Financial Risk Management in the Design of Water Utilization Systems in Process Plants

Anantha P. R. Koppol and Miguel J. Bagajewicz*

University of Oklahoma, 100 E. Boyd Street, T-335, Norman, Oklahoma 73019

This paper discusses techniques to design water utilization systems in process plants when the parameters, especially the loads of the contaminants, are uncertain. Financial risk management in such designs is discussed in detail. It is also managed in a noniterative fashion using a one-step procedure that incorporates financial risk management as a constraint in the design procedure.

Introduction

Water is used in process plants as a means of removing contaminants from different processes. In the majority of cases, this takes place by putting water (or steam) in contact with organic phases from which the contaminants are extracted. Such water utilization systems consist of a network of water reuse and partial regeneration, aimed at the reduction of cost. In a review paper, Bagajewicz[1] offers a detailed description of the different reuse and regeneration schemes, as well as the variety of solution procedures that have been proposed. In addition, Koppol et al.[2] discuss zero-liquid discharge cycles.

One of the biggest criticisms that all of the existing methods had endured, especially from practitioners, is the difficulty in determining the loads of the contaminants to be removed in each process. In addition, the different processes may vary their throughput relative to each other, adding another dimension to the uncertainty. Thus, while many of the methods proposed are robust, the data are uncertain. To address this problem, this paper discusses a methodology that yields a flexible design, which is feasible over a range of values for the uncertain parameters and manages the financial risk at the same time. This is achieved by constructing a mathematical optimization model.

Problem Statement

Given the limiting inlet and outlet concentrations and uncertain loads to be picked up in each unit of a set of water-using/water-disposing processes, it is desired to determine a network of interconnections of water streams among the processes so that the expected cost is minimized while processes receive water of adequate quality, with or without change of flows, for all instances of uncertain parameters in the interval. It is also desired to manage financial risk.

In this paper, we concentrate on the single-contaminant case. The problem is one of design under uncertainty, for which several methodologies exist. The importance of design under uncertainty has been widely recognized and discussed by several researchers.[3−8]

Solution Procedure

The objective of the above problem is to minimize the cost, which includes both capital and operating cost. The capital cost can be considered as directly related to the number of interconnections in the network, while the operating cost is related to the total freshwater consumed. For the deterministic case (no uncertainty), the following MILP formulation was proposed:[9]

$$\text{P2} = \text{Min}\{\sum_j c_j^{\text{w}}\ Y_j^{\text{w}} + \sum_{i,j} c_{i,j} Y_{i,j} + \sum c_j^{\text{out}}\ Y_j^{\text{out}} + \alpha \sum_j F_j^{\text{w}}\} \quad (1)$$

s.t.

$$F_j^{\text{w}} + \sum_i F_{i,j} - \sum_k F_{j,k} - F_{j,\text{out}} = 0 \qquad \forall\, j \in N$$

$$\sum_i F_{i,j}(C_{i,\text{out}}^{\max} - C_{j,\text{in}}^{\max}) - F_j^{\text{w}}\, C_{j,\text{in}}^{\max} \le 0 \qquad \forall\, j \in N$$

$$\sum_i F_{i,j}(C_{i,\text{out}}^{\max} - C_{j,\text{out}}^{\max}) - F_j^{\text{w}}\, C_{j,\text{out}}^{\max} + L_j = 0 \qquad \forall\, j \in N$$

$$F_j^{\text{w}} - Y_j^{\text{w}} U \le 0 \qquad \forall\, j \in N$$

$$F_{i,j} - Y_{i,j} U \le 0 \qquad \forall\, i, j \in N$$

$$F_j^{\text{out}} - Y_j^{\text{out}} U \le 0 \qquad \forall\, j \in N$$

$$Y_j^{\text{w}},\ Y_{i,j} \in \{0, 1\} \qquad \forall\, i, j \in N$$

The binary variables (Y) take value 1 when an interconnection exits and 0 otherwise. Thus, the capital cost in the objective can be represented as the product of the binary variable (Y) and the annualized cost incurred for each connection ($c_{i,j}$), while the operating cost is the product of total freshwater and the annualized cost per unit of freshwater consumed (α).

Uncertainty in contaminant loads can be thought of as being given by a probability distribution (P_1) within a certain known interval [a_i, b_i]. This is a reasonable assumption because it is possible to specify the range within which the uncertain parameters vary, values that come from assessing how large variations in throughput the water utilization process can have. In this paper, uncertainty in the inlet and outlet maximum concentration limits is not considered. Although they can be easily added, they are in many cases hardly uncertain. Maximum outlet concentrations are, in reality, either practical limits established by corrosion prevention or other limitations or maximum solubility values. Maximum inlet concentrations, in turn, are an

* To whom correspondence should be addressed.

