Financial Risk Management in Offshore Oil Infrastructure Planning and Scheduling

Ahmed Aseeri, Patrick Gorman, and Miguel J. Bagajewicz*

School of Chemical Engineering, University of Oklahoma, 100 E. Boyd Street, T-335, Norman, Oklahoma 73019

This paper discusses the financial risk management in the planning and scheduling of offshore oil infrastructure. The problem consists of determining the sequence of platforms to build and the wells to drill as well as how to produce these wells over a period of time. The problem has shown numerical difficulties, and several decomposition methods have been attempted to alleviate these difficulties. We added budgeting constraints to this model, following thus the cash flow of the project, taking care of the distribution of proceeds, and even considering the possibility of taking loans against some built equity. The model was made stochastic, the financial risk, an important aspect of these ventures, is analyzed, and the ability to manage it is discussed. The numerical difficulties resulting from the addition of uncertainty are overcome by using the sampling average algorithm (SAA; Verweij, B.; Ahmed, S.; Kleywegt, A. J.; Nemhauser, G.; Shapiro, A. *Comput. Appl. Optim*. **2001**, *24*, 289−333). We show that the SAA yields a very good solution that can be proven to be very close to the optimum using the concept of an upper bound risk curve. A measurement of the gap of solutions of the SAA is also introduced. The main objective is to emphasize the conceptual aspects of the modeling.

1. Introduction

The planning and scheduling of oil-field infrastructure investment, especially in the offshore, is a complex job. It involves the management of many assets and facilities: fields, reservoirs, wells, platforms (production platforms, well platforms, and tie-in platforms), and connecting flow lines. It also involves the scheduling of resources utilization like drilling rigs, and it requires that the proper order of facility installations be managed. Finally, it involves big dollar investments, the proper management of which can yield substantial savings.

The problem is characterized by long-term planning (years) and is known to have several numerical difficulties. Iyer et al.[2] presented a multiple-integer linear programming (MILP) procedure for this problem. The paper deals with a decomposition procedure and other approximations to solve it in reasonable computation time. Later, Van den Heever and Grossmann[3] proposed an aggregation/disaggregation method to solve the problem. The same group of researchers added tax and royalty calculations to the problem, which increased its numerical complexity,[4] and studied the use of big-M constraints as well as disjunctive programming. Finally, Van den Heever et al.[5] proposed a Lagrangean decomposition procedure.

In this paper we do not intend to present new numerical schemes to the solution of the problem. Rather, we introduce uncertainty and risk management and add budgeting constraints to the model. Thus, we use an MILP model very similar to the one by Iyer et al.[2] These budgeting constraints follow the cash flow of the project, take care of the distribution of proceeds, and consider the possibility of taking loans against some built equity. We then discuss the numerical difficulties resulting from the addition of uncertainty and how the sampling average algorithm (SAA)[1] can be used to overcome these difficulties. We show that the SAA yields a very good solution that can be proven to be very close to the optimum using the concept of upper bound risk curves.[6] A measurement of the gap of solutions of the SAA from the best possible integer solution is also introduced. We use a new approach to the management of financial risk that was recently presented by Barbaro and Bagajewicz.[7,8] The methodology uses a well-known definition of risk through a cumulative probability distribution of profit, which is modeled and added as a constraint to two-stage stochastic models. It also uses alternative measures of risk, such as downside risk.[9]

Problem Statement. A schematic of the field layout for the problem that is considered as an example for this work is illustrated in Figure 1. One field consisting of three reservoirs is assumed. In each reservoir, two wells can be drilled for which estimates of the drilling cost as well as the well expected productivity index are assumed to be known. The wells in reservoirs R_1 and R_2 can be connected to a well platform WP_1, and the wells in reservoir R_3 can be connected to well platform WP_2. Both well platforms are to be connected to a production platform in which crude oil is processed to separate gas from oil and then oil is sent customers.

The objective of this problem is to maximize the net present value (NPV) of the project. The decision variables in the model are reservoir choice, candidate well sites, capacities of the well and production platforms, and fluid production rates from the wells. The problem is solved for a 6-year planning horizon with quarterly time periods (24 time periods).

2. Stochastic Model

Mass Balance. The sum of the flow of oil/gas from all wells associated with a well platform is the total flow of oil/gas at that well platform. Similarly, flows from well platforms are added to determine the flow at the

* To whom correspondence should be addressed. Tel.: (405) 325-5458. Fax: (405) 325-5813. E-mail: bagajewicz@ou.edu.

10.1021/ie034098c CCC: $27.50
Published on Web 04/28/2004

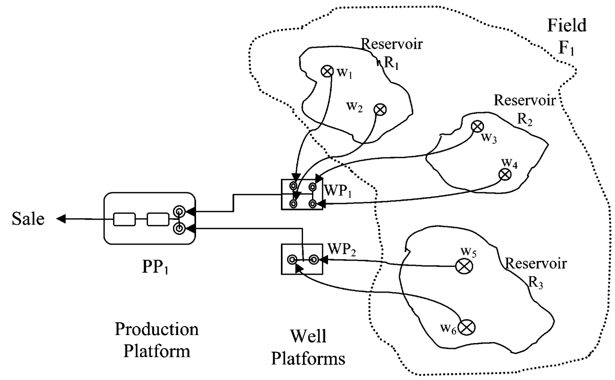

Figure 1. Problem schematic.

production platforms.

$$\sum_{w \in W_{WP}(\pi)} X_{t,s}^{w,\pi,p} = X_{t,s}^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{1}$$

$$\sum_{\pi \in \mathrm{WP}(p)} X_{t,s}^{\pi,p} = X_{t,s}^{p} \qquad \forall\ p \in \mathrm{PP},\ t, \text{ and } s \tag{2}$$

$$\sum_{p \in \mathrm{WPP}} X_{t,s}^{p} = X_{t,s}^{\text{total}} \qquad \forall\ p \in \mathrm{PP},\ t, \text{ and } s \tag{3}$$

The variable X represents the flow rate of crude oil (oil and gas). The subscript t refers to time periods, and s refers to scenarios. The superscript w refers to wells, π refers to well platforms, and p refers to production platforms.

Pressure Balance. The pressure at the well platform is the pressure at the wells associated with that well platform minus the pressure drop in the corresponding pipe. The pressure drop is expressed as a linear function of the oil and gas flow rates.

$$V_{t,s}^{\pi,p} = V_{t,s}^{w,\pi,p} - \alpha X_{t,s}^{w,\pi,p} - \beta g_{t,s}^{w,\pi,p} - \delta_{t,s}^{w,\pi,p} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{4}$$

$$V_{t,s}^{p} = V_{t,s}^{\pi,p} - \alpha X_{t,s}^{\pi,p} - \beta g_{t,s}^{\pi,p} - \delta_{t,s}^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{5}$$

The variable V represents pressure, g represents the gas flow rate, and δ represents the pressure drop across the pressure chokes. The parameters α and β are the pressure drop coefficients for oil flow and the gas-to-oil ratio (GOR), respectively.

