Model Reformulation and Global Optimization of Oil Production Using Gas Lift

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ABSTRACT: Gas lift technology is one of the most common methods used for wells with insufficient reservoir pressure. It makes use of gas reinjection to enhance individual well production. When several wells are used to produce a reservoir, a problem arises regarding the selection about how to produce each well so that overall production can be maximized. To accomplish it, one needs to determine the optimal production of each well and how much gas ought to be injected. In this article, we revise previous efforts and present a new MINLP model, which we solve globally using RYSIA, a recently developed global optimization algorithm (Faria and Bagajewicz, AICHE J. 58 (8) 2320–2335).

1. INTRODUCTION

Oil demand growth and the associated economics forces industry to search for more efficient techniques to increase production. For instance, to avoid the decay of the oil wells production flow, some methods are used to sustain the energy of the pressurized reservoir to make the oil wells economically viable. The gas-lift process is one of the most used methods for artificial lift.

Specifically, gas lift technology is used to increase the oil production in wells with insufficient or none reservoir pressure to be produced. The gas produced with oil, once separated, dried and compressed, can be reinjected in the oil wells. The gas lift process (Figure 1) starts with the continuous pumping of natural gas through the gas lift choke to the annular space around the production column. When the annular pressure in the bottom is bigger than the pressure of the production column, the injection valve opens and the gas injected goes inside the column. The density of the column fluid is reduced allowing the reservoir pressure to push the fluid produced to the surface. The drop pressure in the annular closes the injection valve and it remains closed until the annular bottom pressure increases again to be bigger than the column pressure, restarting the injection cycle. Thus, the gas lift injection is able to raise the oil production. However, a huge gas lift injection may cause friction losses, decreasing the well production. Furthermore, high rates of gas feed influence the process costs and well productivity.

However, in some fields or platforms, there is not always enough gas available, and consequently, an optimal gas lift...
allocation policy needs to be implemented to achieve maximum oil production.

Gas-lift allocation has been the object of previous research. Alarcón et al.² presented an NLP formulation and solved the problem using an SQP method. Teixeira³ studied the same problem solving it using a hybrid strategy: an MILP-SSM algorithm to find the best initial value to use the sequential quadratic programming (SQP) to find the optimum solution. In turn, Camponogara et al.⁴ used a branch-and-cut algorithm, Silva et al.⁵ compared a single and a multiple routing of wells to headers and developed a MILP formulation using multidimensional piecewise-linear functions. Mahmudi and Sadeghi⁶ used a Genetic algorithm in an integrated continuous gas lift model to maximize the net present value of the field and Ray and Sarker⁷ introduced an evolutionary algorithm with a multiobjective formulation. Gunnerud et al.⁸ presented a method for optimization of the oil and gas production from the reservoirs to the inlet of the first separator. The pressure at the separator and the gas and water capacities in this one are the boundary conditions. The optimization used is based on piecewise linearization of all nonlinearities and on decomposition of one problem into smaller subproblems, which is solved by using a column generation in a Branch and Price (B&P) framework. Finally, Silva and Camponogara⁹ developed a computational and simulation analysis for an optimization of proposed framework for pressure drops and well production functions. They proposed a multidimensional piecewise linear model for production optimization of oil fields with gas lift injection. In their method, the MILP reformulations were approximations based on hypercubes and simplexes.

One of the problems in all the aforementioned efforts is that the optimum found is most of the time a local optimum at best or the method used does not even produce a feasible solution. This is a well-known phenomena of local solvers used in nonconvex MINLP problems. In addition, some features that are usually handled with 0–1 variables, such as the activation of a well or not in our case, are sometimes handled using tanh(·) functions, which introduces further nonconvexities and a stiﬀ function in a small domain region.

In this article, we first review the model as it has been presented in a previous work, and then present a new MINLP formulation. Next, we show how this model can be solved to global optimality using a recently developed bound contraction methodology.¹¹

2. MODELS

2.1. Optimization Model. One model used to find the optimum gas lift injection values for each well was presented by Teixeira.¹² The oil production is associated with the gas lift injection in each well, that is limited by its resources.