10.1021/ie0203882 CCC: $25.00
Published on Web 09/12/2003

artifice to limit the flow rate through a process, a constraint that can be added without difficulty.[9] Because probabilities are included, expected cost is used:

$$E(\alpha\sum_j F_j^{\rm w} + \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out}) = \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out} + \alpha\sum_{j=1}^{N}\{\int_{a_1}^{b_1}...\int_{a_N}^{b_N} F_j^{\rm w}(L,Y,Y^{\rm out})\prod_{i=1}^{N}[P_i(L_i)]\ {\rm d}L_1\ ...\ {\rm d}L_N\} \quad (2)$$

In this simplified expression, the freshwater sent to each process ($F_j^{\rm w}$) is a function of the loads in all of the processes (L_j), and the structure of the system (Y, $Y^{\rm out}$). Very frequently, the distribution of the uncertain parameters in the interval of uncertainty is not known; hence, the distribution is assumed to be uniform within the interval. Thus, the expected cost is

$$E(\alpha\sum_j F_j^{\rm w} + \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out}) = \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out} + \alpha\sum_{j=1}^{N}\{\prod_{i=1}^{N}[1/(b_i - a_i)]\int_{a_1}^{b_1}...\int_{a_N}^{b_N} F_j^{\rm w}(L,Y,Y^{\rm out})\ {\rm d}L_1\ ...\ {\rm d}L_N\} \quad (3)$$

The objective is then to determine the set of interconnections that minimizes this expected cost. This can be achieved using the following mathematical model:

$$P2 = {\rm Min}\ E(\sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out} + \alpha\sum_j F_j^{\rm w}) \quad (4)$$

s.t.

$$F_j^{\rm w} + \sum_i F_{i,j} - \sum_k F_{j,k} - F_{j,{\rm out}} = 0 \qquad \forall\ j \in N$$

$$\sum_i F_{i,j}(C_{i,{\rm out}}^{\max} - C_{j,{\rm in}}^{\max}) + F_j^{\rm w}\ C_{j,{\rm in}}^{\max} \le 0 \qquad \forall\ j \in N$$

$$\sum_i F_{i,j}(C_{i,{\rm out}}^{\max} - C_{j,{\rm out}}^{\max}) - F_j^{\rm w}\ C_{j,{\rm out}}^{\max} + L_j = 0 \qquad \forall\ j \in N$$

$$a_j \le L_j \le b_j \qquad \forall\ j \in N$$

$$F_j^{\rm w} - Y_j^{\rm w}U \le 0 \qquad \forall\ j \in N$$

$$F_{i,j} - Y_{i,j}U \le 0 \qquad \forall\ i, j \in N$$

$$F_j^{\rm out} - Y_j^{\rm out}U \le 0 \qquad \forall\ j \in N$$

$$Y_j^{\rm w},\ Y_{i,j},\ Y_j^{\rm out} \in \{0, 1\} \qquad \forall\ i, j \in N$$

As stated above, the contaminant loads (L_j) can vary in the intervals of uncertainty [a_j, b_j]. Because the uncertain parameters are continuous variables, the above formulation has an infinite number of constraints (feasibility is to be ensured for each instance of uncertain parameters). To solve the above problem, one needs to resort to discretization, where a finite sample of scenarios are chosen to solve the above problem. The expected cost in this case is

$$E(\alpha\sum_j F_j^{\rm w} + \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out}) = \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out} + \alpha\sum_{j=1}^{N} P_s F_{j,s}^{\rm w} \quad (5)$$

where s is an element of the set of sample scenarios for contaminant load in the processes. Thus, the optimization problem becomes

$${\rm Min}\ E^* = \sum_j c_j^{\rm w}\ Y_j^{\rm w} + \sum_{i,j} c_{i,j}Y_{i,j} + \sum_j c_j^{\rm out}\ Y_j^{\rm out} + \alpha\sum_s\sum_j p_s F_{j,s}^{\rm w} \quad (6)$$

s.t.

$$F_{j,s}^{\rm w} + \sum_i F_{i,j,s} - \sum_k F_{j,k,s} - F_{j,{\rm out},s} = 0 \qquad \forall\ s, \forall\ j \in N$$

$$\sum_i F_{i,j,s}(C_{i,{\rm out}}^{\max} - C_{j,{\rm in}}^{\max}) - F_{j,s}^{\rm w}\ C_{j,{\rm in}}^{\max} \le 0 \qquad \forall\ s, \forall\ j \in N$$

$$\sum_i F_{i,j,s}(C_{i,{\rm out}}^{\max} - C_{j,{\rm out}}^{\max}) - F_{j,s}^{\rm w}\ C_{j,{\rm out}}^{\max} + L_{j,s} = 0 \qquad \forall\ s, \forall\ i, j \in N$$

$$F_{j,s}^{\rm w} - Y_j^{\rm w}U \le 0 \qquad \forall\ s, \forall\ j \in N$$

$$F_{j,{\rm out},s} - Y_{j,{\rm out}}U \le 0 \qquad \forall\ s, \forall\ j \in N$$

$$F_{i,j,s} - Y_{i,j}U \le 0 \qquad \forall\ s, \forall\ i, j \in N$$

$$Y_j^{\rm w},\ Y_{i,j},\ Y_j^{\rm out} \in \{0, 1\} \qquad \forall\ i, j \in N$$

This problem provides one set of interconnections between processes such that there is a feasible solution for all instances of uncertain parameters. Therefore, the binary variables are independent of the scenarios chosen and are the common variable for the sets of equations written for each scenario. To overcome the drawback of having to deal with a very large number of scenarios, which presents some computational challenges, the vertex critical nature[10] of the problem is first proved and later used. Vertex criticality is a property through which the network of interconnections that is feasible at all of the extreme values of the uncertainty intervals is feasible in the entire interval. The theorem proving the vertex criticality of this problem is presented next.

