On the Impact of Corrective Maintenance in the Design of Sensor Networks†

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This paper discusses the incorporation of the corrective maintenance cost of instruments as part of the objective function. It is also shown that, in the presence of repairable instruments, the availability of a variable, instead of its reliability, should be used as a constraint. The impact of the corrective repair rate on the solution of the problem is investigated.

Introduction

In recent years, the scope of data usage has been enlarged from process monitoring, control, and production accounting goals to fault detection and on-line/off-line optimization. Therefore, the need for quality and availability of data has increased. To accomplish this, a combination of properly placed redundant good instrumentation with the aid of data reconciliation has emerged as an important alternative to reduce cost. Because sensors often fail, gross error detection techniques need to rely on the proper redundancy of measurements to detect these biases and leaks. Furthermore, once the gross errors are detected and eliminated or compensated, variable values should still remain available and be precise. That is, proper variable data assessment should remain possible.

As methods to perform data reconciliation and gross error detection proliferate, the need for systematic procedures to design sensor networks has emerged. Different driving forces have been used for the design of sensor networks that were incorporated in the objective function or in the constraints of different design approaches: observability, precision, purchase cost, reliability, ability of managing gross errors.

For steady-state linear processes, Ali and Narasimhan[1] developed a strategy to design a minimum number sensor network that maximizes the least reliability of estimation among all variables while ensuring the observability of all of them. Later, they extended the approach for the design of linear redundant sensor networks[2] as well as bilinear nonredundant sensor networks.[3] In a recent paper, Sen et al.[4] presented a genetic algorithm that can be applied to the design of nonredundant sensor networks using different objective functions.

In relation to the incorporation of reliability, Luong et al.[5] proposed a method that provides cost-optimal measurement structures featuring minimal observability of those variables required for control and a high degree of redundancy of other variables. They used the system reliability as a means of screening alternatives with equal cost.

These previous strategies do not take into account the fact that sensor networks should be able to handle gross errors effectively and should be simultaneously cost effective. This issue was considered by Bagajewicz,[6] who proposed a procedure to find minimum cost robust networks. In this strategy, the purchase cost of a sensor network is minimized subject to constraints related to data reconciliation requirements and gross error detection capabilities. These requirements are the capacity of the network to maintain the precision of the estimates in the presence of instrumentation failures (residual precision), the ability to detect gross errors (error detectability), and the capacity of controlling smearing due to undetected gross errors (resilience). Later, Bagajewicz and Sánchez[7] proved that there exists equivalence between maximum precision models and minimum cost models. This equivalence was established through the so-called Tuy-type duality of optimization problems. This equivalence states that solving one and the other does not really produce different results. In addition, Bagajewicz and Sanchez[8] showed that cost-optimal networks can be designed by taking into account a variety of observability and redundancy requirements. In particular, the concept of estimability of a variable was introduced, which generalizes the observability, nonredundancy, and redundancy in one single concept. Later, the minimum cost model was extended to perform the retrofit of sensor networks.[9] Concepts such as addition, replacement, and relocation of sensors were introduced, and specific constraints were added to the problem by taking them into account. Notwithstanding the practical importance of these developments, these models do not include any consideration about reliability and maintenance cost.

Reliability was added to the minimum cost design procedure by Bagajewicz and Sánchez,[10] and a Tuy-type equivalency was also proved applicable to these models. The mathematical relation between the minimum cost model constrained by reliability thresholds was proven to. be a generalization of previous methods.[1,2] Nevertheless, this general model does not take into account maintenance.

In this paper, the model developed by Bagajewicz[6] and its extension to address reliability[10] are extended to include the availability of variable estimation as a

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constraint. At the same time, both capital and corrective maintenance instrumentation costs are included in the objective function.

Problem Formulation

The optimal sensor network design involves the solution of the following problem:

$$\text{Min }\{\text{instrumentation cost}\}$$

s.t.

$$\begin{cases}\text{precision constraints}\\ \text{residual precision constraints}\\ \text{error detectability constraints}\\ \text{error resilience constraints}\\ \text{estimation availability constraints}\end{cases}$$

The second, third, and fourth constraints were introduced by Bagajewicz[6] as measures of the robustness of the sensor network. Residual precision was defined in the aforementioned work as availability. We propose to change this nomenclature to the new name because availability is a term that has been traditionally related to a probability. Thus, residual precision of order k is the ability of a network to provide a precision in key variables larger than a prespecified threshold upon the deletion of k measurements.