The objective of the problem is to maximize the total amount of oil production $S$ in wells with insufficient or none reservoir pressure to oil production:

$$\maximize \ S = \sum_{j=1}^{Ns} q_{oj} + \sum_{i=1}^{Ns} q_{oi}$$

where $q_{oi}$ is the wells’ oil production and $N$ is the number of wells. The subscript “$j$” represents wells capable of producing naturally; however, gas lift is injected to improve its performance; and the subscript “$i$” represents wells that are not capable to produce naturally with their own reservation pressure. The gas lift injection in each well is the decision variable, since this is associated with the oil production.

The interaction among wells is neglected and the oil production of wells capable or not to produce with their reservoir pressure, respectively, are given by (Alarcón et al.;³ Teixeira⁴):

$$q_{oj}(q_{ig}) = c_{ij} + c_{zj}q_{ig} + c_{yj}q_{ig}^2 + c_{y} \ln(q_{ig} + 1)$$  

$$q_{oi}(q_{ig}) = 0.5 \left[ 1 + \tanh \left( \frac{q_{ig} - q_{ig}^*}{10^{-6}} \right) \right] c_{i} + c_{z}q_{ig} + c_{y}q_{ig}^2$$

$$+ c_{y} \ln(q_{ig} + 1)$$  

where $c_{ij}, c_{zj}, c_{yj}$ and $c_{y}$ are parameters. The term

$$0.5 \left[ 1 + \tanh \left( \frac{q_{ig} - q_{ig}^*}{10^{-6}} \right) \right]$$

in eq 3 represents the oil production discontinuity in wells that are not capable to produce naturally. This term assumes a value of 1 when the better decision is to activate the well, or zero when the decision is not to produce this well. The division by $10^{-6}$ assures that the change from zero to one happens in a very small interval around $q_{ig}^*$, which is the threshold where the flow goes from zero to a finite value. The term introduces a severe nonconvexity to the model, which we intend to remove.

The optimal gas lift injection to maximize oil production is limited by the operational limits of each well:

$$q_{ig}^{\min} \leq q_{ig} \leq q_{ig}^{\max}$$  

by the platform compression capacity:

$$\sum_{j=1}^{Ns} (RGOq_{oj} + q_{ig}^*) + \sum_{i=1}^{Ns} (RGOq_{oi} + q_{ig}) \leq q_{gtc}$$  

the amount of available gas to inject:

$$\sum_{j=1}^{Ns} q_{oj} + \sum_{i=1}^{Ns} q_{oi} \leq q_{gtc}$$  

the fluid handling capacities:

$$\sum_{j=1}^{Ns} \left( \frac{100}{100 - \text{BSW}_j} q_{oj} \right) + \sum_{i=1}^{Ns} \left( \frac{100}{100 - \text{BSW}_i} q_{oi} \right) \leq q_{ltc}$$  

and the water-treatment processing capacity:

$$\sum_{j=1}^{Ns} \left( \frac{\text{BSW}_j}{100 - \text{BSW}_j} q_{oj} \right) + \sum_{i=1}^{Ns} \left( \frac{\text{BSW}_i}{100 - \text{BSW}_i} q_{oi} \right) \leq q_{wtc}$$

In these equations, $q_{ig}^{\min}$ is the minimum gas lift rate in each well, $q_{ig}^{\max}$ is the maximum gas lift rate in each well, $RGO$ is the gas oil ratio, $q_{gtc}$ is the platform compression capacity, $q_{ig}$ is the amount of available gas to inject, $q_{ltc}$ is the fluid handling capacities, BSW is the basic sediments and water (%), and $q_{wtc}$ the water-treatment processing capacity.