Theorem. Given the limiting inlet and outlet concentrations and the interval of uncertainty for the loads for a set of water-using/water-disposing processes with a single contaminant, a set of interconnections between the processes, which renders a feasible solution at all of the vertices of the hypercube formed in the uncertainty space of loads, will also be feasible throughout the interval of uncertainty.

Proof. The proof stems directly from the polyhedrality of the constrained set. Indeed, for each vertex of the set, the constraints can be represented by $Ax_i \le b$. Hence, every linear combination of two vertices, namely, $x_\lambda = x_i\,\lambda + x_j(1 - \lambda)$, will also satisfy the constraints.

Q.E.D.

As a consequence of the above theorem only two sample points for each uncertain contaminant load are

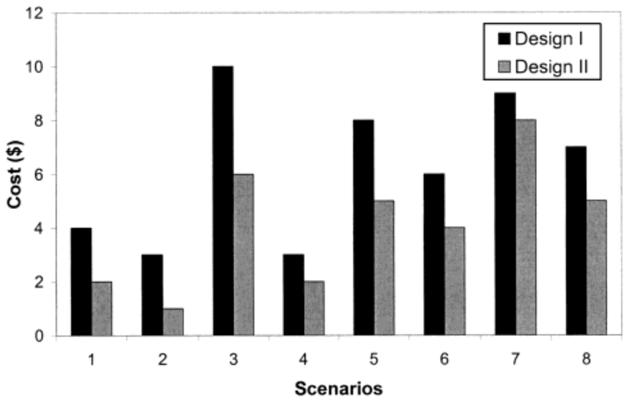

Figure 1. Cost distribution histogram.

needed to ensure feasibility in the entire interval of uncertainty. This results in a large decrease in the number of scenarios and therefore, constraints.

Financial Risk

In this section the procedure to estimate the financial risk associated with a design is outlined. Further, the issues pertaining to risk management in the water allocation planning problem with uncertainty in contaminant loads will be discussed.

Estimation of Risk. Assume that a system has a particular structure (obtained in this case by fixing the binary variables). For each instance or scenario s of the uncertain parameters, one can calculate the total cost (C_s) and, therefore, the risk associated with the scenario is defined as the probability that the cost (C_s) is greater than the aspiration level (γ), that is

$$\mathrm{RISK}_s(\gamma) = P\{C_s \geq \gamma\} \tag{7}$$

This expression can be rewritten using new binary variables (Z_s) as follows:

$$\mathrm{RISK}_s(\gamma) = p_s Z_s \tag{8}$$

where $Z_s = 1$ if $C_s > \gamma$ and $Z_s = 0$ otherwise. The definition for risk given in eq 8 is for a particular scenario s. This concept can be extended to the case where several scenarios are considered by defining cumulative risk, which is the summation of risk for individual scenarios:

$$\mathrm{RISK}(\gamma) = \sum_s p_s Z_s \tag{9}$$

Thus, for a fixed structure that ensures feasibility in the entire interval of uncertainty, one would like to plot risk as a function of the cost aspiration level (γ). When the cost aspiration level is large, the risk is zero. Conversely, as the cost aspiration level tends to 0, the risk tends to 1. This approach follows directly the one presented by Barbaro and Bagajewicz as well as proposed but not used directly by Gupta and Maranas[13].[11,12] It uses one binary variable per scenario, which at some point, if the number of scenarios is large, can result in a numerical problem. In such a case, the use of such binary variables can be avoided by using downside risk.[14] Because downside risk is not a monotone measure of risk,[12] its use is only recommended for cases where numerical computations are cumbersome.

Behavior of Risk Curves. Consider a cost distribution histogram for designs I and II as shown in Figure 1. The risk curve for designs I and II assuming uniform probability for the scenarios is shown in Figure 2. Because in design I the cost of each of the scenarios is higher than that of design II, the risk curve for design I is completely above the risk curve for design II. This implies that a risk curve that is completely beneath the risk curve corresponding to the minimum expected cost is not possible because this will need all of the scenarios to have a lower cost relative to those of the minimum expected cost, which is a contradiction. Therefore, the risk curves corresponding to alternative designs are positioned either completely above or will intersect the risk curve corresponding to the minimum expected cost.

The risk curves, corresponding to any alternative design and the design with the minimum expected cost are exactly the same if and only if the costs at each of the scenarios are the same.

To have risk curves that intersect, one must have designs that have cost distributions such that some scenarios have lower cost relative to the other while some scenarios have higher cost relative to the other, as illustrated in Figures 3 and 4.

