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Computation of Natural Gas Pipeline Hydraulics

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ABSTRACT: When there was a lack of computing power, several approximate formulas (Weymouth, Panhandle, AGA, etc.) were developed to obtain pressure drop in gas pipelines, which are in many instances still being used. As it is well-known, they can be sometimes grossly inaccurate, necessitating the addition of an arbitrary parameter ("pipe efficiency") for each case. The right answer, now that we have computers and numerical integration methods, is to perform integration of the mechanical energy balance at least, if not both the mechanical and the overall energy balance when possible. While we advocate the numerical integration to obtain pressure drop, sometimes hydraulic calculations are embedded in several application procedures (pipe design, leak detection, compressor station operating optimization, etc.), and they require algebraic expressions to be used. We investigate the use of existing approximate formulas, a procedure to adjust the "pipe efficiency" for a given set of conditions, the building of new nonlinear surrogate models, and a Quadratic Metamodel amenable to nonconvex optimization procedures, as opposed to the current rational formulations.

INTRODUCTION

There are several factors that must be considered when designing a modern pipeline system or a gas well such as the depth of the well, the length and the size of the pipeline, the volume and the nature of gas to be transported, the operating temperature and pressure, the elevation change over the route, and others. Optimum design and operations of a gas transmission pipeline require accurate methods for predicting flow rate for a given pressure drop or predicting pressure drop for a specified flow rate in combination with installed energy requirements and compression power (such as in the case of fuel gas with a technical and economic evaluation).[1,2] The most widely used and the most basic relationships in the engineering of gas delivery systems are isothermal steady-state pressure drop or flow rate calculation methods.[1−3]

Dozens of flow equations like the Modified Colebrook−White, AGA, Panhandle A and B, Weymouth, IGT, Spitzglass, Mueller and Fritzsche, Cullender and Smith, Sukker and Cornell equations, etc., have been suggested to relate the gas volume transported along pipes to the different factors that affect this rate during more than 100 years of gas production and transportation in pipes. In the literature, several of the methods and equations can cause nontrivial errors because they have been simplified by approximations, assumptions, and the addition of incorrect friction factor correlations.[3] Many of these correlations are very old (the Weymouth, Panhandle A, and Panhandle B equations are more than 50 years old) and contain adjustable parameters that are to some extent arbitrary. Of all of the aforementioned flow equations, the most widely used and discussed are Panhandle A, Panhandle B, and Weymouth equations.[1,2,4−14]

Several different new methods are available, most of them based on more rigorous pressure drop calculations based on the Mechanical Energy Balance, while others may use the Weymouth, Panhandle, AGA, and other equations. Examples of computer programs used in industry are ProMax, Olga, Pipeflo, FORGAS, Ledaflow, FluidFlow3, PIPESIM, TGNET, and Pipe Flow. Some of these programs even allow for the use of any or most of the above listed approximate pipeline equations. On the other hand, some new methods include modifications of existing flow equations such as in the work of Morssy et al.,[13] Falade et al.,[14] and Usman et al.[15] From these, Morssy et al.[13] use a methodology that has a software package, which allows adding, modifying, and upgrading flow equations (Panhandle A, Panhandle B, AGA, Weymouth, etc.). Moreover, Falade et al.[14] combined the General Flow equation and the Colebrook−White friction factor correlation. Usman et al.[15] adjusted the Weymouth equation to account for change in diameter over the length of the pipe. Of the three (Morssy et al.,[13] Falade et al.,[14] and Usman et al.[15]), only Morssy et al.[13] use pipeline efficiency, while Usman et al.[15] ignore the value, essentially rendering the efficiency equal to a value of 1. Meanwhile, Falade et al.[14] use the friction factor calculated with the Colebrook−White equation and do not regard the efficiency factor at all.

The purpose of this Article is to propose a Metamodel that is better suited for optimization procedures. In the first part of this Article, we review gas hydraulics and perform a certain number of approximations to recreate the appropriate form of a surrogate model (or Metamodel), likely the starting point of all approximate equations. We then develop a Quadratic Metamodel and test its capabilities using certain pipe conditions of pressure, temperature, length, and diameter and compare to the recommended approximate equation for those conditions (Panhandle B) without elevation changes. Our reference for correct values is the integration of the Bernoulli equation. For this, we use Pro/II making sure that the predictions made are accurate, that is, by performing repeated calculations on a number of consecutive short pipeline

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segments as well as making sure that the appropriate equation of state is used. Next, we propose the right procedure to obtain the most accurate Metamodel under certain conditions of pressure temperature, etc. We continue by discussing a Quadratic Metamodel for the case of elevation changes. We finish by exploring the uncertainty in natural gas composition and its effect on the Quadratic Metamodel.

FORMAL INTEGRATION OF THE MECHANICAL ENERGY BALANCE

We start with the differential form of the mechanical energy balance:

$$g\ dz + v\ dp + V\ dV = -2f\frac{V^2}{D}\ dL \tag{1}$$

but $V = v(Q/A)$. Substituting and dividing by v, we get:

$$\frac{g}{v}\ dz + dp + \left(\frac{Q}{A}\right)^2 dv = -2f\left(\frac{Q}{A}\right)^2 v\frac{dL}{D} \tag{2}$$

Now, using $(dz/dL) = \sin\phi$ and reorganizing:

$$\frac{dp}{dL} = -\frac{g}{v}\sin\phi - \left(\frac{Q}{A}\right)^2\frac{dv}{dL} - 2f\left(\frac{Q}{A}\right)^2\frac{v}{D} \tag{3}$$

which is many times presented as the sum of the pressure drop for elevation change plus the acceleration term, plus the friction term, respectively.

Integrating this expression numerically requires computing the molar volume (v) and the friction factor (f) at each integration step. Therefore, a one-step Euler integration formula is

$$(p_{L+\Delta L} - p_L) = -\frac{g}{v_L}\sin\phi\Delta L - \left(\frac{G}{A}\right)^2(v_{L+\Delta L} - v_L) - 2v_L f(v_L, p_L)\left(\frac{G}{A}\right)^2\frac{\Delta L}{D} \tag{4}$$

Recognizing that $v_{L+\Delta L}$ is a function of $p_{L+\Delta L}$, the integration requires iterating until convergence (Euler implicit). For example, assuming $v_{L+\Delta L} = v_L$, we obtain a value of $p_{L+\Delta L}$, which can be used to recalculate $v_{L+\Delta L}$ and so on. One can even make the friction factor a function of the end conditions, making it a completely implicit integration formula. Alternatively, the acceleration term can be neglected, as it is usually very small as compared to the other terms, something that is done in several approximate formulas, anyway, and has a truly explicit integration. Finally, to calculate the specific volume v, a volumetric equation of state is needed ($pv = ZRT$). Thus, a computation of the temperature is also needed. When the pipeline is isothermal, then this is straightforward. However, if the flow is not isothermal, a total heat balance may be needed to recompute the temperature. For that, the first law of thermodynamics for open systems can be applied to the section of pipe of length, ΔL.

$$(\underline{H}_{L+\Delta L} - \underline{H}_L) = \dot{q}\Delta L = UA(T_{\text{out}} - T_L)\Delta L \tag{5}$$

This makes the equation also implicit because enthalpy $\underline{H}$ is a function of pressure and temperature. Thus, under these circumstances, the correct approach is to start with some value of $v_{L+\Delta L}$, which allows computing $p_{L+\Delta L}$ and $\underline{H}_{L+\Delta L}$, from which $T_{L+\Delta L}$ can be computed. With this temperature and the value of $p_{L+\Delta L}$, the new value of $v_{L+\Delta L}$ can be obtained.

APPROXIMATE AND SURROGATE MODELS OR METAMODELS

Existing pressure drop equations and methods need to be generalized to modify, design, or forecast gas pipeline characteristics. We seek here the use of simulation to obtain a new equation that will allow us its use in a noniterative form. In fact, we seek a model similar to the exiting pressure drop correlations. To do this, we now divide eq 1 by v and put it in integral form:

$$g\int\frac{dz}{v^2} + \int\frac{dp}{v} + \left(\frac{Q}{A}\right)^2\int\frac{dV}{V} = -2\left(\frac{Q}{A}\right)^2\frac{1}{D}\int f\ dL \tag{6}$$

Using average values of density and friction factor, one gets:

$$\rho_{\text{av}}^2\cdot g\cdot\Delta z + \int_{\text{in}}^{\text{out}}\frac{dp}{v} + \left(\frac{Q}{A}\right)^2\ln\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = -2\cdot\left(\frac{Q}{A}\right)^2\cdot f_{\text{av}}\cdot\frac{L}{D} \tag{7}$$

where

$$\rho_{\text{av}} = \frac{Mp_{\text{av}}}{2\cdot Z_{\text{av}}RT_{\text{av}}} \tag{8}$$

where the following average for the pressure can be used:

$$p_{\text{av}} = \frac{\int_{\text{in}}^{\text{out}}p^2\ dp}{\int_{\text{in}}^{\text{out}}p\ dp} = \frac{2}{3}\left[\frac{p_{\text{out}}^3 - p_{\text{in}}^3}{p_{\text{out}}^2 - p_{\text{in}}^2}\right] = \frac{2}{3}\left[p_{\text{in}} + p_{\text{out}} - \frac{p_{\text{in}}p_{\text{out}}}{p_{\text{in}} + p_{\text{out}}}\right] \tag{9}$$

In turn:

$$T_{\text{av}} = \frac{T_{\text{in}} + T_{\text{out}}}{2} \tag{10}$$

Also, the average friction factor is

$$f_{\text{av}} = \frac{f(T_{\text{in}}, P_{\text{in}}) + f(T_{\text{out}}, P_{\text{out}})}{2} \tag{11}$$

We now use $(1/v) = (pM/Z_{\text{av}}RT_{\text{av}})$, which leads to

$$\int_{p_{\text{in}}}^{p_{\text{out}}}\frac{dp}{v} = \frac{M}{Z_{\text{av}}RT_{\text{av}}}\int_{p_{\text{in}}}^{p_{\text{out}}}p\cdot dp = \frac{M}{2\cdot Z_{\text{av}}RT_{\text{av}}}(p_{\text{out}}^2 - p_{\text{in}}^2) \tag{12}$$

Therefore:

$$\rho_{\text{av}}^2 g\cdot\Delta z + \frac{M}{2\cdot Z_{\text{av}}RT_{\text{av}}}(p_{\text{out}}^2 - p_{\text{in}}^2) + \left(\frac{Q}{A}\right)^2\ln\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = -2\left(\frac{Q}{A}\right)^2 f_{\text{av}}\frac{L}{D} \tag{13}$$

but

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \left(\frac{Z_{\text{out}}\cdot T_{\text{out}}}{Z_{\text{in}}\cdot T_{\text{in}}}\right)\cdot\frac{p_{\text{in}}}{p_{\text{out}}} \tag{14}$$

Then

$$\rho_{av}^2 g\cdot\Delta z + \frac{M}{2\cdot Z_{av}RT_{av}}(p_{out}^2 - p_{in}^2) + \left(\frac{Q}{A}\right)^2 \ln\left(\cdot\frac{Z_{out}\cdot T_{out}p_{in}}{Z_{in}\cdot T_{in}p_{out}}\right) = -2\left(\frac{Q}{A}\right)^2 f_{av}\frac{L}{D} \quad (15)$$

The values of Z_{av} can be obtained in various ways.

