EFFECT OF PRICING, ADVERTISEMENT AND COMPETITION IN MULTI-SITE CAPACITY PLANNING

M. Bagajewicz

University of Oklahoma
100 E. Boyd St., Norman OK 73019, USA

Abstract

The multi-site capacity planning problem with a long term time horizon, with transportation constraints is expanded to consider pricing of the product, the effect of advertisement and competition. We use a modification of the Cobb-Douglas model for consumer utility and include the effect of advertising.

Keywords: Capacity Planning Problem, Planning, Pricing.

1. Introduction

The investment capacity planning problem has been so far formulated for enterprises where the demand and price were considered as parameters, sometimes uncertain (Sahinidis et al., 1989, Liu and Sahinidis, 1996, Iyer and Grossmann, 1998, Ahmed and Sahinidis, 1998, etc.). Recently, Guillen et al., (2003) have shown that pricing decisions can be added to scheduling problems. The pricing model assumes a certain relationship between prices and demands (the classical price*demand = constant). Extensions of the model presented by Guillen et al. (2003) to capacity planning are straightforward. This approach, however, does not take into account the effect of advertisement, nor does it make any assumption on the influence of competition.

This paper addresses the simultaneous addition of pricing, advertisement cost and competition to investment and capacity planning problems. The problem is formulated as a typical capacity planning problem with expansions, but several different potential locations are considered. In addition, several markets, not necessarily the same sets as the as the locations, are allowed. Sales in each potential market are connected to advertisement expenditures. Maximum market shares as well as sales to advertisement ratios are a function of each potential market. Finally, the model is crafted so that reinvestment is allowed only by using portions of proceeds and no new capital. Multiple locations are also allowed. The model output provides the expansions through the time horizon, the anticipated sales, and the advertising strategies and expenses. We have used a microbrewery capacity planning problem (Spencer et al., 2004) as basis for illustration.

2. Background

Spencer et al. (2004) considered the typical capacity planning problem with expansions, but several different potential locations for the plants and applied it to the location of beer microbreweries. In addition, several markets, not necessarily the same as those in plant locations, were allowed and transportation costs were included in the

model. Finally, the model was crafted so that reinvestment is allowed only by using portions of proceeds and no new capital. Multiple locations were also allowed. The model output provides the expansions through the time horizon, the anticipated sales, and the advertising strategies and expenses. The model maximizes net present value of variable proceeds given back to the investor throughout the years and determines the level of investment needed. The model also considers self financed expansions.

The novel aspect of the model was that sales in each potential market were connected to advertisement expenditures and the effect of competition was considered. Maximum market shares as well as sales to advertisement ratios were considered as a function of each potential market. Competition was modeled in the following way:

Sales< Market Demand *(Market share without competition- Effect of competition)

The effect of competition is to reduce the market share in a form which is proportional to the number of breweries in the market. Advertisement was considered to affect the market share as follows:

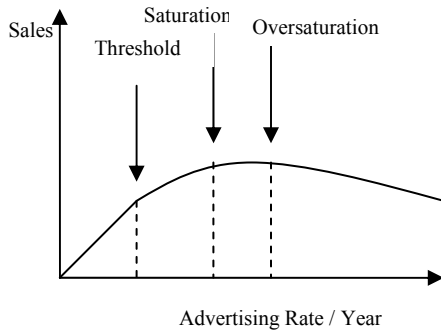Market Share= Market share without competition + b * Advertisement

Figure 1. Sales vs. advertising effort.

where b is the constant relating market share increase per dollar spent in advertisement. The assumption is that the sales to advertisement efforts behave linearly. In advertisement theory (Rao, 1970), sales grow linearly with advertisement until they reach saturation (they level off) after which there is a small decline (oversaturation phase). The results obtained by Spencer et al (2004), show a significant effect of the When advertisement is not present, they showed that location changes to places where a larger market share can be obtained with

changes to places where a larger market share can be obtained with less advertisement efforts and a location closer to more markets.
Spencer et al (2004) considered prices fixed. This paper relaxes this assumption and also considers competition and advertisement in a different way.