Risk Management. Once one has solved the problem given by eq 6, one obtains a structure for which the expected cost is minimum but for which the associated risk may not be according to the decision maker's

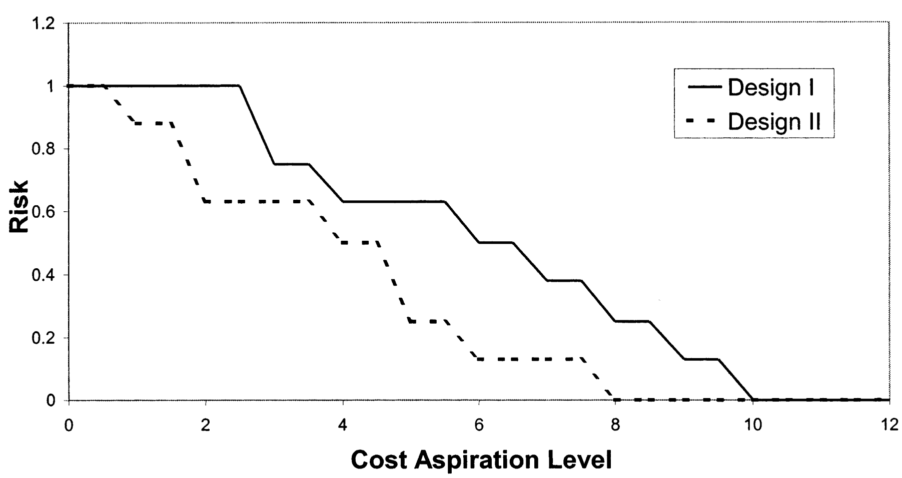

Figure 2. Risk vs aspiration level.

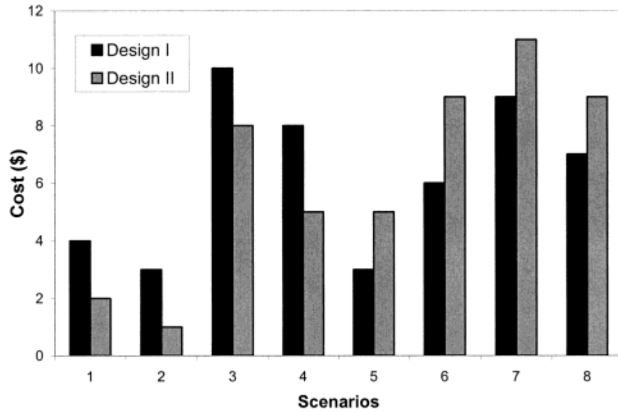

Figure 3. Cost distribution histogram.

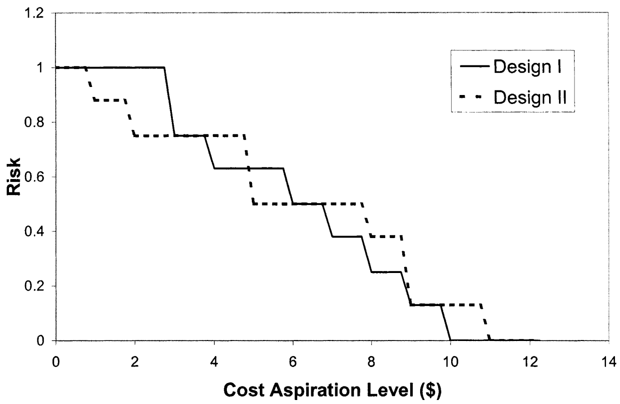

Figure 4. Risk vs aspiration level.

aspiration. Altering risk is, of course, possible only if alternative feasible designs exist. Further, reducing risk is possible only if the alternative designs have scenarios with lower cost compared to the corresponding scenarios in the current design. To obtain alternative structures, problem (6) needs to be modified by adding constraints that will force such a change. For example, if the decision maker seeks a design with a risk level lower than ϵ_r at different aspiration levels γ_r, one needs to solve the following problem:

$$\text{Min } E^* = \sum_j c_j^{\text{w}} Y_j^{\text{w}} + \sum_{i,j} c_{i,j} Y_{i,j} + \sum_j c_j^{\text{out}} Y_j^{\text{out}} + \alpha \sum_s \sum_j p_s F_{j,s}^{\text{w}} \quad (10)$$

s.t.

