Management of Pricing Policies and Financial Risk as a Key Element for Short Term Scheduling Optimization

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In this article the scheduling of batch plants is integrated with pricing decisions. The proposed integrated model simultaneously provides the optimal prices and schedule as opposed to earlier models where prices are usually considered as input data. The main advantages of such formulation are highlighted through a case study where comparison with the traditional approach is carried out. A two-stage stochastic mathematical model is also developed in order to address the uncertainty associated to the demand curve. Finally, financial risk management is discussed.

1. Introduction

With the recent trend of building small and flexible plants that follow the market dynamics closer, there has been renewed interest in batch processes.[1] Thus, a high effort to develop different strategies and tools for modeling, simulation, and optimization of these processes is underway.[2] Nevertheless, in all these developments, significant aspects related to the influence of the currently dynamic environment into the batch plant activity have not been properly studied.

One such aspect is related to the way pricing decisions are made in these batch-chemical companies. Industry managers and academia are realizing the strategic importance of the price variable. Pricing strategies consist of selecting the most appropriate price for a particular economic environment or market situation. Traditionally, pricing decisions were the responsibility of the marketing manager who sets a price within the context of his overall market strategy,[3−5] but an effort to integrate pricing strategies with different decision making process is now underway. For instance, Chen et al.[3] analyze a finite horizon, single product, periodic review model in which pricing and inventory decisions are made simultaneously. They also report some articles where price, inventory control, and quality of service (retail and service industries) are integrated.

Moreover, issues so far related to the integration of decisions at different levels (scheduling, price determination, etc.) and the associated uncertainty have not been considered. Therefore, while many models have been proposed for scheduling[2,6−8] few are devoted to include uncertainty.[9−14] Sources of uncertainty in scheduling can be divided into short-term (processing time variations, equipment breakdowns, etc.) and long-term (market trends, technology changes, etc.). For the short-term, reactive scheduling has been used, while some form of stochastic programming has been considered for the long-term.[15]

Furthermore, although stochastic models optimize the total expected performance measure, they usually do not provide any control on its variability over the different scenarios; i.e., they assume that the decision maker is risk neutral. However, different attitudes toward risk may be encountered. In general, most decision makers are risk averse implying a major preference for lower variability for a given level of return. In relation to this, Bonfill et al.[16] presented some techniques to manage financial risk in scheduling problems similarly to the way it was done by Barbaro et al.[17] for planning problems. Some of these techniques were also used by Guillén et al.[18] for manipulating the financial risk associated to a given supply chain configuration under demand uncertainty.

In this work a new strategy for integrating pricing decisions with the scheduling of batch plants and managing the financial risk associated with the consideration of the uncertainty associated with the demand curve is introduced. The starting point is the modeling and forecasting of the relationship between product prices and demand aiming at the incorporation of pricing as a decision variable instead of treating it as input data. Once this relation is obtained, it is integrated into a scheduling mathematical model in order to simultaneously determine the prices and the associated optimal schedule which maximizes the resulting profit. The capabilities of such model are highlighted through a case study where comparison with the traditional approach applied to fix prices is carried out. A two-stage stochastic formulation is then developed to deal with the uncertainty associated with the demand curve, and the main advantages of the proposed formulation are highlighted through comparison with the deterministic approach. A methodology to managing financial risk which relies on the sample average approximation (SAA) method as a way of generating solutions which perform in dissimilar ways under the uncertain environment is next described and applied to our problem. Some risk performance indicators are finally used for assessing the obtained solutions and guiding the decision maker's choice.

2. Pricing Background

Prices are marketing variables often characterized by quick market responses; that is, any decision on pricing

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has a fast response of the market demand and must be therefore considered at the tactical (short term) level. The decisions related to the determination of prices begin with the available information on the fixed and variable costs, which are easily obtained from accounting and production registers. The relationship of demand and prices is also required as input data, and it is usually obtained by using historical data and, what is less usual, by direct experimentation over the consumer's response to different price levels in several markets. The objectives pursued by pricing need to be consistent with the enterprise goals. The most widely used objective function when determining prices is the maximization of profit. We first start by discussing single product pricing followed by mutliproduct pricing, highlighting some of the shortcomings of these models.

2.1. Single Product Pricing Model. Consider the benefit maximization for a single product. Starting from the benefit equation:

$$B = I - C \tag{1}$$

where B represents the benefits, I, the incomes, and C, the costs. A typical model for I is

$$I = Sales \cdot Price \tag{2}$$

$$Sales \le Q \tag{3}$$

$$Sales \le Demand \tag{4}$$

In these equations *Price* is the product price and *Sales* are the product sales which are constrained to be smaller than the volume of production Q and the demand *Demand* (eqs 3 and 4). In fact, when no capacity constraints are considered, the sales take the same value as the demand and the production volumes. There are several models that reflect the relationship between demand and prices.[4] The most simple one involving one product and assuming that the products of the competition have fixed demands and process is given by the expression:

$$Demand = k \cdot Price^{-e} \tag{5}$$

where, k and e (the demand elasticity) are determined according to the historical information over a commonly bounded range of prices. The elasticity e can be a function of price, but in practice, given the narrow ranges of prices one is able to manipulate and, for the sake of simplicity, one can assume a linear relationship between price and demand:

$$Demand = Demand_0 - m \cdot Price \tag{6}$$

$$Price^{lo} \le Price \le Price^{up} \tag{7}$$

In addition, replacing the demand by its value in the classical model for the production cost C without economy of scale considerations, we obtain:

$$C = C_F + C_v \cdot Demand = C_F + C_v \cdot Demand_0 - C_v \cdot m \cdot Price \tag{8}$$

where C_F represents fixed costs and C_v represents the unitary variable costs. Therefore, to determine the optimal price (denoted by *Price**), the following equation assuming an unlimited production capacity should be solved:

$$\frac{dB}{dp} = \frac{d}{dPrice} \times \left\{ \begin{matrix} I - C = Price \cdot (Demando_0 - m \cdot Price) - \\ -(C_F + C_v + Demand_0 - C_v \cdot m \cdot Price) \end{matrix} \right\} = 0 \tag{9}$$

which leads to

$$Price^* = \frac{\frac{Demand_0}{m} + C_v}{2} \tag{10}$$

$$if \quad Price^{lo} \le Price^* \le Price^{up} \; and \; m > 0 \tag{11}$$

$$Otherwise \; Price^* = Price^{up} \tag{12}$$

Thus, as long as C_v is constant ($Demand_0$ and m can be considered constants), the pricing and the associated scheduling problem are trivial.

2.2. Multiple Product Pricing Model. For the case of several products, one may determine separately the prices for each one using single-product techniques and obtain the desired quantities to produce and sell by adding all the individual terms. However, the shared resources are finite (R_i), and therefore they have to be included explicitly as constraints. Thus, the pricing problem becomes

SIMPLE PRICING MODEL

$$\max B_T = \sum_p B_p = \sum_p (I_p - C_p) = \sum_p (Sales_p \cdot Price_p - Q_p \cdot C_{v_p}) \tag{13}$$

s.t.

$$Sales_p \le Q_p \forall p \tag{14}$$

$$Sales_p \le Demand_p \forall p \tag{15}$$

$$Demand_p = Demand_{p0} - m_p \cdot Price_p \forall p \tag{16}$$

$$Price_p^{lo} \le Price_p \le Price_p^{up} \, \forall p \tag{17}$$

$$\sum_p r_{ip} \cdot Q_p \le R_i \, \forall i \tag{18}$$

where, in this case, $Sales_p$ represent the total sales of product p. Although taxes may play an important role, for the sake of simplicity, they are ignored. Constraint (18) is a summation over all products of the resources used to produce them. In this equation r_{ip} is the amount of resource i needed for the production of one unit of product p. Its summation over p is limited by the availability of i (R_i). In general, R_i refers to any kind of resources which are generally classified as renewable (capacity, human power, etc.) and nonrenewable (raw materials, intermediate products, etc.).

At this point, additional considerations can be introduced in order to obtain a more realistic formulation of the problem. Such formulation is shown below (model DETPRICE):

MODEL DETPRICE

$$\max B_T = \sum_p B_p = \sum_p (I_p - C_p) = = \sum_p \sum_t (Sales_{pt} \cdot \sum_r Z_{pr} \cdot dPrice_{pr} - Q_{pt} \cdot C_{v_p} - ESales_{pt} \cdot EPrice_p) \quad (19)$$

s.t.

$$Sales_{pt} \leq Demand_{pt} \; \forall p, t \quad (20)$$

$$dDemand_{ptr} = dDemand_{pt0} - m_{pt} \cdot dPrice_{pr} \; \forall p, t, r \quad (21)$$

$$Demand_{pt} = \sum_r Z_{pr} \cdot dDemand_{ptr} \; \forall p, t \quad (22)$$

$$Price_p = \sum_r Z_{pr} \cdot dPrice_{pr} \; \forall p \quad (23)$$

$$\sum_r Z_{pr} = 1 \; \forall p \quad (24)$$

$$Sales_{pt} = ESales_{pt} + ISales_{pt} \; \forall p, t \quad (25)$$

$$ISales_{pt} \leq Q_{pt} + \sum_{t'=1}^{t'=t-1} (Q_{pt'} - Sales_{pt'}) + Inv_{p0} \; \forall p, t \quad (26)$$

$$\sum_p \sum_t r_{ip} \cdot Q_{pt} \leq R_i \; \forall i \quad (27)$$

In first place, discrete variables instead of continuous ones can be used to represent prices. This modification tries to reflect industrial practices in which prices are commonly selected from a set of allowable values given by marketing surveys rather than picked from continuous ranges. Such discrete values are commonly obtained by discretizing the demand curve of each product p into r intervals as stated by eqs 21−23. The continuous variable used for representing pricing decisions $Price_p$ is therefore substituted by a binary variable Z_{pr} which takes the value of one if the discrete price $r(dPrice_{pr})$ is selected for product p or zero otherwise (eq 24). Additionally this use of discrete variables avoids the nonlinearities which appeared in the previous formulation and were due to products of continuous variables representing the sales and prices of products.

The possibility of production outsourcing (25), a common practice in industry, is introduced next into the formulation. Specifically, the satisfaction of the demand is obtained either by in house production, i.e., sales of products manufactured at the plant ($ISales_{pt}$), or by paying a premium ($EPrice_p$) to other external producers to deliver the products ($ESales_{pt}$) to the consumer. Therefore, the total sales ($Sales_{pt}$) for each product and time interval comprises the internal and external sales terms.

The mathematical formulation also includes more than one period of time. A subscript t is therefore introduced in those decision variables related to production and sales of materials. The possibility of selling in a certain time interval t stock materials which have been produced in earlier periods is therefore taken into account in eq 26, where Inv_{p0} represents the initial inventory of product p, Q_{pt}, the total amount of product p manufactured during period t, and $ISales_{pt}$, the internal sales, i.e., the amount of product p sold to customers during time interval t coming from internal production carried out by the plant. In general, it is desirable that the internal sales equals the production rate. However, chemical plants are often forced to store materials in order to deal with the variability and uncertainty associated with the demand. For this reason, both terms are different; Q_{pt} is the one that we consider appropriate to be included in the objective function because it represents the total production rate of the plant that originates the variable production costs.

This simple approach, however, does not reflect correct manufacturing costs. Indeed, in discrete or batch manufacturing scenarios, the coefficients C_{v_p} and r_{ip} depend on the way the production is performed, especially the schedule. Therefore, the first one (C_{v_p}) should include inventory costs, the cost of the utilities consumed by labor intensive tasks, and the costs of the wastes generated when changes of batches of different products take place in the production lines. On the other hand, the resource utilization factor (r_{ip}) must constrain the total production rate of each of the items, which is indeed a function of the sequence of batches. Thus, the solution of the problem requires a scheduling model (DETSCHED) in order to check the feasibility of the proposed plan (amount of materials to be produced and their corresponding prices) and compute realistic values of C_{v_p} and r_{ij}. Such model is described next.