The last constraint corresponds to the probability of estimation of a variable, which depends on the sensor service availability. In this work, maintenance instrumentation costs and measures of the availability are especially addressed. Although the derivation for instrumentation costs is general, the availability measure is especially restricted to steady-state processes described by mass balance equations. Thus, this work is restricted to the optimal design of flowmeter networks.

1. Instrumentation Cost. Newly purchased instruments must be maintained for their entire life. In many instances, the equipment purchase cost is lower than the equipment field maintenance cost over the entire life cycle. Therefore, when a new instrument is purchased, the entire life cycle cost rather than the initial cost should be considered.[11]

The life cycle cost (LCC_j) of an instrument j is expressed in terms of the procurement cost (PC_j) and the ownership cost (OC_j) as follows:

$$LCC_j = PC_j + OC_j \tag{1}$$

The procurement cost includes acquisition, transportation, installation, test equipment, training, etc. Because it is a nonrecurring cost, its value may be considered fixed. For this analysis, a discrete set of instruments with fixed features is available for selection.

The ownership cost involves inventory, labor, maintenance, and operating costs. For the case of instruments, we consider only maintenance cost. Therefore, eq 1 reduces to

$$LCC_j = PC_j + MC_j \tag{2}$$

where MC_j is the present value of the maintenance cost of instrument j. In turn, the maintenance cost may be of two types: corrective and preventive. In this work, it is considered that the maintenance work includes only corrective tasks.

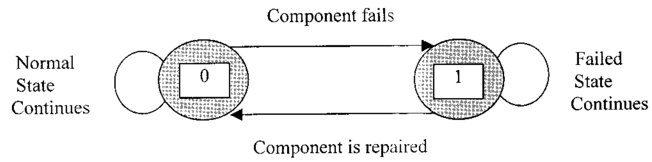

Figure 1. Markov transition diagram for an instrument.

If the life cycle is estimated in n years, the present value of the life cycle cost (2) can be calculated in terms of the present value of the corrective maintenance cost CMC_j as follows:

$$LCC_j = PC_j + MC_j = PC_j + \sum_{p=1}^{p=n} \frac{CMC_j(p)}{(1+i)^p} \tag{3}$$

where i = interest rate, n = operation time of the instrument in years, and $CMC_j(p)$ = corrective maintenance cost for the time interval $[(p-1), p]$.

The corrective maintenance cost $CMC_j(p)$ can be evaluated in a simple form by taking into account the cost of each repair and the expected number of repairs in the interval of time $[(p-1), p]$, that is

$$CMC_j(p) = (CSPC_j + CMPC_j)ENR_j(p) \tag{4}$$

where $CSPC_j$ = spare parts cost/repair, $CMPC_j$ = manpower cost/repair, and $ENR_j(p)$ = expected number of repairs between p and $p-1$.

Instruments have different lifetimes. Thus, the present value of some instruments will reflect maintenance cost of years in which other instruments have exceeded their lifetime. To overcome this difficulty, the capitalized cost of the sensor can be used. In such a case, n will be the common life of the plant and the ownership cost can be calculated using the capitalized cost.[12] In this paper, we will assume the same lifetime for all instruments.

2. Expected Number of Repairs. In order to calculate $ENR_j(p)$, a Markov model is used. It is based on the following assumptions:

i. At any given time a sensor either is functioning normally (normal state) or has failed (failed state).

ii. A repair restores the sensor to a condition as good as new (AGAN).

iii. The sensor is in the normal state at time $t = 0$.

iv. A sensor changes its state simultaneously when a transition takes place. One transition occurs in a sufficiently small time interval, and the possibility of two or more simultaneous transitions is negligible. The state transitions are summarized by the Markov diagram of Figure 1.

To calculate the expected number of repairs in the interval $[(p-1), p]$, $ENR(p)$, some probabilistic parameters and its relations are of importance. They are briefly reviewed in the appendix. The $ENR(p)$ is obtained from eq A-10, replacing t_1 by $(p-1)$ and t_2 by p. Thus,

$$ENR_j(p) = \left\{\frac{d_j}{c_j} + \frac{d_j}{c_j^2} e^{-c_j p}[1 - e^{c_j}]\right\} \tag{5}$$

Replacing $d_j = \lambda_j \mu_j$ and $c_j = \lambda_j + \mu_j$, one obtains

$$ENR_j(p) = \frac{\lambda\mu}{\lambda+\mu} + \frac{\lambda\mu}{(\lambda+\mu)^2} e^{-(\lambda+\mu)p}[1 - e^{\lambda+\mu}] \tag{6}$$

