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On the use of heat pumps in total site heat integration

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Abstract

The purpose of this paper is to discuss the role of heat pumps in total site heat integration. It will be shown how heat pumps can help de-bottleneck heat transfer so that maximum energy savings are attained. Non-linear targeting models to get maximum cost savings and optimum heat pumping temperature levels are presented. Finally, the paper describes optimal solutions with heat pumps that do not cross the pinch, contradicting an old result from pinch technology.

Keywords: Heat pumps; Total site heat integration; Energy savings; Targeting; Process design

1. Introduction

In several practical cases, heat integration across plants can lead to important energy savings (Rodera & Bagajewicz, 1999; Bagajewicz & Rodera, 2000, 2002). These savings can be implemented either directly using process streams or indirectly using an intermediate fluid such as steam or dowtherms. Morton and Linnhoff (1984) used the overlap of grand composite curves to show the maximum possible heat recovery between two plants using steam as an intermediate fluid. Ahmad and Hui (1991) also used the concept of overlapping grand composite curves to study direct and indirect heat integration and proposed a systematic approach for heat recovery schemes for inter-process integration. The concept of "Total Site" was introduced by Dhole and Linnhoff (1992) to describe a set of processes serviced by and linked through a central utility system. Finally, a linear programming model to target maximum energy savings for heat integration across plants was presented by Rodera and Bagajewicz (1999) who introduced the concepts of "assisting" and "effective" heat. They showed that heat transfer among plants can occur in three regions: above pinches, between pinches and below pinches, as shown in Fig. 1.

Rodera and Bagajewicz (1999) proved that heat transfer between pinches leads to energy savings, reducing the total site heating and cooling utilities. The amount of heat transferred in this zone is referred as "effective heat" (Q_E). In addition, they showed that no net savings could be obtained by transferring heat in the regions above or below pinches. However, heat transfer from Plant 1 to Plant 2 in either of these regions (Q_A, Q_B) can facilitate the transfer of heat in the region between pinches, increasing the energy savings, hence Q_A and Q_B are referred as "assisting heat". Rodera and Bagajewicz (1999) also showed that in some cases assisting heat could not be delivered to debottleneck one plant because the receiving plant is limiting such delivery and hence, maximum effective heat cannot be achieved. In this work, it will be shown how a heat pump can eliminate this limitation.

Several authors have addressed the use of heat pumps in the integration of a single plant, especially as refrigeration systems that would eliminate the need of expensive low-temperature cooling utilities (Shelton & Grossmann, 1986). Townsend and Linnhoff (1983) proposed qualitative guidelines for the placement of heat pumps and heat engines in order to minimize utility consumption. Colmenares and Seider (1987) introduced a non-linear programming model for the integration of heat pumps and heat engines in a single plant. In their model, they allowed multiple heat pumps to receive and transfer heat from several temperature intervals, result-

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Nomenclature

c	cost coefficient
C	operating cost
F	heat capacity flowrate
Q	heat flow
T	temperature
W	electric work
δ	heat cascaded down
γ	heat requirement
ΔT_{MIN}	minimum temperature difference
Subscripts	
i, k	cold temperature interval
j, l	hot temperature interval
A	assisting heat pump above the pinch
B	assisting heat pump below the pinch
E	effective heat pump
H	rejection of heat from heat pump to cold streams in a hot temperature interval
C	input to heat pump from hot streams in a cold temperature interval
CU	cooling utility
HU	heating utility
W	electric work

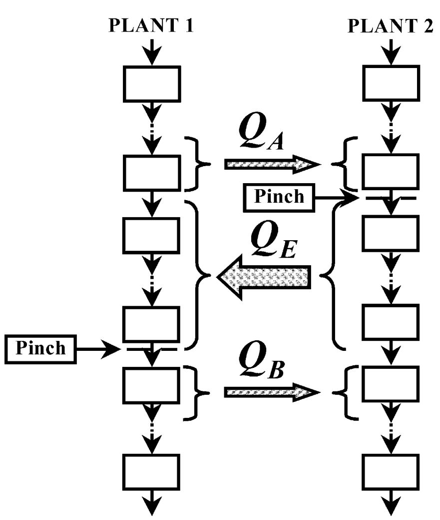

Fig. 1. Directions of heat transfer for the two plants case.

ing in complex and highly cross-linked schemes whose solution to optimality may be difficult to assess. In addition, the solution obtained depends on the temperature interval partition, which requires the division of the original intervals into smaller ones and increases the computational time.

Swaney (1989) introduced a new approach by proposing a transportation model to determine optimum heat loads of heat engines and pumps using also fixed temperature levels. More recently, Holiastos and Manousiouthakis (1999) gave an analytical form of the optimal temperature levels of a single reversible heat pump transferring heat between finite-capacity cold and hot reservoirs. Since they assumed the working fluid matches the temperature change in both hot and cold reservoirs, multiple (formally an infinite number) heat pumps are required to achieve the optimal solution.

Finally, Vaidyaraman and Maranas (1998) proposed a non-linear formulation for the synthesis of complex refrigeration systems with optimal refrigerant selection based on the approach of Shelton and Grossmann (1986). The advantage of this formulation is that under certain hypothesis it can be reduced to a linear problem.

A common weakness of all these models is that they use discrete temperature levels at which a heat pump can receive and send heat. Thus, a ‘‘fine’’ interval partition is needed to ensure that no good solutions are overlooked due to coarseness of the discretization, increasing the computational time required to solve the models. Since these methods are based on transshipment formulations, another drawback is that, in general, more than one heat exchanger will be required to satisfy each resulting heat load. In industrial heat pumps, it is not likely to have more than one condenser and evaporator within a single heat pump due to design considerations; thus, the solution given by those models may be difficult to implement.

In this article, temperature levels are considered decision variables instead of discrete fixed values. In this way, cross-linking of heat transfer like those resulting from transshipment models is avoided, giving therefore more realistic solutions. Additionally, the role of heat pumps on heat debottlenecking in the integration of multiple plants is discussed. For simplicity, the case of two plants is first discussed, including the use of

