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Reallocation and upgrade of instrumentation in process plants

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Abstract

In several process plants the precision of parameter estimates is low because the installed set of instruments do not satisfy the new information requirements for different purposes, from simple data reconciliation to on-line optimization. This paper presents models to perform the upgrading of instrumentation at minimum cost to achieve maximum precision of selected parameters. Alternative equivalent models based on maximizing the precision of the parameters are capable on putting a bound on the capital cost. The intricacies of these apparently conflicting goals are explained and a unique procedure based on an MINLP model is presented.

1. Introduction

The introduction of ever increasing computing capabilities in process plants has prompted the development of several techniques that allow the optimization of their performance. Data of good quality is essential for such optimizations. Thus, the process industry is increasingly relying on the use of data reconciliation to enhance the precision of parameter estimates and screening of gross errors. However, software redundancy, that is the use of redundancy through the use of models, cannot be increased unless new instrumentation is added, and it is desired that this instrumentation be incorporated at a minimum cost. In addition, the reorganization of existing resources like laboratory assay capabilities, relocation of existing sensors can also lead to an increased precision. Therefore, a need for a systematic tool to perform a cost–benefit analysis of such resource reallocation and possible upgrade is needed.

The problem of selecting instruments for optimal parameter estimation has been analyzed by several authors. For on-line process optimization, Krishnan, Barton and Perkins (1992a), Krishnan, Barton, and Perkins (1992b) presented strategies for selecting the degree of model complexity and the best measurement structure and parameters. MacDonald and Howat (1988), as well as Serth, Srinkanth and Maronga (1993) discussed the effect of data reconciliation and gross error detection in parameter estimation. Finally the only paper that deals directly with the optimization of the existing resources allocated to the sensors for improving the precision of a parameter was presented by Alheritiere, Thornhill, Fraser and Knight (1997). The authors reported that this method was successfully applied to a crude distillation unit (Alheritiere, Thornhill, Fraser & Knight, 1998) but offered few details.

In addition to the aforementioned papers, several publications address the more general problem of the design of sensor networks for steady state process, for which the parameter estimation problem is a particular case. The designs satisfied different purposes, such as observability, precision, cost, reliability and robustness. Vaclavek and Loucka (1976) first explored this problem to guarantee observability of a required set of variables in multicomponent process. Ali and Narasimhan (1993) presented a method that maximizes reliability in linear non-redundant systems. This approach was extended to redundant systems by Ali and Narasimhan (1995). Sen, Narasimhan and Deb (1998) presented a genetic algorithm approach to design redundant sensor networks using different objective functions. Madron (1992) presented a strategy based on the construction of Minimum Spanning Trees for sensor networks with minimum cost or maximum overall precision. Meyer,

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Le Lann, Koehret and Enjalbert (1994) presented an algorithm to select the measurements that lead to the total or partial observability of the process, while minimizing the instrumentation cost. Another method based on cost minimization, cycle identification and constraints related to observability, redundancy and reliability was presented by Luong, Maquin, Huynh and Ragot (1994). Kretsovalis and Mah (1987) proposed a combinatorial search based on the effect of the variance of measurements on the precision of reconciled values. Maquin, Darouach, Fayolle and Ragot (1987) analyzed the problem of selecting flow measurements to obtain entire observability at least cost. Madron and Veverka (1992) proposed to categorize the variables of linear systems in required and non-required. Unmeasured variables are later ordered from 'hardly measured' to 'easily measured'. The cost and a measure of the overall precision of the systems are used as objective functions. Bagajewicz (1997) proposed a MINLP problem to obtain cost-optimal network structures for linear systems subject to constraints on precision and robustness. Robustness is defined by certain measures that allow the network of sensors to effectively manage gross errors, that is, to be able to identify them effectively (error detectability), and to reduce their smearing effect (resilience). In addition, the retention of a certain precision in key variables in the presence of gross errors is introduced as another important constraint (residual precision). Finally, Bagajewicz and Sánchez (1999) discuss the connections between maximum precision models and Minimum Cost Models. With the exception of the aforementioned work presented by Alheritiere et al. (1997); Alheritiere et al. (1998), there is no other methodology that deals with the cost–benefit analysis of upgrading instrumentation.

This paper concentrates on a general framework that will allow the reallocation of existing resources and the addition of new instrumentation at the same time and at minimum cost. The model presented by Alheritiere et al. (1997); Alheritiere et al. (1998) is reviewed first, followed by an overview of the Minimum-Cost approach (Bagajewicz, 1997) and its equivalency to a Generalized Maximum Precision Model. In the next sections, MINLP models for sensor upgrading and sensor reallocation are separately considered. Their equivalencies to maximum precision models are also shown. Finally, one model is proposed that minimizes the total cost of the upgrading instrumentation project incorporating both the upgrading and reallocation cost of sensors. Some illustrative examples are solved.

2. Maximum precision models

Maximum precision models contain a measure of the estimation quality of parameters or state variables in the objective function. In some models precision is maximized, in others error estimates are minimized.

For steady state systems with only random measurement noise, Kretsovalis and Mah (1987) developed a combinatorial strategy to incorporate measurements to an observable system. Each time a new instrument k is added, the following objective function is minimized

$$R_k = ac_k^i + (J_0 + \Delta J_k^i) \tag{1}$$

where a is a weighting factor, c_k^i is cost associated with the instrument k placed at position i, J_0 is measure of the estimation error corresponding to the original structure, ΔJ_k^i is change in the estimation error when an instrument k is placed at position i.

The trace of the covariance matrix of error estimates, Σ_x, is used as a measure of the estimation precision, that is

$$J = E[(\hat{x} - x)^T(\hat{x} - x)] = tr(\Sigma_x) \tag{2}$$

where $\hat{x}$ is the least square estimation of the reconciled measurements. This method tries to make a weighted average between the cost of the measurements and the precision of the estimates. In practice, it is very difficult to assess the weights.

Later on, Madron and Veverka (1992) proposed to design sensor networks in order to minimize the mean square error of the required quantities, by resolving the following problem

$$\min\left[\sum_{j=1}^{I_r} \sigma_j^2/I_r\right]^{1/2} \tag{3}$$

where σ_j is the standard deviation of the jth required quantity and I_r is the number of required quantities. This model was later on efficiently solved by Madron (1992), for $I_r = 1$, using the concept of Minimum Spanning Tree.

In a recent paper, Alheritiere et al. (1997) evaluated the contribution of process data to performance measures and proposed a scheme for a cost–benefit analysis. The paper proposes to obtain the optimal redistribution of fixed resources to the different sensors of an existing plant, in order to maximize the precision of a parameter estimate. They propose as a starting point the following nonlinear optimization problem:

$$\left.\begin{array}{l} \min\limits_{c_i} \ \sigma_\theta \\ s.t. \\ \sum\limits_{i \in M_m} c_i(\sigma_m) = c_T \\ c_i^L \le c_i \le c_i^U \qquad \forall i \in M_m \end{array}\right\} \tag{4}$$

where c_T is the total resource allocated to all sensors, σ_m is the vector of measurement standard deviations and M_m corresponds to the set of measured variables.