Equation 15 can be further simplified. First, neglect the acceleration term because it is usually small as compared to the others, to obtain

$$\rho_{av}^2 g\cdot\Delta z + \frac{M}{2\cdot Z_{av}RT_{av}}(p_{out}^2 - p_{in}^2) + 2\left(\frac{Q}{A}\right)^2 f_{av}\frac{L}{D} = 0 \quad (16)$$

From this equation, we can get Q^2, as follows:

$$Q^2 = -\frac{M\pi^2D^5}{64f_{av}Z_{av}RT_{av}L}(p_{out}^2 - p_{in}^2) - \frac{\rho_{av}^2 g\pi^2D^5}{32f_{av}L}\cdot\Delta z \quad (17)$$

After using $\rho_{av} \cong (p_{av}/Z_{av}RT_{av})$, and rearranging we get

$$Q^2 = \frac{M\pi^2D^5}{64f_{av}Z_{av}RT_{av}L}(p_{in}^2 - p_{out}^2) - \frac{p_{av}^2 g\pi^2D^5}{32f_{av}Z_{av}^2R^2T_{av}^2L}\cdot\Delta z \quad (18)$$

Equation 18 can give rise to several approximate expressions, with the appropriate choice of the friction factor expression. Because of the dependence of f_{av} with the Reynolds number, we expect such approximate expressions to be likely more accurate for certain cases.

Alternatively, one can start from eq 2, neglect the acceleration term $V\,dV$, and use $\rho = p/RT$ as well as $V = v(Q/A)$ to get:

$$\frac{dp}{dL} = -p\frac{g}{RT}\sin\phi - f\frac{32RTQ^2}{p\pi^2D^5} \quad (19)$$

Multiplying by p and rearranging, one gets:

$$\frac{p\,dp}{\left[p^2 - f\frac{32R^2T^2Q^2}{g\sin\phi\pi^2D^5}\right]} = -\frac{g}{RT}\sin\phi\,dL \quad (20)$$

Integrating:

$$\ln\left[\frac{p_1^2 + C^2Q^2}{p_2^2 + C^2Q^2}\right] = s \quad (21)$$

where $C^2 = f_{av}(32R^2T_{av}^2/g\sin\phi\pi^2D^5)$ and $s = (gL/RT)\sin\phi$. Therefore, we get

$$P_1^2 = e^sP_2^2 + \left[f_{av}\frac{32RT_{av}^2}{\pi^2D^5}\right]Q^2L\frac{(e^s-1)}{s} \quad (22)$$

Thus, if we call $L_e = L((e^s - 1)/s)$, which in the limit of $s \to 0$ becomes $L_e = L$, and rearrange, we get:

$$Q^2 = \frac{P_1^2 - e^sP_2^2}{\left[f_{av}L_e\frac{32RT_{av}^2}{\pi^2D^5}\right]} \quad (23)$$

One can use eq 18 or 23 as a starting point for further approximations.

■ APPROXIMATE EXPRESSIONS

We first review some of the approximate expressions that were derived through time: AGA, Panhandle A, Panhandle B, and Weymouth are usually recommended these days. All of these equations are derived from the General Flow equation, that is, a simplification of eq 18 or 23. They are

$$Q = 77.54\frac{T_b}{P_b}\left(\frac{P_1^2 - e^sP_2^2}{GT_fL_eZf}\right)^{0.5}D^{2.5} \quad \text{General Flow Equation} \quad (24)$$

$$Q = 435.87E\left(\frac{T_b}{P_b}\right)^{1.0788}\left(\frac{P_1^2 - e^sP_2^2}{G^{0.8539}T_fL_eZ}\right)^{0.5394}D^{2.6182} \quad \text{Panhandle A} \quad (25)$$

$$Q = 737E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1^2 - e^sP_2^2}{G^{0.961}T_fL_eZ}\right)^{0.51}D^{2.53} \quad \text{Panhandle B} \quad (26)$$

$$Q = 433.5E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^sP_2^2}{GT_fL_eZ}\right)^{0.5}D^{2.667} \quad \text{Weymouth} \quad (27)$$

$$Q = 38.77F\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^sP_2^2}{GT_fL_eZ}\right)^{0.5}D^{2.5} \quad \text{AGA} \quad (28)$$

We first point out that these expressions correspond to different choices of the friction factor. In the case of AGA, the transmission factor is used instead of the Darcy friction factor; they are related by $F = 2/\sqrt{f}$. For the Panhandle and Weymouth equations, the corresponding friction factors as reported by Guo and Ghalambor[10] are

$$f = \frac{0.085}{Re^{0.147}} \quad \text{Panhandle A} \quad (29)$$

$$f = \frac{0.015}{Re^{0.0392}} \quad \text{Panhandle B} \quad (30)$$

$$f = \frac{0.032}{D^{1/3}} \quad \text{Weymouth} \quad (31)$$

Because these expressions have been judged to have the right coefficients and exponents for a clean pipe, the so-called dimensionless pipe "efficiency" factor was added to adjust for the inaccuracies. This factor is adjustable to meet "experimental" results in some fashion. The recommended range of values in the literature is from 0.6 to 0.92[9] or from 0.85 to 1.0.[3,6,7,17] The typical value that nearly all of the literature assumes is 0.92.[2,3,6,7,11,16,17]

The AGA transmission factor reported by S. Menon[8] must be the smaller of the following two equations:

$$F = 4\log\left(\frac{3.7D}{e}\right) \quad (32)$$

$$F = 4D_f\log\left(\frac{Re}{1.4125F_t}\right) \quad (33)$$

where

$$F_t = 4\log\left(\frac{Re}{F_t}\right) - 0.6 \tag{34}$$

and D_f is the pipe drag factor, which is between 0.9 and 0.99.

These equations have been further changed over time. Morssy et al.[13] modified these equations by adjusting the exponents and constants. The equations managed to fit their example better, with an assumed efficiency factor of 0.92. There error was adjusted as follows: Panhandle A saw an improvement from 3% error to 0.16%, Panhandle B improved from 3% to 0.159%, and Weymouth improved from 0.5% to 0.157%. Because the efficiency factor was assumed, the entire equation needed to be molded to fit the data.

Further, Falade et al.[14] combined the General Flow equation and the Colebrook–White equation to develop the following equation:

$$Q = 0.15504\frac{T_b}{P_b} \times \frac{B\mu D}{G} \times -\log\left(\frac{e}{3.7065D} + \frac{0.45541}{B}\right) \tag{35}$$

where e is the pipe roughness, and B is equivalent to the following:

$$B = \left[\frac{(P_1^2 - P_2^2)D^3G}{T_fLZ\mu^2}\right]^{0.5}$$

This equation is noniterative when calculating the flow rate with known pressures. It is iterative when calculating pressure though. This means the equation becomes more unwieldy for optimization purposes.

Finally, Usman et al.[15] redesigned the Weymouth equation as follows:

$$Q = 3.23\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - yP_2^2}{GT_fL_eZf}\right)^{0.5}D^{2.5} \tag{36}$$

with y being a correction factor for changes in diameter and f being the Weymouth friction factor. This equation applies to a horizontal pipe only. As the equation is written, the pipeline efficiency is ignored, rendered a value of 1. The purpose of the value y is to adjust the outlet pressure for changes in diameter. The equation must be used in conjunction with an explicit equation for y. If $y = 1$, the diameter is constant, and the original Weymouth equation applies.

When there are changes in elevation, the values of s and L_e in the above-mentioned Panhandle and Weymouth (eqs 24 and 27) equations need to be adjusted. The value of s is calculated as follows:

$$s = 0.0375G\left(\frac{H^2 - H_1}{T_fZ}\right) \tag{37}$$

However, the literature does not agree upon whether to the use the above correction factor e^s or the following equation in Panhandle B form:[4]

$$Q = 737E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{(P_1^2 - P_2^2) - \frac{0.0375G(H_2 - H_1)P_{av}^2}{T_fZ}}{G^{0.961}T_fL_eZf}\right)^{0.51}D^{2.53} \tag{38}$$

where the friction and elevation pressure drop terms are separate. This equation resembles eq 18 a lot better.

■ ADJUSTMENT OF THE PIPE EFFICIENCY

We suggest to fit the data with one value to produce the most simplistic equation, so the common forms listed in eqs 25–28 are just as viable. While most of the literature (and software) leaves the value of efficincy to the discretion of the user, one can actually obtain this value by regression of rigorous results. Consider the Panhandle B equation. Assume no change in elevation, thus allowing the equation to be written as follows:

$$\frac{Q^{1.96078}G^{0.961}T_fL_eZP_b^2}{D^{4.96078}737^{1.96078}T_b^2} = E^{1.96078}(P_1^2 - P_2^2) \tag{39}$$

We can represent the value $E^{1.96078}$ in eq 39 as E_a and then plot $Q^{1.96078}G^{0.961}T_fL_eZP_b^2/D^{4.96078}737^{1.96078}T_b^2$ versus $(P_1^2 - P_2^2)$, to obtain the value of E_a through linear regression for all of the ranges of flow, diameter, length, and initial pressure investigated.

In turn, if we want to absorb the inaccuracies of using the same Z for different conditions, we can use the following:

$$\frac{Q^{1.96078}L_e}{D^{4.96078}} = E_c(P_1^2 - P_2^2) \tag{40}$$

where E_c includes the dependence on all parameters that are not diameter and length dependent (G, T_f, Z, T_b, and P_b). We then plot $Q^{1.96078}L_e/D^{4.96078}$ versus $(P_1^2 - P_2^2)$, and obtain the value of E_c for all of the ranges of flow, diameter, length, and initial pressure investigated. We illustrate this below.