If $\mu \gg \lambda$, then $\lambda + \mu \approx \mu$, and therefore $ENR_j(p)$ can be approximated by

$$ENR_j(p) = \lambda + \frac{\lambda}{\mu}e^{-\mu p} - \frac{\lambda}{\mu}e^{-\mu p}\, e^{\mu} = \lambda + \frac{\lambda}{\mu e^{\mu p}} - \frac{\lambda}{\mu e^{\mu(p-1)}} \cong \lambda \quad (7)$$

If the reliability of the instrument is $R(t) = e^{-\lambda t}$, then, for $t = 1$ year, $R(1\ \text{year}) = e^{-\lambda}$; thus, ENR($p$) can be approximated in terms of the reliability of the instruments as follows:

$$ENR_j(p) \cong \lambda = \ln\left(\frac{1}{R(1\ \text{year})}\right) \quad (8)$$

This formula indicates that, because the repair is very fast, the number of repairs in 1 year equals the number of failures per year.

Thus, to evaluate the life cycle cost, information is required about the following parameters: SPC_j, MPC_j, λ_j, and μ_j.

3. Estimation Availability Constraints. In previous work, instruments are considered nonrepairable systems. Therefore, instrument reliability is used to evaluate variable reliability. However, considering that instruments are repairable systems, the concept of the availability of a variable is more appropriate. Estimation availability at time t is the probability that a variable can be calculated at time t, given that it was calculated at time zero. Thus, it is a less stringent requirement than variable reliability, which requires that the instruments not fail between time zero and time t.

Corrective maintenance tasks allow the instruments to operate after each fault during their life cycle. Thanks to maintenance work, the sensor service availability at time t is higher than its reliability. Furthermore, it tends to a positive constant as time goes to infinity; in contrast, reliability tends to zero. Thus, to design for more realistic scenarios, the cost of corrective maintenance tasks is included in the objective function and estimation availability constraints of key variables, for a time equal to the life cycle, are used.

The estimation availability of a variable at time t given a set of instruments is obtained using the sensor service availability at time t and the cutsets of the graph that represent the process. This is done using a procedure similar to the one presented by Ali and Narasimham[2] for estimating variable reliabilities.

The sensor service availability at a time t depends on two parameters: the failure and repair rates (see eq A-8 in the appendix). The first one is a feature of the selected instrument, but the rate of repair is related to the internal organization of the plant maintenance division. Both will be considered fixed in this work.

The above results can change substantially if the assumption used is that the repair leaves the instrument as good as old (AGAO). In the AGAN assumption the failure rate is considered constant. In the AGAO assumption, the failure rate changes as the instrument is repaired. We leave the analysis of this for future work.

4. Model. Using the formulations derived in items 1–3, a cost-optimal sensor network subject to precision, residual precision, and estimation availability constraints is obtained by solving the following optimization problem:

$$\min \sum_j LCC_j q_j \quad (9)$$

s.t.

$$\sigma_p \le \sigma_p^* \qquad \forall p \in M_P$$

$$\sigma_{R,p}^k \le (\sigma_{R,p}^k)^* \qquad \forall p \in M_{RP}$$

$$A_p \ge A_p^* \qquad \forall p \in M_A$$

where $\mathbf{q}$ is a vector of binary variables such that $q_j = 1$ if flow rate j is measured and $q_j = 0$ otherwise; σ_p represents the standard deviation of the estimate of variable p obtained using data reconciliation procedures; $\sigma_{R,p}^k$ stands for the standard deviation of the variable p estimate, calculated after the elimination of k measurements; A_p corresponds to the availability of variable p; M_P, M_{RP}, and M_A are the sets of variables with precision, residual precision, and availability requirements, respectively. An example of the application of this formulation is included below.

5. Examples. The design of sensor networks for the simplified ammonia network represented in Figure 2 is included in this section. All cutsets for this network are extracted from Ali and Narasimhan.[2]

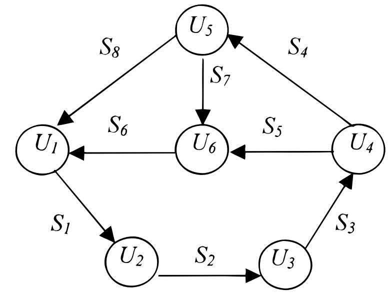

Figure 2. Simplified ammonia plant network.