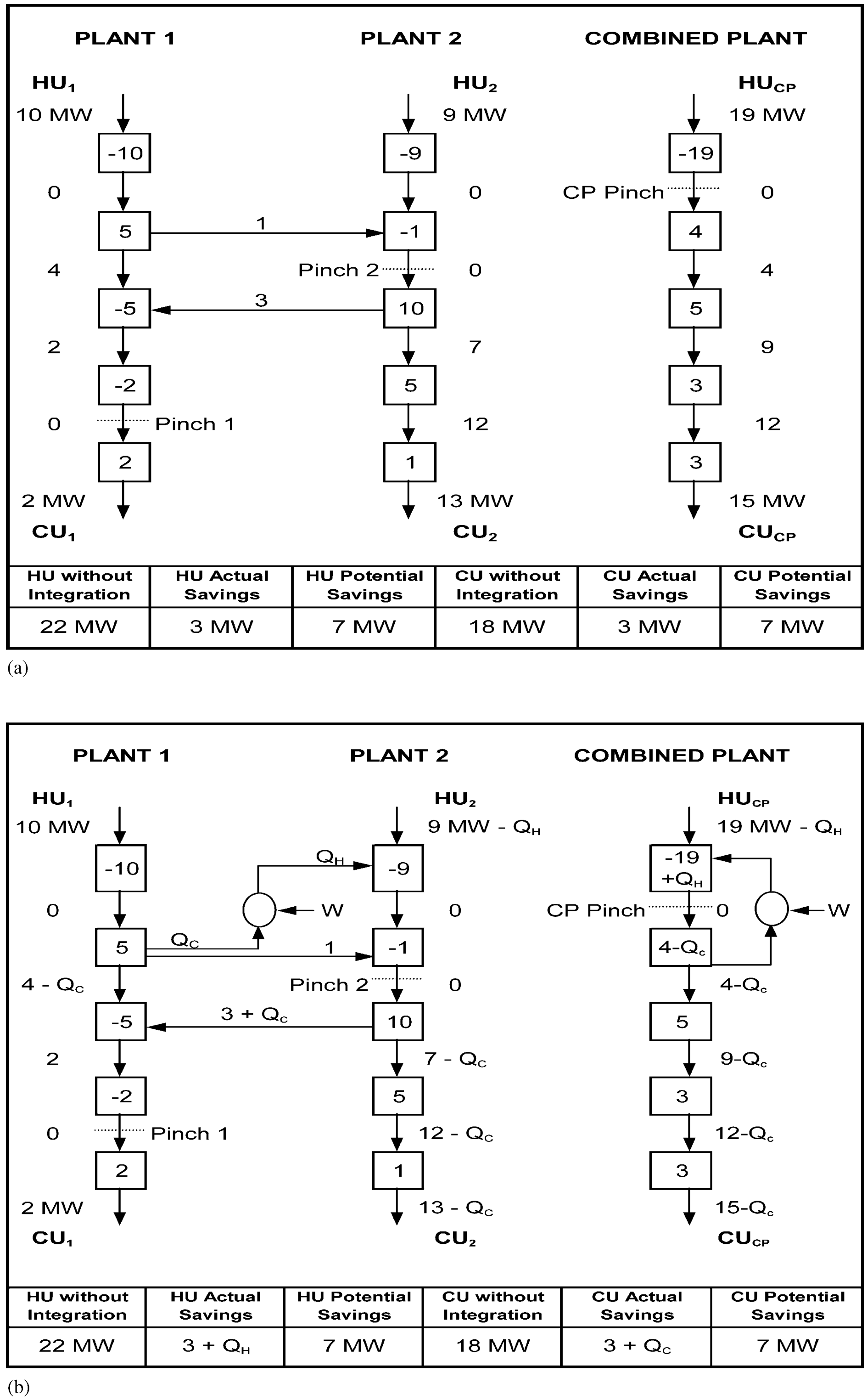

Fig. 2. Cascade diagram for Example 1. (a) No heat pumping. (b) With one heat pump.

grand composite diagrams. It is next shown that the combined plant cascade can be used for targeting, and therefore, results can be easily generalized to many plants. A simple formulation is developed to identify and target cost saving alternatives. One important result is that the old rule of not placing heat pumps that do not cross the pinch is not valid when cost is considered as the objective.

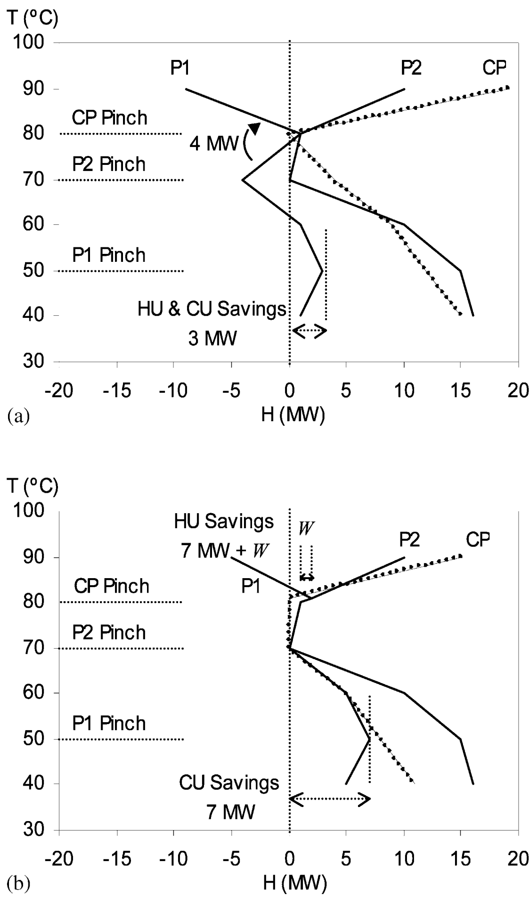

Fig. 3. Grand composite curves for Example 1. (a) No heat pumping. (b) With one heat pump.

2. Motivation for the use of heat pumps in Total Sites

Consider the case of two plants and the cascade diagrams of Fig. 2a, which show the interval heat balances and the current values of heating and cooling utilities for each plant as well as for the combined plant. Heat integration between plants is maximized using the model developed by Rodera and Bagajewicz (1999).

The energy savings obtained by directly integrating plants 1 and 2, with respect to the case without heat integration, are 3 MW. However, the maximum potential savings between pinches are 7 MW, which cannot be achieved because Plant 2 cannot receive assisted heat from Plant 1 in the second temperature interval. Indeed, only 1 MW can be transferred from Plant 1 to Plant 2 as assisting heat in this interval, while 4 MW have to be cascaded down to the next interval in Plant 1. If these 4 MW could be transferred to Plant 2 as assisting heat instead of cascading them down then maximum savings could be achieved. The only possibility to transfer these four units of heat from Plant 1 to Plant 2 is to send them to a higher temperature level, that is, using a heat pump. The cascade diagram for this scheme is showed in Fig. 2b, where heat balances for each interval are expressed in terms of the heat removed from the lower temperature interval, Q_C, and the heat transferred to the higher temperature, Q_H As a consequence of introducing a heat pump, the required cooling utility for the total site is reduced in Q_C units and the reduction attained in heating utility is Q_H units. This can be easily noticed using the combined plant cascade diagram and realizing that the heat pump crosses the combined pinch, leading to effective savings in both heating and cooling utilities.

In conclusion, whenever there is a limitation to transfer assisting heat above pinches, adding a heat pump in that zone can eliminate this restriction and the energy savings are increased. However, as we shall see later, the use of heat pumps is not limited to the case described where a heat pump is used to debottleneck assisting heat.