3. Multiple Product Pricing Model with Scheduling Considerations

As it has been mentioned before, the simplified pricing model (DETPRICE) determines the amount of materials (Q_{pt}) to be produced and their corresponding prices (Z_{pr}). We now present a scheduling model which will allow the determination of the parameters r_{ij} and C_{v_p}. The problem is formally stated as follows:

Given

(a) The amount of product p to be produced and sold in time interval t (Q_{pt} and $ISales_{pt}$).

(b) The cost of the utilities consumed when fabricating product p (C_{u_p})

(c) The cost of the wastes originated when changing products in the manufacturing lines ($PCW_{pp'}$).

(d) The nominal batch size and inventory costs of product p to be fabricated (bs_p and α_p)

(e) The processing time of stage j involved in the fabrication of product p (top_{pj})

(f) The time horizon (H) and number and length of time intervals (T_t).

Determine

The schedule that satisfies the requirements given by the plan (amount of materials to be manufactured and their corresponding prices) computed by means of DETPRICE and that minimizes the total costs (inventory, utilities and changeover waste cost).

The aforementioned model comprises three major sets of constraints which are described in detail next.

Scheduling Constraints: These constraints, which are required in order to introduce scheduling considerations into the formulation, should enable the computation of the initial and finishing times of all the tasks involved in the production of the batches and ensure the feasibility of the resulting schedule.

In this work, a very simple scheduling model, which is suitable for a multiple stage and multiproduct batch plant with one single unit per stage, is applied to illustrate the way scheduling and pricing decisions interact in manufacturing environments. However, such a model can be extended to more complex structures.

The proposed formulation is derived based on a batch slot concept. With this formulation the time horizon is viewed as a sequence of L batches that can be assigned to only one particular product. Concerning the maximum number of batches to be produced it should be pointed out that such value can be either estimated based on capacity limitations or given by the decision maker. Sequence decisions are linked to a binary variable X_{lp} which represents the existence of a batch l of product p and takes the value of 1 in case l belongs to p and 0 otherwise.

$$X_{lp} = \begin{cases} 1 \text{ if batch } l \text{ is of product } p \\ 0 \text{ otherwise} \end{cases}$$

$$\sum_p X_{lp} \le 1 \forall l \tag{28}$$

$$\sum_p X_{lp} \le \sum_p X_{l+1p} \forall l < L \tag{29}$$

Constraint 28 states that a batch l can belong at most to only one product, while equation 29 enforces the condition for which the nonproduced batches are located at the beginning of the schedule. Although constraint 29 is not necessary, it helps computations. Indeed, by fixing the position of the nonproduced batches we get smaller branch-and-bound trees and shorter computational times. The specific position of these nonproduced batches (at the beginning of the schedule) is absolutely arbitrary and is chosen for simplicity.

To reduce the complexity of the formulation, only the case of a zero wait policy (ZW) is here considered. Neither intermediate storage nor waiting times in the processing units are available. Under ZW policy, there can be no delay between the time a product batch finishes processing on stage j (TF_{lj}) and the time it commences processing on stage $j+1$ (TI_{lj+1}) as stated by constraint 30:

$$TI_{lj+1} = TF_{lj} \forall l, j < J \tag{30}$$

The finishing time of stage j (TF_{lj}) involved in the fabrication of batch l is computed from the initial time of l in j (TI_{lj}) and the duration of stage j which is given by the recipe of the product the batch belongs to (top_{pj}) as expressed by constraint 31.

$$TF_{lj} = TI_{lj} + \sum_p top_{pj} \cdot X_{lp} \forall l, j \tag{31}$$

The ZW assumption resulting in the above commented constraints (eqs 30 and 31) can be easily modified in order to consider other transfer policies such as the unlimited intermediate storage and nonintermediate storage ones (UIS and NIS).

For the sake of simplicity it has been also assumed that only one production line with one assigned equipment per stage is available. Constraint 32 reflects the aforementioned policy by forcing stage j involved in the fabrication of batch l' to start after the end of the same stage j performed in any previous batch l. Such policy could be easily modified in order to reflect other equipment-task allocations possibilities.

$$TI_{l'j} \ge TF_{lj} \forall j, l' > l \tag{32}$$

Inventory Constraints: These constraints are introduced in order to compute the inventory costs. We follow here the work developed by Tsiakis et al.[19]

The computation of the total amount of product p manufactured in time interval t (Q'_{pt}) is carried out by adding all the batch sizes b_{sp} of the l batches of p produced during period t as expressed by eq 33.

$$Q'_{pt} = \sum_l \sum_p bs_p \cdot X_{lp} \cdot Y_{lt} \forall p, t \tag{33}$$

In this equation, Y_{lt} is a binary variable used to allocate batches to periods of time and takes the value of one if the last stage of batch l is finished within time interval t and zero otherwise. If the limits of interval $t \in [1, NT]$, where NT represent the total number of periods in which the time horizon is divided, are denoted as $\overline{T_{t-1}}$ and $\bar{T}_t$, the above definition can be represented by the following linear constraints:

$$Y_{lt} = \begin{cases} 1 \text{ if } TF_{lJ} \in [\overline{T_{t-1}} \text{ and } \bar{T}_t] \\ 0 \text{ otherwise} \end{cases}$$

$$\sum_t Y_{lt} = 1 \forall l \tag{34}$$

$$Y_{lt} \cdot T_{t-1} \le TF'_{lJt} \le Y_{lt} \cdot T_t \forall l, t \tag{35}$$

$$\sum_t TF'_{lJt} = TF_{lJ} \forall l \tag{36}$$

Constraint 34 ensures that each batch l is finished within only one period of time; i.e., only one of the variables Y_{lt} (say, for $t = t^*$) takes a value of 1, with all others being zero. Constraint 35 allocates each of the batches to its corresponding time interval using the defined binary variable Y_{lt}. Such an equation forces the auxiliary continuous variable TF'_{lJt} to 0 for all $t \ne t^*$, while also bounding TF'_{lJt^*} in the range $[\overline{T_{t^*-1}}, \overline{T_{t^*}}]$. Finally, constraint 36 expresses the condition for which the summation of the auxiliary variable TF'_{lJt} over t must be equal to the time in which batch l finishes its last stage J, what implies that $TF_{lJ} = TF'_{lJt^*}$ and, therefore, $TF_{lJ} \in [\overline{T_{t^*-1}}, \overline{T_{t^*}}]$, as desired.

Furthermore, the in-house sales ($ISales'_{pt}$) must be higher than the sales requirements ($ISales_{pt}$) and lower than the demand ($Demand_{pt}$) computed by model DETPRICE as expressed by constraints 37 and 38. In fact, these two constraints represent the link between the planning and the scheduling model, i.e., the scheduling model (DETSCHED) must satisfy the sales requirements determined by the planning one (model DETPRICE).

$$ISales'_{pt} \ge ISales_{pt} \forall p, t \tag{37}$$

$$ISales'_{pt} \le Demand_{pt} \forall p, t \tag{38}$$

In addition, the sales derived from in-house manufacturing ($ISales'_{pt}$) must be also smaller than the amount of products available in each time interval t as stated by constraint 39:

$$ISales'_{pt} \le Q'_{pt} + Inv_{pt} \forall p, t \tag{39}$$

Finally, the average inventory of product p in period t ($\overline{Inv_{pt}}$) is computed with the initial inventory of product p kept at the beginning of period t (Inv_{pt}) and the production rate (Q'_{pt}) (constraint 40). In this equation it has been assumed that sales are executed at the end of each time interval and that the real profile of materials can be approximated by a linear equation. The error incurred by the use of such approximation is believed to be negligible for a high number of batches and time intervals.

$$\overline{Inv_{\text{pt}}} = Inv_{\text{pt}} + \frac{Q'_{\text{pt}}}{2} \forall p, t \tag{40}$$

$$Inv_{\text{pt}} = Inv_{\text{pt}-1} + Q'_{\text{pt}-1} - ISales'_{\text{pt}-1} \ \forall p, t \tag{41}$$

Integer Cuts: A way to reduce the computational effort associated to the resolution of the mathematical formulation consists of generating integer cuts that explicitly forbid some infeasible solutions. In our case, some integer cuts can be inferred from the sequence constraints. Equation 42 states that nonproduced batches must be located in the first period, while constraint 43 expresses the condition for which the classification of batches into time intervals must be performed following the sequence of batches in the production lines.

$$Y_{\text{lt}} \geq 1 - \sum_{p} X_{\text{lp}} \ \forall l, t = 1 \tag{42}$$

$$Y_{\text{l't'}} + \sum_{\text{t}>\text{t'}} Y_{\text{lt}} \leq 1 \forall l > l' \tag{43}$$

Objective Function: The presented model accounts for the minimization of the total cost (C') associated to the schedule that satisfies the requirements computed by DETPRICE. It includes the cost of the utilities, the cost of the changeover wastes, and the inventory cost as stated by eq 44.

$$C' = \left[\sum_{p}\sum_{t} Q'_{\text{pt}} \cdot C_{\text{u}_\text{p}} + \sum_{p}\sum_{p'}\sum_{l} X_{lp} \cdot X_{l+1p'} \cdot PCW_{pp'} + \\ + \sum_{p}\sum_{t} \alpha_p \cdot \overline{Inv_{pt}} \right] \tag{44}$$

Finally, the overall problem can be expressed as follows:

MODEL DETSCHED

minimize C' subject to eqs 28−44.

4. Hierarchical Scheduling−Pricing Procedure

In this section, one way by which the pricing and scheduling models can be used to reach an improved solution is discussed.

Pricing policies and scheduling decisions are nowadays taken in many companies through a hierarchical decision-making process involving the financial and the production departments of the companies represented by the chief financial officer (CFO) and the production manager, respectively. The CFO is responsible for the financial area of the enterprise and has therefore to take care of financial-planning decisions with the help of the available decision-support tools (planning models such as DETPRICE). On the other hand, once the financial decisions are taken, the feasibility and cost of the proposed plans are evaluated by the production manager using scheduling models such as DETSCHED. This information is finally reported to the financial department (CFO) and the overall process is sometimes repeated until a final solution which satisfies the involved managers is reached, but in many others these iterations do not even take place.

Table 1. High Level Data (I)

product	$Eprice_{\text{p}}$ (m.u./kg)	Dem_{pt0} (kg)	m_{pt} (kg/m.u.)
P1	110	160	1
P2	70	120	1
P3	50	100	1

Table 2. High Level Data (II)

product	r_{ip} (u.r./kg)	Δr_{ip} (u.r./kg)	C_{v_p} (m.u./kg)
P1	1.3	0.1	45
P2	1.2	0.1	20
P3	1.2	0.1	5

Table 3. Low Level Data (I)

product	bs_{p} (kg)	$\alpha_{\text{p}} \times 10^2$ (m.u./kg h)	C_{v_p} (m.u./kg)
P1	15	4.9	35
P2	12	3.5	10
P3	12	0.7	2

An algorithm that tries to reflect this type of traditional enterprise practices is described below. Such an algorithm is applied with the aim to highlight the drawbacks of such methodologies, and it has not been developed as a way of decomposing the problem in order to overcome numerical difficulties. Moreover, the changeover costs term, which implies the computation of scheduling variables and appears in the objective function of the overall problem, would make it difficult to apply decomposition strategies available in the literature[10,20] which rely on either combining mixed-integer programming (MILP) to model the assignment part and constraint programming (CP) for modeling the sequencing part, or else combining MILP models for both parts. We now present the iterative model.