■ GENERALIZED METAMODEL

One of the problems of all of the approximate formulas is that they make assumptions about the friction factor, which in our simulations is obtained using the Colbrook equation, so we expect all approximations for which the efficiency has been properly adjusted and our bilinear Metamodel to be somehow inaccurate. To obtain the most accurate Metamodel for a given range of flows, pressures, lengths, and diameters, we assume the following form:

$$\frac{Q^{\alpha}L_e^{\beta}}{D^{\gamma}} = E_g(P_1^2 - P_2^2) \tag{41}$$

Also, we use nonlinear regression to obtain the values of the coefficients.

Quadratic Metamodel. In many cases, the hydraulic equations are used as part of an optimization procedure. Examples are data reconciliation, leak detection, compressor station operations, etc. Because the problems are nonlinear (NLP), and sometimes even mixed integer nonlinear (MINLP), they are difficult to solve, especially if the equations contain fractional exponents. Keeping this in mind, one can develop a Quadratic Metamodel that is amenable to the use of global optimization procedures and can guarantee solutions can be found. This is in contrast to the traditional procedures that require a good initial point to provide an answer that many times is not actually the best.

Thus, we propose the following procedure:

$$L\frac{Q^2}{D^5} = A(p_{in}^2 - p_{out}^2) - p_{av}^2B\Delta z \tag{42}$$

where A and B are a function of the gas composition, the average temperature, and indirectly to pressure through the calculation of f_{av}, but not explicitly on the geometry (length and diameter). They are, however, an implicit function of the pipe diameter (through the calculation of the friction factor). Equation 42 is our quadratic suggested Metamodel. For $\Delta z = 0$, a linear regression model suggests a relation between LQ^2/D^5 and $(p_{\text{in}}^2 - p_{\text{out}}^2)$, the slope being A and intercept zero. To improve accuracy, the values can be calculated for specific flow rate ranges and also for specific pressure ranges. All of these options will be discussed below. When Δz is not zero, a linear regression model suggests a relationship of $LQ^2/D^5 p_{\text{av}}^2 \Delta z$ and $p_{\text{in}}^2 - p_{\text{out}}^2/p_{\text{av}}^2 \Delta z$, the slope being A and the intercept B.

Optimization. In the case of using the quadratic equation in the context of optimization procedures, an appropriate A value needs to be selected. Unless an A value is selected generalizing all possible variables (diameter, temperature, pressure, length, composition, and flow), then an optimization method must be used to vary A to each parameter. As there are different values for different ranges (or intervals, i) of flow, then, in addition to eq 36, one would use the following equations:

$$A = \sum a_i \lambda_i \tag{43}$$

$$\sum \lambda_i = 1 \tag{44}$$

$$Q_i^{\min} \lambda_i \leq Q \leq Q_i^{\max} \lambda_i \tag{45}$$

where a_i are the values of A for the different flow intervals, and λ_i are binary variables that assume the value of one for the flow rate falling in the corresponding interval. The values $Q_i^{\min}, Q_i^{\max}$ are the limits of each interval. Equations 43−45 essentially select the appropriate A value for a corresponding flow rate. If, in addition, the value of A is calculated for different pressure ranges or particular natural gas compositions, similar equations need to be added.

■ ILLUSTRATIONS

Consider a natural gas of the following composition: 85% methane, 8% ethane, 4% propane, 1% butane, 0.2% pentane, 0.05% hexane, 0.05% *i*-butane, 0.05% *i*-pentane, 1% nitrogen, and 0.65% carbon dioxide. We performed simulations in Pro/II for different lengths and different diameters, as well as different initial pressures with no change in pipe elevation ($\Delta z = 0$). Specifically, we varied the flow rate from 6.24 Mft^3/s (539 MMSCFD) to 7.879 Mft^3/s (681 MMSCFD) using 16 intermediate points, the initial pressure from 1100 to 1400 psia using 4 intermediate points, and the length from 39 600 ft (7.5 miles) to 79 200 ft (15 miles) using 4 intermediate points.

However, there is error in a simulator as well. To mitigate this error as much as possible, all of the calculations are done over 600 pipe segments regardless of length. The pressure profile calculated over a single long pipe segment is prone to larger error as it has a large pressure change. If the calculations are over smaller segments of the whole, the error reduces significantly.

Among all correlations that have been developed, Panhandle B is the most simplistic equation that can fit our data. Indeed, this equation is recommended to be used when the Reynolds number is from 4 to 40 million.[8] Our example falls around 25−42 million; thus Panhandle B covers the majority of the range of our example.

When analyzing an existing pipeline, the length and diameter are parameters, and the flow rate and pressure drop are variables. If the pipeline is being designed, all of these values are variables. The length of the pipe in the design phase is dependent on restrictions or the maximum pressure drop. We vary the length for the generalized case, but we use a specified length (7.5 miles) when we reduce the range in which the A value or E value is usable.

Something we leave general in the Panhandle B equation is the compressibility factor. The compressibility factor, Z, can be rigorously calculated as it is the average over the length and is dependent on the pressure profile. When using a general compressibility factor (we leave it at 0.84 for all diameters, pressures, lengths, and flows), all of the error from the compressibility factor is put into the E value. We could ignore the compressibility term, but we want the E value to be close to its actual value when compressibility is considered. We do this to make the comparison between the error associated with A and B in the Quadratic Metamodel and only the E value in the Panhandle B equation. If we use the correct compressibility factor for the average pressure and temperature in a pipe, we would reduce the error in the Panhandle B equation.

Figure 1 shows the regression for Panhandle B to obtain E, and Figure 2 shows the regression line for the Quadratic

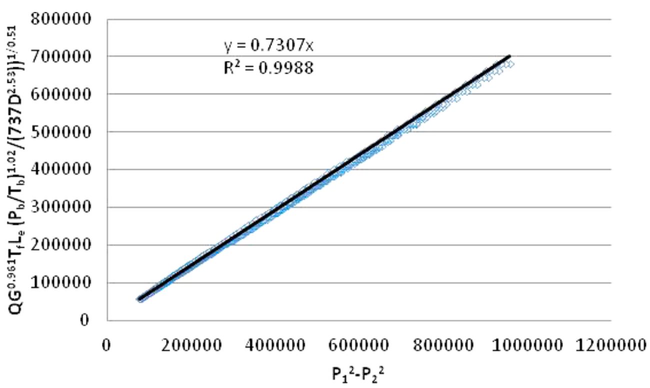

Figure 1. Linear regression of E_c value in Panhandle equation.

Metamodel to calculate A. We recall that the elevation change is zero for this example. This causes the trend line to not go through the center of the points. The majority of the points are in the lower pressure drop region. To provide an idea of this, in the case of Figure 2, 50% of the points have a value of $P_1^2 - P_2^2$ less than 270 000. Meanwhile, 75% of the points have a value of $P_1^2 - P_2^2$ less than 430 000. The maximum value of $P_1^2 - P_2^2$ seen in Figure 2 is approximately 960 000.

Linear regression renders a value of $E_a = 0.74027$ (the value when compressibility is adjusted per case) and $E_c = 0.7307$ (when compressibility is general), or $E = 0.8578$ and $E = 0.8521$, respectively. The quadratic model linear regression gives $A = 0.407072$ ($\text{Mft}^2/\text{s}^2 \cdot \text{psia}^2$). Finally, the nonlinear regression for the generalized Metamodel renders $\alpha = 1.9816$, $\beta = 0.9971$, $\gamma = 5.1312$, and $E_g = 0.3533$.

Using all of the ranges for all of the pressures, diameters, flows, and lengths results in bigger uncertainties for larger pressure drops. As discussed above, to get better results, the values of A can be calculated for smaller ranges. For example, one can divide the flow rate range into five ranges and obtain five different values of A in these flow rate ranges for a particular diameter and length. The resulting A values are shown in Table 1 for a diameter of 20 in. and length of 7.5 miles.

We also looked at the effect of elevation changes on the equations. We performed this analysis for an assumed network with a particular diameter (20 in.) and distance (7.5 miles). For

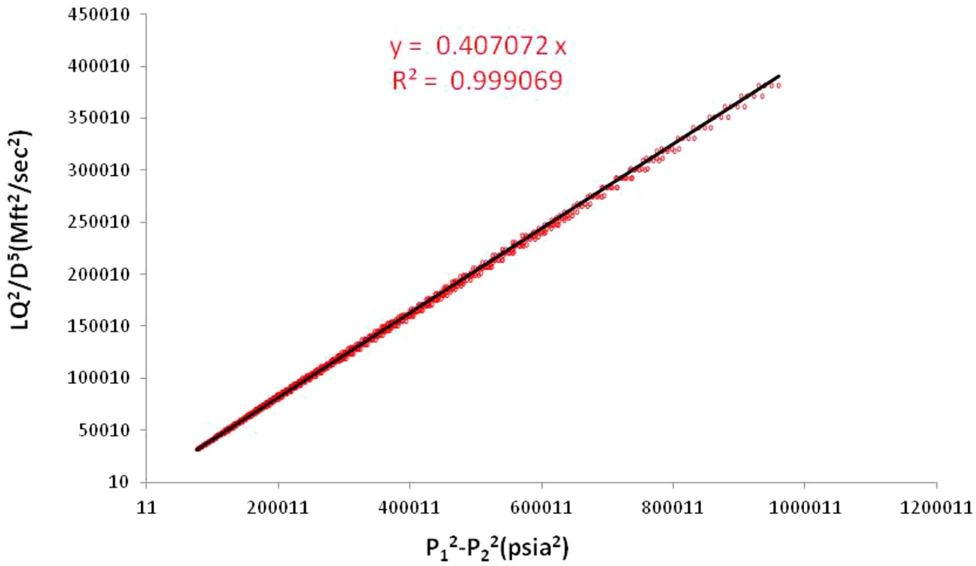

Figure 2. Pressure drop correlation results.

Table 1. Specific *A* Values for Different Ranges of Flow Rates and for Different Pressures

	P (psia)			
Q (Mft^3/s)	1100	1200	1300	1400
from 6.240 to 6.568	0.3967	0.4010	0.4050	0.4087
from 6.568 to 6.896	0.3969	0.4013	0.4054	0.4091
from 6.896 to 7.223	0.3972	0.4016	0.4057	0.4094
from 7.223 to 7.551	0.3974	0.4018	0.4060	0.4097
from 7.551 to 7.879	0.3977	0.4021	0.4063	0.4101

elevation changes and the computation of *A* and *B*, we consider the change in height from 200 m (656.2 ft) to 800 m (2624.8 ft). Figure 3 shows the desired general linear relationship, which considers multiple ranges of flow, pressure, and elevation. As a result, A = 0.399793 and B = 0.00002 (Mft/s^2·psia^4). Although *B* looks like a small number, this does not mean that pressure drop due to elevation changes is negligible. In fact, the pressure drops about 18−19 psia for every 200 m of increased elevation in this example. As stated before, it is possible to divide the data into more specific ranges to increase accuracy at the cost of generality.