The variance of the parameter estimate, is expressed in terms of σ_m and the sensitivity coefficients s_i as follows:

$$\sigma_{\theta}^2 \approx \sum_i s_i^2 \sigma_{m,i}^2 \quad (5)$$

The sensitivity coefficients s_i were calculated using a commercial simulator by varying the inputs around their nominal point and adjusting the parameter in order to fit the measured data. In a similar way, each output measurement is changed in turn and the parameter is adjusted in the least square sense to fit new output data. Alheritiere et al. (1997) suggest that there is no need for screening of gross errors, although it is well known it may be a dangerous practice.

Even though the NLP formulation presented by Alheritiere et al. (1997) is sound, it omits important aspects of the problem, leads to sub-optimal solutions and can provide misleading results for sensor network design or upgrade. They propose to calculate the cost of a certain number of instruments converting a discrete relation into a non-discrete one as follows: assume $N_{0,i}$ sensors of equal variance ($\sigma_{0,i}^2$) are installed to obtain the value of measurement i. The cost associated with the $N_{0,i}$ sensors is $c_{0,i}$ and the variance of measurement i is calculated using the following expression:

$$\sigma_{m0,i}^2 = \frac{\sigma_{0,i}^2}{N_{0,i}} \quad (6)$$

If an upgrade is made, a different number of sensors $N_{1,i}$ will be used, and therefore a similar expression to (6) can be written:

$$\sigma_{m1,i}^2 = \frac{\sigma_{0,i}^2}{N_{1,i}} \quad (7)$$

Since the cost of all instruments installed for measurement i is given by the number of sensors multiplied by the individual sensor cost ($c = Nc_0$), then

$$\frac{\sigma_{m0,i}^2}{\sigma_{m1,i}^2} = \frac{N_{1,i}}{N_{0,i}} = \frac{c_{1,i}}{c_{0,i}} \quad (8)$$

Using (8) as means to relate initial to final cost through the modification of hardware redundancy, problem (4) becomes (Alheritiere et al., 1997):

$$\left.\begin{array}{l} \min \sqrt{\sum_{i \in M_m} s_i^2 \sigma_{m,i}^2} \\ s.t. \\ \sum_{i \in M_m} c_{o,i} \left(\frac{\sigma_{m0,i}}{\sigma_{m,i}}\right)^2 = c_T \\ \sqrt{\frac{c_{0,i}}{c_i^U}}\, \sigma_{m0,i} \leq \sigma_{m,i} \leq \sqrt{\frac{c_{0,i}}{c_i^L}}\, \sigma_{m0,i} \quad \forall i \in M_m \end{array}\right\} \quad (9)$$

A lower bound on cost is needed because now the discrete nature of the problem is lost. By restricting the total cost to be the same as the allocated one, they consider an optimal redistribution of a fixed total resource.

Hardware redundancy is an important tool to enhance process knowledge, but it should be used within reasonable limits. In this regard, Alheritiere et al. (1997) presented unrealistic scenarios. For example, when pressure is considered, they start with 1 sensor to end-up allocating 4.232 sensors, as it was shown in Table 2 of the above reference. Some other inconsistencies arise from the same table: the only possible discrete values of sensors for x_1 before and after the upgrade is five and three, respectively.

If a set of measurements leading to the estimation of the parameter is redundant, the variance of the estimates of the variables obtained using data reconciliation is lower than the corresponding variance of the measurements. This also has an impact on the parameter estimation, which is now performed using more accurate values. Thus, software redundancy helps in increasing the desired precision. In presenting their model Alheritiere et al. (1997) claim that in the absence of data reconciliation procedures the approach 'provides a benchmark against which the benefits of data reconciliation can be assessed in the future'. Moreover, they also claim that when 'two different methods are available, this method (Eq. (4) or Eq. (9)), can be used to determine which one is preferable on a cost–benefit basis' (Alheritiere et al., 1998). This statement clearly refers to a redundant case.

Not only software redundancy helps in obtaining more precise parameter estimation, but it also does it at a smaller cost and provides additional reliability. It is not recommendable, to set a priori the desired set of instruments through which the parameter will be estimated as it is proposed in (4). Different sets of measurements can lead to the estimation of the same parameter, thus several alternatives that could be less expensive will be omitted by setting one a priori.

Finally, the model presented is only useful for one parameter, and there are no guidelines published as of what to do for many parameters at the same time.

In the following sections we will review an alternative model for the design of sensor networks, then this model will be extended to consider the upgrading and reallocation of instrumentation.

3. A sensor network design model based on minimum cost

Consider a given process whose steady state operation is described by the nonlinear algebraic system of equations

$$f(z) = 0 \quad (10)$$

where z contains the vector w of state variables and the vector θ of process parameters

$$z=\begin{bmatrix} w \\ \theta \end{bmatrix} \tag{11}$$

Due to cost or technical feasibility, not all state variables in z are measured so let q be a vector of binary variables such that:

$$q_i=\begin{cases} 1 & \text{if } w_i \text{ is measured} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Consider a set of parameters that is desired to estimate M_p. The selection of instruments such that the cost is minimized and precision constraints on parameters are satisfied, involves the solution of the following optimization problem:

$$\left.\begin{array}{ll} \min\limits_{q_i} & \sum\limits_{i\in M_w} c_i q_i \\ s.t. & \\ & \sigma_j(q)\le\sigma_j^* \quad \forall j\in M_p \\ & q_i=0,1 \quad \forall i\in M_w \end{array}\right\} \tag{13}$$

where for the sake of simplicity no instrument localization restrictions are imposed and it is assumed that there is only one potential measuring device with associated cost and standard deviation $(c_i, \sqrt{\psi_{ii}})$ for each variable $w_i\in M_w$. Furthermore $\sigma_j(q)$ represents the standard deviation of the estimated value of parameter θ_j obtained after data reconciliation. This should be kept lower than the threshold value σ_j^* for each parameter in the set M_p.

This formulation is only a special case of a more general problem, which involves the accurate estimation of a set of variables M, containing parameters and state variables as well as the inclusion of other robustness constraints that deal with the availability of measurements as well as with the effectiveness of the sensor network in dealing with gross errors (Bagajewicz, 1997). The model presented by Bagajewicz (1997) is confined to linear systems and it is generalized for non-linear systems in the following paragraphs.

For a selected set of instruments represented by the vector q, the standard deviation $\sigma_j(q)$ may be estimated by first linearizing the model, making sure the variables are observable and finally, using the canonical form of the system to obtain the variance.

The linearized model is:

$$Cz\cong d \tag{14}$$

where matrix C represents the Jacobian of $f(z)$ around z_0 (the expected operation point) and d is the corresponding constant. Matrix C may be partitioned in sub-matrixes A and B, which are related to the vector of measured variables (x) and unmeasured parameters and state variables (v), respectively

$$[A \quad B]\begin{bmatrix} x \\ v \end{bmatrix}=d \tag{15}$$

Typically, parameters are not measured. Therefore, they should be observable. The observability of unmeasured variables is determined through a variable classification procedure, which can be accomplished by matrix projection (Crowe, Garcia Campos & Hrymak, 1983), QR decomposition (Swartz, 1989; Sánchez & Romagnoli, 1996) or matrix co-optation (Madron, 1992). If the observability analysis indicates that all parameters in M_p are observable, then the set of proposed instruments represented by q is feasible and the following step, the accurate estimation of the parameters, can be undertaken.