TRADITIONAL PROCEDURE:

Do

Solve the simplified pricing model (DETPRICE) assuming initial values of costs (C_{v_p}) and use of resources (r_{ip}).

Use the results ($ISales_{pt}$) as data for the production planning detailed model (DETSCHED).

If DETSCHED is feasible **Then**

Update C_{v_p} and r_{ip} in the pricing model (DETPRICE) using the results from DETSCHED.

Else

Increase r_{ip} if the production of product p is not zero. Keep it constant otherwise.

End if

Until a finalization criterion is met.

The way of updating these coefficients during the iterative search and the starting point of the iterative procedure may have an impact in the final solution. To illustrate the capabilities of the proposed approach, the algorithm shown above is applied to a case study of a multiproduct batch plant manufacturing three different products in three stages. It is assumed a time horizon of 12 days divided into two periods of equal length. Moreover, sales of products are supposed to take place each 6 days, and only one resource Ri is considered. The data of the problem are listed in Tables 1−5. The set of discrete prices and their associated demands are generated discretizing the demand curve into 10 intervals of equal length. The overall problem consists of finding the

Table 4. Low Level Data (II): Processing Times, top_{pj} (h)

	stage		
product	j1	j2	j3
P1	15	21	15
P2	12	18	15
P3	15	18	6

Table 5. Low Level Data (III): Changeover Waste Costs, $PCW_{pp'}$

	product		
product	P1	P2	P3
P1	0	10	20
P2	20	0	40
P3	30	20	0

product prices and plant schedule that maximizes the total enterprise profit. Regarding the details of the sequential approach, such a procedure is carried out applying eqs 45 to 47 to update C_{v_p} and r_{ip} in DETPRICE and considering constant values of Δr_{ip}.

Feasible DETSCHED:

$$C_{v_p} = C_{u_p} + \frac{\sum_{p'}\sum_{p''}\Sigma_l X_{lp'} \cdot X_{l+1p''} \cdot PCW_{p'p''} + \sum_{p'}\Sigma_t \alpha_{p'} \cdot \overline{Inv_{p't}}}{\sum_t Q'_{pt}} \quad (45)$$

$$r_{ip} = r_{ip} - \Delta r_{ip}\, if \sum_t Q_{pt} \neq 0\ \forall i, p \quad (46)$$

Infeasible DETSCHED:

$$r_{ip} = r_{ip} + \Delta r_{ip}\, if \sum_t Q_{pt} \neq 0\ \forall i, p \quad (47)$$

Constraint 45 reflects the heuristic consisting of estimating the production cost of a product by assigning a part of the overall costs generated by the schedule to it.

Table 6. Iterative Method

	initial solution		final solution	
product	$price_p$ (m.u.)	Q_p (kg)	$price_p$ (m.u.)	Q_p (kg)
P1	128	60	128	80
P2	96	0	96	36
P3	80	0	70	0
B_T (m.u.)	7010		9544	

The iterative procedure is therefore implemented assuming initially a low plant capacity (R_i equal to 100 capacity units) and high variable costs (elevated C_{v_p}). The capacity is then progressively increased (lower values of r_{ip}) until infeasibility in DETSCHED is detected. Figure 1 shows the evolution of C_{v_p} and r_{ip}, while Figure 2 and 3 depict the decision variables ($Price_p$ and Q_p) and the profits (the original one given by DETPRICE and the corrected one updated with the results provided by the detailed model DETSCHED) during the iterative procedure. The Gantt charts for the initial and final solutions are given in Figure 4, while Table 6 shows the values of the decision variables and the profits obtained for the initial and final solutions. The first infeasibility is reached in iteration 108. At this point the plant is not capable of manufacturing the amount of materials provided by model DETPRICE. This makes the corrected benefit drop to zero as depicted by Figure 3. The coefficients are next updated, and a second infeasibility is then reached at iteration 110. The iterative parameters are then updated again, and the iterative scheme finally converges at iteration 115. Figure 2 indicates low oscillations of production volumes during the iterative process while prices remain more or less constant. Moreover, Figure 3 shows the difference between the original benefit given by DETPRICE and the corrected one computed once DETSCHED is solved varies from 0.4% to 8% for the feasible plans.

Results indicate that the earliest plans involve the fabrication of small amounts of products thus leading to lower profits. On the other hand, higher amounts of

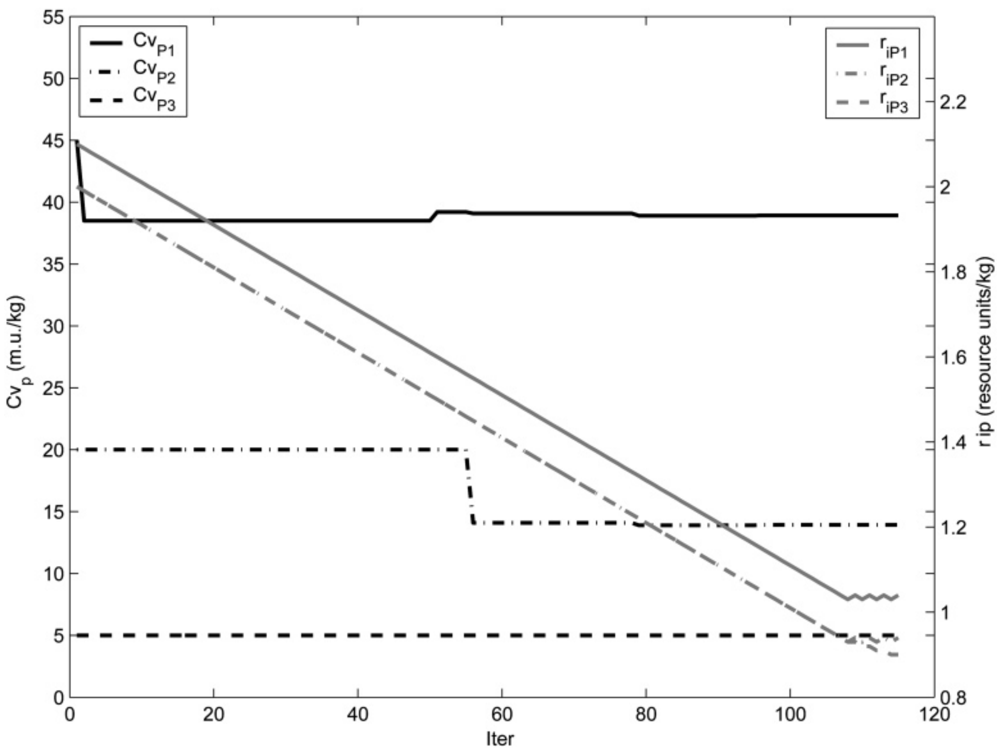

Figure 1. Evolution of C_{v_p} and r_{ip}.

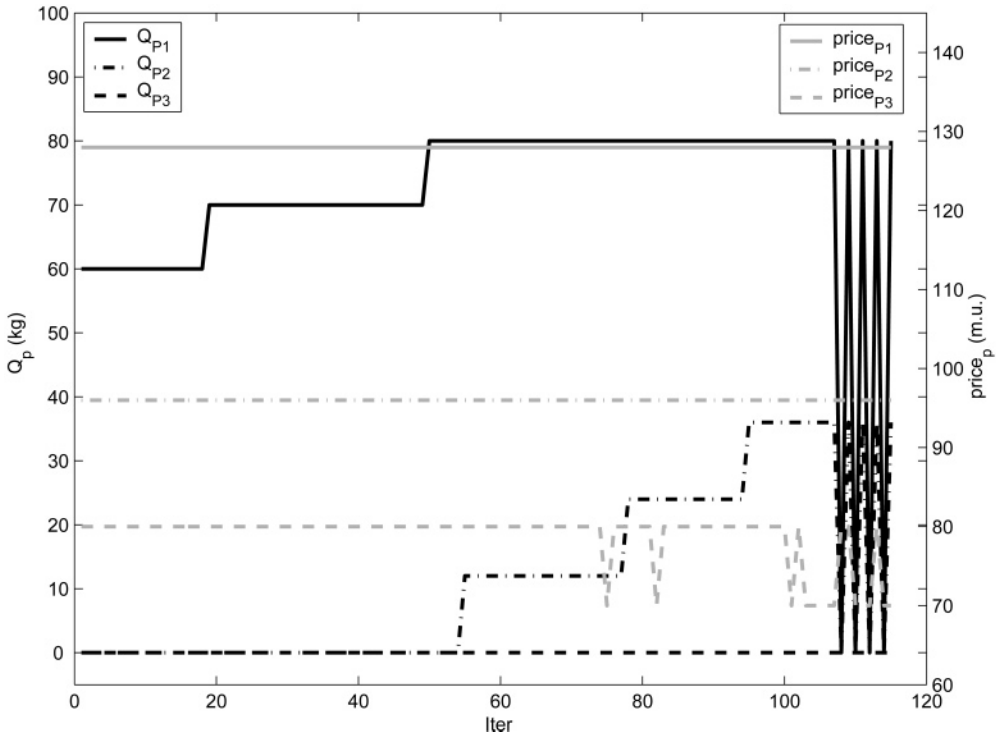

Figure 2. Evolution of $price_p$ and Q_p.

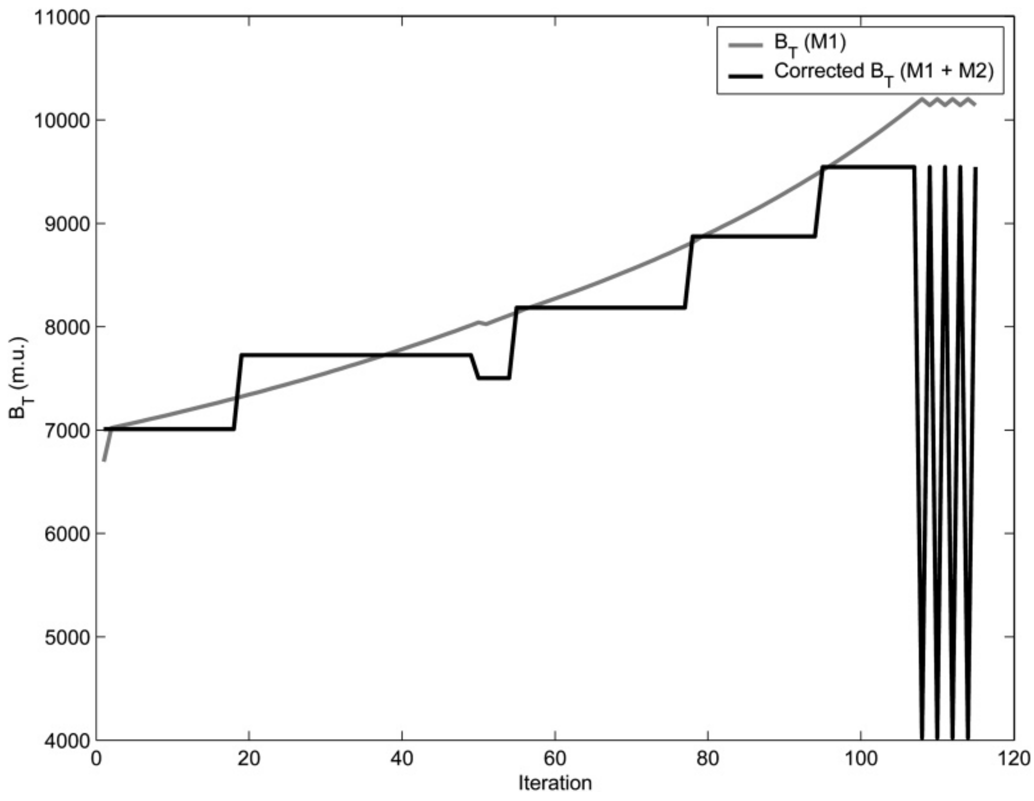

Figure 3. Evolution of B_T.

materials and hence bigger benefits are associated to the last iterations.

