definition of experimental uncertainty may be taken as the possible value the error may have. This uncertainty may vary a great deal depending upon the circumstances of the experiment. Perhaps it is better to speak of experimental uncertainty instead of experimental error because the magnitude of an error is always uncertain. Both terms are used in practice, however, so the reader should be familiar with the meaning attached to the terms and the ways that they relate to each other.

It is very common for people to speak of experimental errors when the correct terminology should be “uncertainty.” Because of this common usage, we ask that the reader accept the faulty semantics when they occur and view each term in its proper context.

At this point we may mention some of the types of errors that may cause uncertainty in an experimental measurement. First, there can always be those gross blunders in apparatus or instrument construction, which may invalidate the data. Hopefully, the careful experimenter will be able to eliminate most of these errors. Second, there may be certain fixed errors, which will cause repeated readings to be in error by roughly the same amount but for some unknown reason. These fixed errors are sometimes called systematic errors. Third, there are the random errors, which may be caused by personal fluctuations, random electronic fluctuations in the apparatus or instruments, various influences of friction, etc. These random errors usually follow a certain statistical distribution, but not always. In many instances it is very difficult to distinguish between fixed errors and random errors.

The experimentalist may sometimes use theoretical methods to estimate the magnitude of a fixed error. For example, consider the measurement of the temperature of a hot gas stream flowing in a duct with a mercury-in-glass thermometer. It is well known that heat may be conducted from the stem of the thermometer, out of the glass body, and into the surroundings. In other words, the fact that part of the thermometer is exposed to the surroundings at a temperature different from the gas temperature to be measured may influence the temperature of the stem of the thermometer. There is a heat flow from the gas to the stem of the thermometer, and, consequently, the temperature of the stem must be lower than that of the hot gas. Therefore, the temperature we read on the thermometer is not the true temperature of the gas, and it will not make any difference how many readings are taken—we shall always have an error resulting from the heat-transfer condition of the stem of the thermometer. This is a fixed error, and its magnitude may be estimated with theoretical calculations based upon known thermal properties of the gas and the glass thermometer.

3-3 ERROR ANALYSIS ON A COMMONSENSE BASIS

We have already noted that it is somewhat more explicit to speak of experimental uncertainty rather than experimental error. Suppose that we have satisfied our-

consideration such factors as instrument accuracy, competence of the people using the instruments, etc. Eventually, the primary measurements must be combined to calculate a particular result that is desired. We shall be interested in knowing the uncertainty in the final result due to the uncertainties in the primary measurements. This may be done by a commonsense analysis of the data, which may take many forms. One rule of thumb that could be used is that the error in the result is equal to the maximum error in any parameter used to calculate the result. Another commonsense analysis would combine all the errors in the most detrimental way in order to determine the maximum error in the final result. Consider the calculation of electric power from

\[ P = EI \]

where \( E \) and \( I \) are measured as

\[ E = 100 \, \text{V} \pm 2 \, \text{V} \]
\[ I = 10 \, \text{A} \pm 0.2 \, \text{A} \]

The nominal value of the power is 100 \times 10 = 1000 \, \text{W}. By taking the worst possible variations in voltage and current, we could calculate

\[ P_{\text{max}} = (100 + 2\times10 + 0.2) = 1040.4 \, \text{W} \]
\[ P_{\text{min}} = (100 - 2\times10 - 0.2) = 960.4 \, \text{W} \]

Thus, using this method of calculation, the uncertainty in the power is + 4.04 percent, - 3.96 percent. It is quite unlikely that the power would be in error by these amounts because the voltmeter variations would probably not correspond with the ammeter variations. When the voltmeter reads an extreme "high," there is no reason why the ammeter must also read an extreme "high" at that particular instant; indeed, this combination is most unlikely.

The simple calculation applied to the electric-power equation above is a useful way of inspecting experimental data to determine what errors could result in a final calculation; however, the test is too severe and should be used only for rough inspections of data. It is significant to note, however, that if the results of the experiments appear to be in error by more than the amounts indicated by the above calculation, then the experimenter had better examine the data more closely. In particular, the experimenter should look for certain fixed errors in the instrumentation, which may be eliminated by applying either theoretical or empirical corrections.

As another example we might conduct an experiment where heat is added to a container of water. If our temperature instrumentation should indicate a drop in temperature of the water, our good sense would tell us that something is wrong and the data point(s) should be thrown out. No sophisticated analysis procedures are necessary to discover this kind of error.

The term "common sense" has many connotations and means different
and results for gross errors and variations. In subsequent sections we shall present methods for determining experimental uncertainties in a more precise manner.