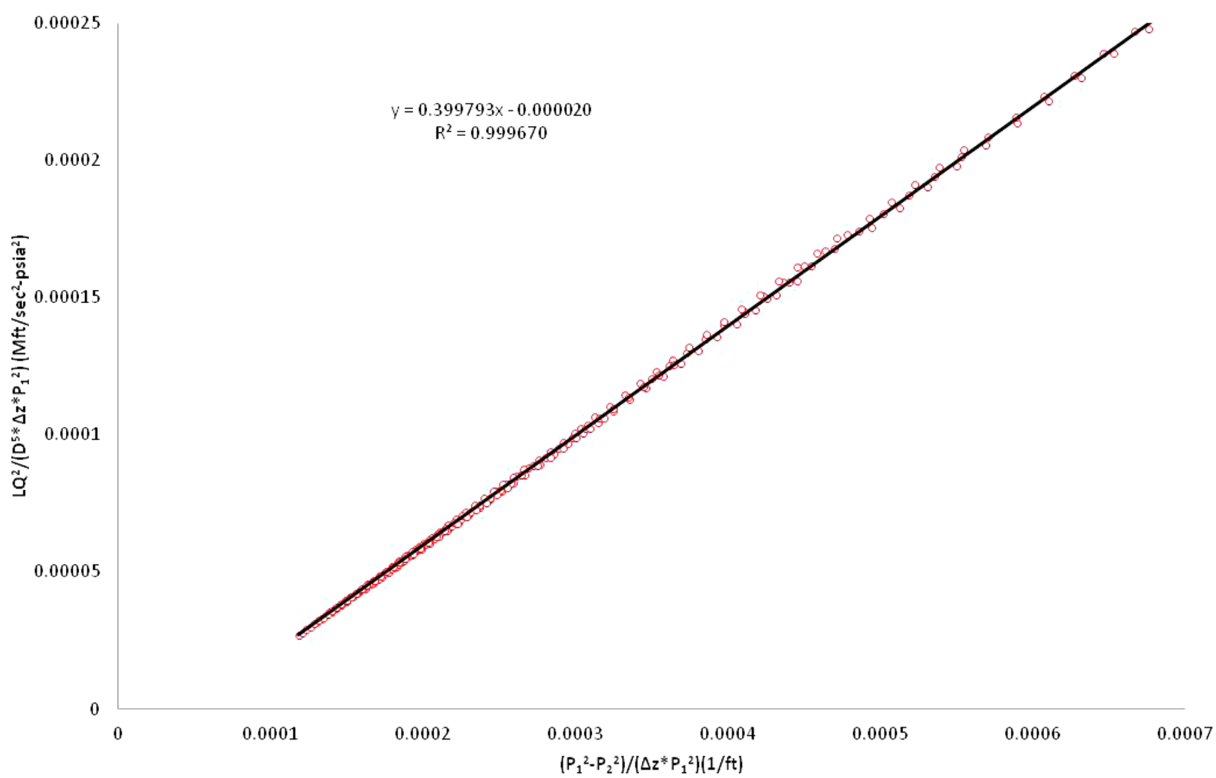

Figure 3. Pressure drop correlation results.

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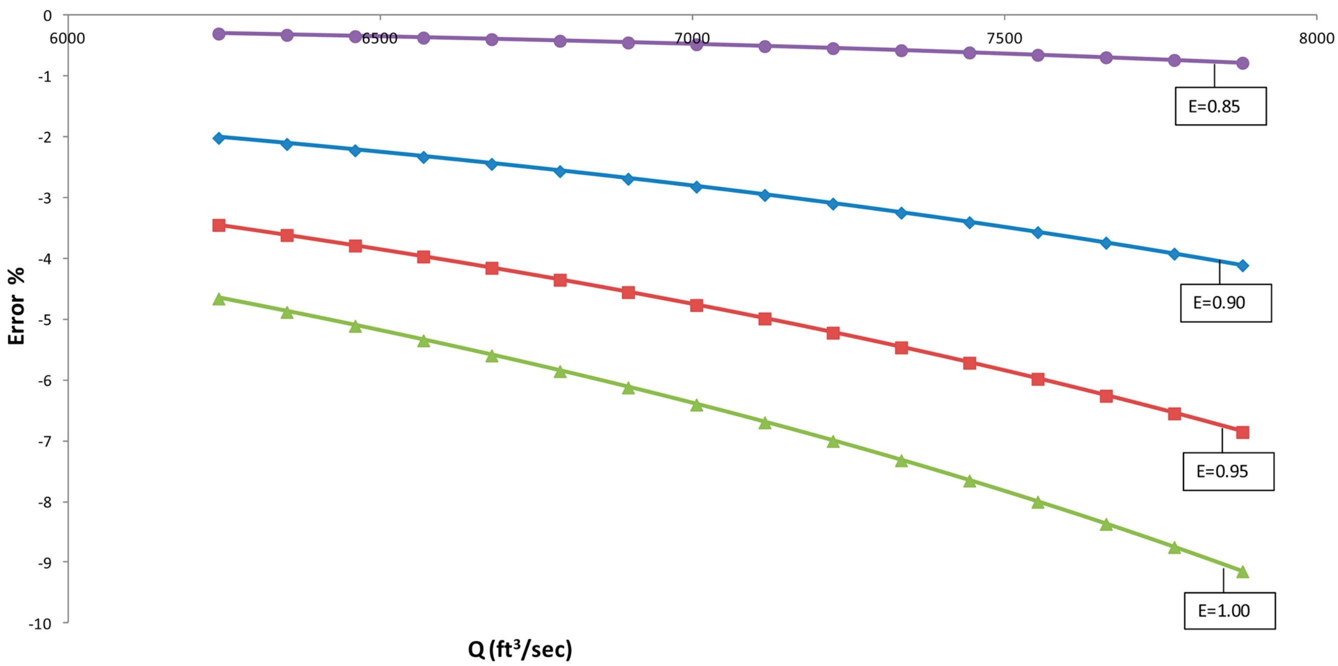

Figure 4. Error from Panhandle B equation for various assumed E values.

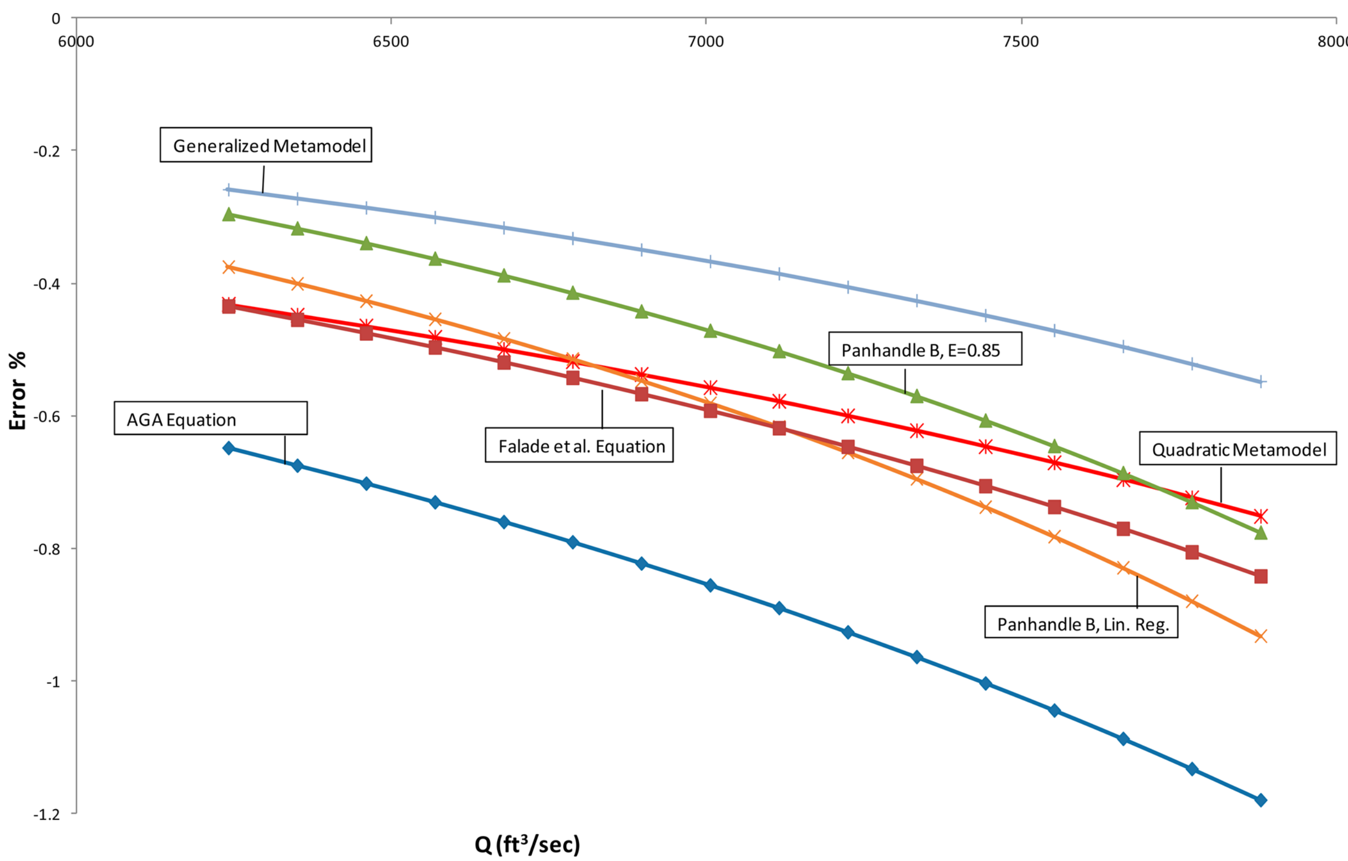

Figure 5. General comparison of the percent error for the various equations at L = 7.5 miles, D = 20 in., and P = 1100 psia.