The grand composite curves for Example 1 are presented next. Fig. 3a shows the starting situation where the limitation to transfer assisting heat from Plant 1 to Plant 2 is evidenced by the contact of both composite curves at 80 °C. On the other hand, Fig. 3b shows the resulting composite curves for the case where 4 MW were pumped across the combined pinch.

3. Targeting model for one heat pump

In the following sections, simple models to determine favorable temperature levels for heat pumping are introduced. The targeting strategy is defined by recognizing that a limited (small) number of heat pumps will likely be more cost effective, due to the large fixed cost associated with compressors and heat exchangers. Thus, the strategy is to add one heat pump at a time and determine the incremental savings, which can then be weighed against the capital costs to decide if more heat pumps are cost-convenient. At a first design stage, the heat cascade of the combined plant is used for targeting purposes. Afterwards, the resulting heat pumping duties should be introduced into the targeting models proposed by Rodera and Bagajewicz (1999) in order to determine plant-to-plant optimum heat transfer which will render maximum energy cost savings.

Allowing the use of a single heat pump, the problem definition is:*Given a set of process heat demands and surpluses at different temperature intervals, find the temperature levels, heat flows, and required electric work for a heat pump in order to minimize the operating cost*.

The strategy proposed to find the problem optimum is as follows: first, a pair of intervals "i,j" is selected and a mathematical programming problem $P_{i,j}$ is solved to obtain optimal temperature levels, heat loads and required work for those intervals. Next, a new pair of intervals is selected based on the solution obtained previously. Whenever the objective function cannot be improved anymore, the optimum has been attained and the search procedure stops. The model equations and search procedure are described next. A schematic representation of the model is given in Fig. 4.

The operating costs for the cases without and with the use of a heat pump (C° and C, respectively) are:

$$C^{\circ} = c_{HU}Q^{\circ}_{HU} + c_{CU}Q^{\circ}_{CU} \tag{1}$$

$$C = c_{HU}Q_{HU} + c_{CU}Q_{CU} + c_{W}W \tag{2}$$

Since C° is a known constant, minimizing the total operating cost is equivalent to maximizing the difference between the cost without and with a heat pump, ΔC:

$$\Delta C = c_{HU}\Delta Q_{HU} + c_{CU}\Delta Q_{CU} + c_{W}W \tag{3}$$

Townsend and Linnhoff (1983) proved that a single heat pump could only lead to utility savings if placed across the pinch. This type of heat pumps will be referred in this work as “effective” heat pumps since they lead directly to energy savings. In later sections, it will be shown that heat pumps that do not cross the pinch (referred as “assisting” heat pumps) can indirectly increase the energy and cost savings.

For an effective heat pump taking heat from interval “i” and delivering it to interval “j” the energy savings and the cost reduction are given by:

$$\Delta Q^{E}_{HU} = Q^{H}_{ij} \tag{4}$$

$$\Delta Q^{E}_{CU} = Q^{C}_{ij} \tag{5}$$

$$\Delta C^{E}_{ij} = c_{HU}Q^{H}_{ij} + c_{CU}Q^{C}_{ij} - c_{W}W_{ij} \tag{6}$$

Considering a heat pump with a working fluid operating in a cycle receiving heat from process streams at the temperature interval “i” (cold heat source) and delivering it at interval “j” (hot heat sink) and using the hot temperature scale, the heat transferred and received at the correspondent intervals is expressed as:

$$Q^{C}_{ij} = F_i(T'_{i-1} - T^{C}_{ij}) \tag{7}$$

$$Q^{H}_{ij} = F_j(T^{H}_{ij} - T'_j) \tag{8}$$

$$F_i = \frac{Q_i}{T_{i-1} - T_i} \tag{9}$$

$$F_j = \frac{Q_j}{T_{j-1} - T_j} \tag{10}$$

where Q_i and Q_j are the heat loads of intervals i and j, F_i and F_j their respective heat capacity flowrates and T'_{i-1} and T'_j auxiliary temperatures defined by:

$$T'_{i-1} = T_{i-1} + \frac{\delta_{i-1}}{F_i} \tag{11}$$

$$T'_j = T_j - \frac{\gamma_{j+1}}{F_j} \tag{12}$$

In turn, δ_i is the heat cascaded down at each interval below the pinch and γ_j the heat requirement below each interval above the pinch and they are given by:

$$\gamma_j = \begin{cases} \gamma_{j+1} + Q_{j+1} & j = 1 \ldots p-1 \\ 0 & j = p \end{cases} \tag{13}$$

$$\delta_i = \begin{cases} 0 & i = p \\ \delta_{i-1} + Q_{i-1} & i = p+1 \ldots n \end{cases} \tag{14}$$

The decision variables T^{H}_{ij} and T^{C}_{ij} are the hot and cold temperatures of the working fluid, respectively. Notice that, above the pinch, the working fluid acts as a hot stream, and therefore, its temperature coincides with the registered in the hot scale, T^{H}_{ij}. On the other hand, below the pinch, the working fluid acts as a cold stream with temperature $T^{C}_{ij} - \Delta T_{MIN}$, since cold streams temperatures are shifted a value of ΔT_{MIN} when constructing the composite curves.

The relationships between heat and work for a reversible heat pump with a working fluid cycling between hot and cold temperatures, T^{H}_{ij} and $T^{C}_{ij} - \Delta T_{MIN}$, respectively, are:

$$W_{ij} = Q^{H}_{ij} - Q^{C}_{ij} \tag{15}$$

$$Q^{H}_{ij} = \frac{T^{H}_{ij}}{T^{C}_{ij} - \Delta T_{MIN}} \cdot Q^{C}_{ij} \tag{16}$$

Equating the definition of Q^{H}_{ij} given in equations (8) and (16) and using the definition of Q^{C}_{ij} given in equation (7), T^{H}_{ij} can be expressed in terms of T^{C}_{ij} for every combination of intervals crossing the combined plant pinch:

$$T^{H}_{ij} = \frac{F_j \cdot T'_j \cdot (T^{C}_{ij} - \Delta T_{MIN})}{(F_i + F_j) \cdot T^{C}_{ij} - (F_i \cdot T'_{i-1} + F_j \cdot \Delta T_{MIN})} \tag{17}$$

The feasible region for the problem is defined by the temperature intervals limits, which are given by the next set of equations.

$$T^{C}_{ij} \le T_{i-1} \tag{18}$$

$$T^{C}_{ij} \ge T_i \tag{19}$$

$$T^{H}_{ij} \le T_{j-1} \tag{20}$$

$$T^{H}_{ij} \ge T_j \tag{21}$$

Using (17) to eliminate T^{H}_{ij}, these temperature constraints can be expressed in terms of only one variable, T^{C}_{ij}:

$$T^{C}_{ij} \ge \max\left\{T_i, \frac{F_iT_{j-1}T'_{i-1} + F_j(T_{j-1} - T'_j)\Delta T_{MIN}}{F_iT_{j-1} + F_j(T_{j-1} - T'_j)}\right\} \tag{22}$$

$$T^{C}_{ij} \le \min\left\{T_{i-1}, \frac{F_iT'_{i-1}T_j + F_j(T_j - T'_j)\Delta T_{MIN}}{F_iT_j + F_j(T_j - T'_j)}\right\} \tag{23}$$