Flow Constraints in the Wells. The maximum flow of oil is related to the productivity index of the well and the allowable pressure drop. The gas flow is limited by the maximum allowable GOR:

$$X_{t,s}^{w,\pi,p} = l_{t,s}^{w,\pi,p} + g_{t,s}^{w,\pi,p} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{6}$$

$$g_{t,s}^{w,\pi,p} \le l_{t,s}^{w,\pi,p}\mathrm{GOR}_{\max} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{7}$$

$$l_{t,s}^{w,\pi,p} \le \rho^{w,\pi,p} P_{\max} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{8}$$

$$l_{t,s}^{w,\pi,p} = \rho_s^{w,\pi,p}(V_{t,s}^{f,r} - V_{t,s}^{w,\pi,p}) \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{9}$$

The variable l represents the liquid oil flow rate, and the parameter ρ is the productivity index for the wells. The superscripts f and r refer to fields and reservoirs, respectively. The parameter $P_{\max}$ is the maximum pressure drop from well bore to well head.

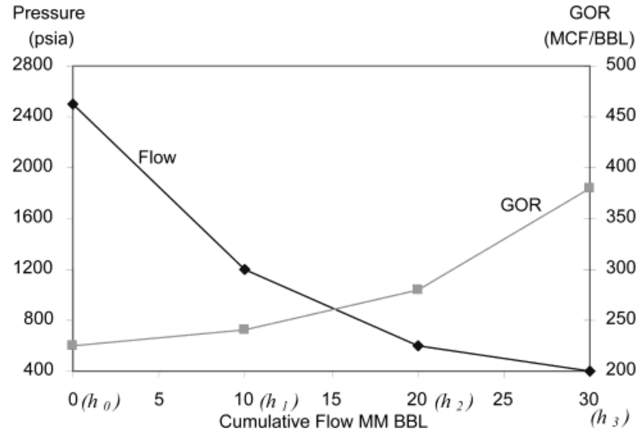

Figure 2. Sample reservoir performance (r2).

Cumulative Flow. The cumulative flow of oil is calculated by the summation over time periods of the flows from that well.

$$\mathrm{Xc}_{t,s}^{w,\pi,p} = \mathrm{Xc}_{t-1,s}^{w,\pi,p} + X_{t-1,s}^{w,\pi,p}\Delta T \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \tag{10}$$

$$\mathrm{Xcr}_{t,s}^{r,f} = \sum_{(w,\pi,p) \in W_{\mathrm{FR}}(f,r)} \mathrm{Xc}_{t,s}^{w,\pi,p} \qquad \forall\ r \in R(f),\ f \in F,\ t, \text{ and } s \tag{11}$$

The variables Xc and Xcr represent the cumulative flow rates from the wells and reservoirs, respectively, and ΔT is the duration of each time period.

Piecewise Linear Interpolation. The pressure and GOR in a reservoir are considered to be nonlinear functions of the cumulative oil produced. Alternatively, in more sophisticated models, one could use reservoir simulation data. They are calculated using piecewise linear interpolation. The oil and gas flow rates from the individual wells in a reservoir are also calculated using piecewise linear interpolation. Equations used for piecewise linear interpolation are as follows:

$$\mathrm{Xcr}_{t,s}^{r,f} = \sum_h m_{t,h,s}^{r,f}\mathrm{Xcri}_h^{r,f} \qquad \forall\ r \in R(f),\ f \in F,\ t, \text{ and } s \tag{12}$$

$$\sum_h m_{t,h,s}^{r,f} = 1 \qquad \forall\ r \in R(f),\ f \in F,\ t, \text{ and } s \tag{13}$$

$$Vr_{t,s}^{r,f} = \sum_h [\overline{V}ri_{h-1}^{r,f}\ m_{t,h,s}^{r,f}] \qquad \forall\ r \in R(f),\ f \in F,\ t, \text{ and } s \tag{14}$$

The variable m represents the piece used for linear interpolation over h pieces (see Figure 2). It is declared as a special ordered set of type 2 (SOS2) for which at most two variables, within the special ordered set, can have nonzero values. The two nonzero values have to be adjacent.

Logical Constraints for Installation. The wells may be drilled only once in one of the time periods. Also, the facilities (well and production platforms) may be installed only in one of the time periods.

$$\sum_{t=1}^{T} Z_t^{w,\pi,p} \le 1 \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \tag{15}$$

$$\sum_{t=1}^{T} Z_t^{\pi,p} \leq 1 \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (16)$$

$$\sum_{t=1}^{T} Z_t^{p} \leq 1 \qquad \forall\ p \in \mathrm{PP} \text{ and } t \quad (17)$$

Z is a binary variable that represents the installation of a facility in a specific time period.

Logical Constraints for the Existence and Flow of Facilities. The flow of oil and gas from a facility can be nonzero only after it is installed. This modified formulation allows for production lag and construction time. A construction time of two time periods (i.e., 6 months) is assumed for production and well platforms and that of one time period (i.e., 3 months) for drilling a well.

$$y_t^{w,\pi,p} = y_{t-1}^{w,\pi,p} + Z_{t-1}^{w,\pi,p} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (18)$$

$$X_{t,s}^{w,\pi,p} \leq \Omega^u y_t^{w,\pi,p} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \quad (19)$$

$$y_t^{\pi,p} = y_{t-1}^{\pi,p} + Z_{t-2}^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (20)$$

$$X_{t,s}^{\pi,p} \leq \Omega^u y_t^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \quad (21)$$

$$y_t^{p} = y_{t-1}^{p} + Z_{t-2}^{p} \qquad \forall\ p \in \mathrm{PP} \text{ and } t \quad (22)$$

$$X_{t,s}^{p} \leq \Omega^u y_t^{p} \qquad \forall\ p \in \mathrm{PP},\ t, \text{ and } s \quad (23)$$

The variable y represents the facility existence in operable condition. The parameter Ω is an upper bound for oil flow.

The well platform associated with a well must be installed before drilling that well. Similarly, production platforms must be installed before the associated well platforms.

$$Z_t^{w,\pi,p} \leq y_t^{\pi,p} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (24)$$

$$Z_t^{\pi,p} \leq y_t^{p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (25)$$

Expansion. The expansion variable e may be nonzero only in one time period, which represents the period when the facility is installed. Also flow from a facility should be limited to the maximum design capacity d.

$$e_t^{\pi,p} \leq \Omega^u Z_t^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (26)$$

$$X_{t,s}^{\pi,p} \leq d_t^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t, \text{ and } s \quad (27)$$

$$d_t^{\pi,p} = d_{t-1}^{\pi,p} + e_t^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (28)$$

Drilling a Well. There is a maximum number of wells that can be drilled at one time, in our case only one assuming that only one drilling rig is available.

$$\sum_{w \in W_{\mathrm{WP}}(\pi)} Z_t^{w,\pi,p} \leq 1 \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (29)$$

Rig Moving Time. It is assumed that it will take one time period (3 months) to relocate the drilling rig from one well platform to the other. This will promote the model to drill wells in the same platform in consecutive time periods to save rig time.