The expressions corresponding to parameter estimations are extracted from the whole set and are:

$$\hat{\theta}=T\hat{x}+b=Sx^{+}+r \tag{16}$$

where T is a matrix that relates parameters with reconciled measurements $\hat{x}$, b and r represent vector of constants and x^{+} stands for the vector of measurements, that satisfies the model

$$x^{+}=x+\varepsilon \tag{17}$$

where ε is assumed to follow a normal distribution, that is, $\varepsilon\sim N(0,\Psi)$.

In turn, the reconciled values are given by:

$$\hat{x}=(I-\Psi G^T(G\Psi G^T)^{-1}G)x^{+}+\Psi G^T(G\Psi G^T)^{-1}d \tag{18}$$

where Q_2^T comes from the QR decomposition of $B(Q_2^TB=0)$and $G=Q_2^TA$

Therefore, the relation between S and T is the following:

$$S=T(I-\Psi GT(G\Psi GT)^{-1}G) \tag{19}$$

S is a $(p\times g)$ matrix of constants that represent the sensitivity of parameters to measurements.

In turn, the relation between b and r is given by:

$$r=b+T\Psi GT(G\Psi GT)^{-1}Q_2^Td \tag{20}$$

Then, since the variance of $\hat{x}$ is

$$\Sigma_{\hat{x}}=[I-\Psi G^T(G\Psi G^T)^{-1}G]\Psi[I-\Psi G^T(G\Psi G^T)^{-1}G]^T \tag{21}$$

the variance matrix of the parameters is given by:

$$\Sigma_{\hat{\theta}}=T\Sigma_{\hat{x}}T^T \tag{22}$$

4. Generalized maximum precision design model

The generalized model for maximum precision (Bagajewicz & Sánchez, 1999) is:

$$\left.\begin{aligned} &\underset{q_i}{Min} \quad \sum_{j \in M_p} a_j \sigma_j^2(q) \\ &s.t. \quad \sum_{i \in M_w} c_i q_i \le c_T \\ &\qquad q_i = 0, 1 \qquad \forall i \in M_w \end{aligned}\right\} \tag{23}$$

where, as above M_p is the set of parameters that is desired to estimate and M_i is the set of state process variables. When hardware redundancy is used then an upper bound on the cost of each type of instrument (or on number of instruments, if they are all equal) can be imposed for each stream. Lower bounds on cost are not needed. One of the major difficulties of this formulation is that it requires weights (a_j) to obtain the objective function. There is no published detailed criteria to obtain them. Some of the main advantages of the generalized model (23) in contrast with problem (4) formulation have been pointed out by Bagajewicz and Sánchez (1999). In particular, it was pointed out that problem (4) is a particular case of (23). In addition, when q is a-priori selected, as in (4), then the problem becomes NLP, the only difference between (23) and (4) is that the cost constraint is an equality in (4). However, it can easily be proven that both problems have the same solution.

Proof. Assume that a solution $\hat{\sigma}_m$ of problem (4) is such that the constraint on cost is not binding, i.e.

$$\sum_{i \in M_m} c_i(\hat{\sigma}_m) < c_T \tag{24}$$

Since the functions $c_i(\sigma_m)$ are continuous functions of the variance σ_m, then there exists a solution $\tilde{\sigma}_m$ in the neighborhood of $\hat{\sigma}_m$ such that

$$\sum_{i \in M_m} c_i(\tilde{\sigma}_m) > \sum_{i \in M_m} c_i(\hat{\sigma}_m) \tag{25}$$

$$\sum_{i \in M_m} c_i(\tilde{\sigma}_m) < c_T \tag{26}$$

If all costs corresponding to $\hat{\sigma}_m$ are at their upper bound, then (4) is infeasible, which completes the proof. **Q.E.D.**

Furthermore the Minimum Cost Model given by (13) is the dual of the Maximum Precision Model given by (23) in the Tuy sense (Bagajewicz & Sánchez, 1999). This duality was established in general by Tuy (1987). In particular, it was established that the solution of one problem is one solution of the other and vice versa.

5. A model for instrumentation upgrade

The upgrading of a sensor network by the simple addition of instrumentation is considered first. This has to be done at minimum cost while reaching certain goals of precision in key parameters. This task can be accomplished using a Minimum Cost Model very similar to the Minimum Cost Model for sensor network grassroots design.

$$\left.\begin{aligned} &\underset{q_{ik}}{\min} && \sum_{i \in M_n} \sum_{k \in K_{n,i}} c_{ik} q_{ik}^N \\ &s.t. \\ &\sigma_j(q^N) \le \sigma_j^* && \forall j \in M_p \\ &\sum_{k \in K_{n,i}} q_{ik}^N + N_{i,0} \le N_i^* && \forall i \in M_n \\ &q_{ik}^N = 0, 1 && \forall i \in M_n, \forall k \in K_{n,i} \end{aligned}\right\} \tag{27}$$

where M_n stands for the set of variables that may be measured using new instruments. For each member $i \in M_n$, a set $K_{n,i}$ of potential sensors is selected. A pair (c_{ik}, σ_{ik}) is associated to each element of $K_{n,i}$, that represents the cost and the standard deviation of the measuring error corresponding to that sensor. The binary variable $q_{i,k}^N = 1$, if the kth element of $K_{n,i}$ has been selected to measure variable i, that was unmeasured in the design stage, $q_{i,k}^N = 0$ otherwise. A variable $q_{i,k}^N$ is an element of the vector q^N of length $\Sigma_{i=1}^{C(\mathrm{Mn})} C(K_{n,i})$, where $C(set)$ represents the set cardinality.

Furthermore σ_j represents the standard deviation of jth parameter $\in M_p$, $N_{i,0}$ is the number of existing sensors measuring variable i and N_i^* stands for the maximum number of sensors that are allowed for measuring variable i.

The first constraint establishes an upper bound on parameter precision, that is a function of the measuring errors of existing and new sensors. The second constraint establishes an upper bound on the number of sensors used to measure each variable. This number is usually one for the case of flow rates, but it can be larger in the case of laboratory measurements of concentrations. In this case, these constraints can be lumped in one constraint for all concentration measurements to express the overall limitation of the laboratory on a daily (or fraction thereof) basis.

When maximum precision is requested and cost is a constraint, then a maximum precision model of the following type can be used.