2.2. Reformulated Model. In our MINLP model, the oil production for wells capable of producing naturally are obtained as described in eq 2. On the other hand, for wells
that are not capable to produce naturally with their own reservation pressure, the oil production should be measured by eq 3. But, instead of relying on the use of the \( \tanh(\cdot) \) function to define whether a well is or not active, which introduces several nonconvexities, we introduce a binary variable \( y_i \) which indicates that the well is producing \( (y_i = 1) \) or not \( (y_i = 0) \). Thus, eq 3 is rewritten as follows:

\[
q_{\text{y}}(q_{ig}^*) = y_i [c_i + c_2 q_{ig} + c_3 q_{ig}^2 + c_4 \ln(q_{ig} + 1)]
\] (9)

The following equations define the value of \( y_i \):

\[
(q_{ig} - q_{ig}^*) - (q_{ig}^{\text{max}} - q_{ig}^*) y_i \leq 0
\] (10)

\[
(q_{ig} - q_{ig}^*) + q_{ig}^{\text{max}} (1 - y_i) \geq 0
\] (11)

\[
q_{ig} - q_{ig}^{\text{max}} y_i \leq 0
\] (12)

Indeed, when \( y_i = 0 \) there is no gas lift injection at this well, as eq 12 reduces to \( q_{ig} \leq 0 \). However, \( q_{ig} \) is a positive variable.

Therefore: \( q_{ig} = 0 \) and eq 9 forces \( q_{ig} (q_{ig}^*) \) to be zero. In turn, eq 10 reduces to \( q_{ig} \leq q_{ig}^{\text{max}} \), which is trivial, and eq 11 reduces to \( q_{ig} - q_{ig}^{\text{max}} \leq q_{ig} \), which is also trivial. When \( y_i = 1 \), gas lift injection takes place. In this case, eq 12 is trivial (it reduces to \( q_{ig}^{\text{max}} \leq q_{ig}^{\text{max}} \), its natural upper bound), eq 10 reduces to \( q_{ig} \leq q_{ig}^{\text{max}} \), also trivial, and eq 11 reduces to \( q_{ig} \geq q_{ig}^{\text{max}} \), the desired limit.

One can write the expression using three big M constraints, but our approach is better because it omits such big M constraints and therefore produces better bounds in any MILP problem used.

2.3. MINLP Model. Equations 2 and 9, however, contain products of integer and continuous variables as functions of variables, which need some reformulation to become a set of equations compatible with classical formulations of MILP models, namely those linear in the integers. Thus, we linearize this expression introducing new positive variables: \( P_i = q_{ig}^2 \) and \( R_i = \ln(q_{ig} + 1) \) in the oil production equation for wells capable of producing naturally, and \( w_i = y_d q_{ig} x_i = y_d q_{ig}^2 \) and \( z_i = y_d \ln(q_{ig} + 1) \) for wells that are not capable to produce naturally with their own reservation pressure. Thus, eqs 2 and 9 are rewritten by eq 13 and 14.

\[
q_{\text{y}}(q_{ig}^*) = c_i + c_2 q_{ig} + c_3 P_i + c_4 R_i
\] (13)

\[
q_{\text{y}}(q_{ig}^*) = y_i c_i + w_i c_j + x_i c_j + z_i c_j
\] (14)

The equations for new variables, \( w_i = y_d q_{ig} \), \( x_i = y_d q_{ig}^2 \), and \( z_i = y_d \ln(q_{ig} + 1) \) are replaced by a linear set of three equations for each, as follows:

\[
w_i - q_{ig}^{\text{max}} x_i \leq 0
\] (15)

\[
w_i \geq 0
\] (16)

\[
x_i - (q_{ig}^{\text{max}})^2 x_i \leq 0
\] (17)

\[
x_i \geq 0
\] (18)

\[
z_i - [\ln(q_{ig}^{\text{max}} + 1)] x_i \leq 0
\] (19)

\[
z_i \geq 0
\] (20)

When \( y_i = 0 \) there is no gas lift injection at this well, so also \( w_i \), \( x_i \), and \( z_i \) are zero. When \( y_i = 1 \), gas lift injection takes place and the values of \( w_i \), \( x_i \), and \( z_i \) can be between zero and the maximum value obtained by the corresponding function, \( q_{ig}^{\text{max}} \), \( (q_{ig}^{\text{max}})^2 \), and \( \ln(q_{ig}^{\text{max}} + 1) \).

3. GLOBAL OPTIMIZATION METHODOLOGY (RYSIA)

Most global optimization solvers use branch and bound as part of their solution strategy. Instead, our method (RYSIA) focuses on a strategy called “bound contraction”.