$$\sum_{w \in W_{\mathrm{WP}}(\pi)} Z_t^{w,\pi,p} + \sum_{w \in W_{\mathrm{WP}}(\pi)} Z_{t-1}^{w,\pi,p} \leq 1 \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (30)$$

$$\sum_{w \in W_{\mathrm{WP}}(\pi)} Z_t^{w,\pi,p} + \sum_{w \in W_{\mathrm{WP}}(\pi)} Z_{t+1}^{w,\pi,p} \leq 1 \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \text{ and } t \quad (31)$$

Well Platforms Scheduling. It is assumed that for limitations on resources two well platforms cannot be constructed at the same time.

$$\sum_{\pi \in W_{\mathrm{WP}}(\pi)} \left[\sum_{\theta=t}^{t+1} Z_{\theta}^{\pi,p}\right] \leq 1 \qquad \forall\ t \quad (32)$$

Budgeting Constraints. Cash flow is based on a maximum available cash (injected) per time period, as well as a potential for borrowing. The introduced positive variables are proceeds (PR), injected cash (IC), borrowed amount (BA), payment (PAY), debt (DT), cash flow (CF), fixed investment (FI), operating costs (OC), and sales (SA). These are explained next in a series of equations:

$$\mathrm{DT}_{t,s} \leq \mathrm{Max\_DT}_t \qquad \forall\ t \text{ and } s \quad (33)$$

This gives a maximum debt allowable, i.e., the largest amount of money that the bank will allow one to borrow. Debt can also be thought of as internal loans within the company

$$\mathrm{DT}_{t,s} = 0 \qquad \forall\ s \text{ and } t = 24 \quad (34)$$

So there is no unpaid debt in the end of the project time frame.

Injected cash is limited, for each time period. This makes the model borrow money if it deems necessary.

$$\mathrm{IC}_{t,s} \leq \mathrm{Max\_IC}_t \qquad \forall\ t \text{ and } s \quad (35)$$

The cash flow must always be greater than a minimum to make sure that some cash is always available for operations.

$$\mathrm{CF}_{t,s} \geq \mathrm{Min\_CF}_t \qquad \forall\ t \text{ and } s \quad (36)$$

The following equations control the cash flow:

$$\mathrm{CF}_{t,s} = \mathrm{CF}_{t-1,s} + \mathrm{IC}_{t,s} + \mathrm{BA}_{t,s} + \mathrm{SA}_{t-1,s} - \mathrm{FI}_t - \mathrm{OC}_{t-1,s} - \mathrm{PR}_{t,s} - \mathrm{PAY}_{t,s} \qquad \forall\ t \text{ and } s \quad (37)$$

$$\mathrm{DT}_{t,s} = \mathrm{DT}_{t-1,s}(1 + i) + \mathrm{BA}_{t,s} - \mathrm{PAY}_{t,s} \qquad \forall\ t \text{ and } s \quad (38)$$

The parameter i is the interest rate per time period for the debt. The fixed capital investment in each time period (eq 39) can be calculated as the sum of the fixed

Table 1. Input Data for Wells

reservoir	well	productivity index (bbl/day/psi) mean	st. dev.	drilling cost ($MM)
R_1	R_1w_1	30	1	5.83
	R_1w_2	30	1	5.76
R_2	R_2w_1	35	1	6.34
	R_2w_2	35	1	6.69
R_3	R_3w_1	40	10	7.00
	R_3w_2	45	10	7.34

Table 2. Interpolation Data for Reservoirs

interp. index	cumul. flow (MMbbl)			pressure (psia)			GOR (MCF/bbl)		
	R_1	R_2	R_3	R_1	R_2	R_3	R_1	R_2	R_3
0	0	0	0	2500	3000	3500	200	225	240
1	10	11	15	1000	1300	1700	210	240	260
2	20	22	30	400	600	900	235	280	300
3	30	35	40	300	400	700	350	380	400

Table 3. Fixed and Variable Cost Factors for Platforms

cost factor	production platforms	well platforms
c_2 (fixed) (MM$)	70	10
c_3 (variable) (MM$/Mbbl/day)	0.470	0.190

Table 4. Budgeting Limits

	amount (MM$)	
maximum debt	100 $\forall$ t < 24	0 for $t = 24$
maximum injected cash	500 for $t = 1$	0 $\forall$ t > 1
minimum cash flow	1 $\forall$ t < 24	0 for $t = 24$

and variable costs of installed platforms and the cost of drilling wells.

$$\mathrm{FI}_t = \sum_P (c_2^p Zp_t^p + c_3^p e_t^p) + \sum_{\mathrm{pi}} (c_2^\pi Z\mathrm{pi}_t^\pi + c_3^\pi e_t^\pi) + \sum_W (c_2^w Zw_t^w) \qquad \forall\ t \text{ and } s \quad (39)$$

The parameters c_2 and c_3 are the fixed and variable cost coefficients for facilities.

$$\mathrm{SA}_{t,s} = c_{1s}\, \mathrm{d}t \sum_w l_{t,s}^{w,\pi,p} \qquad \forall\ t \text{ and } s \quad (40)$$

The parameter c_1 is the oil price and dt is the duration of each time period.

$$\mathrm{OC}_{t,s} = c_{4s} \sum_w l_{t,s}^{w,\pi,p} + \sum_w c_{5s}^p\, y_t^p + \sum_w c_{5s}^{\pi,p}\, y_t^{\pi,p} \qquad \forall\ t \text{ and } s \quad (41)$$

The parameters c_4 and c_5 are the variable and fixed operating cost coefficients.

$$\mathrm{NPV}_s = \sum_t [\mathrm{df}_t {*} (\mathrm{PR}_{t,s} - \mathrm{IC}_{t,s})] \qquad \forall\ t \text{ and } s \quad (42)$$

$$\mathrm{ENPV} = \sum_s \mathrm{Pr}_s \mathrm{NPV}_s \quad (43)$$

The parameter df is the discount factor for each time period. ENPV is the expected NPV over all scenarios, and Pr is the probability of each scenario.

Objective Function. The discounted profit, which includes proceeds from the sale of oil as well as investment cost for the creation of facilities, is maximized in this function.

$$\text{maximize profit} = \sum_{t,s} \{\mathrm{Pr}_s \mathrm{df}_t [\mathrm{Pr}_{t,s} - (1 + \epsilon) IC_{t,s}]\} \quad (44)$$

The parameter ϵ is a small penalty term (0.00001, for example) on the injected cash to prevent the model from injecting cash when it is not needed; this makes the trade-off between the injected cash and proceeds nontrivial.

3. Results

The MILP planning model was implemented in GAMS (General Algebraic Modeling System, GAMS Development Corp.) using CPLEX 7.5.

Input Data. The performance of wells characterized by the productivity index and their drilling costs is shown in Table 1.

The reservoir pressure and GOR interpolation parameters are shown in Table 2 (refer to Figure 2 for a corresponding sample reservoir performance profile). The cost factors (both fixed and variable) for the installation costs of wells and production platforms are listed in Table 3. Budgeting limits used in the runs are listed in Table 4. The price of oil was assumed to be $22/bbl with a standard deviation of 5.