There are four drawbacks of this procedure:

(a) The number of iterations. For instance, in the proposed case study the first solution reached after 50 iterations exhibits a profit which is nearly 21.4 lower than the final one. The initial solution also leads to 26.6% less benefit than the last one.

(b) The way of updating C_{v_p} and r_{ip} in DETPRICE. Different starting points and updating processes may lead to dissimilar solutions. Therefore, good solutions may not be found, although they may exist.

(c) The infeasibilities which arise when solving model DETSCHED. Such infeasibilities appear when DETPRICE generates rather optimistic plans that assume higher capacities than the available ones.

(d) Even if these two issues are resolved, the optimality of the final solution cannot be guaranteed.

To ameliorate all the problems derived from this approach an integrated model is suggested. Such a model is presented in the next section.

5. Pricing Policies Model

In this section a mathematical model is proposed in order to simultaneously determine the amount of products to be manufactured, their corresponding prices, and the associated scheduling. The proposed model can be derived from the scheduling mathematical formulation and, in general, from any of the existing scheduling formulations,[6−8] by eliminating the constraints which link DETSCHED with DETPRICE (eqs 37 and 38) and adding eqs 48 to 53, which are in fact taken from DETPRICE:

$$dDemand_{\mathrm{ptr}} = dDemand_{\mathrm{pt0}} - m_{\mathrm{pt}} \cdot dPrice_{\mathrm{pr}} \; \forall p, t, r \quad (48)$$

$$Demand_{\mathrm{pt}} = \sum_r Z_{\mathrm{pr}} \cdot dDemand_{\mathrm{ptr}} \; \forall p, t \quad (49)$$

$$Price_{\mathrm{p}} = \sum_r Z_{\mathrm{pr}} \cdot dPrice_{\mathrm{pr}} \; \forall p \quad (50)$$

$$\sum_r Z_{\mathrm{pr}} = 1 \; \forall p \quad (51)$$

$$Sales'_{\mathrm{pt}} = ESales'_{\mathrm{pt}} + ISales'_{\mathrm{pt}} \; \forall p, t \quad (52)$$

$$Sales'_{\mathrm{pt}} \le Demand_{\mathrm{pt}} \; \forall p, t \quad (53)$$

The new objective function is

$$MaxB_{\mathrm{T}} = \sum_p \sum_t Sales'_{\mathrm{pt}} \cdot Price_{\mathrm{p}} - \left[\begin{array}{l} \sum_p \sum_t Q'_{\mathrm{pt}} \cdot C_{\mathrm{u_p}} + \sum_p \sum_{p'} \sum_l X_{lp} \cdot X_{l+1p'} \cdot PCW_{pp'} + \\ \sum_p \sum_t \alpha_p \cdot \overline{Inv_{pt}} + ESales'_{\mathrm{pt}} \cdot EPrice_{\mathrm{p}} \end{array} \right] \quad (54)$$

And finally the overall problem can be expressed as follows:

MODEL DETINT

maximize B_{T} subject to eqs 28−36, 39−43, and 48−54.

To illustrate the capabilities of this approach, the proposed case study is solved with the integrated model and the results are compared with those obtained by means of the iterative procedure. For the problem discussed above, the resulting formulation has 1169 constraints, 474 continuous variables, and 241 binary variables. It was implemented in GAMS[21] and solved using the MIP solver of CPLEX (7.0). It takes about 26 s to reach a solution with 0% integrality gap on an AMD Athlon 3000 computer. Table 7 shows the obtained results, while Figure 5 depicts the Gantt chart associated with the resulting schedule. The profit achieved by the integrated model is nearly 14.6% higher than the best solution provided by the iterative scheme (11 170 m.u. for the integrated vs 9544 m.u. for the hierarchical-iterative method). Moreover, this percentage increases up to 37.2% when comparing the integrated solution with the first solution given by the iterative method (11 170 m.u. for the integrated vs 7010 m.u. for the iterative approach).

Table 7. Integrated Model

product	$price_{\mathrm{p}}$ (m.u.)	Q_{p} (kg)
P1	128	60
P2	96	48
P3	80	12
B_{T} (m.u.)		

There are two main reasons for why the integrated model yields higher profit than the sequential one. In first place, the DETPRICE model used in the sequential approach does not take into account that production is carried out in a discontinuous way; i.e., a continuous variable is used for representing the production rate instead of taking into account the batch sizes associated to the existing equipments. This issue makes the DETSCHED model compute schedules which either are infeasible or require higher amounts of products than the ones requested by DETPRICE, which results in higher storage costs. Second, the DETPRICE model does not consider the changeover costs, which are sequence dependent, and therefore cannot properly assess the profitability of the different products thus leading to suboptimal solutions.

6. Integrated Model under Uncertainty

A stochastic programming approach based on a recourse model with two stages is proposed to incorporate the uncertainty associated with the relationship between demand and price. In a two-stage stochastic optimization approach, the uncertain model parameters are considered random variables with an associated probability distribution and the decision variables are classified into two stages. The first-stage variables correspond to those decisions which need to be made here-and-now, prior to the realization of the uncertainty. The second-stage or recourse variables correspond to those decisions made after the uncertainty is unveiled and are usually referred to as wait-and-see decisions. After the first-stage decisions are taken and the random events realized, the second-stage decisions are made subject to the restrictions imposed by the second-stage problem. Due to the stochastic nature of the performance associated with the second-stage decisions, the objective function consists of the sum of the first-stage performance measure and the expected second-stage performance. Birge et al.[22] provides an overview of this kind of stochastic techniques.

In our problem, product demands are represented by a set of scenarios with a given probability of occurrence, which are obtained by applying a Monte Carlo sampling. Decision variables related to the production schedule (X_{lp}) and the pricing policies (Z_{pr}) are considered as first-stage decisions, since it is assumed that they have to be taken at the scheduling stage before the uncertainty is unveiled. On the other hand, the sales ($Sales_{\mathrm{pts}}$, $ISales_{\mathrm{pts}}$, and $ESales_{\mathrm{pts}}$) and thus the inventory profiles ($\overline{Inv}_{\mathrm{pts}}$ and Inv_{pts}) are second-stage variables. Therefore, at the end of the scheduling horizon, a different profit value is obtained for each particular realization of demand uncertainty (B_{Ts}). The proposed model accounts for the maximization of the expected value of this profit distribution ($E[B_{\mathrm{T}}]$). The overall stochastic for-

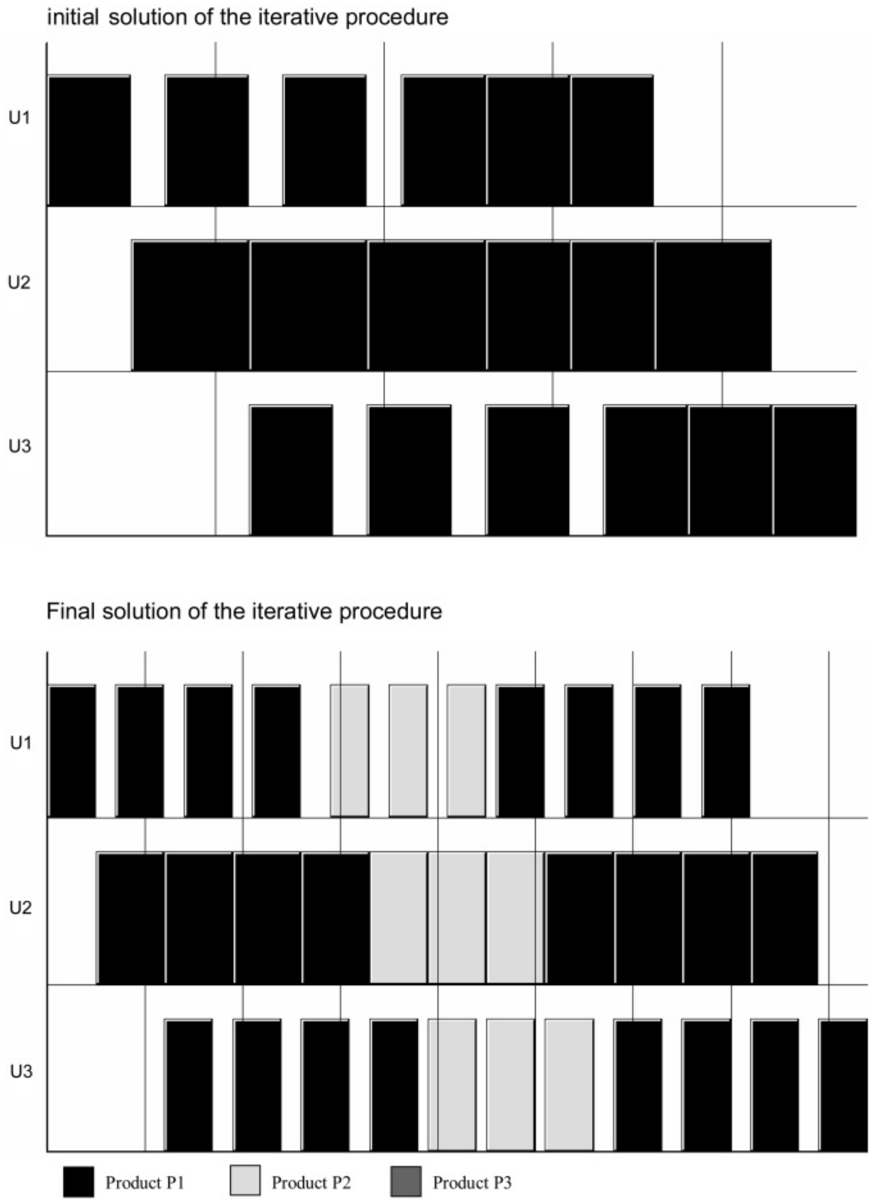

Figure 4. Gantt charts of the iterative approach.