$$F_{j,s}^{\text{w}} + \sum_i F_{i,j,s} - \sum_k F_{j,k,s} - F_{j,\text{out},s} = 0 \qquad \forall\, s, \forall\, j \in N$$

$$\sum_i F_{i,j,s}(C_{i,\text{out}}^{\max} - C_{j,\text{in}}^{\max}) - F_{j,s}^{\text{w}} C_{j,\text{in}}^{\max} \le 0 \qquad \forall\, s, \forall\, j \in N$$

$$\sum_i F_{i,j,s}(C_{i,\text{out}}^{\max} - C_{j,\text{out}}^{\max}) - F_{j,s}^{\text{w}} C_{j,\text{out}}^{\max} + L_{j,s} = 0 \qquad \forall\, s, \forall\, i, j \in N$$

$$F_{j,s}^{\text{w}} - Y_j^{\text{w}} U \le 0 \qquad \forall\, s, \forall\, j \in N$$

$$F_{j,\text{out},s} - Y_{j,\text{out}} U \le 0 \qquad \forall\, s, \forall\, j \in N$$

$$F_{i,j,s} - Y_{i,j} U \le 0 \qquad \forall\, s, \forall\, i, j \in N$$

$$\text{Cost}_s - \gamma_i \le U Z_{s,r} \qquad \forall\, s, \forall\, r \in R$$

$$\text{Cost}_s - \gamma_i \ge U(Z_{s,r} - 1) \qquad \forall\, s, \forall\, r \in R$$

$$\text{Risk}(\gamma_r) = \sum_{\forall s} p_s Z_{s,r} \le \epsilon_r \qquad \forall\, r \in R$$

$$Y_j^{\text{w}}, Y_{i,j}, Z_{s,r} \in \{0, 1\} \qquad \forall\, s, \forall\, i, j \in N, \forall\, r \in R$$

A goal programming version of the above problem is

$$\text{Min } E^* = \sum_j c_j^{\text{w}} Y_j^{\text{w}} + \sum_{i,j} c_{i,j} Y_{i,j} + \sum_j c_j^{\text{out}} Y_j^{\text{out}} + \alpha \sum_s \sum_j P_s F_{j,s}^{\text{w}} + \sum_{r \in R} \rho_r \text{Risk}(\gamma_r) \quad (11)$$

s.t.

$$F_{j,s}^{\text{w}} + \sum_i F_{i,j,s} - \sum_k F_{j,k,s} - F_{j,\text{out},s} = 0 \qquad \forall\, s, \forall\, j \in N$$

$$\sum_i F_{i,j,s}(C_{i,\text{out}}^{\max} - C_{j,\text{in}}^{\max}) - F_{j,s}^{\text{w}} C_{j,\text{in}}^{\max} \le 0 \qquad \forall\, s, \forall\, j \in N$$

$$\sum_i F_{i,j,s}(C_{i,\text{out}}^{\max} - C_{j,\text{out}}^{\max}) - F_{j,s}^{\text{w}} C_{j,\text{out}}^{\max} + L_{j,s} = 0 \qquad \forall\, s, \forall\, i, j \in N$$

$$F_{j,s}^{\text{w}} - Y_j^{\text{w}} U \le 0 \qquad \forall\, s, \forall\, j \in N$$

$$F_{j,\text{out},s} - Y_{j,\text{out}} U \le 0 \qquad \forall\, s, \forall\, j \in N$$

$$F_{i,j,s} - Y_{i,j} U \le 0 \qquad \forall\, s, \forall\, i, j \in N$$

$$\text{Cost}_s - \gamma_i \le U Z_{s,r} \qquad \forall\, s, \forall\, r \in R$$

$$\text{Risk}(\gamma_r) = \sum_{\forall s} p_s Z_{s,r} \qquad \forall\, r \in R$$

$$Y_j^{\text{w}}, Y_{i,j}, Y_j^{\text{out}}, Z_{s,r} \in \{0, 1\} \qquad \forall\, s, \forall\, i, j \in N, \forall\, r \in R$$

Note that in eq 11 the constraint that forces $Z_{s,r}$ to be zero when cost is less than the aspiration level has not been included because $Z_{s,r}$ is naturally tending to zero when the cost of a scenario is smaller than the aspiration level because risk is maximized in the objective. Note that using the formulation for risk management in eq 11 the relative importance for each aspiration level γ_r can be set by choosing the appropriate penalty ρ_r. By solving problem (11), one can obtain different solutions by changing the value of the penalty parameter. Barbaro and Bagajewicz[11] prove that the model in eq 12 guarantees Pareto optimal solutions.

In water allocation planning problems, usually the freshwater cost and the final treatment cost are much higher than the capital cost (which is mostly based on piping). Thus, the operating cost is the major component of the total cost. Now, consider a particular structure. If the structure is such that all scenarios can realize maximum reuse, the minimum possible operating cost is achieved for each scenario. An alternative structure featuring the same property of being a maximum reuse structure will, therefore, have the same exact operating

Table 1. Process Data (Example 1)

process	$C_{\mathrm{in}}^{\mathrm{max}}$ (ppm)	$C_{\mathrm{out}}^{\mathrm{max}}$ (ppm)	load (g/h)
1	25	80	2000
2	10	90	2880
3	25	200	4000
4	50	100	3000
5	400	800	5000
6	150	300	6500

cost, differing only in capital. This result is not a surprise because the existence of different designs that are maximum reuse structures for a particular scenario of parameters was already pointed out by Bagajewicz and Savelski.[15] Therefore, the difference in the total expected cost between the designs is the same as the difference between the capital costs. As a consequence, the risk for the designs with comparable expected cost is almost the same. On the other hand, for the designs with an expected cost higher than that of the minimum expected cost, the risk will also be high. One can therefore make an important conclusion: if there exists a process structure that is a maximum reuse structure for all scenarios, risk can hardly be altered.