Table 1
Set of possible instruments for the flash tank

Measured variable	Instrument cost	Instrument standard deviation
F_1	250	3.0
y_{11}	700	0.015
y_{12}	700	0.015
y_{13}	700	0.015
F_2	250	1.515
y_{12}	700	0.01
y_{22}	700	0.01
y_{32}	700	0.01
F_3	300	1.418
y_{13}	800	0.01
y_{23}	800	0.01
y_{33}	800	0.01
P	100	14

Table 2
Minimum cost optimal solutions for flash sensor upgrading

Case	σ_η^*	σ_η	Minimum cost	Optimal set of new measurements
1	0.2	0.00866	0	
2	0.006	0.00574	1200	F_1 F_2 y_{22}
3	0.005	0.00480	2550	F_1 y_{22} y_{13} y_{23}
4	0.0046	0.00459	4900	F_1 y_{11} y_{21} F_2 y_{22} y_{32} y_{13} y_{23}
5	0.004	–	–	–

$$\min_{q_{ik}^N} \quad \sum_j a_j \sigma_j^2(q^N)$$

$s.t.$

$$\sum_{i \in M_n} \sum_{k \in K_{n,i}} c_{ik} q_{ik}^N \leq c_T$$

$$\sum_{k \in K_{n,i}} q_{ik}^N + N_{i,0} \leq N_i^* \qquad \forall i \in M_n$$

$$q_{ik}^N = 0, 1 \qquad \forall i \in M_n, \forall k \in K_{n,i} \tag{28}$$

where c_T is the bound on the capital expenditure. The two models are equivalent in the Tuy sense (Bagajewicz & Sánchez, 1999).

It is interesting to notice that the models presented in this work may be extended to consider inferential measurements. These are unmeasured process variables whose value may be calculated in terms of reconciled measurements through process model equations or empirical relations. Consequently both inferential measurements and parameters behave as unknown variables that belong to vector v in Eq. (15). The procedures developed to enhance parameters' estimation by reallocation and upgrade may be successfully applied to other unmeasured variables as inferential measurements.

5.1. Example 1

Appendix A contains a simplified flash tank model and nominal variable values from Van Winkle (1967). This example has been also considered by Alheritiere et al. (1997).

In this case a set of three initial instruments is installed on the unit, measured variables are [y_{12}, y_{33}, P]. New instruments are necessary in order to satisfy precision requirements on the vaporization efficiency coefficient η. Measurements are obtained installing instruments chosen from the set presented in Table 1. In this case, hardware redundancy is not considered and only one type of instrument is available to measure a variable. The optimal set of new instruments is determined by solving both the Minimum Cost Model and the maximum precision model. Results from each model are provided in Tables 2 and 3, respectively. Data reconciliation is applied to estimate the standard deviation of the parameter η, which is calculated using the procedure described previously.

Two examples are included where the minimum cost for the Minimum Cost Model is considered as a bound for the Maximum Precision Model, both models were run with the same bound for the parameter standard deviation. The optimal cost, parameter standard deviation and new instrument set are the same for both models.

In some examples, higher requirements in precision are satisfied incorporating a large set of instruments and increasing the cost. But in other cases, there is no a feasible set of instruments that fulfils precision constraints.

Table 3
Maximum precision optimal solutions for flash sensor upgrading

Case #	C_T	σ_η^*	Minimum σ_η	Total cost	Optimal set of new measurements
1	1700	∞	0.0087	0	–
2	1200	0.006	0.00574	1200	F_1 F_2 y_{22}
3	2550	0.005	0.00480	2550	F_1 y_{22} y_{13} y_{23}
4	3000	0.005	0.00474	3000	y_{22} y_{32} y_{13} y_{23}
5	5500	0.0046	0.00459	5200	F_1 y_{11} y_{21} F_2 y_{22} y_{32} F_3 y_{13} y_{23}
5	7000	0.004	–	–	–

Table 4
Existing instrumentation for the heat exchanger network

Streams	Number of flowmeters	S.D. flowmeters (%)	Number of thermocouples	S.D. thermocouples (°F)
S_1	1	3	1	2
S_3	–	–	1	2
S_4	–	–	1	2
S_5	1	3	1	2
S_7	1	3	1	2
S_9	–	–	1	2

5.2. Example 2

Data for an industrial heat exchanger network is presented in Appendix B. It corresponds to a set of heat exchangers where crude is heated up using hot gas–oil coming from a column. In this case, the heat transfer coefficients for the heat exchangers are estimated in terms of temperature and flow rate measurements. Table 4 presents the existing instrumentation.

The standard deviations of heat transfer coefficients calculated using the installed set of instruments are [12.27 2.96 3.06], when data reconciliation is applied to estimate them. In order to enhance parameter precision's, new instruments should be added. In this example, hardware redundancy is considered. Furthermore, different types of new instruments are available to measure some temperatures. Data for new instrumentation is presented in Table 5, where cost and standard deviation are shown. Flowmeter precision is expressed in percentage of the nominal values and sensor temperature precision in degrees Fahrenheit.

The maximum number of allowed instruments for measuring each variable is given in Table 6, a zero value indicates there is a restriction for measuring the corresponding variable.

Tables 7–9 present results for the upgrade problem using both types of models. When there are two possible instruments to measure a variable, the type of instrument is indicated between parenthesis in the optimal solution set. The weights for the maximum precision model are assumed equal to one. The equivalency between models is shown using Cases 1/3 and 3/4 from Tables 7–9, respectively.

The simplified examples presented here show how the proposed models work. From a practical point of view, the pairs (c_{ik}, σ_{ik}) associated with each element of $K_{n,i}$ should be obtained from manufacturer data for the particular range of process variables. The maximum number of allowable instruments for measuring a variable, N_i^*, is selected by considering location restrictions and other issues, such as reliability. The bounds on parameter standard deviation for the Minimum Cost Model should be determined making a sensitivity analysis on the effect of these parameters on the values of a set of key unmeasured variables chosen beforehand. This can be performed using the model equations or experimental relationships. This paper does not focus on these procedures. Finally, for the objective function of the Maximum Precision Models, different weights may be selected reflecting the relative importance of the parameter precision. Sometimes this decision is based on engineering judgment but it is helpful to perform a sensitivity analysis to quantify the relative effect on revenue of the parameter value spread. This requires the availability of an economic model. As it was anticipated above, the selection of these weights is still a matter of research. However, since the usage of maximum precision models is not here recommended this issue is not discussed further.

Table 5
Availability of new instrumentation for the heat exchanger network

Streams	Flowmeters		Temperature sensors	
	Standard deviation (%)	Cost	Standard deviation (°F)	Cost
S_1	3	2250	2/0.2	500/1500
S_2	3	2250	2	500
S_3	3	2250	2	500
S_4	3	2250	2/0.2	500/1500
S_5	3	2250	2	500
S_6	3	2250	2	500
S_7	3	2250	2	500
S_8	3	2250	2	500
S_9	3	2250	2/0.2	500/1500

Table 6
Maximum number of instruments for the heat exchanger network

Variable	Ni	Variable	Ni
F_1	1	T_1	2
F_2	1	T_2	1
F_3	1	T_3	1
F_4	1	T_4	2
F_5	1	T_5	1
F_6	0	T_6	1
F_7	1	T_7	1
F_8	0	T_8	0
F_9	1	T_9	2

Table 7
Results for the Minimum Cost Model for the heat exchanger network

Case number	$\sigma^*_{U_1}$	$\sigma^*_{U_2}$	$\sigma^*_{U_3}$	σ_{U_1}	σ_{U_2}	σ_{U_3}	Minimum cost	Optimal set of new measurements
1	4.0	4.0	4.0	3.6160	1.9681	2.7112	500	$T_6(1)$
2	3.5	2.0	2.5	2.7746	1.6892	2.3833	1500	$T_2(1)T_4(1)T_6(1)$
3	3.0	1.5	2.5	2.7230	1.4972	2.2844	6500	$F_2F_3T_2(1)T_4(1)T_6(1)T_9(1)$ $F_3F_4T_2(1)T_4(1)T_6(1)T_9(1)$ $F_3F_4T_2(1)T_4(1)T_6(1)T_9(1)$
4	3.5	2.0	2.0	–	–	–	–	–

6. A model for resource reallocation

In many cases, measurements can be easily transferred at no cost from one stream to another. This is the case of concentration measurements that are performed in the laboratory. Pressure gauges and thermocouples can also be transferred from one place to another at a relatively very small cost. However, flowmeters are probably an exception. Since one can consider that the resource reallocation does not involve cost, or eventually neglect it, the Minimum Cost Model would reduce to a set of MINLP algebraic inequalities. However, even in the case where cost is not a matter of consideration, one would like to minimize the number of changes. Now the derivation of such a model is presented.