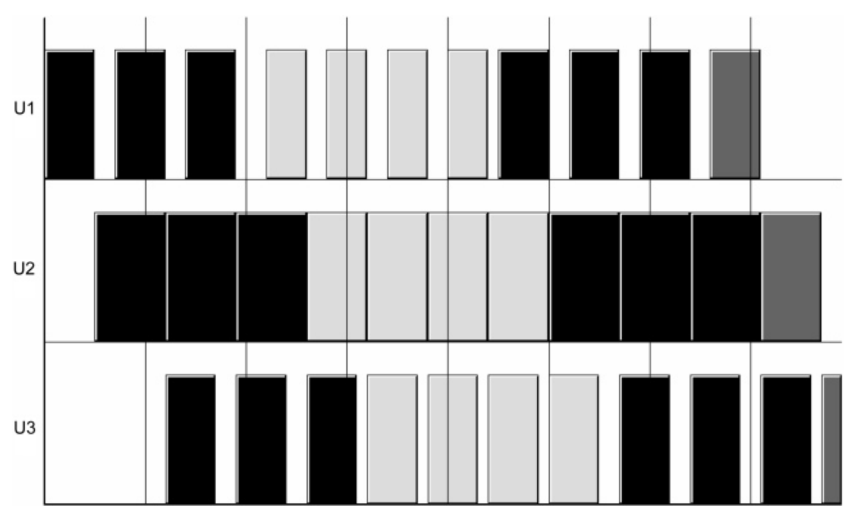

Figure 5. Gantt chart of the integrated solution.

mulation can be derived from the deterministic one by modifying those constraints which are scenario-dependent; that is, deterministic constraints which include second-stage variables 39−41, 48, 49, 52, and 53 are

Table 8. Uncertainty

product	$MeanDem_{p0}$ (kg)	$SDevDem_{p0}$ (%)	K_p (m.u.)
P1	160	32.5	45
P2	120	20	20
P3	100	5	5

replaced by the following equations which must be satisfied for each scenario s:

$$ISales'_{pts} \leq Q'_{pt} + Inv_{pts} \; \forall p, t, s \tag{55}$$

$$\overline{Inv_{pts}} = Inv_{pts} + \frac{Q'_{pt}}{2} \forall p, t, s \tag{56}$$

$$Inv_{pts} = Inv_{pt-1s} + Q'_{pt-1} - ISales'_{pt-1s} \; \forall p, t, s \tag{57}$$

$$dDemand_{ptrs} = Demand_{pt0s} - m_{pts} \cdot dPrice_{pr} \; \forall p, t, r, s \tag{58}$$

$$Demand_{pts} = \sum_r Z_{pr} \cdot dDemand_{ptrs} \forall \; p, t, s \tag{59}$$

$$Sales'_{pts} = ESales'_{pts} + ISales'_{pts} \; \forall p, t, s \tag{60}$$

$$Sales'_{pts} \leq Demand_{pts} \; \forall p, t, s \tag{61}$$

The resulting formulation accounts for the maximization of the expected profit as stated by eq 62:

$$MaxE[B_T] = \sum_s prob_s \cdot B_{Ts} = \\ = \sum_s prob_s \cdot \left[\sum_p \sum_t Sales'_{pts} \cdot Price_p - \sum_p \sum_t Q'_{pt} \cdot C_{up} - \sum_p \sum_t \alpha_p \cdot AvInv_{pts} + \\ - \sum_p \sum_{p'} \sum_l X_{lp} \cdot X_{l+1p'} \cdot PCW_{pp'} - ESales'_{pts} \cdot EPrice_p \right] \tag{62}$$

The overall problem can be therefore formulated as follows:

MODEL STOCINT

maximize $E[B_T]$ subject to eqs 28−36, 42, 43, 50, 51, and 55−62.

To highlight the convenience of using the stochastic approach, the proposed formulation is applied to the case study described before considering the uncertainty associated to the parameters of the demand curve. Uncertainty is represented by 250 scenarios, each of them comprising a certain demand value $Demand_{ptrs}$ for the same discrete price $dPrice_{pr}$. These scenarios are computed assuming that the parameter $Demand_{pt0}$ follows a normal probability distribution with mean and standard deviation given in Table 8 and that there exists a constant rate between such coefficient and m_{pts} for each scenario s (eq 63).

$$m_{pts} = \frac{Demand_{pt0s}}{Kp} \forall p, t, s \tag{63}$$

This stochastic formulation involves 55 205 constraints, 24 622 continuous variables, and 241 binary variables, and it is also implemented in GAMS[21] and solved using the MIP solver of CPLEX (7.0). It takes about 18 844 s to reach a solution with 0% integrality gap on an AMD Athlon 3000 computer. All the scenarios where considered simultaneously in this model.

Table 9. Deterministic vs Stochastic

product	deterministic $price_p$ (m.u.)	deterministic Q_p (kg)	stochastic $price_p$ (m.u.)	stochastic Q_p (kg)
P1	128	60	128	40
P2	96	48	96	36
P3	80	12	70	48

	deterministic	stochastic
$E[B_T]$ (m.u.)	10 243	10 583
VaR (m.u.)	3322	1646
OV (m.u.)	1553	786
RAR (adim.)	0.31	

For comparison purposes, the solution which was originally obtained by applying the deterministic model for the mean demand scenario is evaluated against the same 250 scenarios by fixing the first-stage variables (prices and schedule) and computing the second-stage ones with the stochastic formulation. Table 9 shows the decision variables and some risk performance indexes, which are described in the next section, associated to both solutions. The stochastic Gantt chart is given in Figure 6, while the deterministic and stochastic risk curves are also shown in Figure 7. Such curves are none other than cumulative profit probability curves, which can be used to assess risk.[1]

As it can be observed, the stochastic solution produces a less amount of items with higher demand uncertainty (P1) and higher profitability and also decreases the production of materials characterized by demands with low standard deviation (P3) but which are on the other hand less profitable. The stochastic approach suggests also variations in prices in order to adjust the demand to the new production volumes. For instance, the price of P3 is reduced in order to increase its demand thus allowing higher sales.

The expected profit associated to the stochastic solution is nearly 3% higher than the deterministic one (10 243 m.u. for the deterministic solution and 10 583 m.u. for the stochastic one). Moreover, both approaches lead to rather different solutions from the risk management point of view. In addition to providing higher profits, the stochastic solution yields lower probabilities of low profits, making the production also less risky in economic terms. For instance, a 10% probability of scenarios with earnings below 10 000 m.u. is achieved in the stochastic formulation, while this probability increases up to 35% in the deterministic approach (see Figure 7). On the other hand, the deterministic solution yields higher probabilities of larger benefits. For instance, a 40% probability of earnings above 11 000 m.u. is reported by this solution, while the stochastic one provides only a value of 20%. This makes the former more attractive for risk-taker decision makers.

7. Financial Risk

As it has been previously mentioned, although stochastic models optimize the total expected performance measure, they usually do not provide any control of its variability over the different scenarios; i.e., they assume that the decision maker is risk neutral. However, different attitudes toward risk may be encountered. In this section, financial risk is reviewed and a recent approach to manage it is described and applied to our problem.

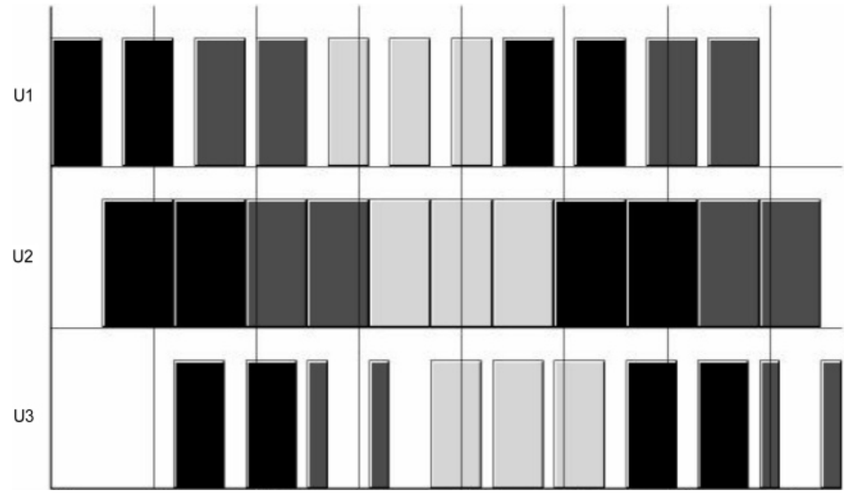

Figure 6. Gantt chart of the stochastic solution.

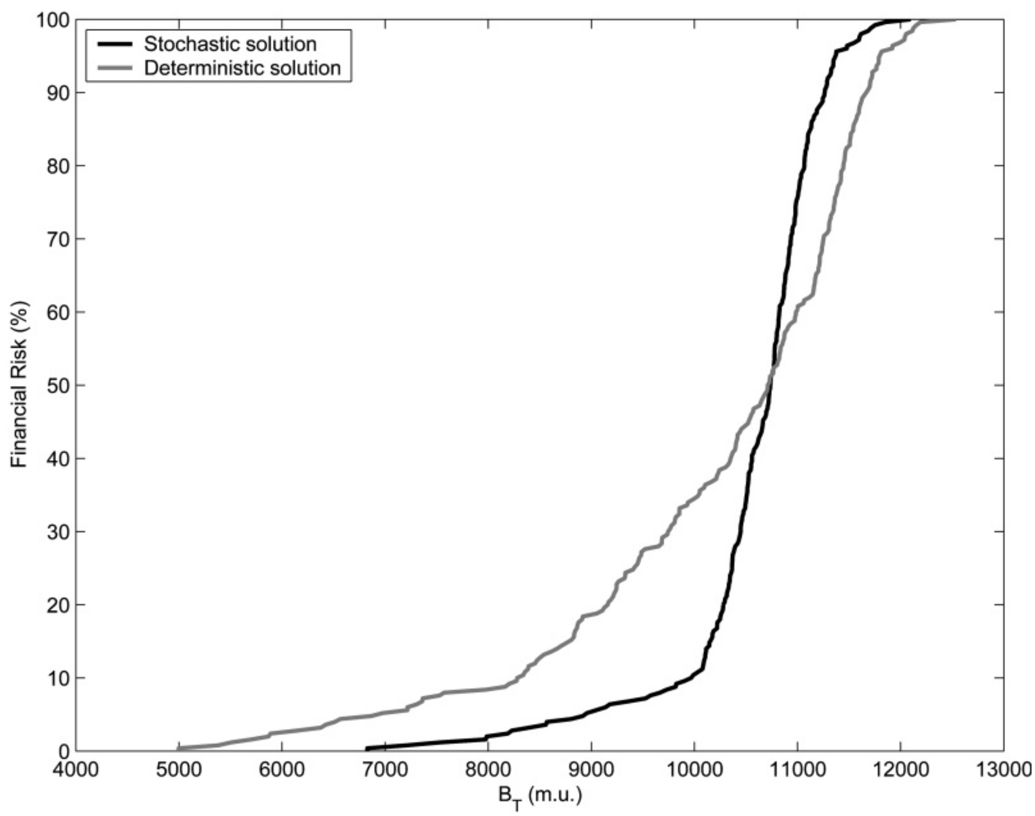

Figure 7. Deterministic vs stochastic solution.

The financial risk associated with a plan under uncertainty is defined as the probability of not meeting a certain target profit (maximization) or cost (minimization) level referred to as Ω.[17] For the two-stage stochastic problem, the financial risk associated with a design x and target profit Ω is therefore expressed by the following probability:

$$Risk(x,\Omega) = P(Profit(x) < \Omega) \tag{64}$$

where $Profit(x)$ is the $Profit$ after the uncertainty has been unveiled and a scenario is realized. The definition of $Risk(x,\Omega)$ can be rewritten with the help of binary variables as follows:

$$Risk(x,\Omega) = \sum_s prob_s \cdot z_s(x,\Omega) \tag{65}$$

where z_s is a new binary variable which equals 1 in case $Profit_s < \Omega$ and 0 otherwise:

$$z_s = \begin{cases} 1 \text{ if } Profit_s < \Omega \\ 0 \; otherwise \end{cases}$$

In the case of a discrete scenario, financial risk is given by the cumulative frequency obtained from the *Profit* histogram.

Barbaro et al.[17] proved that minimization of risk at some profit levels renders a tradeoff with expected profit. A risk-averse decision maker will feel comfortable with low risk at low values of Ω, while a risk taker will prefer to lower the risk at high values of Ω. The tradeoff lies in the fact that minimizing risk at low values of Ω (e.g., a loss) is in conflict with the minimization of risk at high values of Ω (e.g., large profits) and vice versa.

From a mathematical programming point of view, minimizing $Risk(x,\Omega)$ for a continuous range of profit targets Ω results in an infinite multiobjective optimization problem. Even though this model would be able to reflect the decision maker's intention, having an infinite optimization problem is computationally prohibitive.