■ COMPARISON OF APPROXIMATE EQUATIONS AND METAMODELS

The Panhandle B equation is dependent on the pipe efficiency value, E. As we discussed, this value should be less than 1; however, a means of properly calculating the value is not easily available, unless one does the regression shown above. Typically, the value is assumed for the particular pipe (usually 0.92 on average). Figure 4 indicates the significant errors in the final pressure when it is assumed between 0.85 and 1.00. The error is obtained over a range of flows but for a constant pipe

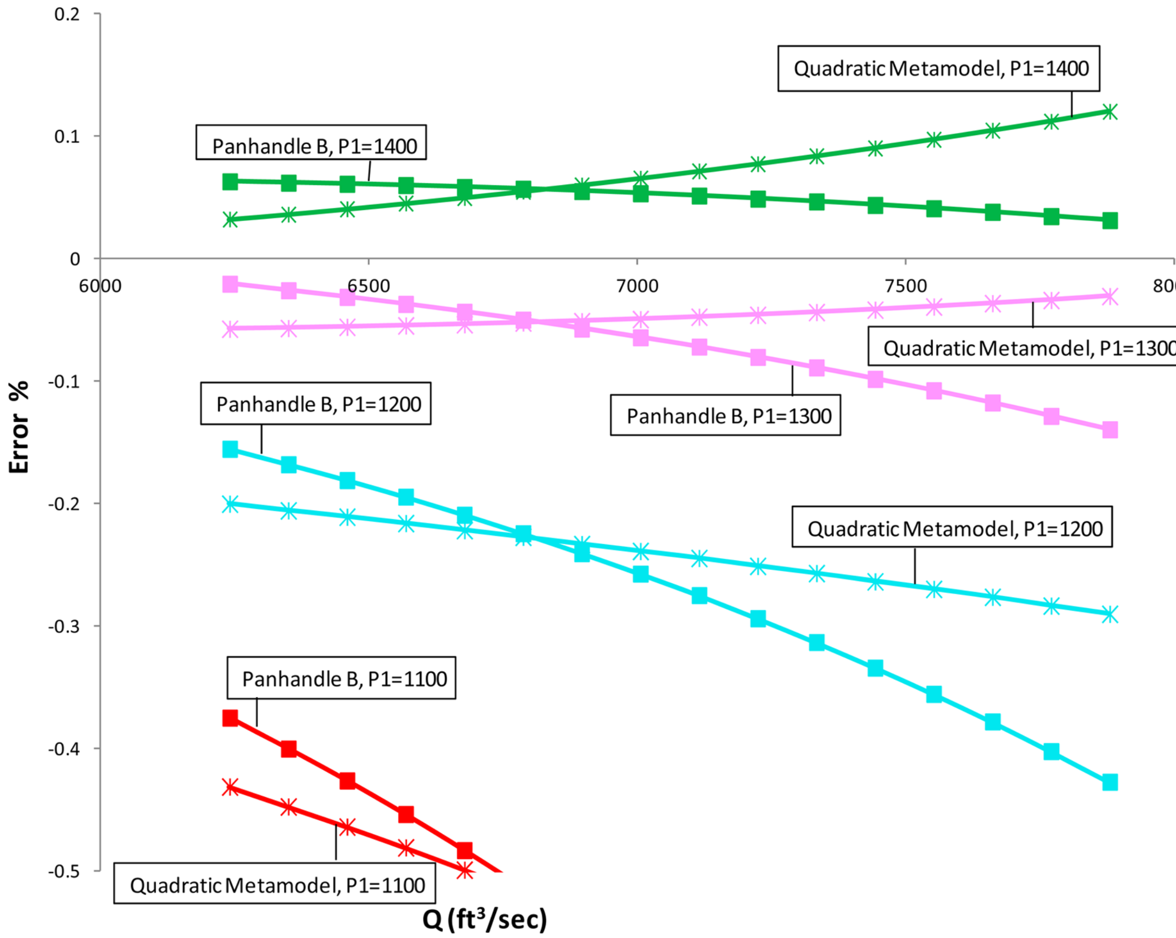

Figure 6. General comparison of percent error of the Quadratic Metamodel and Panhandle B with varying inlet pressures for L = 7.5 miles and D = 20 in.

diameter of 20 in., a constant pipe length of 7.5 miles, and a constant initial pressure of 1100 psia.

For this example, the value of E should actually be closer to the lower end around 0.85, but one does not know unless rigorous values are provided to calculate the error. As the value of E approaches 1, the error increases to as much as 5−10%.

In Figure 5, we compare the best assumption with a general linear regression of E for Panhandle B, a general linear regression of A in the Quadratic Metamodel, and a general nonlinear regression of the Metamodel equation. We also include the AGA equation and the equation from Falade et al.[14] The linearly regressed E value is 0.8571, which considers all flows, pressures, lengths, and diameters. What can also be noticed is the Quadratic Metamodel using a linearly regressed A value, at 0.407072 ($\mathrm{Mft^2/s^2 \cdot psia^2}$), performs better than the linearly regressed Panhandle B equation. The assumed value of 0.85 outperforms the Panhandle B equation using a linearly regressed value of E for this particular case (L = 7.5 miles, D = 20 in., and $P1$ = 1100 psia). One must remember that the value of E for all Panhandle B cases was obtained as a compromise using a regression for all pressures, diameters, and lengths. One should expect then that for different conditions of diameter, length, and initial pressure, the equation using the linearly regressed value of E for all values will be outperformed by some particular value of E. In turn, the generalized nonlinear regression Metamodel performs the best, as we indicated it would. The problem with using typical values for E is not a new problem and has been discussed by some authors.[17]

Figure 5 highlights the importance of properly determining the E or A values. Both methods tend to increase in error as flow increases, which is not always the case as we show next. There are a few other details about Figure 5 that can be determined. First, AGA does not adequately calculate the outlet pressure, having the worst error. The Falade et al.[14] equation performs about as well as Panhandle B and the Metamodel. Both of these equations were designed using the absolute pipe roughness directly from Pro/II at 0.00075 in. Because the AGA equation underperforms and both the AGA and the Falade et al.[14] equations are complex equations containing logarithms and iterations, we continue the work focusing on Panhandle B and the Metamodel.

Figure 6 is a comparison of the Panhandle B equation and the Metamodel with the E and A values linearly regressed across multiple flow rates, diameters, lengths, and pressures. The trends displayed in Figure 6 are the results for a constant diameter (20 in.), constant length (7.5 miles), and varying pressures (1100−1400 psia). The lines associated with $P1$ = 1100 psia will extend to Q = 7879 cfs, with the error for Panhandle B ranging from −0.38% to −0.93% and the error for the Metamodel ranging from −0.43% to −0.75.

Figure 7 illustrates the case where the diameter is also variable when the length is constant at 7.5 miles and the initial pressure is held constant at 1100 psia. It is not present, but the

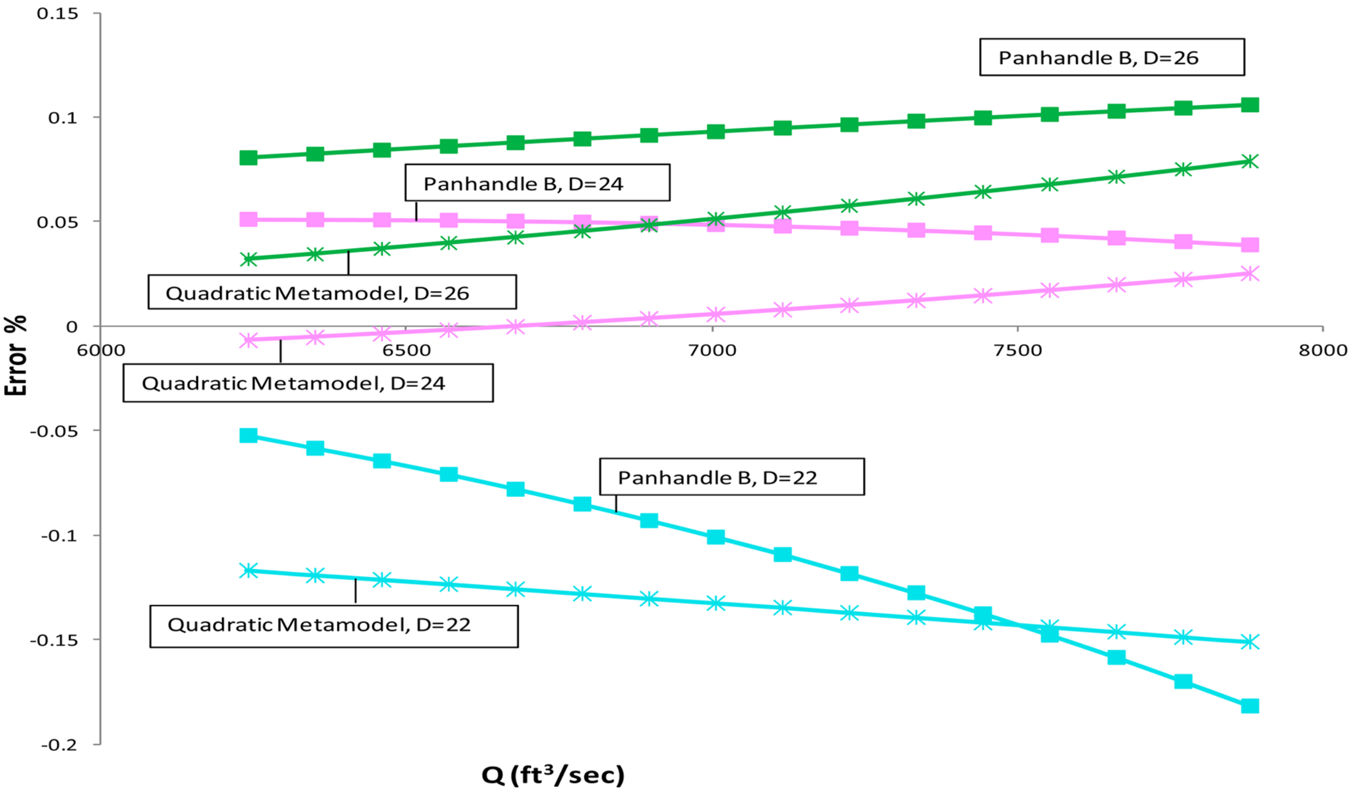

Figure 7. General comparison of Metamodel and Panhandle B with varying diameter for L = 7.5 miles and P = 1100 psia.