3-4 UNCERTAINTY ANALYSIS

A more precise method of estimating uncertainty in experimental results has been presented by Kline and McClintock [1]. The method is based on a careful specification of the uncertainties in the various primary experimental measurements. For example, a certain pressure reading might be expressed as

\[ p = 100 \text{ kN/m}^2 \pm 1 \text{ kN/m}^2 \]

When the plus or minus notation is used to designate the uncertainty, the person making this designation is stating the degree of accuracy with which he or she believes the measurement has been made. We may note that this specification is in itself uncertain because the experimenter is naturally uncertain about the accuracy of these measurements.

If a very careful calibration of an instrument has been performed recently, with standards of very high precision, then the experimentalist will be justified in assigning a much lower uncertainty to measurements than if they were performed with a gage or instrument of unknown calibration history.

To add a further specification of the uncertainty of a particular measurement, Kline and McClintock propose that the experimenter specify certain odds for the uncertainty. The above equation for pressure might thus be written

\[ p = 100 \text{ kN/m}^2 \pm 1 \text{ kN/m}^2 \text{ (20 to 1)} \]

In other words, the experimenter is willing to bet with 20 to 1 odds that the pressure measurement is within \( \pm 1 \text{ kN/m}^2 \). It is important to note that the specification of such odds can only be made by the experimenter based on the total laboratory experience.

Suppose a set of measurements is made and the uncertainty in each measurement may be expressed with the same odds. These measurements are then used to calculate some desired result of the experiments. We wish to estimate the uncertainty in the calculated result on the basis of the uncertainties in the primary measurements. The result \( R \) is a given function of the independent variables \( x_1, x_2, x_3, \ldots, x_n \). Thus,

\[ R = R(x_1, x_2, x_3, \ldots, x_n) \quad (3-1) \]

Let \( w_R \) be the uncertainty in the result and \( w_1, w_2, \ldots, w_n \) be the uncertainties in the independent variables. If the uncertainties in the independent variables are all given with the same odds, then the uncertainty in the result having these odds is given in Ref. [1] as

If this relation is applied to the electric power relation of the previous section, the expected uncertainty is 2.83 percent instead of 4.04 percent.

**Example 3-1** The resistance of a certain size of copper wire is given as

\[ R = R_0[1 + \alpha(T - 20)] \]

where \( R_0 = 6 \Omega \pm 0.3 \) percent is the resistance at 20°C, \( \alpha = 0.004 \text{°C}^{-1} \pm 1 \) percent is the temperature coefficient of resistance, and the temperature of the wire is \( T = 30 \pm 1 \)°C. Calculate the resistance of the wire and its uncertainty.

**Solution** The nominal resistance is

\[ R = (6)[1 + (0.004)(30 - 20)] = 6.24 \Omega \]

The uncertainty in this value is calculated by applying Eq. (3-2). The various terms are:

- \( \frac{\partial R}{\partial R_0} = 1 + \alpha(T - 20) = 1 + (0.004)(30 - 20) = 1.04 \)
- \( \frac{\partial R}{\partial \alpha} = R_0(T - 20) = (6)(30 - 20) = 60 \)
- \( \frac{\partial R}{\partial T} = R_0 \alpha = (6)(0.004) = 0.24 \)
- \( w_{R_0} = 6(0.003) = 0.18 \Omega \)
- \( w_\alpha = (0.004)(0.01) = 4 \times 10^{-5} \text{°C}^{-1} \)
- \( w_T = 1 \text{°C} \)

Thus, the uncertainty in the resistance is

\[ w_R = [(1.04)^2(0.18)^2 + (60)^2(4 \times 10^{-5})^2 + (0.024)^2(1)^2]^{1/2} = 0.0305 \Omega \text{ or } 0.49\% \]

Particular notice should be given to the fact that the uncertainty propagation in the result \( w_R \) predicted by Eq. (3-2) depends on the squares of the uncertainties in the independent variables \( w_x \). This means that if the uncertainty in one variable is significantly larger than the uncertainties in the other variables, say, by a factor of 5 or 10, then it is the largest uncertainty that predominates and the others may probably be neglected.

To illustrate, suppose there are three variables with a product of sensitivity and uncertainty \([\frac{\partial R}{\partial x}w_x]\) of magnitude 1, and one variable with a magnitude of 5. The uncertainty in the result would be

\[ (5^2 + 1^2 + 1^2 + 1^2)^{1/2} = \sqrt{28} = 5.29 \]
of instrumentation, etc. Very little is gained by trying to reduce the "small" uncertainties. Because of the square propagation it is the "large" ones that predominate, and any improvement in the overall experimental result must be achieved by improving the instrumentation or experimental technique connected with these relatively large uncertainties. In the examples and problems that follow, both in this chapter and throughout the book, the reader should always note the relative effect of uncertainties in primary measurements on the final result.