The authors, however, suggested approximating the ideal infinite optimization approach by a finite multiobjective problem that only minimizes risk at some finite number of profit targets and maximizes the expected profit. This approach gives rise to the following finite multiobjective formulation:

$$maximise E[B_T] = \sum_s prob_s \cdot q^T{}_s \cdot y_s - c^T \cdot x$$

$$Min Risk(x,\Omega_1) = \sum_s prob_s \cdot z_{s1}$$

$$Min Risk(x,\Omega_i) = \sum_s prob_s \cdot z_{si}$$

$$\dots$$

subject to

$$q^T{}_s \cdot y_s - c^T \cdot x \geq \Omega_i - U_s \ \forall i, s \tag{66}$$

$$q^T{}_s \cdot y_s - c^T \cdot x \leq \Omega_i + U_s \cdot (1 - z_{si}) \ \forall i, s \tag{67}$$

$$z_{si} \in \{0, 1\} \ \forall i, s \tag{68}$$

where y_s represents the optimal second-stage solution associated with the design x that corresponds to scenario s. In the above formulation, constraints 66 and 67 force the new integer variable z_{si} to take a value of zero if the profit for scenario s is greater than or equal to the target level (Ω_i) and a value of one otherwise. To do this, an upper bound of the profit of each scenario (U_s) is used. The value of the binary variables is then used to compute and penalize financial risk in the objective function. Such a procedure generates a set of Pareto optimal solutions behaving in dissimilar ways under the uncertain environment from which the decision maker should choose the best one according to his/her preferences. Nevertheless, the inclusion of new integer variables represents a major computational limitation of the resulting formulation.

In this work, a novel approach is introduced aiming at the reduction of the computational expenses associated with the previous procedure. Within this framework, the multiobjective optimization approach is replaced by the sample average approximation method (SAA) as a way of generating the set of candidate solutions that exhibit different risk performances. Furthermore, the concept of dominance in terms of expected profit and risk associated to a discrete set of targets, in the way introduced by Barbaro et al.,[17] and some standard risk performance indexes are also applied for evaluating the solutions obtained and providing decision support in finding the one that represents the right compromise between expected profit and risk. The proposed framework to manage risk has similar features to the one suggested by Aseeri et al.[23]

The SAA method is an approach for solving stochastic optimization problems by using Monte Carlo simulation.[24] In the SAA technique, the expected second-stage profit (recourse function) in the objective function is approximated by an average estimate of NS independent random samples of the uncertain parameters, and the resulting problem is called *approximation problem*. Here, each sample corresponds to a possible scenario, and so NS is the total number of scenarios considered. Then, the resulting approximation problem is solved repeatedly for M different independent samples (each of size NS) as a deterministic optimization problem. In this way, the average of the objective function of the approximation problems provides an estimate of the stochastic problem objective. Notice that this procedure may generate up to M different candidate solutions. To determine which of these M (or possibly less) candidates is optimal in the original problem, the values of the first-stage variables corresponding to each candidate solution are fixed and the problem is solved again using a larger number of scenarios $NS' >> NS$ in order to distinguish the candidates better. After solving these new problems, the optimal solution of the original problem ($\hat{x}^*$) is determined. Therefore, $\hat{x}^*$ is given by the solution of the approximate problems that yields the highest objective value for the approximation problem with NS' samples.

For our specific problem this algorithm would be as follows:

Select NS, NS', M
For $m = 1$ to M
For $s = 1$ to NS
Use Monte Carlo sampling to generate an independent observation of the uncertain parameters. Define $prob_s = 1/NS$.
Next s
Solve problem STOCINT with NS scenarios. Let the $\hat{x}^m$ be the optimal first-stage solution.
Next m
For $m = 1$ to M
For $s = 1$ to NS'
Use Monte Carlo sampling to generate an independent observation of the uncertain parameters. Define $prob_s = 1/NS'$.
Next s
Solve problem STOCINT with NS' scenarios, fixing $\hat{x}^m$ as the optimal first-stage solution.
Next m
Use $\hat{x}^* = argmax\{Obj(\hat{x}^m)|\ m = 1, 2, \dots, M\}$ as the estimate of the optimal solution to the original problem where $Obj(\hat{x}^m)$ is the estimate of the optimal objective value.
End.

In our approach it is proposed that the problem for each scenario be solved and these deterministic results be used to obtain an upper bound of the problem, in other words, $NS = 1$ and $M = |NS'|$. Then, according to the SAA, the problem is solved fixing the first-stage variables (schedules and prices) obtained in the previous runs, and the second-stage variables (sales and inventory profiles) are computed using the stochastic model for the NS' scenarios. After that, the solutions are screened as follows.

First, all the dominated solutions, namely those which are completely dominated by at least another one, are disregarded. A solution x_1, which comprises a set of scheduling and pricing decision variables, associated to the objective function vector $u = [u_1, u_2, \dots, u_n]$, dominates other solution x_2, with its corresponding objective function vector $v = [v_1, v_2, \dots, v_n]$, if and only if

$$\forall i \in \{1, 2, \dots, n\}, v_i \leq u_i \wedge \exists i \in \{1, 2, \dots, n\} | v_i < u_i$$

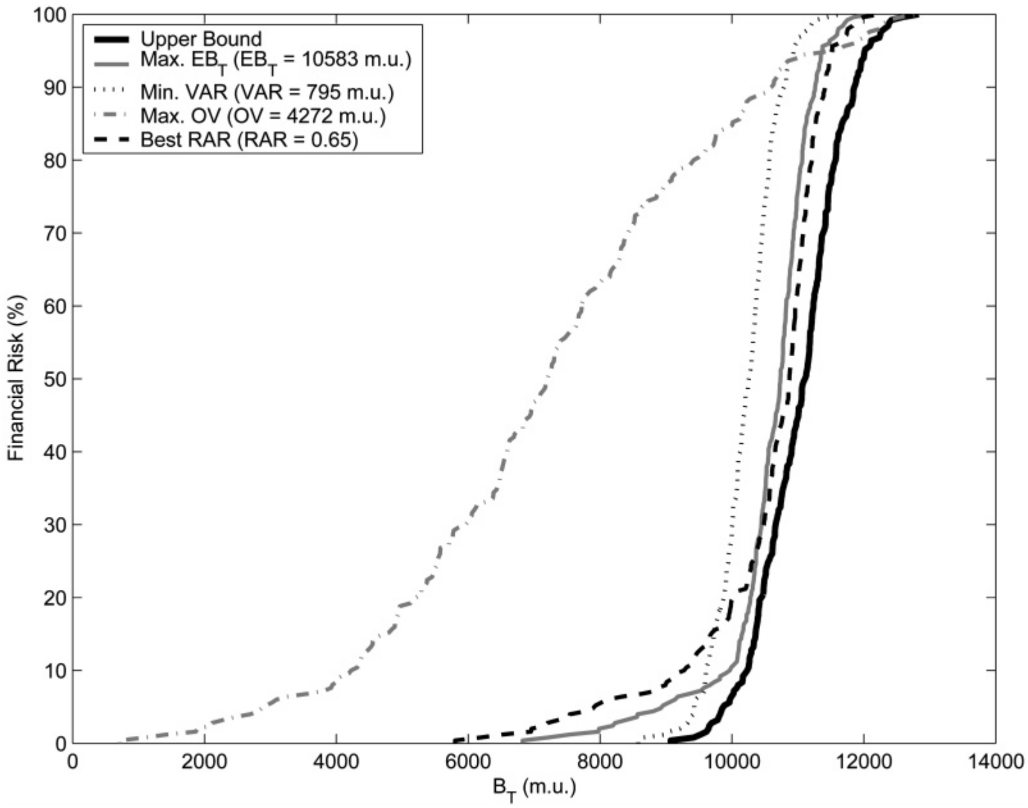

Figure 8. Best risk curves.

In other words, x_1 yields better or equal objective function values than those reported by x_2 for all the objectives, and it performs strictly better than the latter in at least one of them. In our case, we consider as objectives the expected profit and the risk associated to a set of $n - 1$ targets; therefore

$$u = [E[B_T], -Risk(x,\Omega_1),... -Risk(x,\Omega_i),... -Risk(x,\Omega_{n-1})]$$

This implies that if solution x_1 dominates x_2, its risk curve lies entirely to the right side of the risk curve of x_2. This property can be therefore used for discarding the dominated solutions from the original set of solutions computed by the SAA. A risk curve belonging to the set of nondiscarded solutions (nondominated solutions) must satisfy the condition of intersecting at least at one point all the nondominated curves.[17] As a result of this calculations, a set of nondominated solutions are obtained.

Second, various measures used and/or introduced by Aseeri et al.[23] are automatically calculated, and finally, the solutions which perform better in terms of these measures as well as the one which exhibits maximum expected profit are identified. The risk measures considered are as follows:

(a) The *Value at Risk* or *VaR*, defined as the difference between the expected value and the profit for a certain confidence interval usually set at 5%.[25,26] Solutions with low *VaR* perform better for low targets and therefore are likely to be chosen by risk-averse decision makers.

(b) The *Upside Potential* (UP) or *Opportunity Value* (*OV*) proposed by Aseeri et al.,[23] defined in a similar way to *VaR* but at the other end of the risk curve with a quantile of (1-*p*) as the difference between the benefit corresponding to a risk of (1-*p*) and the expected value of the profit. Solutions with high *OV* may be chosen by risk-taker decision makers, since they exhibit better performance for high targets.

(c) The *Risk Area Ratio* (*RAR*) proposed also by Aseeri et al.,[23] calculated as the ratio of the *OpportunityArea* (*O_Area*), enclosed by the two curves (the curve under analysis and the curve corresponding to maximum expected profit) above their intersection, to the *RiskArea* (*R_Area*), enclosed by the two curves below their intersection as stated by the following equation:

$$RAR = \frac{O\_Area}{R\_Area} \qquad (69)$$

The areas can be calculated by integrating the difference of risk between the two curves over B_{Ts}. The closer this ratio is to one, the better the alternative solution is. This is indeed an indication of how significant the reduction in opportunity is compared to the small reduction in risk.