Table 1. Instrumentation Data

	1	2	3
purchase cost	350	250	200
precision (%)	1.5	2.5	3
failure rate (failure/year)	0.3	0.6	0.7

Table 2. Constraints of the Optimization Problem

stream	precision requirements (%)	residual precision requirements (%)	estimation availability requirements
S_1			0.9
S_2	1.5	2	
S_5	2.5	3	
S_7			0.9

In this example, sensors for each stream may be selected from a set of three instruments with different precision, purchase cost, and failure rate. These data are included in Table 1.

Maintenance corrective costs are evaluated considering spare part cost SPC = 10, manpower cost MPC = 40 for all instruments, life cycle = 5 years, and annual interest = 6%. Constraints of precision, residual precision, and estimation availability are requested for only two flow rates. The limits on these variables are presented in Table 2.

Table 3. Optimization Results for the Simplified Ammonia Process Flowsheet

repair rate	measured variables	instrument precision (%)	cost	precision for S_2 and S_5 (%)	residual precision for S_2 and S_5 (%)	estimation availability for S_1 and S_7
0.7	S_1, S_4, S_5, S_6, S_7, S_8	1, 1, 3, 1, 1, 3	2063.0	0.8443, 1.7069	0.9215, 1.7120	0.9050, 0.9050
1.5	S_1, S_4, S_5, S_6, S_7	1, 3, 3, 1, 3	1633.4	1.1937, 1.9000	1.2327, 2.6702	0.9314, 0.9105
3.5	S_4, S_5, S_6, S_7, S_8	3, 3, 1, 3, 3	1670.0	1.2313, 1.9963	1.9712, 2.0086	0.9544, 0.9882
20	S_4, S_5, S_6, S_7, S_8	3, 3, 1, 3, 3	1775.2	1.2313, 1.9963	1.9712, 2.0086	0.9983, 0.9999

The repair rate of instruments, a parameter that is a characteristic of the plant in consideration, has been varied between 1 and 20. For each case, the results of the optimization problem are presented in Table 3.

In the first case, the repair rate is comparatively low. Consequently, the availability of instruments in the life cycle is also low. To satisfy the precision availability of key variables, the optimal solution includes a set of six instruments. Four of these instruments are of type 1, which are sensors of low failure rate, high precision, and high cost.

When the repair rate is assumed to be 1.5, an optimal solution exists that consists of five instruments. Two of these instruments are of type 1, and the rest are of type 3. Consequently, the total instrumentation cost decreases. Furthermore, the residual precision for variable S_5 and the estimation availability for variable S_7 are near the bounds.

If the repair rate changes from 1.5 to 3.5, a little increase in the optimum instrumentation cost results. Although capital cost decreases for the optimum solution, corrective maintenance cost increases in relation to case 2. The reduction in capital cost originates that now residual precision of variable S_1 is binding, The estimation availability constraints are satisfied in excess, because of the high repair rate.

The results of the last case confirm that the influence of sensor availability constraints decreases for high repair rates. The cost increases because of the effect of increasing the repair rate μ (from 3.5 to 20) in the maintenance cost model. In this case, only the residual precision of variable S_2 is binding.

6. Preventive Maintenance. If preventive maintenance wants to be considered, then the Markov model to calculate the expected number of repairs needs to be revisited. Such a model will now have a new state, which is not a failed state, but rather a state of preventive maintenance. Instead of a failure rate, a rate at which sensors are taken for maintenance needs to be used. This is now a design variable and not an instrument anymore. The impact on the availability is the same.

Conclusions

In this work, the design of minimum life cycle cost sensor networks has been presented. Unlike previous works, the cost is now composed of capital and maintenance costs. Precision and data reconciliation robustness requirements are used as constraints. In addition, lower bounds on the estimation availability for key variables are incorporated as another constraint in the design problem.

As a conclusion, the repair rate has a direct influence on sensor service availability. If the repair rate is high, instruments are working during almost all of the life cycle, and therefore the estimation availability is always satisfied. In these cases, the design follows the requirements of precision and residual precision constraints. The estimation availability is more likely to be a binding constraint for lower repair rates. In this situation, cost may increase, because it is necessary to incorporate more instruments to calculate the variable using alternative ways.

Notation

CMC = corrective maintenance cost
CMPC = manpower cost/repair
CSPC = spare part cost/repair
ENR = expected number of repairs
i = interest rate
LCC = life cycle cost
MC = maintenance cost
n = life cycle
OC = ownership cost
PC = procurement cost
P_0 = availability
P_1 = unavailability
R = reliability
t = time

Greek Symbols

λ = conditional failure intensity
μ = conditional repair intensity
ν = unconditional repair intensity

Appendix A

Availability $P_0(t)$ at time t is the probability of the component of being normal at time t given that it is as good as new at time zero.