The original problem is relaxed in such a way that a global solution can be obtained. For minimization, the solution of this relaxed problem is called a lower bound (upper bound for a maximization one, as in this article). The original MINLP problem constitutes a source for an upper bound (lower bound in maximization). We note that the original MINLP problem can be solved using standard MINLP local solvers with the sometimes very important aid of initial points provided by solutions of the lower (upper) bound. In addition, any other ad-hoc technique to obtain feasible points of the original problem, like an experienced guess made with or without information on the lower (upper) bound is acceptable, as long as it respects the current box bounds of variables.

One way of constructing the lower (upper) bound is relaxation to a linear MILP. One could actually leave some convex constraints if one wants, but in our experience this leads to a larger computational time. The way to relax a model is to isolate the nonlinearities and use some form of partitioning of some of the variables involved in intervals. This partitioning, accompanied by allowing the variable in question float between the interval bounds is what leads to a linear model. Examples of how this is done for bilinear terms were presented by Faria and Bagajewicz. Examples of partitioning techniques for other type of nonlinear terms (square roots of bilinear terms, power functions, etc.) were introduced by Faria et al. and Kim and Bagajewicz. In fact, in this article we resort to linearization/relaxation of nonlinear terms such as powers and logarithms using our special bounding techniques to obtain our linear bounding problem.

The global optimization strategy is now summarized as follows: We run the lower-bound model first. Then we use the result of the lower-bound model as initial values for the upper-bound model (or as stated by some ad-hoc procedure). At this point there are two options: (a) use a small number of intervals in the partitioning procedure, or (b) use a larger number of intervals compatible with memory and time, first.

In the former case, the gap will be larger, while in the latter it is small. If option b is feasible and allows an acceptable small gap in the first iteration, then it is preferred for its simplicity. Otherwise, if bound contraction is needed, then option a is preferred because it has larger chance of bound contracting.

The bound contraction procedure used by RYSIA is an interval elimination strategy presented by Faria and Bagajewicz and Faria et al. We summarize the basic strategy for the case of objective function minimization next. Further details of different strategies can be found in the original paper.

1. Run the lower bound model to calculate a lower bound (LB) of the problem and identify the partitions containing the solution of the lower bound model.
2. Run the original MINLP initialized by the solution of the lower-bound model to find an upper-bound (UB) solution. If there is failure use ad-hoc alternatives.
3. Calculate the gap between the upper-bound solution and the lower-bound solution. If the gap is larger than the
tolerance, the solution was found. Otherwise go to the step 4.

4. Run the lower-bound model forbidding one of the partitions identified having the solution (this is done in step 1). This is called single elimination. Other options exist (see Faria and Bagajewicz). If the solution is infeasible or if it is feasible but larger than the upper bound, then all the partitions that have not been forbidden for this variable are eliminated. The surviving region between the new bounds is repartitioned. If the solution is feasible but lower than the upper bound, one cannot bound contract.

5. Repeat step 4 for all the other variables, one at a time.

6. Go back to step 1 (a new iteration using contracted bounds starts).

The detailed illustration of the partition elimination using the bound contraction procedures was introduced in our previous publications using examples (Faria and Bagajewicz).

4. RELAXED MODEL

A relaxed upper-bound model (the problem is one of maximization here) is obtained by partitioning certain variables and then using it to linearize nonconvex functions. We first discretize \( q_{ij} \), \( P_j \), \( R_j \), \( q_{ij} \) and \( x_i \) and \( z_i \) as follows:

\[
\sum_{d=1}^{D-1} \tilde{q}_{ij,d} \leq q_{ij} \leq \sum_{d=1}^{D-1} \tilde{q}_{ij,d+1} \quad (21)
\]

\[
\sum_{d=1}^{D-1} \bigg[(\tilde{q}_{ij,d})^2 \nu_{jd} \bigg] \leq P_j \leq \sum_{d=1}^{D-1} \bigg[(\tilde{q}_{ij,d+1})^2 \nu_{jd} \bigg] \quad (22)
\]