Finally, the one-variable optimization problem set to obtain the maximum cost savings between a pair of temperature intervals “i,j” crossing the combined plant pinch is detailed below equation (24).

$$P_{ij}^{E} = \begin{cases} \max \Delta C_{ij}^{E} = \left(c_{CU} + c_{W} - \dfrac{(c_{HU} - c_{W})F_j T_j'}{(F_i + F_j)T_{ij}^{C} - (F_i T_{i-1}' + F_j \Delta T_{MIN})} \right) F_i (T_{i-1}' - T_{ij}^{C}) \\ \text{s.t. } T_{ij}^{C} \le \min\left\{ T_{i-1}, \dfrac{F_i T_{i-1}' T_j + F_j (T_j - T_j')\Delta T_{MIN}}{F_i T_j + F_j (T_j - T_j')} \right\} \\ T_{ij}^{C} \ge \max\left\{ T_i, \dfrac{F_i T_{j-1} T_{i-1}' + F_j (T_{j-1} - T_j')\Delta T_{MIN}}{F_i T_{j-1} + F_j (T_{j-1} - T_j')} \right\} \end{cases} \tag{24}$$

In spite of being a non-linear problem, an analytical solution to P_{ij}^{E} can be obtained. Indeed, the objective function is concave. Therefore, the unique unconstrained optimum is a candidate for a solution, together with the solutions obtained by considering one or both constraints binding. Fig. 5 illustrates the possible cases. If the unconstrained maximum is located inside the feasible region of P_{ij}^{E}, as in Case I, then it automatically becomes the optimum for P_{ij}^{E}. However, if the unconstrained maximum is outside the feasible region of P_{ij}^{E}, the optimum for the constrained problem is placed at either the upper or the lower limit of the feasible region, as in cases II and III.

The first order necessary condition is used to find the unconstrained optimum of ΔC_{ij}^{E}. This condition is:

$$\frac{d\Delta C_{ij}^{E}}{dT_{ij}^{C}} = \frac{(c_W - c_{HU}) \cdot T_j' \cdot F_j^2 \cdot (T_{i-1}' - \Delta T_{MIN})}{((F_i + F_j) \cdot T_{ij}^{C} - (F_i \cdot T_{i-1}' + F_j \cdot \Delta T_{MIN}))^2} - (c_{CU} + c_W) = 0 \tag{25}$$

Solving (25) for T_{ij}^{C}, the candidate for unconstrained optimum of ΔC_{ij}^{E}, referred here as $T_{ij}^{C\circ}$, is found. Notice that the first order optimality condition is a quadratic equation, and therefore, two solutions exist.

$$T_{ij}^{C\circ +} = \frac{T_{i-1}' \cdot F_i + \left(\Delta T_{MIN} + \sqrt{\left(\dfrac{c_W - c_{CU}}{c_{CU} + c_W} \right) \cdot T_j' \cdot (T_{i-1}' - \Delta T_{MIN})} \right) \cdot F_j}{(F_i + F_j)} \tag{26}$$

$$T_{ij}^{C\circ -} = \frac{T_{i-1}' \cdot F_i + \left(\Delta T_{MIN} - \sqrt{\left(\dfrac{c_W - c_{CU}}{c_{CU} + c_W} \right) \cdot T_j' \cdot (T_{i-1}' - \Delta T_{MIN})} \right) \cdot F_j}{(F_i + F_j)} \tag{27}$$

However, only the first of these solutions is feasible. The fact that solution $T_{ij}^{C\circ -}$ is infeasible can be seen by replacing this solution in equation (17) and noting that this yields a negative value for the denominator $((F_i + F_j) \cdot T_{ij}^{C} - (F_i \cdot T_{i-1}' + F_j \cdot \Delta T_{MIN}))$ and consequently for the hot temperature T_{ij}^{H}. Given that temperatures in the Kelvin scale cannot be negative, one concludes that $T_{ij}^{C\circ -}$ is infeasible. For the sake of notational simplicity, given that only one solution is left, we eliminate the superscript $(+)$ to refer to the unconstrained optimum temperature.

To prove that $T_{ij}^{C\circ}$ is a maximizer of ΔC_{ij}^{E}, the second order optimality condition is checked:

$$\frac{d^2\Delta C_{ij}^E}{dT_{ij}^{C2}} = \frac{-2\cdot(c_W - c_{HU})\cdot T'_j\cdot F_j^2\cdot(F_i+F_j)\cdot(T'_{i-1}-\Delta T_{MIN})}{((F_i+F_j)\cdot T_{ij}^C-(F_i\cdot T'_{i-1}+F_j\cdot\Delta T_{MIN}))^3} < 0 \tag{28}$$

The second order condition is satisfied since all the terms are positive. Thus, equation (26) gives the global unconstrained maximum since both the first and second order optimality conditions are met. Moreover, the solution to P_{ij}^E will coincide with the unconstrained optimum if and only if $T_{ij}^{C^\circ}$ is inside its feasible region. Therefore, the solution to P_{ij}^E can be presented as follows:

$$T_{ij}^{C*} = \begin{cases} \min\left\{T_{i-1}, \dfrac{F_iT'_{i-1}T_j + F_j(T_j - T'_j)\Delta T_{MIN}}{F_iT_j + F_j(T_j - T'_j)}\right\} & \text{If } C_{ij}^1 > 0 \vee C_{ij}^2 > 0 \\ \dfrac{T'_{i-1}\cdot F_i + \left(\Delta T_{MIN} + \sqrt{\left(\dfrac{c_W - c_{CU}}{c_{CU}+c_W}\right)\cdot T'_j\cdot(T'_{i-1}-\Delta T_{MIN})}\right)\cdot F_j}{(F_i+F_j)} & \text{If } C_{ij}^1 \le 0 \wedge C_{ij}^2 \le 0 \wedge C_{ij}^3 \le 0 \\ \max\left\{T_i, \dfrac{F_iT_{j-1}T'_{i-1} + F_j(T_{j-1} - T'_j)\Delta T_{MIN}}{F_iT_{j-1} + F_j(T_{j-1} - T'_j)}\right\} & \text{If } C_{ij}^1 \le 0 \wedge C_{ij}^2 \le 0 \wedge C_{ij}^3 > 0 \end{cases} \tag{29}$$

with the parameters C_{ij}^k defined as follows.