3. Pricing and Competition

We assume first that there is an established market for the new product (in this case, beer) and that what we are looking is for substitutes, so, the total demand is assumed constant. The question is what price will be the right one to attract the optimal number of customers and the new demand one will create associated to it. In other words, one needs to determine what the demand will be and how that demand for this new product will be related to the price and the advertisement efforts. We begin with the following basic equation that relates price and demand. Let p_1 be the targeted new product selling price, d_1 its demand, p_2 the average competitor's product price, and d_2 is the corresponding competitor demand. We start assuming that the following relation is valid:

$$p_1 d_1 = c\ p_2 d_2 \tag{1}$$

In this relation c is a constant. This relation can be derived using the concept of indifference curves (Hirshleifer and Hirshleifer, 1998), which express how much equal utility (happiness) a combination of product 1 and 2 will provide to a customer. Such an expression, compatible with (1) is $d_1^{r_1} d_2^{r_2} = U$, where U is the utility and $c = r_1 / r_2$. This form of the utility function is known as the Cobb-Douglas utility, which requires $r_1 + r_2 = 1$. Thus, (1) is obtained by maximizing U subject to a budget constraint $p_1 d_1 + p_2 d_2 \leq Y$, where Y is the total consumer budget. In the case where the prices of the competition (p_2) and d_2 are fixed, the expression reduces to $p_1 d_1 = const$, which corresponds to a market structure of monopolistic competition, (Dixit and Stiglitz, 1977) which is in turn based on two assumptions: a) the firm maximize it's own profit by choosing an optimal price for given prices by other firms, and b) each demand is small relative to entire market.

The effect of using the above formula for pricing is troublesome sometimes. Consider the case where $d_2 >> d_1$. In such case, if an equality is used, p_1 will be very small, and likely incompatible with the economics. If we use an inequality (e.g. $p_1 d_1 \leq c\, p_2 d_2$) the restriction becomes meaningless and the price chosen will be too high. To overcome this issue, we propose to use the plant capacity instead of d_2. This is equivalent to assuming that the market size is of the capacity of the plant. Indeed, if such a market is shared 50/50, then prices will be equal (for $c=1$).

4. Advertising

Equation (1) does not express the dynamics of the competition process, but rather the state of equilibrium achieved among competitors at some point in time. When $c=1$, equation (1) reveals that equal prices of the new product and that of the competitors' product, results in an evenly shared market. Such outcome would be realistic if the following conditions are met: a) The new product and the existing ones have been in the market for a long time. b) The quality of each product is the same. c) Advertising campaigns are equally effective, and d) Production capacities of all competitors can satisfy the demands

Since these conditions are hardly met for a new product, the above model needs to be improved. First, the competitors will have a clear advantage over the new product because they will have been established in the market for a number of years. They will have earned loyal customers and will have successful advertising campaigns in place. On the other hand, the new product will have the advantage of being a superior product, which will either increase convenience of use or save money and time to the customer. To account for influences from advertising, two correcting functions are introduced into the model as follows:

$$\beta(t,a) \cdot p_1 d_1 = c\, p_2 d_2 \cdot \alpha(t,a) \tag{2}$$

This still corresponds to an indifference curve of the form $d_1^{r_1} d_1^{r_2} = U$, where now r_1 and r_2 are functions of $\alpha(t,a)$ and $\beta(t,a)$. The function $\alpha(t,a)$ is a function of time and the advertisement campaign efforts for the new product. This function ranges between zero and one. At the beginning ($t=0$), $\alpha(0,a) = \alpha_0$ (a small number)

indicating that the demand of the new product d_1 is small, no matter what the price is. As time goes by, the function approaches one, reflecting equal opportunities for all competitors in terms of advertisement. Thus, before the value of $\alpha(t,a)$ reaches one, the competitors have a competitive advantage by virtue of their longer standing in the market, with an established customer base. Thus, we call $\alpha(t,a)$ the Inferiority Function for the new product. In turn, $\beta(t,a)$ is also a function of time and the advertisement campaign of the new product (a). At the beginning ($t = 0$), $\beta(0,a) = 1$ indicating that there is no initial advantage for the new product. As time increases, β approaches zero asymptotically, and becomes zero only if the competition disappears. The function represents the superiority of the new product and ultimately its competitive edge.
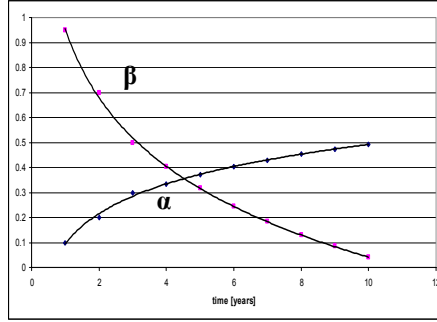

Figure 2. Inferiority and superiority functions.