\[
\sum_{d=1}^{D-1} \bigg[\ln(\tilde{q}_{ij,d} + 1) \nu_{jd} \bigg] \leq R_j \leq \sum_{d=1}^{D-1} \bigg[\ln(\tilde{q}_{ij,d+1} + 1) \nu_{jd} \bigg] \quad (23)
\]

\[
\sum_{d=1}^{D-1} \tilde{q}_{ij,d} \nu_{jd} \leq \tilde{q}_{ij} \leq \sum_{d=1}^{D-1} \tilde{q}_{ij,d+1} \nu_{jd} \quad (24)
\]

\[
\sum_{d=1}^{D-1} \tilde{q}_{ij,d} \nu_{jd} \leq \nu_i \leq \sum_{d=1}^{D-1} \tilde{q}_{ij,d+1} \nu_{jd} \quad (25)
\]

\[
\sum_{d=1}^{D-1} \bigg[(\tilde{q}_{ij,d})^2 \nu_{jd} \bigg] \leq x_i \leq \sum_{d=1}^{D-1} \bigg[(\tilde{q}_{ij,d+1})^2 \nu_{jd} \bigg] \quad (26)
\]

\[
\sum_{d=1}^{D-1} \bigg[\ln(\tilde{q}_{ij,d} + 1) \nu_{jd} \bigg] \leq z_j \leq \sum_{d=1}^{D-1} \bigg[\ln(\tilde{q}_{ij,d+1} + 1) \nu_{jd} \bigg] \quad (27)
\]

where \( \tilde{q}_{ij} \) and \( \tilde{q}_{ij} \) is a set of \( D \) interval points defined as follows:

\[
\tilde{q}_{ij,d} = q_{ij}^{\min} + (d - 1) \left( \frac{q_{ij}^{\max} - q_{ij}^{\min}}{D - 1} \right), \quad \forall \ d = 1, \ldots, D
\]

\[
\tilde{q}_{ij,d} = q_{ij}^{\min} + (d - 1) \left( \frac{q_{ij}^{\max} - q_{ij}^{\min}}{D - 1} \right), \quad \forall \ d = 1, \ldots, D
\]

In turn, \( \nu_{jd} \) and \( \nu_{jd} \) are binary variables, one of which is 1 and the rest zero. This is guaranteed by the following equations:

\[
\sum_{d=1}^{D} \nu_{jd} = 1 \quad (30)
\]

\[
\sum_{d=1}^{D} \nu_{id} = 1 \quad (31)
\]

Having defined the lower- and upper-bound models, the bound contraction algorithm we use is the one proposed by Faria and Bagajewicz. However, as we shall see, the algorithm actually converges in the first iteration when the number of intervals is high (\( D > 1000 \)) (option b above) using a reasonable amount of time. For larger problems one might find that increasing the number of intervals consumes too much time and one might want to use a smaller number of intervals and perform bound contraction. One such case is encountered when using RYSIA to solve heat exchanger network superstructure problems (Kim and Bagajewicz).

5. RESULTS AND DISCUSSION

The platform capacities values are given in Table 1 and the RGO and BSW values for each well are shown in Table 2.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value (m³/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{1%} )</td>
<td>2 000 000</td>
</tr>
<tr>
<td>( q_{2%} )</td>
<td>6 000</td>
</tr>
<tr>
<td>( q_{3%} )</td>
<td>12 000</td>
</tr>
<tr>
<td>( q_{4%} )</td>
<td>1 000 000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>well</th>
<th>RGO (m³/m³)</th>
<th>BSW (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
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<td>4</td>
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<td>60</td>
<td>60</td>
</tr>
<tr>
<td>11</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>12</td>
<td>114</td>
<td>35</td>
</tr>
</tbody>
</table>

The parameters \( c_1 \), \( c_2 \), \( c_3 \) and \( c_4 \) are obtained when the data are compared with the mathematic fit of eq 2 (Table 3). The data are presented in Figure 2 where the gas lift performance curves for each well are depicted.