$$C_{ij}^1 = \frac{F_iT_{j-1}(T'_{i-1} - T_{i-1})}{F_j(T_{j-1} - T'_j)(T_{i-1} - \Delta T)} - 1 \tag{30}$$

$$C_{ij}^2 = 1 - \frac{\min\left\{T_{i-1}, \dfrac{F_iT_jT'_{i-1} + F_j(T_j - T'_j)\Delta T_{MIN}}{F_iT_j + F_j(T_j - T'_j)}\right\}(F_i+F_j)}{F_iT'_{i-1} + F_j\Delta T_{MIN} + F_j\sqrt{R\cdot T'_j\cdot(T'_{i-1}-\Delta T_{MIN})}} \tag{31}$$

$$C_{ij}^3 = \frac{\max\left\{T_i, \dfrac{F_iT_{j-1}T'_{i-1} + F_j(T_{j-1} - T'_j)\Delta T_{MIN}}{F_iT_{j-1} + F_j(T_{j-1} - T'_j)}\right\}(F_i+F_j)}{F_iT'_{i-1} + F_j\Delta T_{MIN} + F_j\sqrt{R\cdot T'_j\cdot(T'_{i-1}-\Delta T_{MIN})}} - 1 \tag{32}$$

Notice that if $C_{ij}^1 > 0$ or $C_{ij}^2 > 0$ then placing a heat pump will not be convenient and the solution to P_{ij}^E is located at the lower limit of the feasible region. On the other hand, if C_{ij}^1, C_{ij}^2, and C_{ij}^3 are less or equal than zero then the optimal solution coincides with the unconstrained maximum. Finally, if C_{ij}^1 and C_{ij}^2 are less or equal zero and $C_{ij}^3 \ge 0$ the optimal solution is located at the upper limit of the feasible region.

So far, a solution for the problem has been developed for a given pair of temperature intervals. The next step is to outline a search procedure to explore different combinations of intervals and attain the global optimum, which corresponds to the combination "i,j" that yields maximum cost savings. The flow diagram of this procedure is outlined in Fig. 6. Before implementing this strategy, a lumping procedure like the one described by Colmenares and Seider (1987) is needed in order to reduce the heat cascade only to intervals with positive hot deficit above the pinch and positive cold deficit below it.

The procedure starts by setting the intervals for heat pumping equal to the first interval below the pinch (i) and above it (j), respectively. Afterwards, problem P_{ij}^E is solved and its optimal solution is compared with the interval limits. If T_{ij}^C coincides with the lower limit of the cold interval then interval i is expanded and the next interval in the cascade is added. Similarly, if T_{ij}^H coincides with the upper limit of the hot interval j then the previous interval in the cascade is added. This loop is repeated until $T_i \le T_{ij}^H < T_{i-1}$ and $T_j < T_{ij}^H \le T_{j-1}$ where the optimal solution is found.

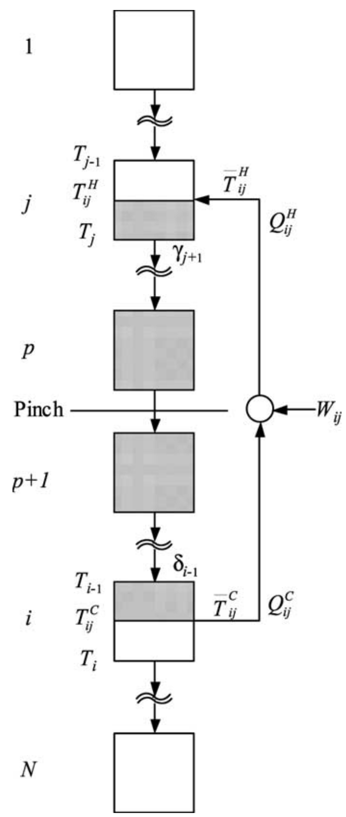

Fig. 4. Schematic representation of a heat pump using a combined plant heat cascade.

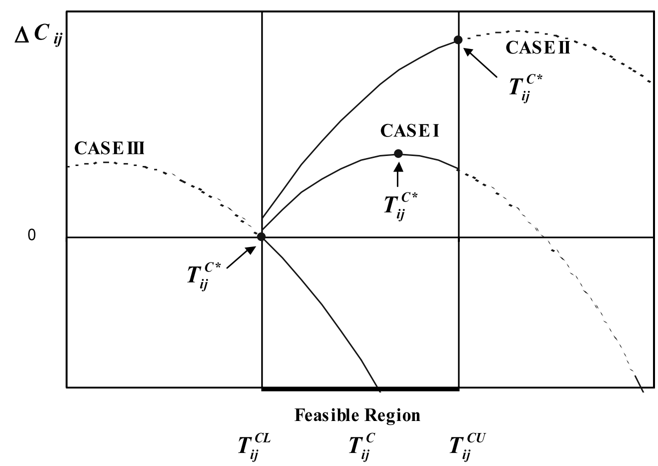

Fig. 5. Location of the optimal solution for problem P^E_{ij}.

4. Targeting model for two heat pumps

The possible arrangements of two heat pumps are shown in Fig. 7. Case (a), shows two effective heat pumps arranged in series and case (b) illustrates a crossed scheme of two effective heat pumps. Cases (c) and (d) show the use of "assisting" heat pumps, which is explained next.

By definition, an "effective" heat pump crosses the combined plant pinch and directly leads to cost savings. On the other hand, an "assisting" heat pump does not transfer heat across the pinch and is placed entirely either above or below it. In previous work, the use of assisting heat pumps has only been explored as a mean of avoiding the use of low temperature cold utilities. Here, it will be shown that a combination of an assisting and an effective heat pump may lead to higher savings than one or even two effective heat pumps. This fact has not been explored before, mainly because of a misunderstanding of the cross-pinch rule initially established by Townsend and Linnhoff (1983).

Above the pinch, an assisting heat pump is utilized to transfer heat from intervals with heat surplus to intervals with heat deficit located at higher temperatures, as shown in Fig. 7c. In this way, a larger amount of heat deficit at lower temperature (but still above the pinch) becomes available. This can be advantageous for two reasons. First, this deficit can now be serviced by heat coming from an effective heat pump increasing, therefore, the savings in both heating and cooling utilities. Notice that this deficit was not available before because it was being serviced by heat coming from the hotter intervals. The original limiting temperature from where heat could no longer be received from an effective heat pump is denoted as $T^H_{j_p}$. Secondly, the assisting heat pump reduces the amount of heating utility required at higher temperatures and facilitates the use of a lower temperature heating utility, which is generally less expensive. Thus, a cost reduction may also be obtained by placing an assisting heat pump appropriately.

Similarly, an assisting heat pump below the pinch allows heat surplus at higher temperatures to be transferred above the pinch by an effective pump (Fig. 7d), increasing the energy savings. The original limiting temperature, from where heat could no longer be sent to an effective heat pump is denoted $T^H_{i_p}$. In addition, the requirement of lower temperature cooling utility is reduced and facilitates the use of a higher-temperature, and thus cheaper, cooling utility.