Deterministic Model Results. When the model is run deterministically with the given data at the mean oil price and productivity index, a solution was obtained with a NPV of $389.5 million with 5 min of execution time in a computer having a 2.1 GHz processor and 2 GB memory, running a Linux operating system. The results are shown in Table 5 and Figure 3.

Table 5 shows the installation/drilling schedule, where the row labeled "P" is for the production platform, rows labeled "pi1" and "pi2" are for the two well platforms, and the rows labeled "r1w1" through "r3w2" are for the wells. This schedule indicates the time period in which the platform is to be constructed or the well to be drilled. In this solution, the production platform and platform pi2 are to be constructed on the first time period. After a construction time of two time periods, the two wells associated with pi2 are drilled on the third and fourth time periods (r3w2 and then r3w1). The well platform pi1 is to be constructed on the fourth time

Table 5. Installation/Drilling Schedule for the Results of the Deterministic Model

	t 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	34
p1	1																							
pi1				1																				
pi2	1																							
r1w1									1															
r1w2								1																
r2w1						1																		
r2w2							1																	
r3w1			1																					
r3w2				1																				

Table 6. Installation/Drilling Schedule for the Results of d70, Which Maximizes ENPV, from the SAA Runs

	t																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	34
p1	1																							
pi1				1																				
pi2	1																							
r1w1									1															
r1w2								1																
r2w1						1																		
r2w2							1																	
r3w1				1																				
r3w2			1																					

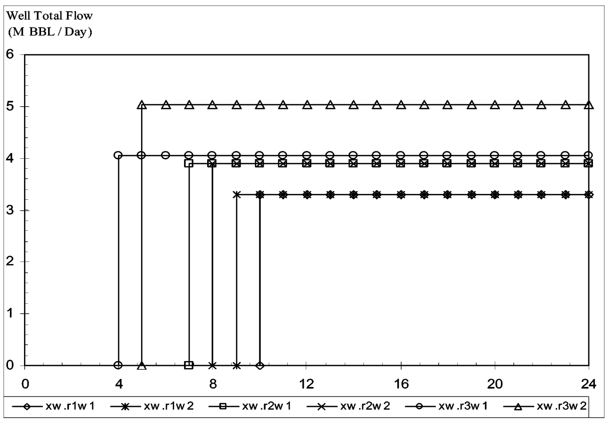

Figure 3. Well flow profile for the results of the deterministic model (xw represents the oil flow of the well).

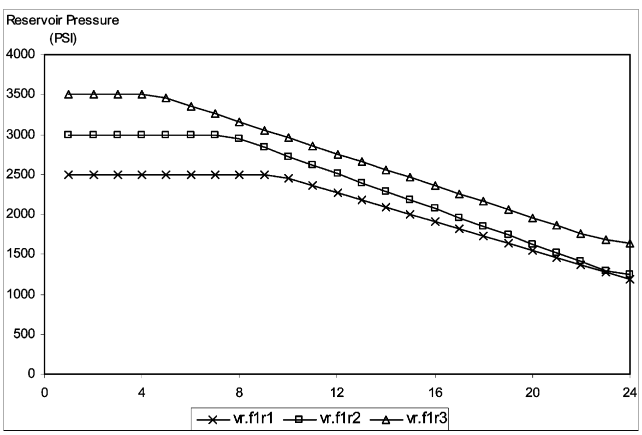

Figure 4. Pressure profile of reservoirs for the deterministic solution (vr represents the reservoir pressure).

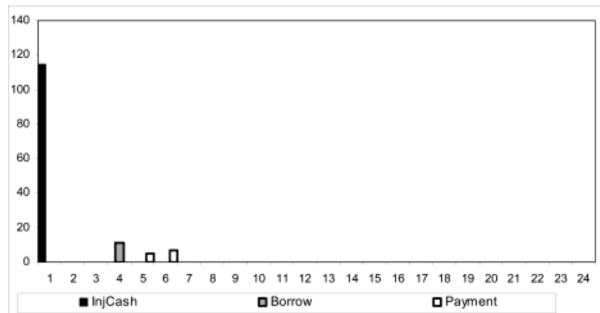

Figure 5. Cash flow for the deterministic problem (injected cash, borrow, and payments).

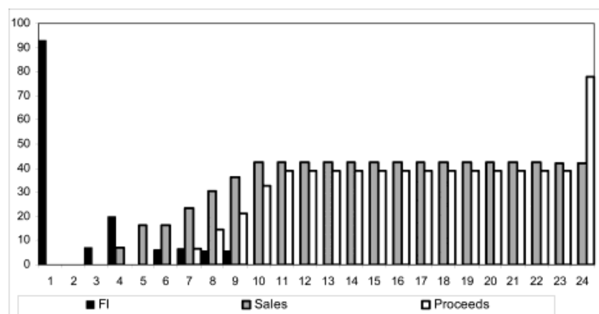

Figure 6. Cash flow for the deterministic problem (fixed investment, sales, and proceeds).

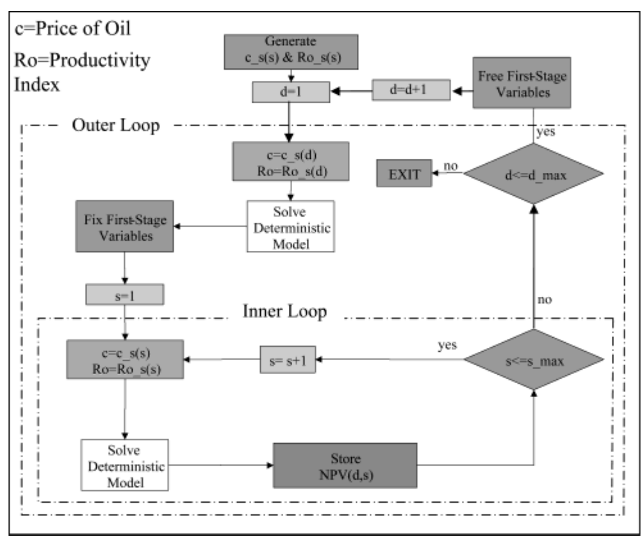

Figure 7. SAA (scenario by scenario).

period. After a construction time of two time periods, the four wells associated with pi1 are drilled: r2w1 on the sixth, r2w2 on the seventh, r1w2 on the eighth, and r1w1 on the ninth time period. The reason the construction of pi1 and consequently the drilling of its wells is delayed by one time period is the scheduling of the drilling rig, which has to take one time period to move from one platform to the other (i.e., the fifth).

The curves in Figure 3 show the flow profile of the wells in 1000 bbl of crude oil/day, which starts on the time period following the construction time period. They have a constant flow profile because the reservoir pressures (Figure 4) do not drop to the region when the pressure differential becomes the controlling factor for oil flow (i.e., the pressure drop becomes less than the maximum allowed). Choking of the wells is used in the beginning to limit the flow from the wells. The time horizon for this problem is only 6 years, so at these conditions, the decrease in the flow profile of the wells will take place after the sixth year.