Therefore, we call it the Superiority function. We envision these two functions to have concave and convex forms as shown in figure 2. One gets some estimated values for $\alpha(t,a)$ and $\beta(t,a)$ using data from the performance of similar novel products in the past.

We note that this is a simplified model ignores the advertisement efforts of the competition.

The assumption that the inferiority and superiority functions are linear with advertisement efforts is first made. For simplicity, we also assume $\beta(t,a) = 1$, that is the existing product is as good as the new one in the consumer minds for a long time The following form of the inferiority function can be assumed: $\alpha(t,a) = \left[\alpha_1 - (\alpha_1 - \alpha_0)e^{-[\gamma_t + \gamma_{at}a]t}\right]$, where α_0 and α_1 are the inferiority function values at time zero and infinity, respectively. It is also assumed that within the time horizon, $[\gamma_t + \gamma_{at}a]t \ll 1$. Under these conditions, (2) renders a linear relationship between d_1 and advertisement efforts consistent with figure 1. This inferiority function does not penalize higher prices and therefore, it leads to answers where the price chosen is always the highest. This can be easily verified by substituting (2) in the following simplified profit function *Profit*=$p_1 d_1 - (g_1 + g_2 d_1)$-$a$ where ($g_1 + g_2 d_1$) is the operating cost and realizing that reducing d_1 and increasing p_1 leads to higher profits. One needs to realize that as much as advertisements changes consumer attitude towards buying the product, price has the adverse effect, so we propose to use $\alpha = \alpha(t,a,p_1) = e^{\gamma_p(p_1 - p_2)}\left[\alpha_1 - (\alpha_1 - \alpha_0)e^{-[\gamma_t + \gamma_{at}a]t}\right]$. Thus, if the price is higher than the market, then revenues are higher, but the inferiority function is also lower.

5. Model

The model is presented succinctly because of space reasons. As stated above, the Spencer et al's (2004) model is used, which considers multiple potential sites for plants. Each plant can send its products to different markets. We also consider several different locations for raw materials. The transportation costs of raw materials as well as products are taken into account. Budgeting constraints are set so that the capital investment is

used at the beginning and all the expansions are financed by the proceeds of the project. Thus, the model decides if it is profitable to reinvest. Finally, the following condition is used to model advertisement and pricing.
$p_1 d_1 = c p_2 Cap \left[\alpha_1 + (\alpha_1 - \alpha_0)(\gamma_t + \gamma_{at} a) \, t \, e^{\gamma_p (p_2 - p_1)} \right]$, which comes from the assumptions of monopolistic competition. (p_2 and d_2 fixed) and a linearization of the inferiority function allowed by the assumption that $[\gamma_t + \gamma_{at} a] t << 1$ over the time horizon. This condition can be relaxed. Although time is already discretized into periods in this model, the expression $p_1 d_1$ is non-linear. It was chosen to discretize prices to be able to linearize the expression and use MILP solvers. Thus, assuming that a set of n_m values of prices per market m are chosen ($p_{m,i}$, i=1.. n_m), the following equations are used:

$$\sum_{i \in P_m} z_{m,i} p_{m,i} sales_m \le c \, p_{c,m} Cap \cdot \left[1 + (1 - \alpha_{0,m})(\gamma_{m,t} + \gamma_{m,at} a_m) t e^{\gamma_p (p_{c,m} - p_{m,i_1})} \right] \quad (3)$$

$$\sum_{i \in P_m} z_{m,i} = 1 \quad (4)$$

where Cap is the final capacity, $p_{c,m}$ is the average price of the competition, and $z_{m,i}$ a binary variable that forces the model to choose only one price. This price stays constant through the life of the project (although this condition is also easy to relax). This is the same strategy as the one used by Guillen et al. (2003). Linearization of (3) is straightforward. We notice that we use sales instead of demand in equation (3) together with a less than or equal sign instead of an equal one, simply because sales (which are lower than demand) may be limited by capacity.