Let us introduce the binary variable $u_{t,k,r}$ that indicates sensor k from variable t is relocated to variable r. This sensor is the kth element of the set K_t of originally allocated sensors for measuring variable t. Following, a model in terms of $u_{t,k,r}$ variables is proposed that minimized the number of reallocations

$$\left.\begin{array}{ll}
\min \sum_{t\in M_t}\sum_{k\in K_t}\sum_{r\in M_r} u_{t,k,r} & \\
s.t. & \\
\sigma_j(u)\le\sigma_j^* & \forall j\in M_p \\
\sum_{r\in M_r}\sum_{k\in K_t} u_{t,k,r}\le u_t^T & \forall t\in M_t \\
\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,r}\le u_r^R & \forall r\in M_r \\
\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i}+\left(N_{o,i}-\sum_{r\in M_r}\sum_{k\in K_i} u_{i,k,r}\right)\le N_i^* & \forall i\in M_w \\
u_{t,k,r}=0,1 & \forall t\in M_t,\ \forall r\in M_r,\ \forall k\in K_t
\end{array}\right\} \quad (29)$$

where $N_{0,i}$ is number of originally allocated sensors for measuring variable i, $N_{0,i}=C(K_i)$. M_r is set of variables that may receive measurements from others; M_t is set of variables whose measurements can be reallocated

In this model bounds are imposed on:

1. the maximum number of instruments that can be reallocated from variable t to other variables (u_t^T);
2. the maximum number of instruments that can be reallocated from other variables to variable $r(u_r^R)$;
3. the total number of instruments allocated for each variable (N_i^*). This is needed because one may allow a large number of instruments to be reallocated to a certain variable r, but this change has to be accompanied with the reallocation to some of the instruments from K_r to some other variables to maintain the total number of sensors limited.

The value of the parameter standard deviation $\sigma_j(t)$ is obtained using (22). Let us consider that the kth element of K_i has a certain precision σ_{ik}, then the variance of each measurement i is now

$$\sigma_{m,i}^2=\frac{\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i}\sigma_{t,k}^2+\sum_{\in K_i}\left(1-\sum_{r\in M_r} u_{i,k,r}\right)\sigma_{i,k}^2}{\left[\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i}+\left(N_{0,i}-\sum_{r\in M_r}\sum_{k\in K_i} u_{i,k,r}\right)\right]^2} \quad (30)$$

As in the case of instrumentation upgrading, a maximum precision problem, that is the dual of the Minimum Cost Model, can be defined as follows:

Table 8
Results for the maximum precision model for the heat exchanger network

Case	$\sigma^*_{\eta_1}$	$\sigma^*_{\eta_2}$	$\sigma^*_{\eta_3}$	C_T	σ_{U_1}	σ_{U_2}	σ_{U_3}
1	–	–	–	6500	2.5379	1.5962	2.1341
2	–	–	–	3000	2.6635	1.6854	2.2068
3	4.0	4.0	4.0	500	3.6160	1.9681	2.7112
4	3.0	1.5	2.5	6500	2.7230	1.4972	2.2844
5	3.0	1.5	2.5	4500	–	–	–

Table 9
Results for the maximum precision model for the heat exchanger network

Case	Total cost	$\Sigma_i\,\sigma^2_{Ui}$	Optimal set(s) of new measurements
1	6250	13.4578	F_2 $T_1(2)$ $T_2(1)$ $T_4(2)$ $T_6(1)$
2	3000	14.8048	$T_1(1)$ $T_2(1)$ $T_4(2)$ $T_6(1)$
3	500	24.2999	$T_6(1)$
4	6500	14.8749	Alternatives: F_2 F_3 $T_2(1)$ $T_4(1)$ $T_6(1)$ $T_9(1)$; F_2 F_4 $T_2(1)$ $T_4(1)$ $T_6(1)$ $T_9(1)$; F_3 F_4 $T_2(1)$ $T_4(1)$ $T_6(1)$ $T_9(1)$

Table 10
Existing instrumentation for the heat exchanger network

Streams	Number of flowmeters	S.D. flowmeters (%)	Number of thermocouples	S.D. thermocouples (°F)
S_1	1	3	2	2/0.2
S_3	–	–	1	2
S_4	–	–	2	2/0.2
S_5	1	3	2	2/2
S_7	1	3	1	2
S_9	–	–	1	2

$$\left.\begin{array}{ll}
\min \sum_{j\in M_p} a_j\sigma_j^2(u) & \\
s.t. & \\
\sum_{t\in M_t}\sum_{k\in K_t}\sum_{r\in M_r} u_{t,k,r} \le u_t^T & \\
\sigma_j(u) \le \sigma_j^* & \forall j\in M_p \\
\sum_{r\in M_r}\sum_{k\in_t} u_{t,k,r} \le u_t^T & \forall t\in M_t \\
\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,r} \le u_r^R & \forall r\in M_r \\
\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i} + \left(N_{o.i} - \sum_{r\in M_r}\sum_{k\in K_i} u_{i,k,r}\right) \le N_i^* & \forall i\in M_z \\
u_{t,k,r} = 0, 1 & \forall t\in M_t,\ \forall r\in M_r \\
 & \forall k\in K_t
\end{array}\right\} \quad (31)$$

where N_r is the maximum number of reallocations allowed.

This model has several differences with the one presented by Alheritiere et al. (1997).

- It includes binary variables to handle hardware redundancy, and therefore it does not reallocate fractions of sensors.
- It considers software redundancy through data reconciliation, something that can be easily added to (4).
- It does not commit to a specific set of sensors, by fixing the sensitivity coefficients. It does not include cost as a bound.

The last issue of cost is important to be addressed in more detail. It makes perfect sense that the constraint on cost is superfluous, as the problem is the reallocation of existing resources, which already have a fixed cost. The reason why Alheritiere et al. (1997); Alheritiere et al. (1998) included this constraint in the first place is because they relate sensor variances to sensor costs, something that the present model does not need.

Thus, when the values of parameters precision bounds are known, the algebraic model (30) may be applied to evaluate measurements' precision and then the value of constraints may be calculated using (22). When the bounds are too tight, the set of equations will become infeasible and the reallocation will not be possible. When precision bounds are not known problem (31) may be solved, settings them to a large value. As stated, maximum precision model are not recommended because the weights are not easily obtained.