The pressure in each reservoir drops to almost half the original value. This indicates that the reservoir still has the potential to produce. For the purpose of this study, we are not concerned with managing the reservoir life and productivity but rather with showing how financial risk can be managed in the scheduling of oil well drilling. Also, this run is at the mean values of the

Table 7. Installation/Drilling Schedule for the Results of D187, Which Minimizes Risk

	t																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	34
p1	1																							
pi1	1																							
pi2						1																		
r1w1					1																			
r1w2						1																		
r2w1			1																					
r2w2				1																				
r3w1									1															
r3w2								1																

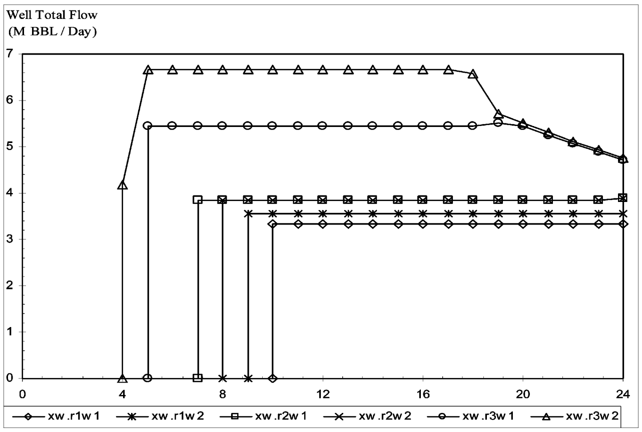

Figure 8. Well flow profile for d70, which maximizes ENPV, from the SAA runs.

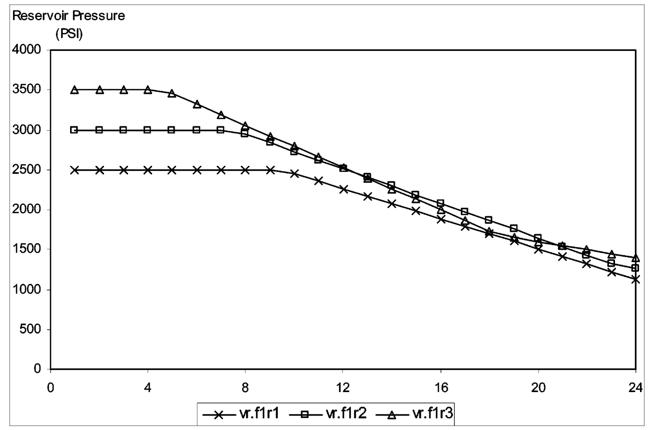

Figure 9. Pressure profile of reservoirs for d70, which maximizes ENPV, from the SAA runs.

productivity indexes, which are only one instance of the sampling region of the stochastic problem, which will be shown next. Higher productivity index values can occur, in which case the reservoir performance will be more complex.

In this solution, \$114.4 million is injected in the first time period (\$500 million is the limit). From that, \$92.8 million is used to install the production platform and the well platform Pi2, \$1 million is secured as operating capital, and the remaining \$21.6 million is kept for use in the other time periods. In the third time period, \$7 million is used to drill the well r3w1. In the fourth time period, in addition to the remaining \$14.6 million, \$11 million is borrowed to install the well platform Pi1, to drill the well r3w2, and to pay operating expenses. From the fifth time period onward, the costs of drilling are taken from the revenues and the debt is completely paid back by the sixth time period. Proceeds start generating from the seventh time period onward. The last time period, the 24, shows a higher proceed because it takes into account revenues from both the 23 and 24 time periods because of the time lag that was assumed. The cash flow details are shown in Figures 5 and 6.

Stochastic Model Results. Running the model stochastically for a large number of scenarios cannot be done with the available computational resources. To obtain some stochastic solutions to the problem, we used the SAA,[1] running one scenario at a time, which has been shown by Aseeri and Bagajewicz[6] to be a valid alternative to obtain solutions and an upper bound of the whole problem. The algorithm followed in these runs is shown in the diagram in Figure 7.

In this algorithm, samples for oil prices and productivity indexes are first generated. The deterministic model is then solved for each scenario. The first stage variables are then fixed and the deterministic model solved for all scenarios to find the NPV of the fixed design under each scenario (inner loop). The same process is repeated for all of the other scenarios (outer loop). After obtaining the results for enough designs, on can compare both the ENPV of each design and the risk curve performance.

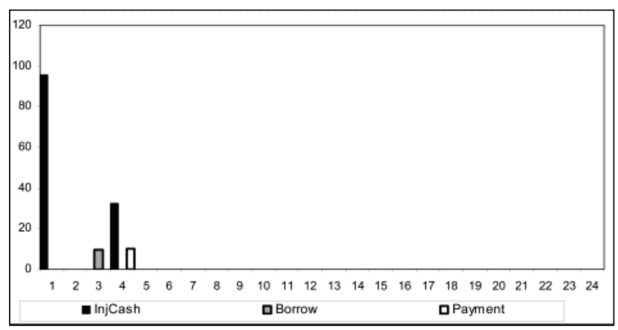

Figure 10. Cash flow for d70, which maximizes ENPV, from the SAA runs (injected cash, borrow, and payments).

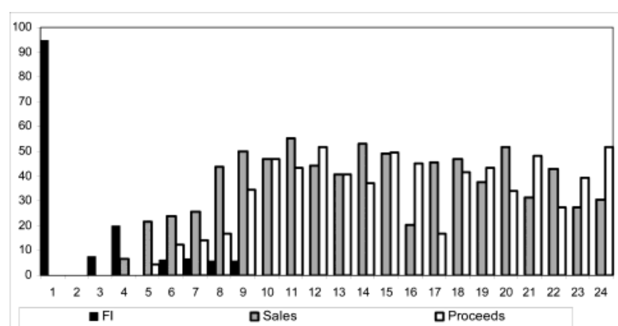

Figure 11. Cash flow for d70, which maximizes ENPV, from the SAA runs (fixed investment, sales, and proceeds).

This algorithm was applied for 1000 scenarios. The design that was found to give the best *ENPV* was design (d70), which is the solution when the seventieth scenario is solved. The *ENPV* of this design is 383 million dollars. The results for this solution are shown in Table 6 and

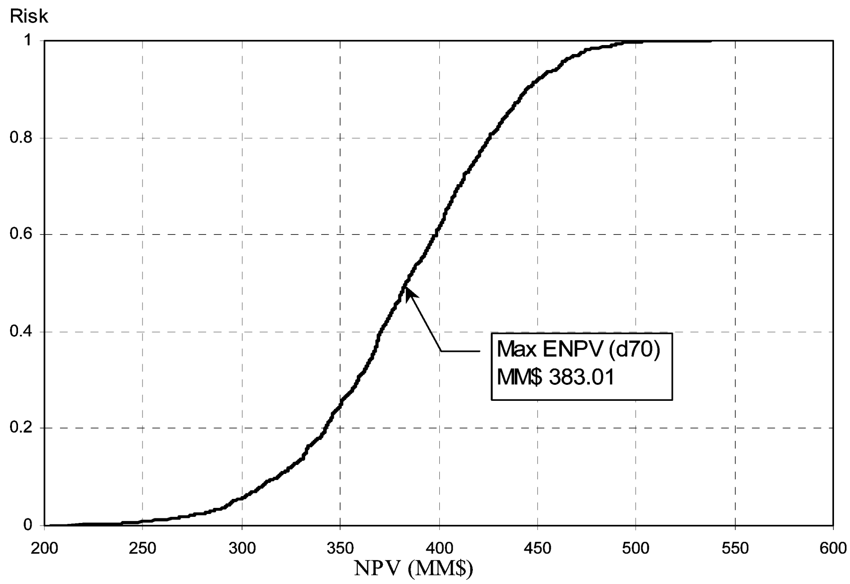

Figure 12. Risk curve for d70, which maximizes ENPV.