To develop a targeting model that takes into account the use of assisting heat pumps, one must first compute the cost functions. For an assisting heat pump above the pinch, the cost difference is given by:

$$\Delta C^A_{kl} = (c_{HU} - c_W) \cdot W_{kl} \tag{33}$$

Since $c_W > c_{HU}$, then ΔC^A_{kl} is always negative. Using (7), (15) and (16), this function becomes:

$$\Delta C^A_{kl} = (c_W - c_{HU})\left(1 - \frac{T^H_{kl}}{T^C_{kl} - \Delta T_{MIN}}\right) F_k(T'_{k-1} - T^C_{kl}) \tag{34}$$

On the other hand, for an assisting heat pump below the pinch, the cost difference function is:

$$\Delta C^B_{kl} = -(c_{CU} + c_W) \cdot W_{kl} \tag{35}$$

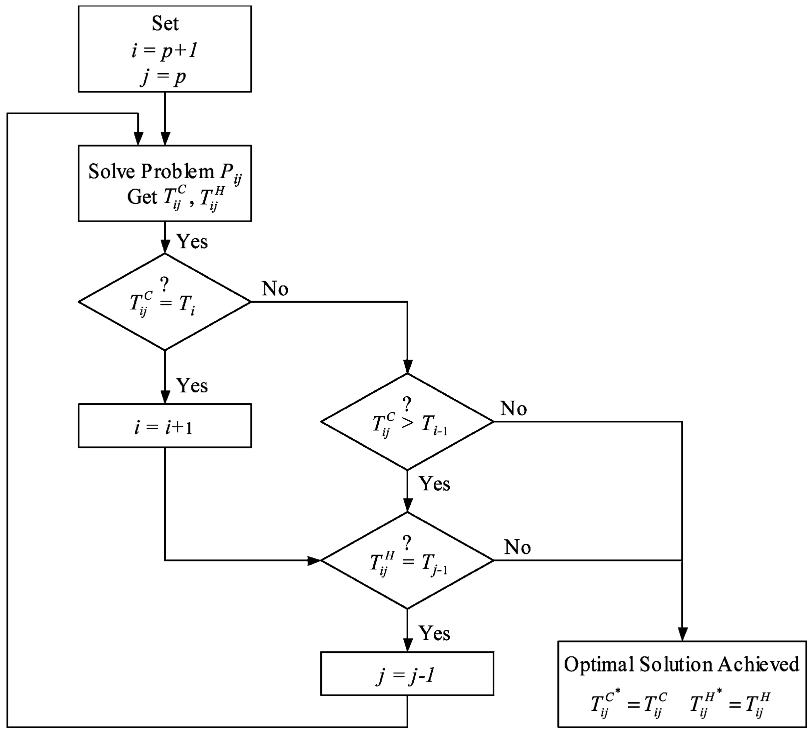

Fig. 6. Search algorithm to find the global optimum.

Notice that ΔC_{kl}^B is also negative since c_W, c_{CU} and W_{kl} are positive. The final expression for the cost difference function, ΔC_{kl}^B, is given by the following equation:

$$\Delta C_{kl}^B = (c_{CU} + c_W)\left(1 - \frac{T_{kl}^H}{T_{kl}^C - \Delta T_{MIN}}\right) F_k (T'_{k-1} - T_{kl}^C) \tag{36}$$

Combining these expressions with the costs functions for effective heat pumps, the total cost savings for arrangements of two heat pumps can be obtained. These results are detailed in the optimization problems shown next.

Notice that all the optimization problems presented below have only four variables: T_{ij}^C, T_{ij}^H, T_{kl}^C, and T_{kl}^H. All the remaining quantities are parameters that are known or can be calculated before running the optimization for each combination of intervals i,j,k,l.

In the case of two pumps, the solution strategy consists of identifying the combination of intervals i, j, k, l that yields the maximum cost savings. A search algorithm to explore all these combinations can be implemented easily and the number of cases to be screened remains small. Thus, there is no need for building complex MNLP schemes to screen them more efficiently. The problems to solve are outlined next.

Two effective heat pumps series scheme:

$$P_{ij,kl}^{EEs} = \begin{cases} \max\ \Delta C_{ij,kl}^{EEs} = \left(c_{CU} + c_W - \dfrac{(c_{HU} - c_W)T_{ij}^H}{T_{ij}^C - \Delta T_{MIN}}\right) F_i (T'_{i-1} - T_{ij}^C) + \left(c_{CU} + c_W - \dfrac{(c_{HU} - c_W)T_{kl}^H}{T_{kl}^C - \Delta T_{MIN}}\right) F_k (T'_{k-1} - T_{kl}^C) \\ \text{s.t. } T_{ij}^C \le T_{i-1} \quad T_{ij}^C \ge T_i \quad T_{ij}^H \le T_{j-1} \quad T_{ij}^H \ge T_j \quad T_{kl}^C \le T_{k-1} \quad T_{kl}^C \ge T_k \quad T_{kl}^H \le T_{l-1} \quad T_{kl}^H \ge T_l \\ T''_{k-1} = T'_{k-1} + \dfrac{F_i}{F_k}(T_{ij}^C - T'_{i-1}) \qquad T''_l = T'_l + \dfrac{F_j}{F_l}(T_{ij}^H - T'_j) \\ T_{ij}^H = \dfrac{F_j T'_j (T_{ij}^C - \Delta T_{MIN})}{(F_i + F_j)T_{ij}^C - (F_i T'_{i-1} + F_j \Delta T_{MIN})} \qquad T_{kl}^H = \dfrac{F_l T''_l (T_{kl}^C - \Delta T_{MIN})}{(F_k + F_l)T_{kl}^C - (F_k T''_{k-1} + F_l \Delta T_{MIN})} \end{cases} \tag{37}$$