The MINLP model exhibits 25 continuous variables, 7 binary variables, and 38 equations. The relaxed model, in turn, has 1000 partitions, 56 continuous variables, 11 995 binary variables, and 178 equations. We tried to solve our reformulated MINLP problem using DICOPT directly. When no initial values or different initial random values were used, the problem was feasible. However, the answer found was not the optimum. The optimum value was found when the problem was solved by RYSIA, using DICOPT for the LB and CPLEX for the UB.
Taking advantage of the fact that the program runs relatively fast, we used the number of partitions \((D)\) from 10 to 20,000, using 0 and 250,000 m\(^3/d\) as limits. In this example, the LB model (original MINLP model) has 38 equations and 32 variables (7 binaries). For \(D = 100\), the UB (relaxed linear model) has 178 equations, 1251 variables (1195 binaries). For \(D = 1000\), the UB has 12,051 variables (11,995 binaries) and when \(D = 10,000\), the UB has 120,051 variables (119,995 binaries). The problems were solved in a Windows 7 Home Premium workstation, using an Intel Core i5-2450 M CPU at 2.50 GHz, 64 Bits, and 6 GB of RAM.

Of these, the results with 1% gap or smaller are reported in more detail (Table 4). The execution time is mostly consumed by the UB (the LB takes a small fraction of a second). As a result, we find that the difference between lower bound and upper bound are very small, and therefore no bound contraction is needed. The optimum value for lower and upper bound models are presented in Table 4, and the gas lift injection values are shown in Table 5. For comparison, we show the results of UB and LB for 10, 100, 1000, and 10,000 intervals, so that the progression of time is noted.

Inspecting Table 5 it is possible to say that for a larger number of intervals, the gap between UB and LB is smaller. Good results, with a gap smaller than 0.1% are found for \(D\) equal or bigger than 1000 using a very small amount of time. One could start with a small number of intervals and actually enter into the bound contraction steps that RYSIA would use, but we find it will take a longer time because the UB, the time-consuming part, would have to be solved several times in each iteration.

We also used the method proposed by Teixeira,\(^4\) which uses MILP with linearization by parts applied in this example. We do not discuss here issues such as the number of piece-wise sections used vs the number of intervals, etc., and concentrate on the type of solutions obtained. In our case, the optimum solution found is near the optimum solution found by RYSIA methodology (Table 6). However, the values for some wells are very different, indicating that this problem has several suboptimal solutions. Although they can be enumerated in decreasing order of objective function, using several different approaches, it is best to enrich the problem with new constraints. For example, one could add a minimum value for each well, when the well is being produced. For example if we set a minimum of 1000 m\(^3/d\) one would at the very least get the result of well 11 in Table 6 to a number larger than or equal to 1000 m\(^3/d\), while maintaining well 6 at zero. The important point is that with the method proposed by Teixeira\(^4\) one arrives at a solution, but does not know if it is global or not.

Comparing with other solvers, BARON\(^15\) identifies the solution early (after 54 s), but spends a lot of time trying to close the gap between upper and lower bound. Thus, after 1000 CPU seconds the gap was still at 11.45%. In turn, ANTIGONE\(^16\) solves in a very small time (0.14 s), but it finds a slightly lower solution (7,661,003 m\(^3/d\)) with very different flows in the wells as compared to those shown in Table 6. Table 7 summarizes this comparison.

### Table 3. Parameters \(c_1, c_2, c_3,\) and \(c_4\) for Each Well

<table>
<thead>
<tr>
<th>well</th>
<th>(c_1 \times 10^{-2})</th>
<th>(c_2 \times 10^3)</th>
<th>(c_3 \times 10^8)</th>
<th>(c_4 \times 10^5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>4</td>
<td>7.4203</td>
<td>0.3880</td>
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<td>0.2843</td>
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<tr>
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<td>1.6117</td>
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<tr>
<td>6</td>
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<tr>
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<td>0.6350</td>
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</tr>
<tr>
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<td>-14.0625</td>
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<td>-0.2422</td>
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</tr>
<tr>
<td>9</td>
<td>-14.0625</td>
<td>0.2105</td>
<td>-0.2422</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>-20.1385</td>
<td>3.2610</td>
<td>149.7788</td>
</tr>
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</table>