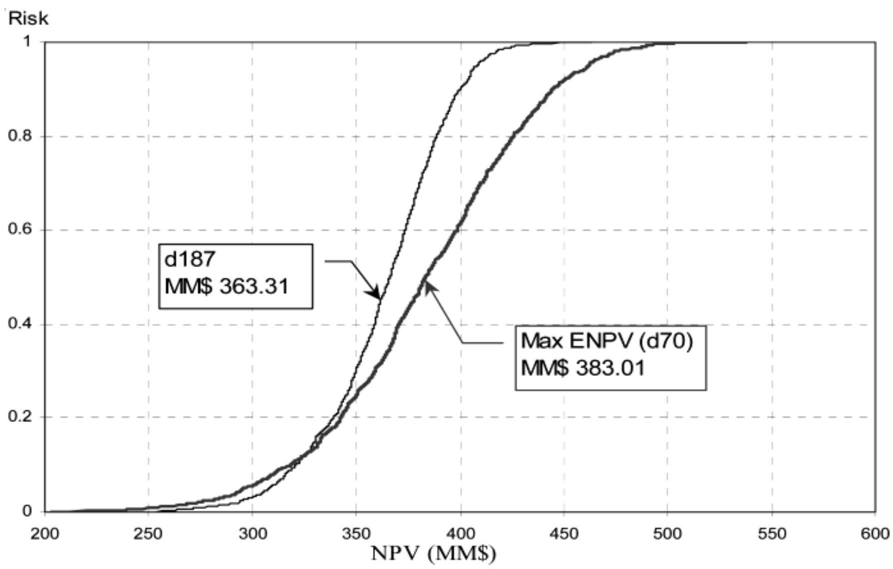

Figure 13. Risk curve for d187 (minimum risk) compared to d70 (maximum ENPV).

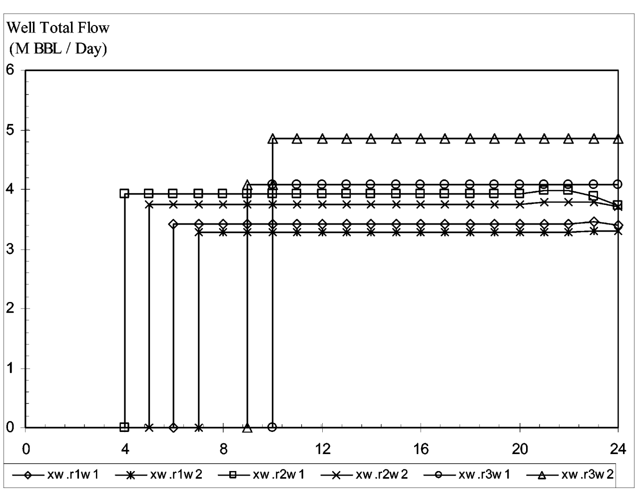

Figure 14. Well flow profile for d187, which minimizes risk.

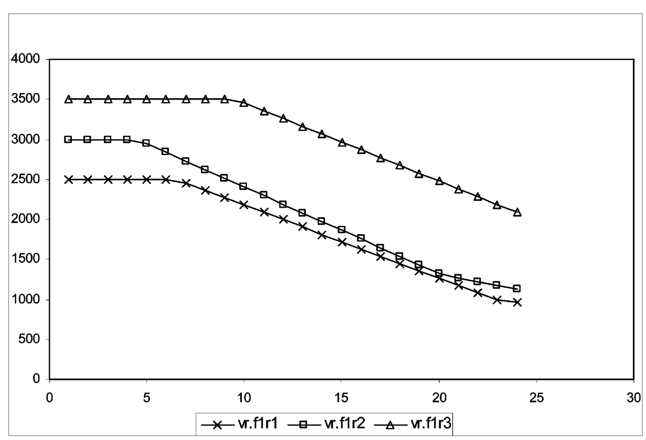

Figure 15. Pressure profile of reservoirs for d187, which minimizes risk, from the SAA runs.

Figure 8. The schedule of drilling is quite similar to that in Table 5 except for reversing the order of drilling wells r3w1 and r3w2. We notice from Figure 8 that the flow profile of the wells associated with reservoir r3 starts to decline from the 17th time period. This is due to the large reduction in pressure (Figure 9), which becomes the controlling factor over the productivity index. Figures 10 and 11 show the cash flow for the solution suggested by the design d70 in a format similar to that

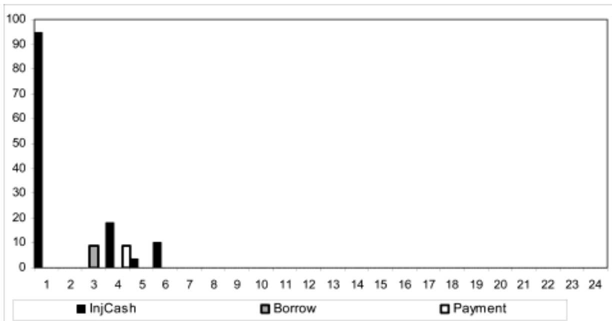

Figure 16. Cash flow for d187, which minimizes risk (injected cash, borrow, and payments).

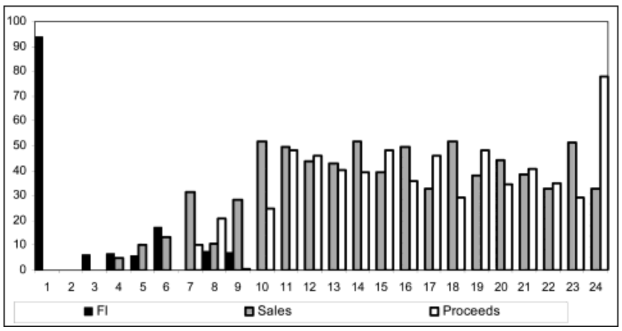

Figure 17. Cash flow for d187, which minimizes risk (fixed investment, sales, and proceeds).

in Figures 5 and 6. Unlike Figures 5 and 6, the sales and proceeds in Figure 10 do not follow a smooth profile. This is due to the difference in the prices of oil from one time period to another in this instance of the sampling region. For the deterministic run, the price is fixed at $22/bbl. The risk curve for this solution is shown in Figure 12.