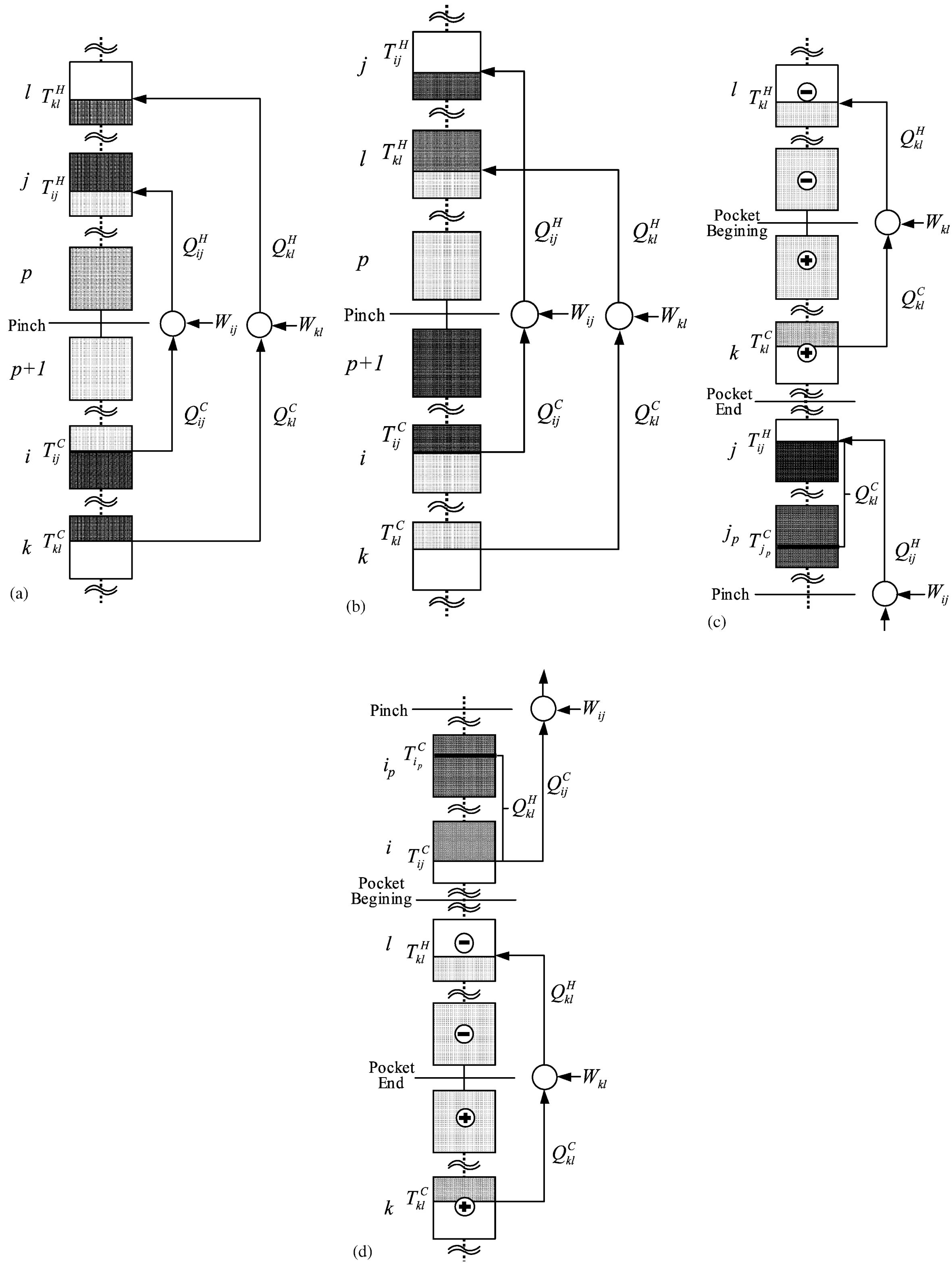

Fig. 7. Arrangements of two heat pumps.

Two effective heat pumps crossed series scheme:

$$P^{EEs}_{ij,kl}=\begin{cases}\max\ \Delta C^{EEc}_{ij,kl}=\left(c_{CU}+c_W-\dfrac{(c_{HU}-c_W)T^H_{ij}}{T^C_{ij}-\Delta T_{MIN}}\right)F_i(T'_{i-1}-T^C_{ij})+\left(c_{CU}+c_W-\dfrac{(c_{HU}-c_W)T^H_{kl}}{T^C_{kl}-\Delta T_{MIN}}\right)F_k(T'_{k-1}-T^C_{kl})\\ \text{s.t. } T^C_{ij}\le T_{i-1}\quad T^C_{ij}\ge T_i\quad T^H_{ij}\le T_{j-1}\quad T^H_{ij}\ge T_j\quad T^C_{kl}\le T_{k-1}\quad T^C_{kl}\ge T_k\quad T^H_{kl}\le T_{l-1}\quad T^H_{kl}\ge T_l\\ T''_{k-1}=T'_{k-1}+\dfrac{F_i}{F_k}(T^C_{ij}-T'_{i-1})\qquad T''_j=T'_j+\dfrac{F_l}{F_j}(T^H_{kl}-T'_l)\\ T^H_{ij}=\dfrac{F_jT''_j(T^C_{ij}-\Delta T_{MIN})}{(F_i+F_j)T^C_{ij}-(F_iT'_{i-1}+F_j\Delta T_{MIN})}\qquad T^H_{kl}=\dfrac{F_lT'_l(T^C_{kl}-\Delta T_{MIN})}{(F_k+F_l)T^C_{kl}-(F_kT''_{k-1}+F_l\Delta T_{MIN})}\end{cases}\tag{38}$$

One effective heat pump and one assisting heat pump above the pinch:

$$P^{EA}_{ij,kl}=\begin{cases}\max\ \Delta C^{EA}_{ij,kl}=\left(c_{CU}+c_W-\dfrac{(c_{HU}-c_W)T^H_{ij}}{T^C_{ij}-\Delta T_{MIN}}\right)F_i(T'_{i-1}-T^C_{ij})+(c_W-c_{HU})\left(1-\dfrac{T^H_{kl}}{T^C_{kl}-\Delta T_{MIN}}\right)F_k(T'_{k-1}-T^C_{kl})\\ \text{s.t. } T^C_{ij}\le T_{i-1}\quad T^C_{ij}\ge T_i\quad T^H_{ij}\le T_{j-1}\quad T^H_{ij}\ge T_j\quad T^C_{kl}\le T_{k-1}\quad T^C_{kl}\ge T_k\quad T^H_{kl}\le T_{l-1}\quad T^H_{kl}\ge T_l\quad T^H_{ij}\ge T^{H'}_{j_p}\\ T^C_{kl}=T'_{k-1}-\dfrac{F_j}{F_k}(T^H_{ij}-T^{H'}_{j_p})\qquad T^{H'}_{j_p}=T_j+\dfrac{F_{j_p}}{F_j}(T^H_{j_p}-T_j)-\dfrac{1}{F_j}(\gamma_j-\gamma_{j_p})\\ T^H_{ij}=\dfrac{F_jT'_j(T^C_{ij}-\Delta T_{MIN})}{(F_i+F_j)T^C_{ij}-(F_iT'_{i-1}+F_j\Delta T_{MIN})}\qquad T^H_{kl}=\dfrac{F_lT'_l(T^C_{kl}-\Delta T_{MIN})}{(F_k+F_l)T^C_{kl}-(F_kT'_{k-1}+F_l\Delta T_{MIN})}\end{cases}\tag{39}$$