The proposed approach is therefore applied to the case study presented before. Of the 250 runs, only 34 render nondominated solutions, one of them being the deterministic solution computed for the mean scenario to these nondominated set of solutions. The solution with highest expected profit obtained by means of the decomposition approach is equal to that computed by the stochastic model for the given 250 scenarios. Moreover, the solution obtained by means of the iterative procedure is also evaluated against the set of 250 scenarios and turns out to be dominated by the ones generated by the SAA. The risk curves of the best solutions in terms of the aforementioned risk measures are plotted together with the maximum expected profit one and the upper bound curve as depicted by Figure 8. The associated Gantt charts are given in Figure 9, while Tables

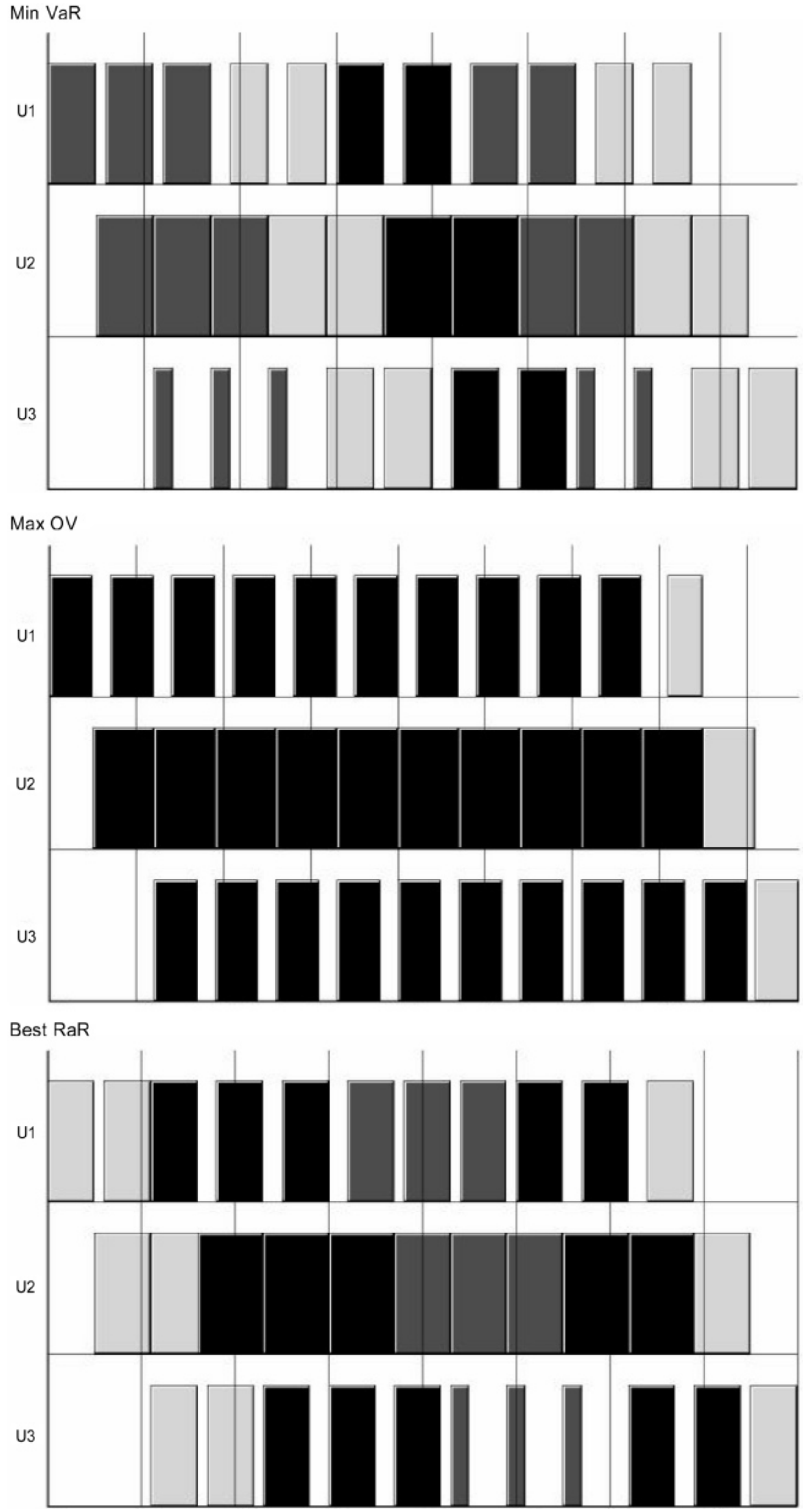

Figure 9. Gantt charts of the risk curves.

10 and 11 show the value of the risk measures and the decision variables for each curve.

Best solutions in terms of the predefined risk performance criteria:

As it can be observed in Table 10, the solutions behave in very different ways under the uncertain environment. Solutions with low *VaR* exhibit high *OV* and vice-versa, thus showing the tradeoff between policies of risk-taker and risk-averse decision makers. Besides, the solutions lead to dissimilar values of *RAR* which can be used so as to properly evaluate the different alternatives. For instance, the *RAR* index suggests not to select the maximum *OV* solution since it exhibits a very high reduction in opportunity compared to the small increase in probabilities of high profits. This conclusion can be also derived from the low expected profit of the solution. Figure 8 shows also how solutions with low *VaR* lead to low probabilities of small benefits, while those with

Table 10. Risk Curves, Risk Performance Indexes

solution	performance indexes $E[B_T]$ (m.u.)	VaR (m.u.)	OV (m.u.)	RAR
$MaxE[B_T]$	10 583	1646	786	0
MinVaR	10 222	795	719	7.15
$VaR = 1018$	10 328	1018	735	7.28
$VaR = 1403$	10 407	1403	924	8.98
MaxOV	7244	4342	4272	7.5×10^{-3}
$OV = 2764$	9087	4125	2764	5.8×10^{-2}
$OV = 3720$	8139	4363	3720	5.8×10^{-2}
BestRAR	10 534	2621	980	0.65

Table 11. Risk Curves, Decision Variables

	$price_p$ (m.u.)			Q_p (kg)		
product	P1	P2	P3	P1	P2	P3
$MaxE[B_T]$	128	96	70	40	36	48
MinVaR	128	84	70	20	48	60
$VaR = 1018$	128	84	70	20	60	48
$VaR = 1403$	128	96	70	30	48	48
MaxOV	128	96	70	100	12	0
$OV = 2764$	128	96	70	80	24	12
$OV = 3720$	128	96	80	100	12	0
BestRAR	128	96	70	50	36	36

high OV give high probabilities of huge benefits. For instance, the solution with lowest VaR yields a 0% probability of profits lower than 8000 m.u., while the curve with highest OV, which may be selected by a risk-taker manager, increases this value up to 62%. Between, the maximum expected profit solution gives a 2, while the best RAR curve shows 7%. On the other hand the best OV option yields a 5% probability of benefits above 11 000 m.u., while the others cannot achieve profits higher than this amount. These differences in risk for different targets of the obtained curves, and not only the aforementioned risk indexes, must be carefully evaluated by the decision maker in order to reach a final solution to be implemented.

Risk-Averse Decision Makers: However, it may occur that none of the selected curves satisfies the involved managers. In this last case it is necessary to identify, from the original set of nondominated curves, "better solutions" in accordance with their risk preferences. Therefore, for risk-averse decision makers, curves with low VaR may be identified and plotted together with the maximum expected profit solution, while, for risk-taker managers, solutions with high OV can be generated. In both cases, the RAR criteria may be used in order to evaluate the compromise between expected profit and risk. To identify these solutions, it is proposed usinf the risky preferences of the decision maker in order to avoid plotting the entire set of nondominated curves with low VaR. As these preferences are usually expressed in terms of profit targets beneath which or above which the decision maker wants to improve, the following procedure to identify conservative solutions is suggested:

(a) First, the curve with maximum expected profit is selected.

(b) A low profit target below which the decision maker wants to improve this solution is next identified.

(c) Curves which perform better than the first solution for the given target and cross the maximum expected profit curve above this target are then generated. These solutions will perform better than the first one not only for the given target but also for the entire profit range since they must cross the maximum expected curve in only one point.[17]

The resulting set of curves is next plotted, and the decision maker chooses finally the best one according to his risky preferences.

Figure 10 shows some curves obtained for a target equal to 9000 m.u., for which the maximum expected profit solution gives a 6% probability, while Tables 11

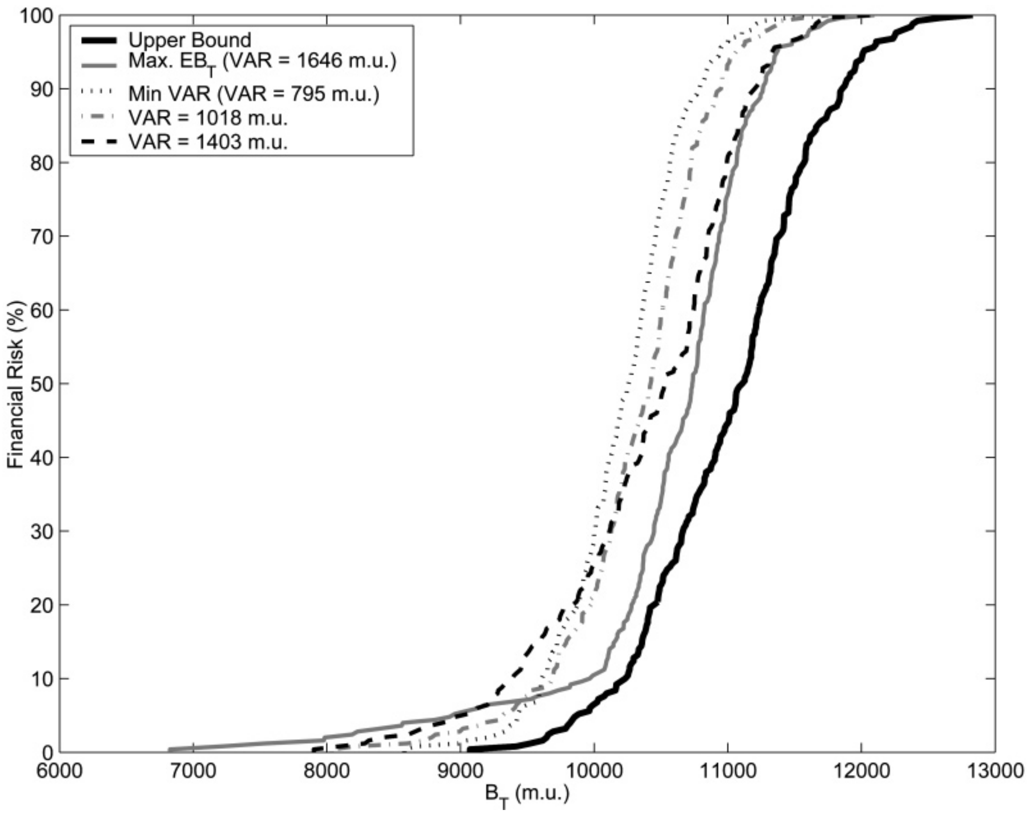

Figure 10. Risk-averse curves.

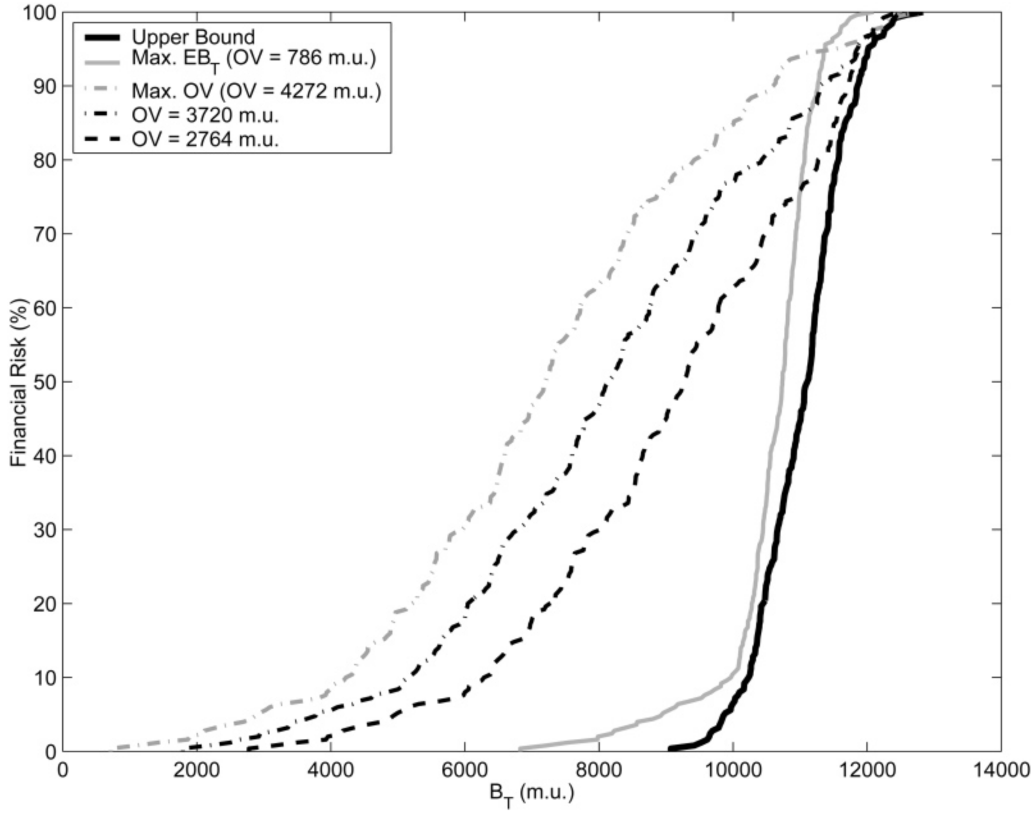

Figure 11. Risk-taker curves.