Risk Management. To see if a solution with a smaller risk can be obtained for this problem, the SAA result curves were compared against design d70. Design d187 was found to reduce the risk in the downside region. The risk curve for this solution is compared to d70 in Figure 13. The ENPV of design d187 is $363.3 million. The results for this solution are shown in Table 7 and Figure 14.

The schedule of installing the well platforms and consequently drilling the wells is inverted compared to that of d70 (Table 7). We notice from the histogram that the flow profile of the wells associated with reservoir r2 and r1 start to decline on the 22nd time period. This is due to the reduction in pressure (Figure 15), in which case it became the controlling factor over the productivity index. Enough reduction in r3 pressure was not achieved as it was in d70 because in this case the wells in r3 were not drilled until five time periods later. Figures 16 and 17 show the cash flow for the solution suggested by the design d187 in a format similar to that in Figures 5 and 6. Similar to Figure 10, the sales and proceeds in Figure 16 do not follow a smooth profile for the same reason.

Value at Risk (VaR) and Upside Potential (UP). A widely used measure of risk used in the literature is VaR[10,11] and is defined as the expected loss for a certain confidence level[12] usually set at 5%. A more general definition of VaR is given by the difference between the mean value of the profit and the profit value corresponding to the p quantile (value at p risk). Aseeri and Bagajewicz[6] proposed that VaR be compared to a similar measure, the UP or opportunity value (OV), defined in a way similar to that of VaR but at the other end of the risk curve with a quantile of $1 - p$ as the difference between the NPV corresponding to a risk of $1 - p$ and the expected value.

The VaRs (at 5% or 0.05 quantile) for the two curves in Figure 13 are illustrated in Figure 18.

From this histogram, we see that the VaR decreased from 87.12 to 55.39 or 36.4% in the design d187 versus that of design d70. Looking at the curves, we see that there is also a more significant reduction in opportunity in the low risk curve, and unless this loss is also taken into account, the comparison of risk will be incomplete. The OV decreased from 78.81 to 45.19 or 42.7% in the design d187 versus that of design d70. This indicates that there is a great loss in opportunity resulting from the lower risk design.

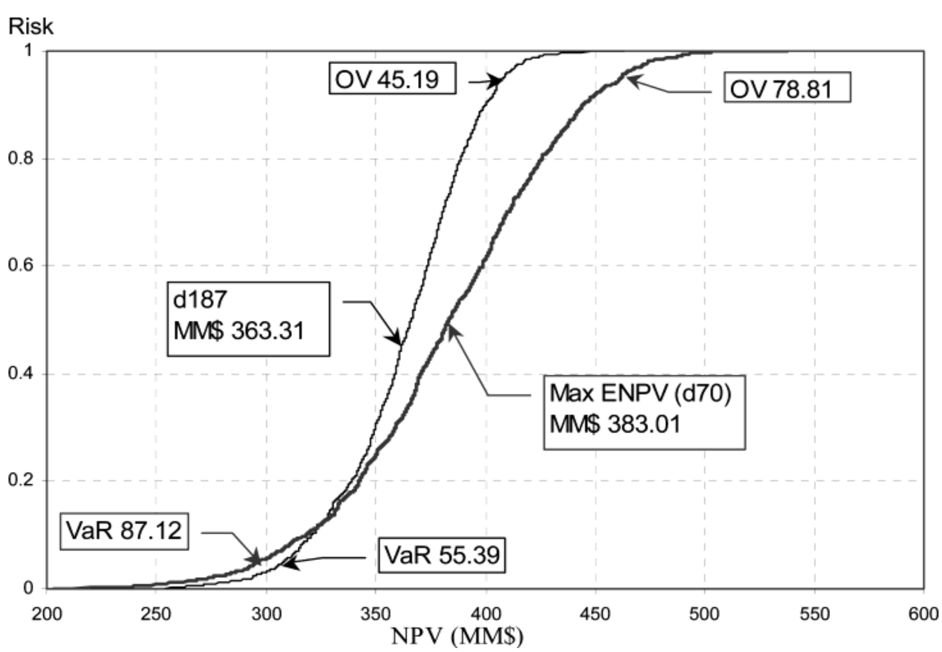

Figure 18. VaR and OV for d187 (minimum risk) compared to d70 (maximum ENPV).

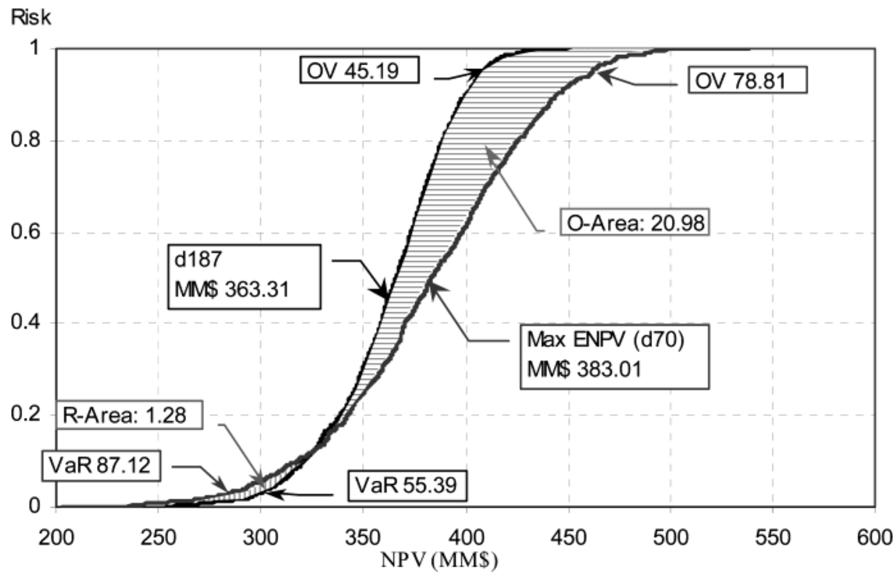

Figure 19. RAR for d70 and d187.

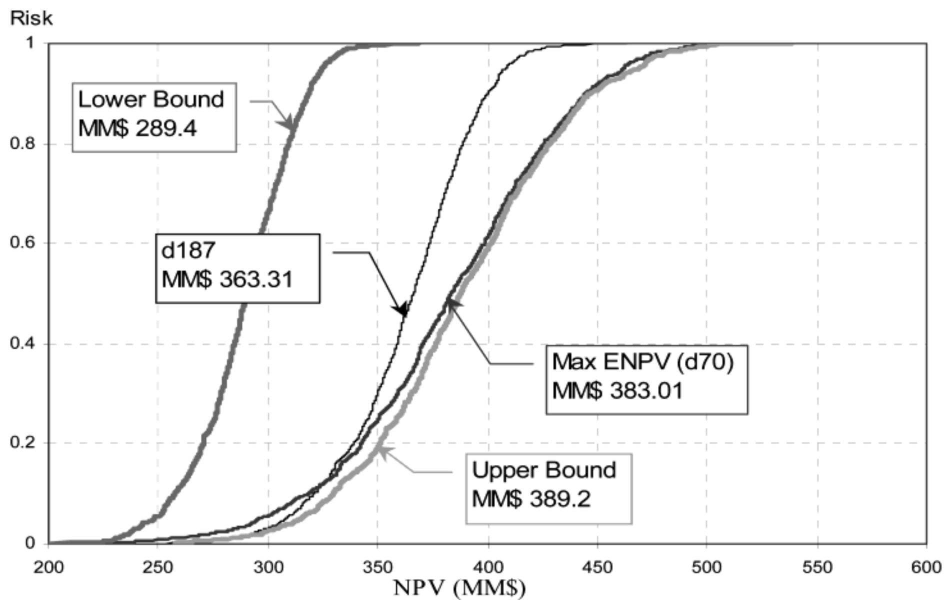

Figure 20. Upper and lower bound risk curves for the oil scheduling problem.