6.1. Example 3

Let us consider the reallocation of duplicate existing

Table 11
Bounds for the reallocation problem

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
u_t^T	1	0	0	1	1	0	0	0	0
u_r^R	0	2	1	0	0	1	1	1	1
N^*	2	2	2	2	2	1	2	1	2

Table 12
Reallocation model — minimum number of reallocations results for the heat exchanger network

Case	$\sigma^*_{U_1}$	$\sigma^*_{U_2}$	$\sigma^*_{U_3}$	σ_{U_1}	σ_{U_2}	σ_{U_3}	Minimum number of reallocations	Optimal reallocations (t, r, k)
1	4.00	1.80	2.40	3.1683	1.5458	2.2124	1	T_1, 2, T_2
				1.9019	1.5458	2.2124	1	T_5, 2, T_2
2	1.35	1.53	2.70	1.3487	1.5236	2.6623	2	[T_1, T_4][a1], [2 ,2], perm[T_6, T_2]
				1.3455	1.5188	2.6610	2	[T_4, T_5], [2, 2], perm[T_2, T_6]
3	2.00	1.40	2.00	1.9057	1.3904	1.9411	2	[T_4, T_5], [2, 2], perm[T_2, T_8]
4	1.80	1.40	2.00	1.7901	1.3730	1.9297	3	[T_4, T_4, T_5], [2, 2, 2], perm[T_2, T_6, T_8]

[a] perm[n_1, n_2] means all possible permutations of the variables between brackets.

Table 13
Reallocation model — maximum precision model results for the heat exchanger network

Case number	$\sigma^*_{U_1}$	$\sigma^*_{U_2}$	$\sigma^*_{U_3}$	N^*_r	σ_{U_1}	σ_{U_2}	σ_{U_3}
1	1.50	1.50	2.00	1	–	–	
2	1.80	1.40	2.00	3	1.7901	1.3730	1.9297
3	4.00	1.80	2.40	1	1.9019	1.5458	2.2124
4	4.00,	4.00 1	4.00	1	1.6973	1.5694	1.95 3
5	4.00	4.00	4.00	3	1.7901	1.3730	1.92 7
6	2.00	1.40 1	2.00	2	1.9057	1.3904	1.9411

thermocouples is an alternative to fulfill precision requirements for the heat exchanger network presented in Example 2. The existing instrumentation is included in Table 10, the standard deviation of the heat transfer coefficients with these sensors is [11.65 2.87 2.46]. The bounds u^T_t u^R_r N^* are shown in Table 11 (see also Tables 12–14).

The example shows that higher requirements in precision may be fulfilled increasing the number of reallocations of existing duplicate thermocouples. Furthermore, three examples are included where the minimum number of reallocations for the Minimum Number Model is considered as a bound for the Maximum Precision Model and both models were run with the same bounds for the parameters' standard deviation. The same solution is obtained for both models in terms of the set of reallocations and parameters' standard deviation.

7. A generalized model for resource reallocation and upgrade

In this section a resource reallocation model based on cost minimizing is presented. Switching from one sample to another in the laboratory may prompt changes of reagents, changes in sampling costs, etc. The change of positions of thermocouples and pressure gauges have also rewiring costs as well as recalibration costs. Finally, one may want to seriously consider reallocating flowmeters, although this may not be a likely outcome because of the relatively higher cost. In addition, these reallocation costs may overcome the simple addition of new instrumentation. Therefore, any reallocation and upgrade program should consider the trade off between all these decisions. This trade off is taken into account if the following upgrade cost function is used:

$$\sum_{i\in M_n}\sum_{k\in K_{n,i}} c_{ik}q^N_{ik} + \sum_{t\in M_t}\sum_{k\in K_t}\sum_{r\in M_r} h_{t,k,r}u_{t,k,r} \tag{32}$$

where $h_{t,k,r}$ represents the cost of reallocation of the type k instrument from variable t to variable r.

The complete generalized reallocation and upgrading model, which is a generalization of (29) is the following:

Table 14
Reallocation model-maximum precision model results for the heat exchanger network (continued)

Case	N_r	$\Sigma_i\,\sigma^2_{Ui}$	Optimal reallocations (t, k, r)
1	–	–	–
2	3	8.8130	[T_1, T_4, T_5], [2, 2, 2], perm[T_2, T_6, T_5]
3	1	10.9016	T_5, 2, T_2
4	1	9.1751	[T_1, T_4], [2, 2], perm[T_6, T_8]
5	3	8.8130	[T_1, T_4, T_5], [2, 2, 2], perm[T_2, T_6, T_8]
6	2	9.3325	[T_4, T_5], [2, 2], perm[T_2, T_8]

Table 15
Instrumentation data

Variable	N_{0i}	$\sigma_{0i,k}$		$c_{i,k}$	$\sigma_{i,k}$
$F1$	2	2.5	2.5	350	2.
$f1$	2	0.015	0.01	2700	0.01
$F2$	1	1.515	–	350	1.48
$f2$	1	0.01	–	2700	0.01
$F3$	1	1.418	–	400	1.38
$f3$	1	0.01	–	2700	0.01
P	1	14.	–	100	14.

Table 16
Flash drum: constraints bounds for sensor reallocation and upgrade

Variable	$F1$	$f1$	$F2$	$f2$	$F3$	$f3$	P
N^*	2	2	2	2	2	2	2
u_i^T	2	1	1	0	0	0	0
u_i^R	1	0	2	1	0	1	0

Table 17
Flash drum: relocation pattern

M_t/M_r	F_1	F_2	f_2	f_3
F_1		$x(80)$		
f_1			$x(0)$	$x(50)$
F_2	$x(80)$			

$$\left.\begin{array}{ll}
\min \sum_{i\in M_n}\sum_{k\in K_{n,i}} c_{ik}q_{ik}^N + \sum_{t\in M_t}\sum_{k\in K_t}\sum_{r\in M_r} h_{t,k,r}u_{t,k,r} & \\
s.t. & \\
\sigma_j(q^N,u) \le \sigma_j^* & \forall j\in M_p \\
\sum_{r\in M_r}\sum_{k\in K_t} u_{t,k,r} \le u_t^T & \forall t\in M_t \\
\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,r} \le u_r^R & \forall r\in M_r \\
\sum_{k\in K_{n,i}} q_{ik}^N + \sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i} + \left(N_{o,i} - \sum_{r\in M_r}\sum_{k\in K_i} u_{i,k,r}\right) \le N_i^* & \forall i\in M_z \\
u_{t,k,r} = 0,1 & \forall t\in M_t,\ \forall r\in M_r \\
q_{ik}^N = 0,1 & \forall k\in K_t \\
 & \forall i\in M_n,\ \forall k\in K_{n,i}
\end{array}\right\} \quad (33)$$

The value of the parameter standard deviation is also obtained using (22). The variance of each measurement i is now

$$\sigma_{m,i}^2 = \frac{\sum_{k\in K_{n,i}} q_{ik}^N\sigma_{i,k}^2 + \sum_{t\in M_t}\sum_{k\in K_i} u_{t,k,i}\sigma_{t,k}^2 + \sum_{k\in K_i}\left(1-\sum_{r\in M_r} u_{i,k,r}\right)\sigma_{i,k}^2}{\left[\sum_{k\in K_{n,i}} q_{ik}^N + \sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i} + \left(N_{0,i} - \sum_{r\in M_r}\sum_{k\in K_i} u_{i,k,r}\right)\right]^2} \quad (34)$$

where new instrumentation can have a variance different from the already installed ones.