4. Example

Microbreweries are defined by the industry as small breweries that produce less than 15,000 barrels of beer per year and distribute the product for consumption off-premise. A pale ale has been chosen for the recipe of the beer to be produced because it is lighter in taste than other microbrews, but it has more taste than the watered-down national brands. The main raw materials used in the production of microbrewed beer are hops, malted barley (yellow dots in figure 3), and yeast (blue dots). Several locations in the US were considered (black dots). The Markets are each of the 48 US contiguous states.

Using a maximum of 1.5 million of initial investment, fixed prices and the aforementioned advertisement and competition model, Spencer et al. (2004) found a solution that chooses to build only one brewery (Phoenix, AZ) for only one market (AZ), with expansions in year two, three and four. For a $3 million initial investment, two breweries are built (Phoenix, AZ and Las Vegas, NV) and expanded afterwards.

When the effect of competition was eliminated and the advertisement was eliminated and the market share considered fixed (slightly higher than the one without advertisement) the model used by Spencer et al (2004) chose to build only one brewery in Milwaukee, Wisconsin. Since the model was not allowed to increase its market percentage through advertising, it was forced to sell to more markets than before. The net present worth came out to be about half of that with advertising included. The new model was run using α_1 =0.95, α_0 =1/7, γ_t=0.029, $\gamma_p = ln\{p_2 1.5 / p_{1max}\}/(p_{1max} - p_2)$, which corresponds to 66% of the

Figure 3. Raw materials and potential plant locations

maximum potential achieved if $p_1=p_2$, and γ_{at} such that the negative effect of the highest price ($p_1=p_{1,max}$) is equivalent to a=\$50,000 at year 10. Only 3 choices of price were given (0.975p_2, p_2 and 1.025p_2). With these parameters, when considering Phoenix, the model puts one plant with a small capacity at first, building up to full capacity (15,000) in year 7, using only about 1.25 million of the 1.5 million

million available. It sells and advertises in Arizona with the lowest price possible, and decides to sell in New Mexico at a highest price *without any advertising,* for only two years, after which it abandons that market. One interesting result is that it advertises in Arizona until year 9, time at which decides to stop, because of the natural growth of the inferiority function. When γ_t was halved, Nevada was added as a market for a few years to later abandon it: the investment is slightly lower (1.175 million).

5. Conclusions

A multi-site capacity planning problem recently developed, which contained location dependent transportation costs and budgeting constraints as well as sales, prices and advertisement costs as variables, was developed. A combination of low prices and advertisement can be a good strategy coupled with capacity planning expansions. Future work will include larger initial investment, uncertainty and risk.

References

Ahmed S. and Sahinidis N.V., "Robust process planning under uncertainty", Industrial & Engineering Chemistry Research, 37(5):1883–1892, (1998).

Dixit, A. K. and J. E. Stiglitz, "Monopolistic Competition and Optimum Product Diversity." The American Economic Review, 67, pp. 297-308 (1977).

Guillén G., Bagajewicz M., S. E. Sequeira, R. Tona, A. Espuña and L. Puigjaner. *Integrating Pricing Policies and Financial Risk Management into Scheduling of Batch Plants.* PSE 2003. 8th International Symposium on Process Systems Engineering. China, June , (2003).

Hirshleifer J., D. Hirshleifer, *Price Theory and Applications.* Prentice Hall, New Jersey, (1998).

Iyer R.R. and Grossmann I.E., "A bilevel decomposition algorithm for long-range planning of process networks", Industrial and Engineering Chemistry Research, 37:474-481, 1998.

Liu M.L. and Sahinidis N.V., "Optimization in Process Planning under Uncertainty", Industrial Engineering and Chemistry Research, 35: 4154-4165, 1996.

Rao, A G., Quantitative theories in Advertisement, Wiley, New York, (1970).

Sahinidis N.V., Grossmann I.E., Fornari R.E., Chathrathi M., "Optimization Model for Long Range Planning in the Chemical Industry", Comp. and Chem.Eng., 13: 1049-1063, 1989.

Spencer N., J. Godwin, M. Bagajewicz, S. Powell and S. Hodge Simultaneous Modeling of Location, Advertisement and Competition in Investment/Capacity Planning With Risk Management. AIChE Annual Meeting, Austin, Nov. (2004).