Reliability $R(t)$ at time t is the probability that the component is in the normal state in the interval $(0, t)$.

Unavailability $P_1(t)$ is the probability that the component is in a failed state at time t given that it jumped into the normal state at time zero.

Unconditional repair intensity $\nu(t)$ at time t is the probability that the component is repaired per unit time at time t given that it jumped into the normal state at time zero.

Expected number of repairs in the interval (t_1, t_2), ENR(t_1,t_2), given that it jumped into the normal state at time zero

$$\mathrm{ENR}(t_1,t_2) = \int_{t_1}^{t_2} \nu(t)\,\mathrm{d}t \tag{A-1}$$

Conditional repair intensity $\mu(t)$ is the probability that a component is repaired per unit time at time t given that it jumped into the normal state at time zero and failed at time t.

Relations for Calculating the Conditional Repair Intensity $\mu(t)$. Using the chain rule, the simultaneous occurrence of events A and C is equivalent to the occurrence of event C followed by event A

$$\Pr(\mathrm{A} \cap \mathrm{C}|\mathrm{W}) = \Pr(\mathrm{A}|\mathrm{C} \cap \mathrm{W})\Pr(\mathrm{C}|\mathrm{W}) \tag{A-2}$$

At most, one repair occurs during a small interval, and the event A implies event C. Thus, the simultaneous occurrence of A and C reduces to the occurrence of A, and formula (A-2) can be written as

$$\mathrm{Pr(A|W) = Pr(A|C \cap W)Pr(C|W)} \qquad \text{(A-3)}$$

Considering the following events in formula (A-3), we have C = component is failed at time t A = component repaired during (t, $t + dt$] and W = component that jumped into the normal state at time zero

$$\nu(t) = \mu(t)\, P_1(t) \qquad \text{(A-4)}$$

Calculation of the Unconditional Repair Intensity $\nu(t)$. When the component has a constant repair rate (m), the conditional repair intensity $\mu(t)$ becomes m and is known. In this case, formula (A-4) is applied to calculate the unconditional repair intensity $\nu(t)$ given μ and $P_1(t)$, using the Markov analysis of the process with constant rates of failure and repair. This analysis is based on the following:

(a) If $P_k(t)$ *is the probability that the system is in state* k *at time* t*, then the derivative* $\dot{P}_k(t)$ *is given by*

$$P_0'(t) = \mu P_1(t) - \lambda P_0(t) \qquad \text{(A-5)}$$

$$P_1'(t) = \lambda P_0(t) - \mu P_1(t) \qquad \text{(A-6)}$$

(b) For calculating the system unavailability, the previous system of differential equations is resolved with the initial conditions $P_0(0) = 1$ and $P_1(0) = 0$, which gives

$$P_1(t) = \frac{\lambda}{\lambda + \mu}(1 - e^{-(\lambda+\mu)t}) \qquad \text{(A-7)}$$

Consequently, system availability is

$$P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu}e^{-(\lambda+\mu)t} \qquad \text{(A-8)}$$

(c) The unconditional repair intensity can be obtained from eq A-4 as follows:

$$\nu(t) = \mu\left\{\frac{\lambda}{\lambda + \mu}(1 - e^{-(\lambda+\mu)t})\right\} = \frac{\mu\lambda}{\lambda + \mu} - \frac{\mu\lambda}{\lambda + \mu}e^{-(\lambda+\mu)t} \qquad \text{(A-9)}$$

The expected number of repairs for the interval (t_1, t_2) is obtained by integrating $\nu(t)$ between (t_1, t_2)

$$\mathrm{ENR}(t_1,t_2) = \int_{t_1}^{t_2}\nu(t)\,\mathrm{d}(t) = \int_{t_1}^{t_2}\left(\frac{d}{c} - \frac{d}{c}e^{-ct}\right)\mathrm{d}t = \frac{d}{c}(t_2 - t_1) + \frac{d}{c^2}e^{-ct_2}[1 - e^{c(t_2-t_1)}] \qquad \text{(A-10)}$$

where $d = \lambda\mu$ and $c = \lambda + \mu$.

Reliability for a Repairable Instrument. To calculate the system reliability, no in-flow from state 1 to state 0 is considered, so the system consisting of eqs A-5 and A-6 is reduced to

$$P_0'(t) = -\lambda P_0(t) \qquad P_0(0) = 1 \qquad \text{(A-11)}$$

which has the following solution:

$$R(t) = P_0(t) = e^{-\lambda t} \qquad \text{(A-12)}$$

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