An equivalent maximum precision problem can be obtained by finding the dual in the Tuy sense:

$$\left.\begin{array}{ll}
\min \sum_{j\in M_p} a_j\sigma_j^2(q^N,u) & \\
s.t. & \\
\sum_{i\in M_n}\sum_{k\in K_{n,i}} c_{ik}q_{ik}^N + \sum_{t\in M_t}\sum_{k\in K_t}\sum_{r\in M_r} h_{t,k,r}u_{t,k,r} \le c_T & \\
\sigma_j(q^N,u) \le \sigma_j^* & \forall j\in M_p \\
\sum_{r\in M_r}\sum_{k\in K_t} u_{t,k,r} \le u_t^T & \forall t\in M_t \\
\sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,r} \le u_r^R & \forall r\in M_r \\
\sum_{k\in K_{n,i}} q_{ik}^N + \sum_{t\in M_t}\sum_{k\in K_t} u_{t,k,i} + \left(N_{o,i} - \sum_{r\in M_r}\sum_{k\in K_i} u_{i,k,r}\right) \le N_i^* & \forall i\in M_z \\
u_{t,k,r} = 0,1 & \forall t\in M_t,\ \forall r\in M_r \\
 & \forall k\in K_t \\
q_{ik}^N = 0,1 & \forall i\in M_n,\ \forall k\in K_{n,i}
\end{array}\right\} \quad (35)$$

The instrumentation team should select the relocation pattern (sets M_t and M_r) based on engineering judgment and process knowledge. For example, duplicate measurements or laboratory analysis may be selected for reallocation. The costs associated with these tasks, $h_{t,k,r}$, are estimated in terms of the manpower and material costs necessary to change measurement location. The bounds for sensor relocation and the maximum allowable amount of instruments for each measurement depend on location restrictions and reliability issues.

7.1. Example 4

Consider that N_0 instruments are installed on the flash tank presented in Appendix A. The number of existing instruments to measure each variable (N_{0i}) and the corresponding standard deviations ($\sigma_{0i,k}$) for different types of installed instruments are included in Table 15. In this example the mass fractions of all the components of a stream are measured on line, a laboratory analysis may be done as a second alternative to know their values.

Table 18
Flash drum: Minimum Cost Model results for sensor reallocation and upgrade

Case	σ_η^*	σ_η	Minimum cost	Reallocations	New instruments
1	–	0.004380	–	–	–
2	0.0038	0.003515	100	–	P
3	0.003467	100	$u_{f1,2,f_2}$	P	
3	0.0033	0.003286	2800	$u_{f1,2,f_2}$	f_3, P
4	0.0031	–	–	–	–

Table 19
Flash drum: maximum precision model results for sensor reallocation and upgrade

Case number	σ_η^*	C_T	Minimum σ_η	Total cost	Reallocations	New instruments
1	–	–	0.004379	–	–	–
2	0.0038	100	0.003467	100	$u_{f1,2,f_2}$	P
3	0.0033	2050	–	–	–	–
4	0.0033	2800	0.003286	2800	$u_{f1,2,f_2}$	f_3, P
5	0.0033	3550	0.003286	3550	$u_{f1,2,f_2}$	F_2, F_3, f_3, P

Table 20
Existing instrumentation for the heat exchanger network

Streams	Number of flowmeters	S.D. flowmeters (%)	Number of thermocouples	S.D. thermocouples (°F)
S_1	1	3	2	2/2
S_3	–	–	1	2
S_4	–	–	2	2/0.2
S_5	1	3	2	2/2
S_7	1	3	1	2
S_9	–	–	1	2

The standard deviation of the vaporization efficiency coefficient estimated using the existing instrumentation is 0.00434 using reconciliation. As this value is not satisfactory, a reallocation and possibly an incorporation of new flowmeters and laboratory composition analysis is proposed using the upgrading model developed first. Data for new measurements ($c_{i,k}$, $\sigma_{i,k}$) is also provided in Table 15.

For this case, the constraint bounds are presented in Table 16. The sets of transferred and received measurements are $M_t = \{F_1, f_1, F_2)$ and $M_r = \{F_1, F_2, f_2, f_3\}$, respectively. In Table 17 feasible reallocations between sets M_t and M_r are indicated using a cross and the relocation costs are included between parenthesis. This relocation pattern is constructed based on engineering judgment.

Table 18 shows the results for the Minimum Cost Model. The first row represents the case for the existing instrumentation. It is interesting to notice that a reduction of the standard deviation from 0.00438 to 0.00347 results if the laboratory analysis for the feed stream is relocated to the liquid stream and a pressure sensor is added. The cost of this case is $100. Higher precision is obtained by means of the reallocation and addition of instruments.

Table 21
Heat exchanger network: bounds for reallocation and upgrade problems

Variable	N$_i$	u_t^T	u_r^R	Variable	N$_i$	u_t^T	u_r^R
F_1	1	1	0	T_1	2	1	0
F_2	1	0	0	T_2	1	0	1
F_3	1	0	1	T_3	1	0	0
F_4	1	0	0	T_4	2	1	0
F_5	1	0	0	T_5	1	1	0
F_6	0	0	0	T_6	1	0	1
F_7	1	1	0	T_7	1	0	0
F_8	0	0	0	T_8	0	0	0
F_9	1	0	1	T_9	2	0	0

Table 22
Heat exchanger network: relocation pattern

M_t/M_r	F_3	F_9	T_2	T_6
F_1	x(1000)			
F_7		x(1000)		
T_1			x(100)	x(100)
T_4			x(100)	x(100)
T_5			x(100)	x(100)

Table 23
Heat exchanger network: available new instrumentation

M_n	S.D.	Cost
F_4	2.5%	2250
F_9	2.5%	2250
T_2	0.2	1500
T_6	0.2	1500

In Table 19 the results from the application of the Maximum Precision Model are presented. Here two examples are included to show the duality between the Minimum Cost Model for Reallocation and Upgrade and the Maximum Precision Model.

7.2. Example 5

Here the reallocation and upgrading of instrumentation is illustrated using the heat exchanger network presented in Example 2. The process instrumentation is provided in Table 20 where it is shown that some temperatures are measured using two thermocouples. The relocation bounds and the maximum number of instruments per stream are included in Table 21.

In this example two flowmeters and three thermocouples may be reallocated. The allowable new positions for these instruments and the reallocation costs are indicated in Table 22. Furthermore some new instruments are available to be incorporated in the instrumentation scheme. Their cost and standard deviation are in Table 23.