In the case where the capital cost is much higher than the operating cost, the optimal amount of water reused could be smaller than what the process can actually reuse; in other words, the design will not be a maximum reuse structure. Clearly, in this case the expected cost will be minimum using the design with the minimum capital cost. Therefore, the cost for each of the scenarios in alternative designs will be higher compared to the cost for the scenarios corresponding to the minimum expected cost. Hence, the risk for the alternative designs will be higher than the risk associated with designs corresponding to the minimum expected cost.

As mentioned earlier, analyzing the risk for various designs helps a decision maker to select a design based on the aspiration level; this is particularly useful when the decision making involves tradeoff, that is, when there is no design that comprehensively outperforms other designs. These kinds of situations arise in the design of water utilization systems when the capital cost for a reuse connection between the processes is comparable to the reduction in operating cost achieved by using such a connection, which, however, is not common. In this case, for some scenarios the total cost by reusing the water could be lower compared to not reusing, while the opposite could be true in some other scenarios.

In the next section the issues discussed above are illustrated. For each of the examples, the minimum expected cost is obtained solving problem (6) using the extreme values of uncertain contaminant loads to construct the scenarios. The design corresponding to the minimum expected cost is compared with other feasible designs obtained using problem (11) for different values of γ_r. Furthermore, the risk associated with these designs is assessed considering sufficient scenarios, evenly distributed throughout the interval of uncertainty.

Examples

Example 1. Consider six processes. The limiting inlet and outlet concentrations and the contaminant loads are presented in Table 1. The intervals of uncertainty for the contaminant loads are assumed to be 20% at each side of the mean values of Table 1. Table 2 shows the distances between the processes. Using a capital cost of \$150/ft, the annualized process interconnection cost (assuming 10 years of operation) is \$15/ft. Thus, the cost for the interconnections is shown in Table 3. Finally, the freshwater cost and the cost for the final treatment was assumed to be \$1.5/ton.

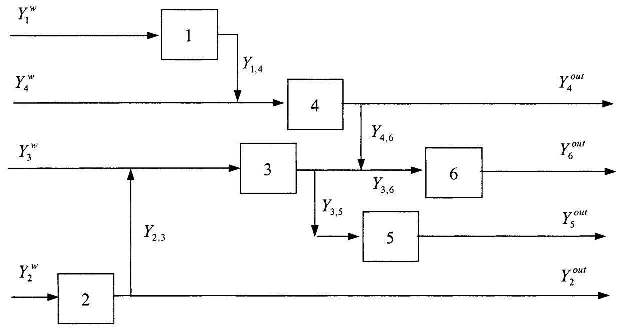

Figure 5. Minimum expected cost solution (example 1).

Table 2. Distance between the Processes (ft)

$d_{i,j}$	water source	1	2	3	4	5	6	water treatment
1	1000	0	1200	1000	400	1200	1600	400
2	400	1200	0	400	2000	1200	1400	2000
3	1800	1000	400	0	1600	600	800	1600
4	1600	400	2000	1600	0	1600	1800	2000
5	600	1200	1200	600	1600	0	1000	1600
6	800	1600	1400	800	1800	1000	0	1800

Table 3. Capital Cost for Piping between the Processes (\$1000/year)

$C_{i,j}$	water source	1	2	3	4	5	6	water treatment
1	15	0	18	15	6	18	24	6
2	6	18	0	6	30	18	21	30
3	27	15	6	0	24	9	12	24
4	24	6	30	24	0	24	27	30
5	9	18	18	9	24	0	15	24
6	12	24	21	12	27	15	0	27

Table 4. Capital Cost for Piping between the Processes (\$1000/year)

$C_{i,j}$	water source	1	2	3	4	5	6	water treatment
1	2.5	0	7.24	62.1	72.5	90.6	248.4	3.0
2	3.0	7.24	0	50.6	42.1	90.6	284.6	2.5
3	2.0	62.1	50.6	0	41.4	90.6	96.6	4.5
4	4.5	72.5	42.1	41.4	0	90.6	265.7	2.0
5	3.5	90.6	90.6	90.6	90.6	0	90.6	4.0
6	4.0	248.4	284.6	96.6	265.7	90.6	0	5.0

Using γ = \$1.35 million/year and $\rho = 2 \times 10^6$ in the risk management model (11), the same solution as in the minimum expected cost model (6) is obtained. The expected cost is \$1.491 million/year, and the expected freshwater consumption is 99.05 ton/h. Figure 5 shows the network.

For γ = \$1.61 million/year and $\rho = 9 \times 10^6$, the same solution is obtained. Further, using γ_1 = \$1.35 million/year, $\rho_1 = 2 \times 10^6$ and γ_2 = \$1.61 million/year, $\rho_2 = 9 \times 10^6$, the same design is obtained. Thus, it can be concluded that for this example the design corresponding to the minimum expected cost has the lowest risk. To illustrate and confirm this claim, the risk curves corresponding to designs with expected costs higher than the minimum expected cost are shown in Figure 6. As anticipated, the minimum expected cost curve lies beneath all of the others.