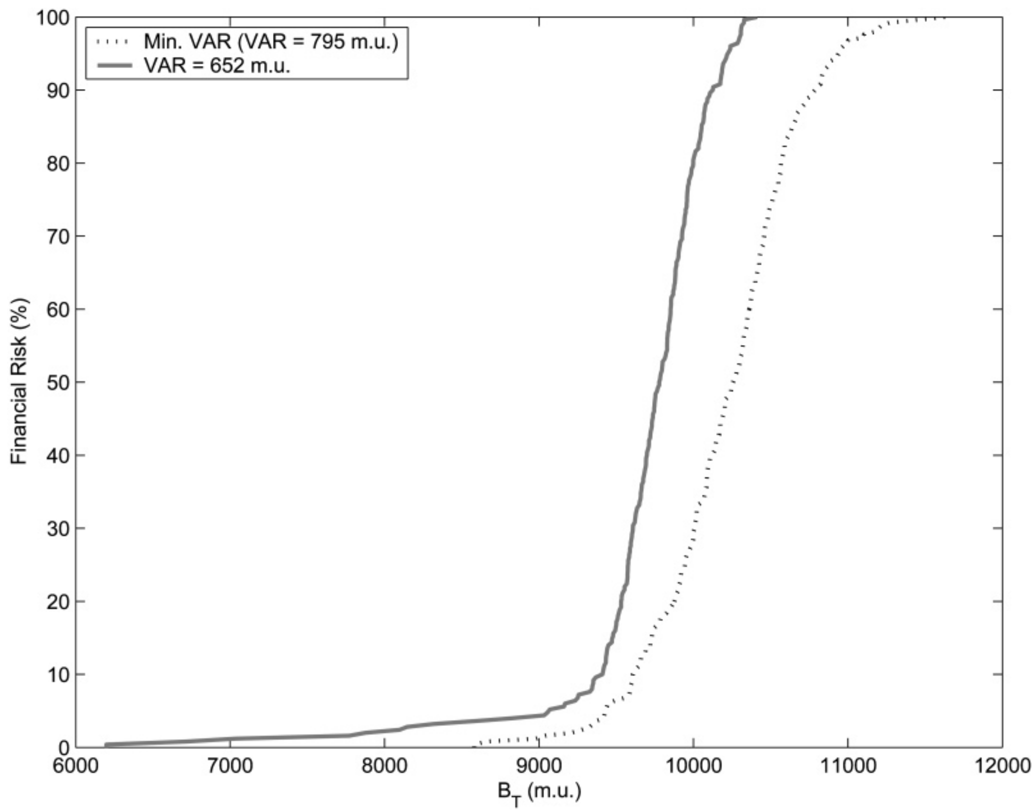

Figure 12. Minimum *VaR* solutions.

and 10 give the associated decision variables and risk indexes. The obtained curves which are sorted according to the *VaR* criteria (1403, 1018, and 795 m.u.) reduce the risk to obtain a profit below 9000 m.u. to 5.8, 4.0, and 2.0, respectively. As it can be observed, the solution with the lowest *VaR*, which was already identified before, turns out to be the safest one, since it lies entirely beneath the others under the selected profit target. On the other hand, this curve performs worst for higher targets (it yields low values of *OV*) which must be also considered when taking the final decision.

Risk-Taker Decision Makers: For risk-taker decision makers a similar procedure can be applied for generating risky solutions. In this case, a high benefit target, instead of a low one, above which the maximum expected solution wants to be improved, must be selected. The curves generated by means of this scheme for a target equal to 11 500 m.u for which the original solution shows a 3% probability of higher benefits are depicted in Figure 11, while Tables 10 and 11 show the values of the decision variables and risk indexes. The obtained curves give *OV* values equal to 4272 (maximum *OV* solution), 3720, and 2764 m.u. and increase the probability of benefits above 11 500 m.u. up to 5%, 8%, and 20% respectively. Although they perform better above the selected target, they also perform poorly for low profits (they yield high values of *VaR*) which makes them less attractive from this perspective.

As it can be observed, there is a strong relationship between the decision variables (prices and scheduling decisions variables) and the shapes of the risk curves. The most conservative solutions (low *VaR*) involve lower prices and lower production volumes of materials with higher uncertainty (P1). On the other hand, the risky or risk-taker solutions (high *OV*) lead to higher prices and amounts of uncertain items. For instance, the solution with the minimum *VaR* involves higher amounts and lower prices of P2 and P3 than the solution with highest *OV*. This pricing policy and the resulting schedule try therefore to reduce the impact of the uncertainty associated to the demand. On the other hand, the solution with highest *OV* exploits the benefits achieved in the most favorable scenarios (those with very high demands), thus leading to elevated prices and huge productions of the most profitable items (P1) despite their high uncertainties.

In addition, the obtained results highlight the importance of discarding the dominated solutions from the entire set of curves. Consider a decision maker driven by the *VaR* criteria. Figure 12 shows two risk curves, the dashed line represents the lowest *VaR* solution (*VaR* equal to 652 m.u.) identified from the initial set of 250 solutions generated by the SAA, while the continuous one corresponds to the lowest *VaR* solution (*VaR* equal to 795 m.u) of the nondiscarded set of curves. It can be observed how the first solution is dominated by the second one, since it lies entirely above it. At first glance, a risk-averse manager would choose the first curve owing to its better performance in terms of the given criteria. However, it is obvious that the nondominated solution is with no doubt better, since its associated risk curve exhibits lower cumulative probabilities than the dominated one for all the possible targets. For instance, the dominated solution gives a 82% probability of profits lower than 1000 m.u, while the nondominated one reduces this probability to 20%. This positive difference in risk varies with the selected profit target but is nevertheless achieved over the entire range. This fact is indeed of special interest, since it shows the inadequacy of evaluating decisions under uncertainty considering partial risk measures and obviating the behavior of the corresponding risk curves.

8. Conclusions

In this work a strategy for integrating pricing decisions with the scheduling process of batch plants in the batchwise production environment has been introduced. The starting point is the modeling and forecasting of the relationship between product prices and demand. Although there is a simplification in the case studied used for illustrations purposes, it is clearly shown how the integration provides better solutions and avoids infeasible solutions. In addition, since future predictions related to the market behavior cannot be perfectly forecasted, several parameters of this schedule problem such as products demands have been considered uncertain. The used risk management approach has allowed for properly handling this situation, which is commonly found in practice, thus helping to reach a final solution in accordance with the decision maker's preferences. The obtained results have highlighted the effect of pricing and scheduling decision variables in risk management and the convenience of studying the shapes of the risk curves coupled with some risk performance indexes when evaluating the solutions to be implemented. The proposed approach seems to offer a larger potential for results improvement as the complexity of the production process grows, because the simplification of the upper level in a sequential approach is likely to be inferior.

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Nomenclature

i = Resources
j = Stages
l = Batches
p = Products
r = Discrete prices
s = Scenarios
t = Time intervals
B = Benefits
B_{T} = Total benefit
B_{Ts} = Total benefit in scenario s
B_{p} = Benefit of product p
bs_{p} = Batch size of product p
C_{p} = Cost of product p
$C_{\mathrm{u_p}}$ = Unitary utility cost of product p
$C_{\mathrm{v_p}}$ = Unitary variable cost of product p
$dDemand_{\mathrm{ptr}}$ = Demand of product p in t associated to discrete price r
$dDemand_{\mathrm{ptrs}}$ = Demand of product p in time interval t in scenario s associated to discrete price r
$Demand_{\mathrm{pt0s}}$ = Demand coefficient of product p in time interval t in scenario s
$Demand_{\mathrm{pt0}}$ = Demand coefficient of product p in time interval t
$Demand_{\mathrm{pt}}$ = Demand of product p in time interval t
$Demand_{\mathrm{pts}}$ = Demand of product p in time interval t in scenario s
$Demand_{\mathrm{p}}$ = Demand of product p
$Demand_{\mathrm{p0}}$ = Demand coefficient of product p
$dPrice_{\mathrm{pr}}$ = Discrete price r of product p

$E[B_T]$ = Expected total benefit
$EPrice_p$ = External price of product p
$ESales_{pt}$ = External sales of product p in time interval t (used in DETPRICE)
$ESales'_{pt}$ = External sales of product p in time interval t (used in DETSCHED and DETINT)
$ESales'_{pts}$ = External sales of product p in time interval t in scenario s (used in STOCINT)
H = Time horizon
I_p = Incomes due to product p
Inv_{pt} = Initial inventory of product p in time interval t
$\overline{Inv}_{pt}$ = Average inventory of product p in time interval t
Inv_{pts} = Initial inventory of product p in time interval t in scenario s
$\overline{Inv}_{pts}$ = Average inventory of product p in time interval t in scenario s
Inv_{p0} = Initial inventory of product p
$ISales_{pt}$ = In house sales of product p in time interval t (used in DETPRICE)
$ISales'_{pt}$ = In house sales of product p in time interval t (used in DETSCHED and DETINT)
$ISales'_{pts}$ = In house sales of product p in time interval t in scenario s used in STOCINT)
K_p = Coefficient rate between demand parameters
m_{ps} = Demand coefficient of product p in scenario s
m_p = Demand coefficient of product p
O_{Area} = Opportunity Area
OV = Opportunity Value
$PCW_{pp'}$ = Cost of the wastes generated when changing from product p to p'
$Price_p{}^{lo}$ = Lower bound of price of product p
$Price_p{}^{up}$ = Upper bound of price of product p
$Price_p$ = Price of product p
$prob_s$ = Probability of scenario s
Q = Total production volume
Q_{pt} = Amount of product p manufactured in time interval t (used in DETPRICE)
Q'_{pt} = Amount of p manufactured in t (used in DETSCHED, DETINT and STOCINT)
Q_p = Production volume of product p
R_{Area} = Risk Area
r_{ip} = Capacity factor of product p with respect to resource i
R_i = Total amount of resource i
RAR = Risk Area Ratio
$\overline{T}_t$ = Final time of time interval t
TF_{lj} = Finishing time of stage j of batch l
TF_{lJ} = Finishing time of final stage J of batch l
TF'_{lJt} = Finishing time of final stage J of batch l in time interval t
TI_{lj} = Initial time of stage j of batch l
top_{pj} = Processing time of stage j of product p
$Sales_p$ = Total sales of product p
$Sales_{pt}$ = Total sales of product p in time interval t (used in DETPRICE)
$Sales'_{pt}$ = Total sales of product p in time interval t (used in DETINT)
$Sales_{pts}$ = Total sales of product p in time interval t in scenario s (used in STOCINT)
VaR = Value at Risk
X_{lp} = Binary variable (1 if batch l belongs to product p, 0 otherwise)
Y_{lt} = Binary variable (1 if batch l is finished in time interval t, 0 otherwise)
Z_{pr} = Binary variable (1 if discrete price r of product p is selected, 0 otherwise)

Greek characters

α_p = Unitary inventory cost of product p
Δr_{ip} = Increment of capacity factor of product p with respect to resource i

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