Table 26
Heat exchanger network: maximum precision model results for the reallocation and upgrade problem (continued)

Case	Total cost	$\Sigma_i \sigma^2_{Ui}$	Reallocations	New instruments
1	1600	10.6663	$u_{T5,2,T_6}$	T_2
2	3000	9.0354	–	T_2, T_6
3	–	–	–	–
4	5250	8.1503	–	F_4, T_2, T_6

Using previous information, the Minimum Cost Model and the Maximum Precision Model are run with different bounds in the constraints. For some cases, reallocation is sufficient to fulfill precision requirements, but the solution contains only new instruments for higher precision requirements. A mixed alternative of medium cost (reallocation and new instrumentation) is achieved in other cases. Two examples are provided to show the duality between both models (see Tables 24–26).

8. Conclusions

The estimation of plant parameters may be enhanced by modifying the instrumentation structure. Different

Table 24
Heat exchanger network: Minimum Cost Model results for the reallocation and upgrade problem

Case	$\sigma^*_{U_1}$	$\sigma^*_{U_2}$	$\sigma^*_{U_3}$	σ_{U_1}	σ_{U_2}	σ_{U_3}	Minimum cost	Reallocation	New instruments
1	4.00	4.00	4.00	3.2826	1.9254	2.2168	100	$u_{T_1,2,T_6}$	–
				2.3947	1.5482	2.6690	100	$u_{T_4,2,T_2}$	–
				2.0620	1.8277	2.6698	100	$u_{T_4,2,T_6}$	–
				3.6035	1.9122	2.2140	100	$u_{T_5,2,T_6}$	–
2	2.00	2.00	2.00	–	–	–	–		
3	2.00	2.00	2.20	1.3891	1.5148	2.1935	3000	–	T_2, T_6
4	1.50	1.50	2.20	1.3492	1.3664	2.1125	5250	–	F_4, T_2, T_6
				1.3839	1.4995	2.1606	5250	–	F_9, T_2, T_6
5	2.40	2.30	2.20	2.0587	1.8174	2.1938	1500	–	T_6
6	2.20	1.80	2.40	1.7890	1.6827	2.2014	1600	$u_{T_1,2,T_2}$	T_6
				2.1503	1.5306	2.2031	1600	$u_{T_1,2,T_6}$	T_2
				2.0980	1.6816	2.1993	1600	$u_{T_5,2,T_6}$	T_6
				1.8644	1.5295	2.2025	1600	$u_{T_1,2,T_6}$	T_2

Table 25
Heat exchanger network: maximum precision model results for the reallocation and upgrade problem

Case	$\sigma^*_{U_1}$	$\sigma^*_{U_2}$	$\sigma^*_{U_3}$	C_T	σ_{U_1}	σ_{U_2}	σ_{U_3}
1	2.20	1.80	2.40	2000	1.8644	1.5295	2.2025
2	2.80	1.80	2.30	3000	1.3891	1.5148	2.1935
3	1.50	1.50	2.20	5000	–	–	–
4	1.50	1.50 1	2.20	525011	1.3492	1.3664	2.1125

alternatives are presented in the paper: sensor reallocation, sensor upgrade and the combined approach. For each alternative, two types of models are presented: Minimum Cost or Minimum Number of Reallocations and Maximum Precision. The mathematical relationship between these models has been established and illustrated with different examples.

Appendix A. Nomenclature

a weighting factor
A matrix of measured variables
B matrix of unmeasured variables
b vector of constants
C Jacobian matrix of the model
c vector of instrument cost
c^k vector of instrument cost for stream k
c_T total resource allocated to all sensors
c_{0i} initial cost of measurement i considering hardware redundancy
c_i optimal cost of measurement i considering hardware redundancy
d vector of model constants
f non-linear model equations
g number of measurements
h reallocation cost
Ir number of required quantities
J trace of covariance matrix of the estimation error
$K_{n,i}$ set of new available sensors to measure variable i
M set of state variables and parameters with precision requirements
M_m set of measured variables
M_n set of new measured variables
M_p set of parameters with precision requirements
M_r set of variables that receive instruments
M_t set of variables that transfer instruments
M_w set of state variables
N^* maximum number of instruments per measured variable
Nr number of reallocations
p number of parameters
q vector of binary variables defined by (12)
r vector of constants
R_k objective function. defined by equation (1)
s sensitivities coefficients
S matrix of sensitivity coefficients
$u_{t,k,r}$ reallocation binary variable
u_t^T maximum number of instruments that may be transferred from a measured variable
u_r^R maximum number of instruments that may be transferred to measure a variable r
v vector of unmeasured variables
w vector of state variables
x vector of measured state variables
x^+ vector of measurements
z vector of state variables and parameters
θ vector of parameters
σ standard deviation of the estimates
σ^2 variance of the estimates
σ_m standard deviation of the measurements
$\sigma_{m0,i}$ initial standard deviation of measurement i considering hardware redundancy
$\sigma_{m,i}$ optimal standard deviation of measurement i considering hardware redundancy
ε vector of measurement errors
ΔJ_i^k change in the estimation error when an instrument i is placed at position k
Ψ covariance matrix of the measurement errors
Σ_x trace of the covariance matrix of the estimation error for state variables

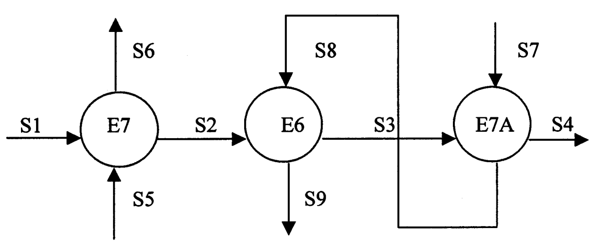

Fig. 1.

Appendix B. Example

The following flash tank model is taken from Van Winkle (1967).

Simplified Flash Model

$$F_1 = F_2 + F_3 \tag{B-1}$$

$$F_1 yi_l = F_2 y_{i2} + F_3 y_{i3} \tag{B-2}$$

$$\Sigma\, y_{i1} = \Sigma\, y_{i2} = \Sigma\, y_{i3} \tag{B-3}$$

$$y_{i3} = \eta y_{i2} P_i(\text{sat})/P \tag{B-4}$$

Appendix C. Example

Data for a heat exchanger network example is presented in Fig. 1 and Tables 27–29.

Table 27
Flash model data[a2]

Stream	Saturation pressures (mmHg)	Feed	Vapor	Liquid
Flow (mol h^{-1})		100	49.50	50.50
Ethanol molar fraction (1)[a]	5287	0.2	0.233	0.167
1-Propanol molar fraction (2)[a]	2932	0.5	0.436	0.563
2-Propanol molar fraction (3)[a]	4651	0.3	0.331	0.270
Temperature (°C)		137		
Pressure (mmHg)		3600		
Vaporization efficiency		95%		

[a] Molar fractions.

Table 28
Flow rate information

Stream	Flow rate (Mlb h^{-1})	Temperature (°F)
S_1	224.677	542.854
S_2	224.677	516.295
S_3	224.677	448.279
S_4	224.677	402.219
S_5	217.019	307.637
S_6	217.019	339.806
S_7	398.008	191.182
S_8	398.008	221.636
S_9	398.008	266.876

Table 29

Heat exchanger	Area (ft^2)	f	$c\bar{p}_h$ (BTU/lb °F)	$c\bar{p}_c$ (BTU/lb °F)
E7	500	0.997	0.6656	0.5689
E6	1100	0.991	0.6380	0.5415
E7A	700	0995	0.6095	0.52

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