Risk Area Ratio (RAR). Another metric for comparing the results' risk curves is a comparison of the areas between the two curves.[6] The proposed ratio, the RAR, is calculated as the ratio of the opportunity area (O_Area), enclosed by the two curves above their intersection, to the risk area (R_Area), enclosed by the two curves below their intersection (eq 45).

$$\mathrm{RAR} = \frac{\mathrm{O\_Area}}{\mathrm{R\_Area}} \tag{45}$$

The areas can be calculated by integrating the difference of risk between the two curves over NPVs. The closer this ratio is to 1, the better is the alternative solution. The RAR for the solutions in Figure 18 is illustrated in Figure 19 and is equal to 16.4. This is an indication of how significant the reduction in opportunity is compared to the small reduction in risk.

Upper and Lower Risk Curve Bounds. Another way of comparing solutions obtained from the SAA, proposed by Aseeri and Bagajewicz,[6] is with the use of the upper and lower bound risk curves. The upper bound risk curve is defined as the curve constructed by plotting the set of NPVs for the best design under each scenario, that is, by using all "wait and see" solutions. They proved that this curve cannot be crossed by any feasible solution. The lower bound risk curve is defined as the curve constructed by plotting the highest risk of the set of designs used to construct the upper bound risk curve at each NPV abscissa. The lower bound risk curve, unlike the upper bound risk curve, can be crossed by feasible solutions, but these would not be Pareto optimal solutions in the downside risk and expected NPV space.

Figure 20 shows the upper and lower bound risk curves for the SAA solutions of the oil scheduling problem as well as the solution that maximizes ENPV and the one that reduces risk.

Aseeri and Bagajewicz[6] proved that any feasible design is positioned entirely above (to the left of) the upper bound risk curve. Hence, the ENPV (the objective function value) of any feasible solution is smaller than or equal to that of the upper bound risk curve. Therefore, the gap (in MILP terminology) between any solution and the *best possible integer solution* will always be less than or equal to the gap between that solution and the upper bound risk curve.

From Figure 40, we see that the SAA solution of the oil scheduling problem that maximizes ENPV (i.e., d70) is very close to the upper bound risk curve with a gap of only 1.6%. This gives an indication that if this is not the best stochastic solution for this problem, it is very close to it and that it is equivalent to a stochastic solution with a gap of less than or equal to 1.6%.

4. Conclusions

In this paper, we discussed the financial risk management in the planning and scheduling of offshore oil infrastructure. We added budgeting constraints to this model, following the cash flow of the project, taking care of the distribution of proceeds, and considering the possibility of taking loans. The model was made stochastic, and financial risk was found to be manageable to some extent. Extension to include taxation and royalties, as was proposed by Van den Heever et al.,[4] is part of future work.

The numerical difficulties resulting from the addition of uncertainty were discussed, and the SAA[1] was used to overcome these difficulties. We showed that the SAA yields very good solutions that were proven to be very close to the optimum using the concept of the upper bound risk curve. It was proven that a SAA solution with a certain gap from the upper bound risk curve is equivalent to a stochastic solution with a gap less than or equal to that gap.

The solutions were compared using the concepts of VaR, OV, and RAR to show how these concepts handle the trade-off between opportunity and risk.

Acknowledgment

We are grateful to the University of Oklahoma Supercomputing Center for Education & Research (OSCER) for allowing us to make many of our runs. This paper is dedicated by M.B. to George Gavalas, who taught him important things, like thinking properly, among many others.

Literature Cited

(1) Verweij, B.; Ahmed, S.; Kleywegt, A. J.; Nemhauser, G.; Shapiro, A. The Sample Average Approximation Method Applied to Stochastic Routing Problems: A Computational Study. *Comput. Appl. Optim*. **2001**, *24*, 289–333.

(2) Iyer, R. R.; Grossmann, I. E.; Vasantharajan, S.; Cullick, A. S. Optimal Planning and Scheduling of Offshore Oil Field Infrastructure Investment and Operations. *Ind. Eng. Chem. Res.* **1998**, *37*, 1380–1397.

(3) Van den Heever, S.; Grossmann, I. E. An Iterative Aggregation/Disagregation Approach for the Solution of the Mixed Integer Nonlinear Oilfield Infrastructure Planning Model. *Ind. Eng. Chem. Res.* **2000**, *39* (6), 1955.

(4) Van den Heever, S.; Grossmann, I. E.; Vasantharajan, S.; Edwards, K. Integrating Complex Economic Objectives with the Design and Planning of Offshore Oilfield Infrastructures. *Comput. Chem. Eng.* **2000**, *24*, 1049–1055.

(5) Van den Heever, S.; Grossmann, I. E.; Vasantharajan, S.; Edwards, K. A Lagrangean Decomposition Heuristic for the Design and Planning of Offshore Field Infrastructures with Complex Economic Objectives. *Ind. Eng. Chem. Res.* **2001**, *40*, 2857–2875.

(6) Aseeri, A.; Bagajewicz, M. New Measures and Procedures to Manage Financial Risk with Applications to the Planning of Gas Commercialization in Asia. *Comput. Chem. Eng.* **2003**, submitted for publication.

(7) Barbaro, A.; Bagajewicz, M. Financial Risk management in Planning under Uncertainty. Proceedings of FOCAPO: Foundations of Computer Aided Process Operations, Coral Springs, FL, Jan 2003.

(8) Barbaro, A.; Bagajewicz, M. Managing Financial Risk in Planning under Uncertainty. *AIChE J.* **2004**, *50* (5), 963–989.

(9) Eppen, G. D.; Martin, R. K.; Schrage, L. A Scenario Approach to Capacity Planning. *Oper. Res.* **1989**, *37*, 517–527.

(10) Jorion, P. *Value at Risk. The New Benchmark for Managing Financial Risk*, 2nd ed.; McGraw-Hill: New York, 2000.

(11) Guldimann, T. *The Story of Risk Metrics*, Risk 13, No 1 (Jan), **2000**, 56–58.

(12) Linsmeier, T. J.; Pearson, N. D. Value at Risk. *Financ. Anal. J.* **2000**, *56* (No. 2), Mar/Apr.

Received for review August 28, 2003
Revised manuscript received February 16, 2004
Accepted February 16, 2004

IE034098C