Example 2. Consider the same set of processes as those in example 1. Further, assume the intervals of

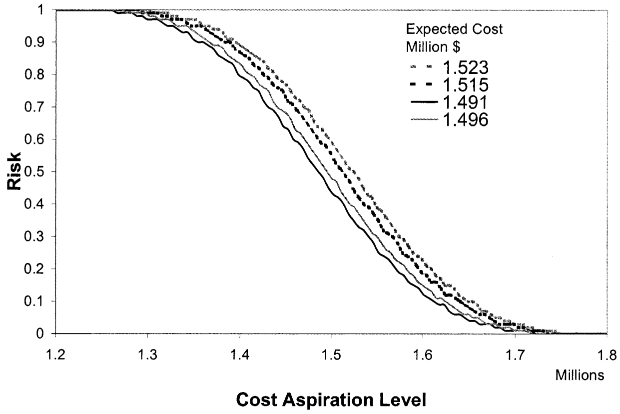

Figure 6. Risk curve for different expected costs.

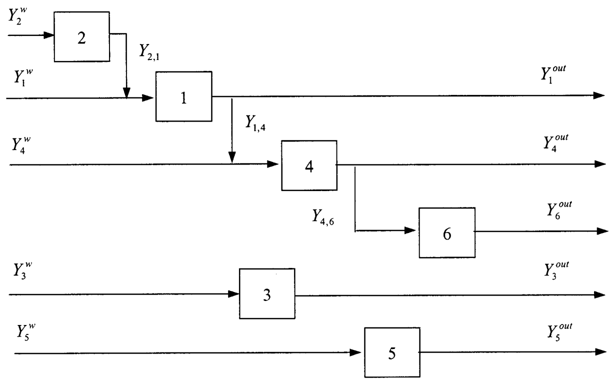

Figure 7. Minimum expected cost network for example 2.

uncertainty for the contaminant loads also to be 20% about the mean values as in example 1. The freshwater cost and the final treatment cost are assumed to be the same as those in example 1, that is, 1.5 \$/ton. However, consider that the capital cost is such that they are comparable to the reduction in operation cost achieved by using reuse connections. Table 4 presents the capital cost data for this example:

The capital cost for this example was chosen such that for some scenarios reuse of wastewater is favored, while it is not favored in some scenarios because of higher capital cost compared to savings achieved in operating cost. Using γ = \$1.9 million/year and $\rho = 1 \times 10^6$, the design of Figure 7 is obtained, which is also the one corresponding to the minimum expected cost solution. The expected cost for this design is \$1.73 million/year, and the expected freshwater consumption is 107.54 ton/h. On the other hand, using γ = \$1.6 million/year and $\rho = 1 \times 10^6$, a design is obtained in which water is not reused (Figure 8). The expected cost for this design is \$1.74 million/year, and the expected freshwater consumption is 134.92 ton/h.

When risk aspiration levels γ_1 = \$1.6 million/year, $\rho_1 = 2 \times 10^6$ and γ_2 = \$1.9 million/year, $\rho_2 = 1 \times 10^6$ are used, the same design as that in Figure 7 is obtained. The risk curves for both designs are shown in Figure 9. The curves intersect simply because there is no design with a risk curve that lies completely below the risk curve corresponding to the minimum expected cost design not constrained by risk.[11,12]

Conclusions

An approach for single-contaminant water allocation planning in the presence of uncertainty in contaminant

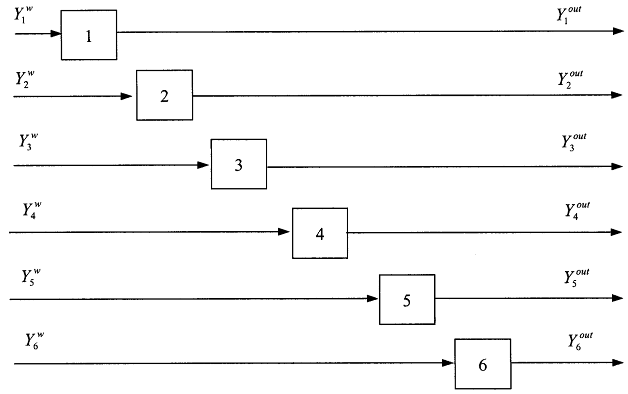

Figure 8. Smaller risk solution network for example 2.

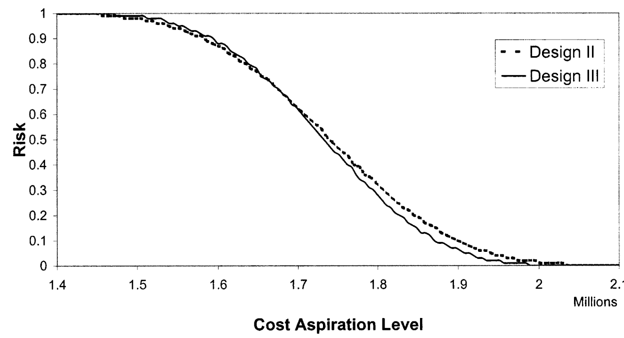

Figure 9. Risk curves for alternative designs of example 2.

loads is presented. Risk management in problems where the operating cost is much higher than the capital cost is not possible; that is, the design corresponding to the minimum expected cost usually has the lowest risk. On the other hand, when the capital cost is comparable to operating cost, reuse of wastewater can be favorable for reducing risk at some aspiration levels.