### Table 4. LB and UB Optimum Values

<table>
<thead>
<tr>
<th>(q_o (m^3/d))</th>
<th>(D)</th>
<th>UB</th>
<th>LB</th>
<th>difference (%)</th>
<th>execution time (sec)</th>
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<tr>
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<td>7785.856</td>
<td>0.0493</td>
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</tr>
<tr>
<td>10000 7786.246</td>
<td>10000</td>
<td>7785.856</td>
<td>0.0050</td>
<td>26.052</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Gas lift performance curves.
In this article we developed a fast and reliable model for the optimization of gas injection in wells. We replace old NLP formulations with a new MINLP model, which we solve globally. We intended to use RYSIA bound contraction methodology, but the runs were so fast in a normal PC for a large number of partitions that we decided to build a large lower bound model that allows convergence without the need of bound contraction. We also found that RYSIA outperforms Baron and Antigone.

Larger problems may need to rely on a small number of partitions and bound contraction because a large number of intervals may take too long. Extensions to models requiring more complex restrictions, such as inclusion of pressure drops, two phase fluid flow, and optimal back pressures, and kick off wells, can be made. We believe that at some point bound contraction will be needed and we do not know whether these additions will change the answers radically or just slightly, something to explore in future work.

## ASSOCIATED CONTENT

Supporting Information
The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.iecr.6b00223.

Stages/substages-wise superstructure model; lower bound model; lifting partition; solution strategy used by RYSIA; examples (PDF)

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**Notes**
The authors declare no competing financial interest.

## NOMENCLATURE

BSW = basic sediments and water
D = interval points
LB = lower bound
RGO = gas-oil ratio
$q_{gtc}$ = platform compression capacity
$q_{ig}$ = gas lift injection rate
$q_{igtc}$ = gas lift available to inject
$q_{ltc}$ = fluid handling capacities
$q_o$ = oil production
$w_t$ = water-treatment processing capacity
UB = upper bound
$\nu$ = binary variable

## REFERENCES


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### Table 5. Gas Lift Injection Rate Found by the UB and LB Models

<table>
<thead>
<tr>
<th>well</th>
<th>UB (m³/d)</th>
<th>LB (m³/d)</th>
<th>difference (%)</th>
<th>UB (m³/d)</th>
<th>LB (m³/d)</th>
<th>difference (%)</th>
<th>UB (m³/d)</th>
<th>LB (m³/d)</th>
<th>difference (%)</th>
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<tr>
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### Table 6. Comparison of Current Results (RYSIA) with Those of Teixeira

<table>
<thead>
<tr>
<th>well</th>
<th>$q_{ig}$ (m³/d)</th>
<th>RYSIA ($D = 10000$)</th>
<th>Teixeira 4</th>
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</thead>
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<td>60 150.000</td>
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<td>158 071.998</td>
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<tr>
<td>4</td>
<td>8 806.094</td>
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</tr>
<tr>
<td>5</td>
<td>139 381.476</td>
<td>120 030.000</td>
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<tr>
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<td>76 126.906</td>
<td>59 677.000</td>
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<td>99.263</td>
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<tr>
<td>12</td>
<td>109 383.817</td>
<td>120 300.000</td>
<td></td>
</tr>
<tr>
<td>objective $q_o$ (m³/d)</td>
<td>7 779.856</td>
<td>7 779.980</td>
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### Table 7. Comparison with Antigone and Baron

<table>
<thead>
<tr>
<th>remarks</th>
<th>execution time</th>
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</thead>
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<tr>
<td>RYSIA</td>
<td>gap smaller than 0.05%</td>
</tr>
<tr>
<td>ANTIGONE</td>
<td>lower optimum different flows</td>
</tr>
<tr>
<td>BARON</td>
<td>identifies solution after 54 s, cannot verify it is the global optimum</td>
</tr>
</tbody>
</table>

---

### 6. CONCLUSIONS

In this article we developed a fast and reliable model for the optimization of gas injection in wells. We replace old NLP formulations with a new MINLP model, which we solve globally. We intended to use RYSIA bound contraction methodology, but the runs were so fast in a normal PC for a large number of partitions that we decided to build a large lower bound model that allows convergence without the need of bound contraction. We also found that RYSIA outperforms Baron and Antigone.

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