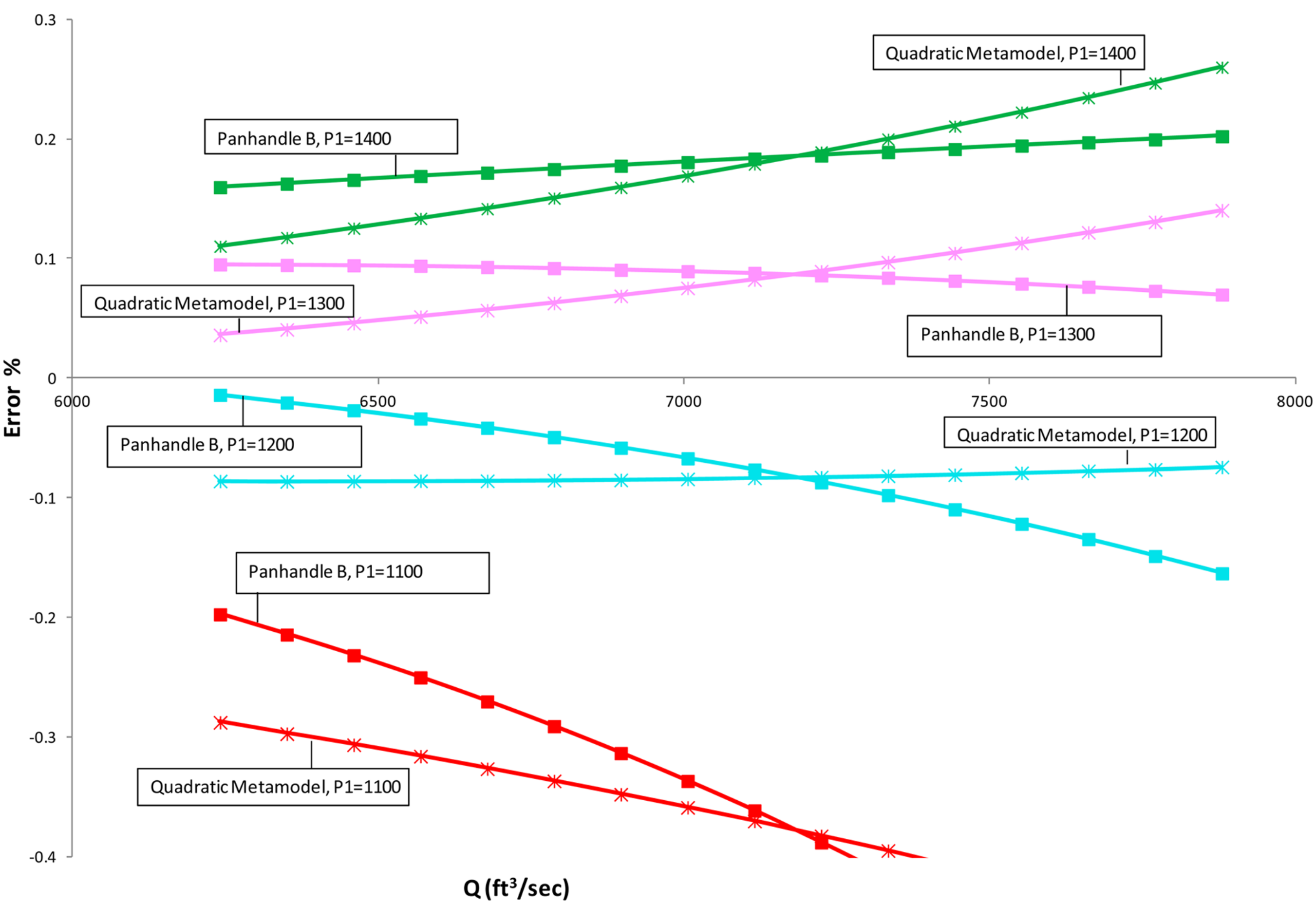

Figure 8. Comparison of percent error of Metamodel and Panhandle B for existing design with L = 7.5 miles and D = 20 in.

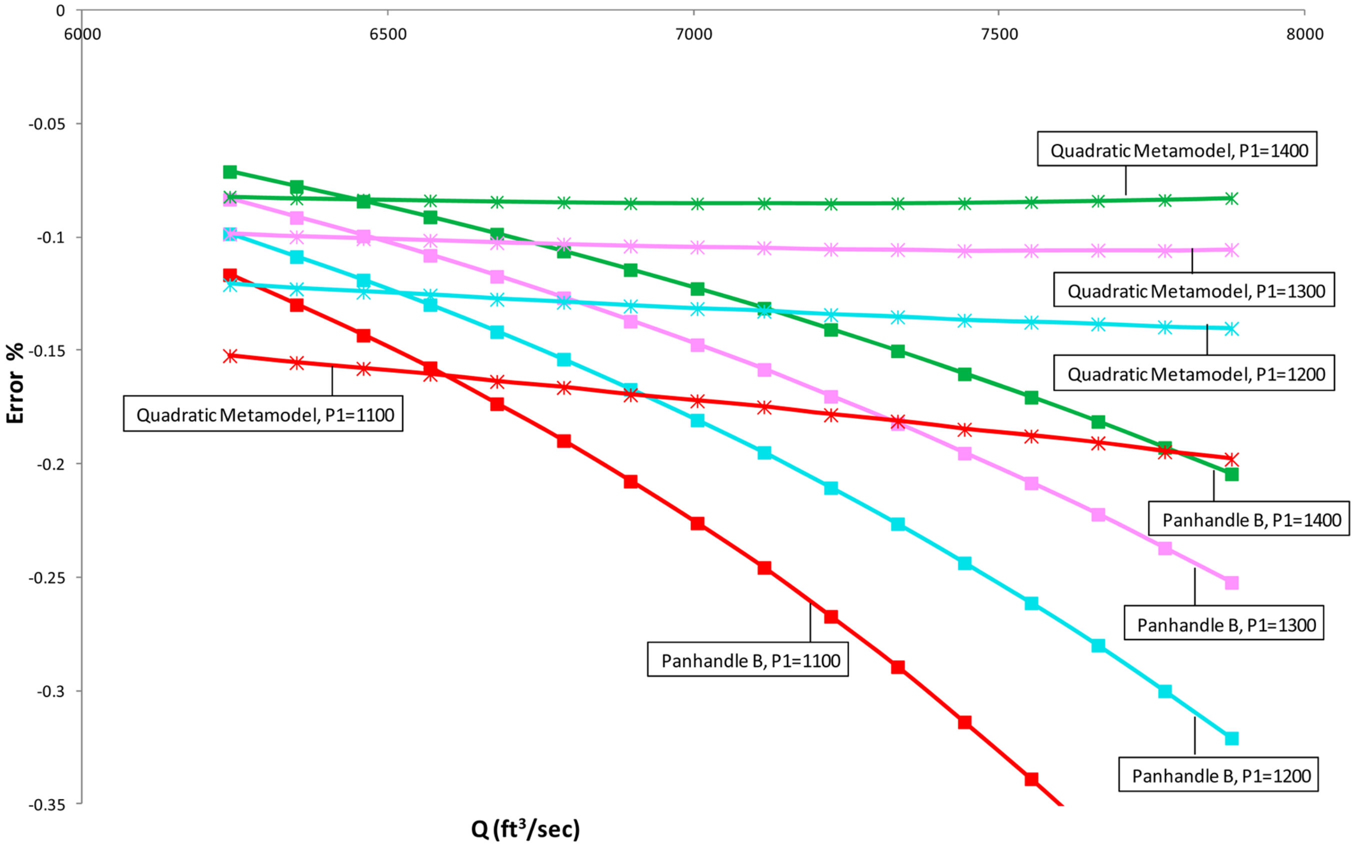

Figure 9. Comparison of percent error between Metamodel and Panhandle B for L = 7.5 miles and D = 20 in. Linearization over particular pressures.

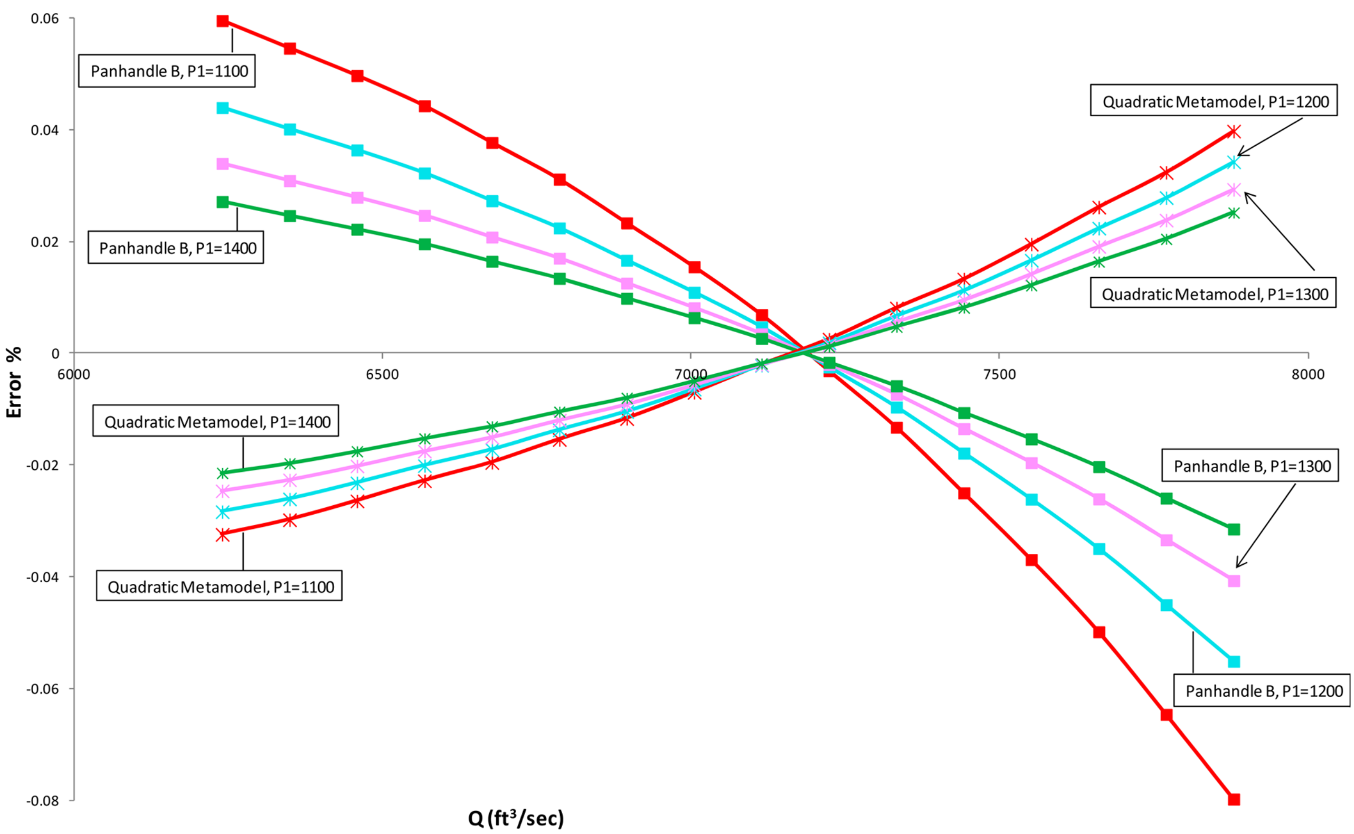

Figure 10. Comparison of percent error between Metamodel and Panhandle B for L = 7.5 miles and D = 20 in. Linearization over particular pressures, diameters, and length.