Acknowledgment

The authors are grateful for the support provided by EPA (Grant R828210).

Nomenclature

α = cost of freshwater and its final treatment per ton
γ = cost aspiration level
ϵ_r = risk at aspiration γ_r
ρ_r = penalty at aspiration γ_r
Cost_s = cost of scenario s
c_j^{w} = freshwater connection for process j
$c_{i,j}$ = cost of interconnection between processes i and j
c_j^{out} = wastewater connection for process j
$C_{j,\mathrm{in}}^{\max}$ = inlet limiting concentration in process j
$C_{j,\mathrm{out}}^{\max}$ = outlet limiting concentration in process j
F_j^{w} = freshwater flow rate to process j
$F_{i,j}$ = water flow from process i to process j
$F_{j,h}$ = water flow from process j to process h
$F_{j,\mathrm{out}}$ = wastewater discharged from process j to treatment
k = index representing process k
i = index representing process i
j = index representing process j
L_j = mass load in process j
N = set of processes in the network
p_s = probability of a scenario s
R = set of levels at which risk is to be controlled
U = upper bound for flows
$Y_{i,j}$ = binary variable representing the connection between processes i and j

Y_j^{w} = binary variable representing the freshwater connection to process j

Y_j^{out} = binary variable representing the wastewater connection from process j

Z_s = binary variable representing the risk for scenario s

$Z_{s,r}$ = binary variable representing the risk for scenario s at γ_r

Literature Cited

(1) Bagajewicz, M. A Review of Recent Design Procedures for Water Networks in Refineries and Process Plants. *Comput. Chem. Eng.* **2000**, *24* (9), 2093−2115.

(2) Koppol, A. M.; Bagajewicz, M.; Dericks, B. J.; Savelski, M. J. On Zero Water Discharge Solutions in the Process Industry. *Adv. Environ. Res.* **2002**, in press.

(3) Murphy, F. H.; Sen, S.; Soyster, A. L. Electric Utility Expansion Planning in the Presence of Existing Capacity: a Nondifferentiable, Convex Programming Approach. *Comput. Oper. Res.* **1987**, *14*, 19−31.

(4) Berman, O.; Ganz, Z. The Capacity Expansion Problem in the Service Industry. *Comput. Oper. Res.* **1994**, *21*, 557−572.

(5) Pistikopoulos, E. N.; Ierapetritou, M. G. Novel Approach for Optimal Process Design under Uncertainty. *Comput. Chem. Eng.* **1995**, *19*(10), 1089−1110.

(6) Liu, M. L.; Sahinidis, N. V. Optimization in Process Planning under Uncertainty. *Ind. Eng. Chem. Res.* **1996**, *35*, 4154−4165.

(7) Biegler, L. T.; Grossmann, I. E.; Westerberg, A. W. *Systematic Methods of Chemical Process Design*; Prentice Hall Inc.: New York, 1997.

(8) Ahmed, S.; Sahinidis, N. V. Robust Process Planning under Uncertainty. *Ind. Eng. Chem. Res.* **1998**, *37* (5), 1883−1892.

(9) Savelski, M.; Bagajewicz, M. On the Use of Linear Models for the Design of Water Utilization Systems in Refineries and Process Plants. *Chem. Eng. Res. Des.* **2001**, *79* (A5), 600−610.

(10) Swaney, R. E.; Grossmann, I. E. An Index for Operational Flexibility in Chemical Process Design. *AIChE J.* **1985**, *31* (4), 621−630.

(11) Barbaro, A.; Bagajewicz, M. Financial Risk Management in Planning under Uncertainty. FOCAPO 2003, Coral Springs, FL, Jan 2003.

(12) Barbaro, A.; Bagajewicz, M. Managing Financial Risk in Planning Under Uncertainty. *AIChE J.* **2003**, in press.

(13) Gupta, A.; Maranas, C. Market-Based Pollution Abatement Strategies: Risk Management Using Emission Option Contracts. *Ind. Eng. Chem. Res.* **2003**, *42*, 802−810.

(14) Eppen, G. D.; Martin, R. K.; Scharge, L. A Scenario Approach to Capacity Planning. *Oper. Res.* **1989**, *37*, 517−527.

(15) Bagajewicz, M.; Savelski, M. On the Optimality Conditions of Water Utilization Systems in Process Plants with Single Contaminants. *Chem. Eng. Sci.* **2000**, *55*, 5035−5048.

Received for review May 24, 2002
Revised manuscript received July 7, 2003
Accepted July 11, 2003

IE0203882