One effective heat pump and one assisting heat pump below the pinch

$$P^{EB}_{ij,kl}=\begin{cases}\max\ \Delta C^{EB}_{ij,kl}=\left(c_{CU}+c_W-\dfrac{(c_{HU}-c_W)T^H_{ij}}{T^C_{ij}-\Delta T_{MIN}}\right)F_i(T'_{i-1}-T^C_{ij})+(c_{CU}+c_W)\left(1-\dfrac{T^H_{kl}}{T^C_{kl}-\Delta T_{MIN}}\right)F_k(T'_{k-1}-T^C_{kl})\\ T^C_{ij}\le T_{i-1}\quad T^C_{ij}\ge T_i\quad T^H_{ij}\le T_{j-1}\quad T^H_{ij}\ge T_j\quad T^C_{kl}\le T_{k-1}\quad T^C_{kl}\ge T_k\quad T^H_{kl}\le T_{l-1}\quad T^H_{kl}\ge T_l\\ T^C_{ij}\le T_{i-1}+\dfrac{F_{i_p}}{F_i}(T^C_{i_p}-T_{i_p-1})+\dfrac{1}{F_i}(\delta_{i-1}-\delta_{i_p-1})\qquad T^H_{ij}=\dfrac{F_jT'_j(T^C_{ij}-\Delta T_{MIN})}{(F_i+F_j)T^C_{ij}-(F_iT'_{i-1}+F_j\Delta T_{MIN})}\\ T^H_{kl}=\dfrac{F_lT'_l(T^C_{kl}-\Delta T_{MIN})}{(F_k+F_l)T^C_{kl}-(F_kT'_{k-1}+F_l\Delta T_{MIN})}\qquad T^H_{kl}=T'_k-\dfrac{F_i(T_{i-1}-T^C_{ij})+F_{i_p}(T^C_{i_p}-T_{i_p-1})+\delta_{i-1}-\delta_{i_p-1}}{F_l}\end{cases}\tag{40}$$

In order to determine plant-to-plant optimum heat transfer, the resulting heat pumping duties obtained solving these models for the combined plant should be introduced directly into the LP targeting models developed by Bagajewicz and Rodera (2000, 2002). Additionally, these models can be extended to consider three and more pumps, task that is left for future work.

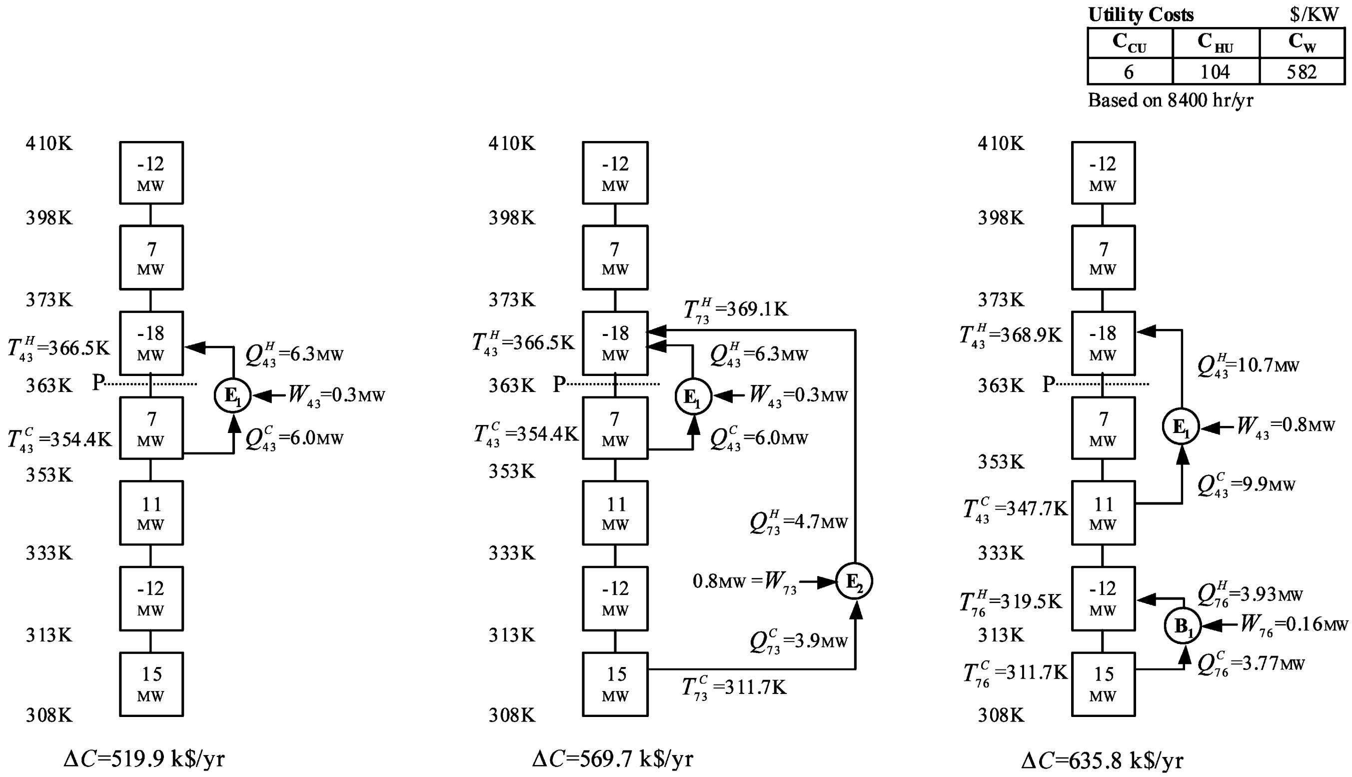

Fig. 8. Optimal solutions for Example 2.

5. Example

Fig. 8 shows the heat cascade for an example case and the optimal solutions obtained with different heat pumps arrangements. The heat requirements for each interval as well as the cost coefficients are also included in Fig. 8.

First, the maximum cost savings using a single heat pump were determined using the targeting procedure described in Section 4. Potential cost savings of 519.9 k$/year were determined in that case. Then, the targeting models for two heat pumps were solved. Using "effective + effective" heat pump arrangement the potential savings increased 9.6% up to 569.7 k$/year. However, the most convenient option in this case is an "assisting+effective" scheme, which gives potential cost savings 635.8 k$/year, 11.6% higher than the other two-heat pump arrangement.

6. Conclusions

The use of heat pumps in the context of heat integration across plants has been examined. As a result, it has been shown that pumps can be utilized to debottleneck the transfer of assisting heat, leading to increased energy and cost savings. A simple targeting procedure that gives optimal cost savings and temperature levels for heat pumping was developed. The advantage of this procedure is that it does not require performing interval partitions or having preset, fixed temperature levels. This procedure could be extended and adapted to consider more rigorous heat pump thermodynamic models that take into account the efficiency of each step of the cycle, similar to those used by Shelton and Grossmann (1986) and Vaidyaraman and Maranas (1998). In addition, it was shown that the combined use of "assisting" and "effective" heat pumps may lead to higher cost savings than the case where only effective heat pumps are used.

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