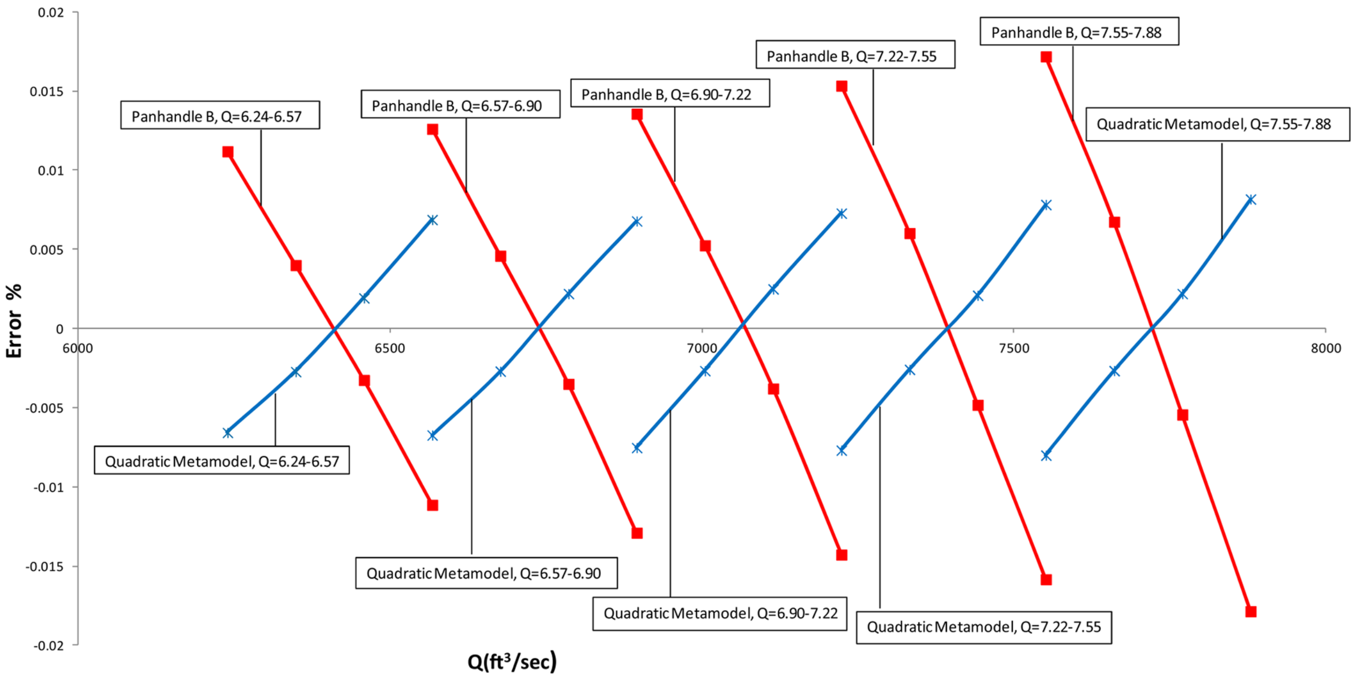

Figure 11. Percent error for Quadratic Metamodel and Panhandle B for L = 7.5 miles, D = 20 in., and P = 1100 psia. Linearization over particular pressures, diameters, lengths, and flow ranges.

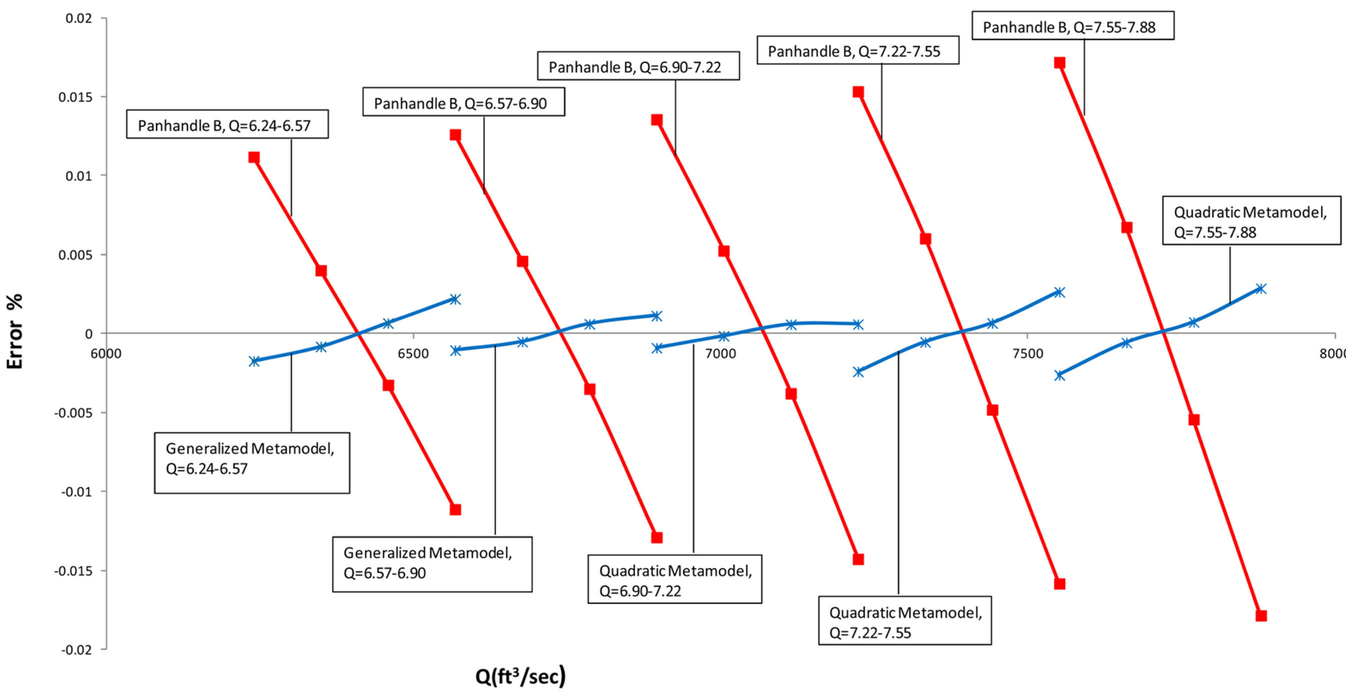

Figure 12. Percent error for Generalized Metamodel and Panhandle B for L = 7.5 miles, D = 20 in., and P = 1100 psia. Linearization over particular pressures, diameters, lengths, and flow ranges.

percent error for a diameter of D = 20 ranges from −0.38% to −0.93% for Panhandle B and −0.43% to −0.75% for the Metamodel.

If the pipeline is already designed, and thus the length and diameter are parameters, Figure 8 can be constructed for the Metamodel and Panhandle B equations. This is a general case where the pressure and flow are considered over a broad range, while the diameter (20 in.) and length (7.5 mi) are held constant. As Figure 8 indicates, using either equation in the general case is sufficient for an existing pipeline and plenty of data. The error for P = 1100 ranges from −0.2 to 0.98 for Panhandle B and from −0.29% to −0.47% for the Metamodel.

We next specify the pressure to decrease the error in the equations. Figure 9 compares the Quadratic Metamodel with Panhandle B with the A value and pipe efficiency regressed for a particular inlet pressure. We used a length of 7.5 miles and estimated the A and E values for each inlet pressure. As can be seen in Figure 9, the error between each initial pressure

Table 2. Errors of the Different Correlations[a]

equation:	Panhandle B		Quadratic Metamodel	
formula:	$Q = 737E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1^2 - e^s P_2^2}{G^{0.961} T_f L_e Z}\right)^{0.51} D^{2.53}$		$\frac{LQ^2}{D^5} = A(P_1^2 - P_2^2)$	
range	E	average error	A	average error
D = 20−24 in., L = 7.5−15 mi, P = 1100−1400 psia, Q = 6.24−7.88 Mft^3/s	0.85	0.264%	0.4071	0.224%
	0.92	2.219%		
	1.00	4.206%		
	0.8521	0.261%		
D = 20 in., L = 7.5 mi, Q = 6.24−7.88 Mft^3/s, $P1$ = 1100 psia, $P2$ = 1200 psia, $P3$ = 1300 psia, $P4$ = 1400 psia	0.8537	0.037%	0.3973	0.019%
	0.8516	0.027%	0.4017	0.017%
	0.8497	0.020%	0.4058	0.015%
	0.8477	0.016%	0.4095	0.013%
D = 20 in., L = 7.5 mi, P = 1100, $Q1$ = 6.24−6.57 Mft^3/s, $Q2$ = 6.57−6.9 Mft^3/s, $Q3$ = 6.9−7.22 Mft^3/s, $Q4$ = 7.22−7.55 Mft^3/s, $Q5$ = 7.55−7.88 Mft^3/s	0.8538	0.007%	0.3967	0.005%
	0.8538	0.008%	0.3969	0.005%
	0.8538	0.009%	0.3972	0.005%
	0.8539	0.011%	0.3974	0.005%
	0.854	0.012%	0.3977	0.005%

[a] Δz = 0; no elevation change.

