bound Risk Curve (Envelope):**
The curve constructed by plotting the set of net present values (NPV) for the best design under each scenario.
SAMPLING ALGORITHM

**Outer Loop**
- Generate $h(s)$
- $d = 1$
- $h = h(d)$
- **Fix First-Stage Variables**
- $s = 1$
- $h = h(s)$
- **Solve Deterministic Model**
- $s = s + 1$
- **Solve Deterministic Model**
- **Store NPV(d,s)**

**Inner Loop**
- $d = d + 1$
- **EXIT**
- $d <= d_{max}$

**This generates several solutions**
**Upper and Lower Bound Risk Curve**

**Lower Bound Risk Curve:**

The curve constructed by plotting the set of net present values (NPV) for the worst (of the set of best designs) under each scenario.

<table>
<thead>
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<th>$s \backslash d$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
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<td>$d_1$</td>
<td>$NPV_{d_1,s_1}$</td>
<td>$NPV_{d_2,s_1}$</td>
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<td>$\ldots$</td>
<td>$NPV_{d_n,s_1}$</td>
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<td>$NPV_{d_2,s_2}$</td>
<td>$NPV_{d_3,s_2}$</td>
<td>$\ldots$</td>
<td>$NPV_{d_n,s_2}$</td>
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<td>$d_3$</td>
<td>$NPV_{d_1,s_3}$</td>
<td>$NPV_{d_2,s_3}$</td>
<td>$NPV_{d_3,s_3}$</td>
<td>$\ldots$</td>
<td>$NPV_{d_n,s_3}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$d_n$</td>
<td>$NPV_{d_1,s_n}$</td>
<td>$NPV_{d_2,s_n}$</td>
<td>$NPV_{d_3,s_n}$</td>
<td>$\ldots$</td>
<td>$NPV_{d_n,s_n}$</td>
</tr>
<tr>
<td>$Min$</td>
<td>$NPV_{Min}^{s_1}$</td>
<td>$NPV_{Min}^{s_2}$</td>
<td>$NPV_{Min}^{s_3}$</td>
<td>$\ldots$</td>
<td>$NPV_{Min}^{s_n}$</td>
</tr>
</tbody>
</table>
Results

Upper and Lower Bound Risk Curve

![Diagram showing upper and lower envelope curves with ENPV values for Mala-GTL (Ships: 4 & 3, ENPV: 4.540), Lower Envelope (ENPV: 3.654), Upper Envelope (ENPV: 4.921), and Indo-GTL (Ships: 6 & 3, ENPV: 4.63).]
This is very useful because it allows nice decomposition,
That is, there is no need to solve the full stochastic problem
Conclusions

- A probabilistic definition of Financial Risk has been introduced in the framework of two-stage stochastic programming. Theoretical properties of related to this definition were explored.

- New formulations capable of managing financial risk have been introduced. The multi-objective nature of the models allows the decision maker to choose solutions according to his risk policy. The cumulative risk curve is used as a tool for this purpose.

- The models using the risk definition explicitly require second-stage binary variables. This is a major limitation from a computational standpoint.

- To overcome the mentioned computational difficulties, the concept of Downside Risk was examined, finding that there is a close relationship between this measure and the probabilistic definition of risk.

- Using downside risk leads to a model that is decomposable in scenarios and that allows the use of efficient solution algorithms. For this reason, it is suggested that this model be used to manage financial risk.

- An example illustrated the performance of the models, showing how the risk curves can be changed in relation to the solution with maximum expected profit.