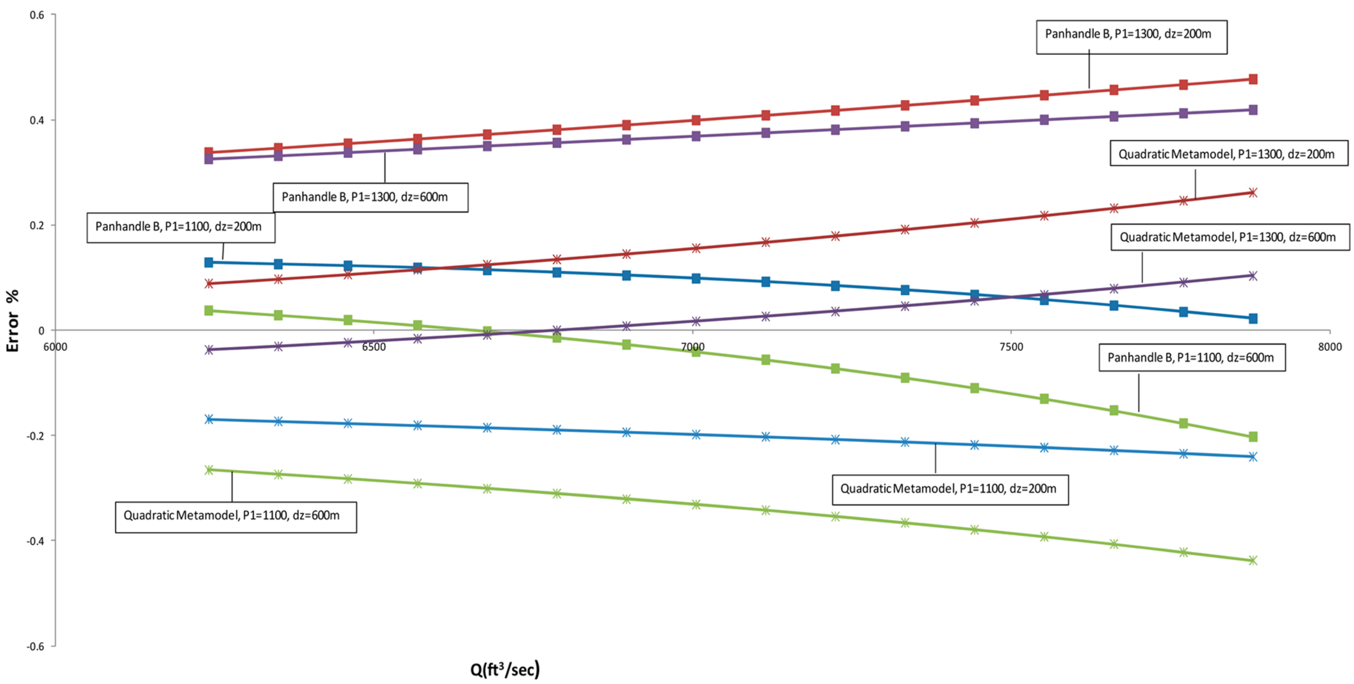

Figure 13. Comparison of % error between Metamodel and Panhandle B for L = 7.5 miles, D = 20 in., and P = 1100 psia. Linearization over particular elevations, diameter, and length.

becomes significantly less. Panhandle B at $P1$ = 1100 psia is not completely visible in the figure and has a range of percent error from −0.12% to −0.43%.

We can make the equation more accurate by calculating the E value for Panhandle B and the A value for the Quadratic Metamodel linearly regressed for a particular pressure (1100, 1200, 1300, or 1400 psia) and a particular diameter (20, 22, 24, or 26 in.), with the length remaining 7.5 miles. Figure 10 shows the result of this analysis for a 20 in. diameter. As can be seen, the error reduces to less than ±0.1%, with the Metamodel still performing better than the Panhandle B equation as the Metamodel is more horizontal.

To reduce the error in our Metamodel further, we calculated the errors corresponding to different flow rate ranges for a pipe of 7.5 miles length and 20 in. diameter for a particular inlet pressure. We also did the same for the Panhandle B equation. Figure 11 shows a comparison between the Metamodel and Panhandle B for inlet pressures at 1100 psia. The error reduces to less than ±0.02%.

Considering the nonlinearly regeressed Metamodel for a particular pressure, diameter, length, and flow range, we can produce Figure 12 indicating even better accuracy from the Metamodel. The exponents and the value E_g do adjust to the following ranges dependent on the flow rate: α = 0.9667−1.9895, β = 1.1342−1.1398, γ = 2.0002−2.0268, and E_g = 7.452−7.8481. As indicated earlier, this equation would be overly complex and would not be very usable for optimization.

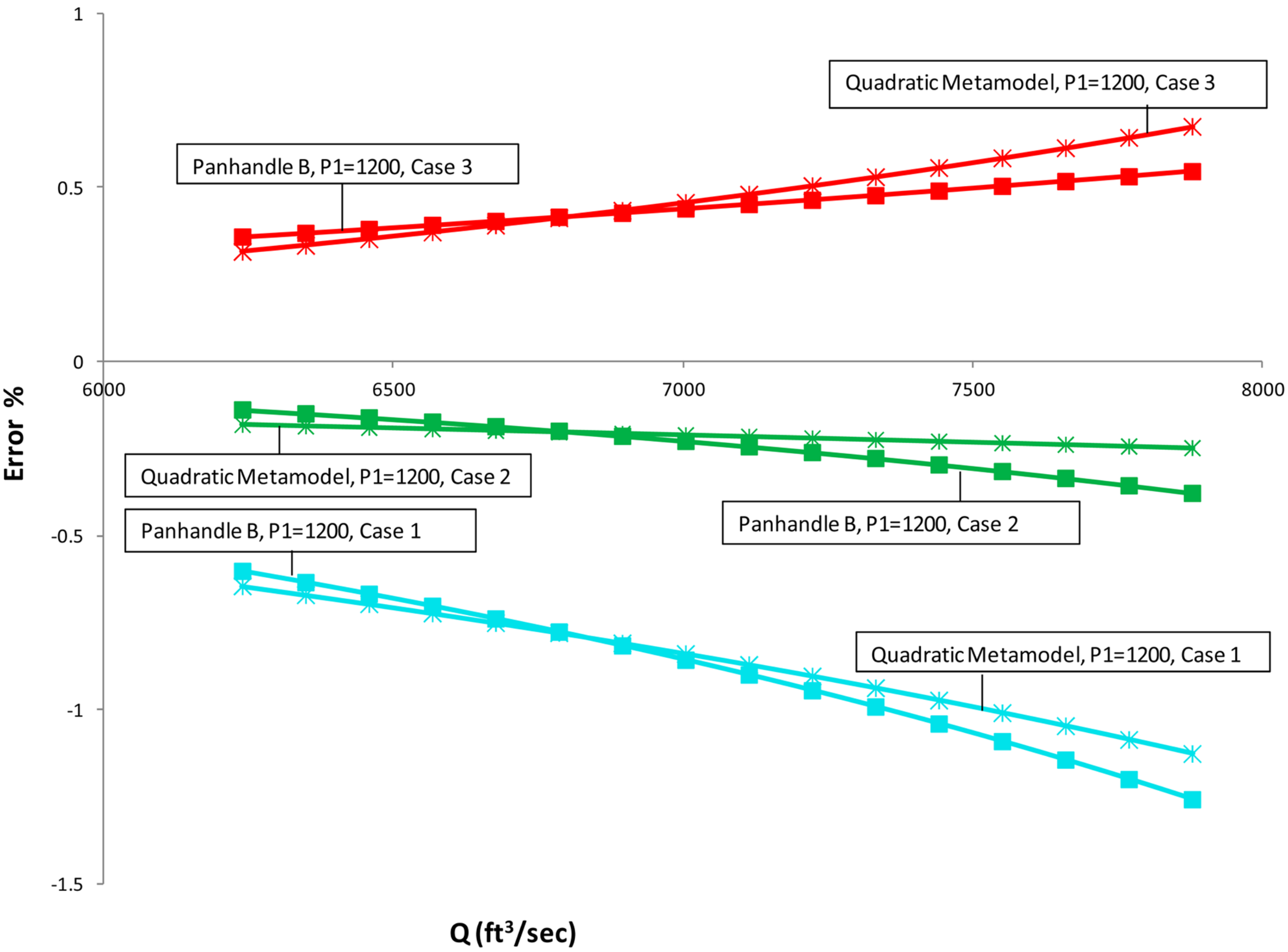

Figure 14. Errors for three natural gas compositions with L = 7.5 mi, D = 20 in., and P = 1200 psi.

Table 2 shows the different correlations tested and some of the results for the Panhandle B and Quadratic Metamodel. As Table 2 indicates and the previous figures indicate, the error on both equations is negligible. The major problem with the Panhandle equation is that it requires pressure and flow data from measurements or by simulation to determine the E value. There is no way around this issue without making assumptions that lead to excessive error as indicated in Figure 2. The Metamodel experiences this problem; however, unlike the Panhandle equation, the value can be determined through rigorous calculation. As pressure and flow measurements may have existing bias and would require data over time, simulation is really the best way to linearly regress these values. Thus, when trying to perform optimization, either equation should be usable with less than 2% error in the general case and less than 0.05% percent error when particular ranges are considered. However, the simplicity of our equation is undeniable and allows for mathematical programming to determine a global optimum.

■ ANALYSIS FOR THE CASE OF ELEVATION CHANGES

We also examined the example with the diameter and length held constant (still at 20 in. and 7.5 miles) and a variation in elevation (between 200 and 800 m). This produces the results as seen in Figure 13 comparing Panhandle B (eq 28) and the Metamodel (eq 32) for a general case. What can be noticed is that neither equation outperforms the other, with both less than 0.6% error. The tendency for Panhandle B to cluster as compared to the Metamodel (as seen with P = 1300 psia) is due to the B term. The compressibility factor is adjusted for each pressure, flow, and elevation in Figure 13. A more generalized compressibility factor would deteriorate the Panhandle equation.

■ UNCERTAINTY IN NATURAL GAS COMPOSITION

The parameters A and E discussed above correspond to a particular natural gas composition. We now develop a Metamodel where, in addition to the diameter, length, flows, and pressures, compositions also vary. We used the same parameters as before, and we varied the composition of the natural gas across three different cases. The compositions are listed in Table 3.

After compiling the data, we then linearly regressed the A value for the Quadratic Metamodel and the E value for the Panhandle B equation. As specific gravity varies with the composition, we used an average specific gravity of 0.618 for the regression of the E value, and we obtained an A value of 0.4217 and an E value of 0.8384. Figure 14 provides the results for the Quadratic Metamodel and the Panhandle B equations. We only provide the data for a pressure of 1200 psia, length of 7.5 miles, and diameter of 20 in. As it indicates, the error does

Table 3. Natural Gas Compositions

	case 1	case 2	case 3
methane	85%	90%	95%
ethane	8%	5.40%	3.20%
propane	4%	1%	0.20%
butane	1%	0.20%	0.03%
i-butane	0.05%	0.06%	0.03%
pentane	0.20%	0.02%	0.02%
i-pentane	0.05%	0.01%	0.01%
hexane	0.05%	0.01%	0.01%
nitrogen	1%	2.30%	1%
CO_2	0.65%	1%	0.50%

increase with the range in the quadratic model between +0.67 and −1.13. Meanwhile, the range in the Panhandle B equation is between +0.55 and −1.26. Both methods remain low in error (less than 2% for this pressure), and thus remain viable equations for the range of diameters, lengths, pressures, flows, and compositions used in this Article.

■ CONCLUSIONS

The methodology presented in this work provides comparable results by using appropriate parameters, which are *A* and *B* values. As indicated, the Metamodel is more simplistic than old pressure drop equations and is more accurate for a generalized or specified case. The models compare very well with the generalized case with variation in height or composition. Therefore, for the purpose of optimization models, it is more advisable to use the Metamodel than the old methods for gas pipelines.

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Notes

The authors declare no competing financial interest.

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