In Sec. 2-11 (Table 2-7) the reader was cautioned to examine possible experimental errors before the experiment is conducted. Equation (3-2) may be used very effectively for such analysis, as we shall see in the sections and chapters that follow. A further word of caution may be added here. It is equally as unfortunate to overestimate uncertainty as to underestimate it. An underestimate gives false security, while an overestimate may make one discard important results, miss a real effect, or buy much too expensive instruments. The purpose of this chapter is to indicate some of the methods for obtaining reasonable estimates of experimental uncertainty.

In the previous discussion of experimental planning we noted that an uncertainty analysis may aid the investigator in selecting alternative methods to measure a particular experimental variable. It may also indicate how one may improve the overall accuracy of a measurement by attacking certain critical variables in the measurement process. The next three examples illustrate these points.

**Example 3-2** A resistor has a nominal stated value of 10 Ω ± 1 percent. A voltage is impressed on the resistor, and the power dissipation is to be calculated in two different ways: (1) from \( P = E^2/R \) and (2) from \( P = EI \). In (1) only a voltage measurement will be made, while both current and voltage will be measured in (2). Calculate the uncertainty in the power determination in each case when the measured values of \( E \) and \( I \) are:

\[
E = 100 \text{ V} \pm 1\% \\
I = 10 \text{ A} \pm 1\%
\]

\[ \text{Figure 3.2 Power measurement across a resistor.} \]

**SOLUTION** The schematic is shown in the accompanying figure. For the first case we have

\[
\frac{\partial P}{\partial E} = \frac{2E}{R} \quad \frac{\partial P}{\partial R} = -\frac{E^2}{R^2}
\]

and we apply Eq. (3-2) to give

\[
w_p = \left[ \left( \frac{2E}{R} \right)^2 w_E^2 + \left( \frac{E^2}{R^2} \right) w_R^2 \right]^{1/2}
\]

Dividing by \( P = E^2/R \) gives

\[
\frac{w_p}{P} = \left[ 4 \left( \frac{w_E}{E} \right)^2 + \left( \frac{w_R}{R} \right)^2 \right]^{1/2}
\]

Inserting the numerical values for uncertainty,

\[
\frac{w_p}{P} = \left[ 4(0.01)^2 + (0.01)^2 \right]^{1/2} = 2.236\%
\]

For the second case we have

\[
\frac{\partial P}{\partial E} = 1 \quad \frac{\partial P}{\partial I} = E
\]

and after similar algebraic manipulation, we obtain

\[
\frac{w_p}{P} = \left[ \left( \frac{w_E}{E} \right)^2 + \left( \frac{w_I}{I} \right)^2 \right]^{1/2}
\]

Inserting the numerical values of uncertainty,

\[
\frac{w_p}{P} = \left[ (0.01)^2 + (0.01)^2 \right]^{1/2} = 1.414\%
\]

Thus, the second method of power determination provides considerably less uncertainty than the first method, even though the primary uncertainties in each quantity are the same. In this example the utility of the uncertainty analysis is that it affords the individual a basis for selection of a measurement method to produce a result with less uncertainty.

**Example 3-3** The power measurement in Example 3-2 is to be conducted by measuring voltage and current across the resistor with the circuit shown in the accompanying figure. The voltmeter has an internal resistance \( R_m \), and the value of \( R \) is known only approximately. Calculate the nominal value of the power dissipated in \( R \) and the uncertainty for the following conditions:

\[
R = 100 \Omega \quad \text{(not known exactly)}
\]

\[
R_m = 1000 \Omega \pm 5\%
\]

\[
I = 5 \text{ A} \pm 1\%
\]

\[
E = 500 \text{ V} \pm 5\%
\]
SOLUTION A current balance on the circuit yields

\[ I_1 + I_2 = I \]
\[ \frac{E}{R} + \frac{E}{R_m} = 1 \]
and

\[ I_1 = I - \frac{E}{R_m} \]  \hauline

The power dissipated in the resistor is

\[ P = EI_1 = EI - \frac{E^2}{R_m} \]  \hauline

The nominal value of the power is thus calculated as

\[ P = (500)(5) - \frac{500^2}{1000} = 2250 \text{ W} \]

In terms of known quantities the power has the functional form \( P = f(E, I, R_m) \), and so we form the derivatives

\[ \frac{\partial P}{\partial E} = I - \frac{2E}{R_m}, \quad \frac{\partial P}{\partial I} = E \]
\[ \frac{\partial P}{\partial R_m} = -\frac{E^2}{R_m^2} \]

The uncertainty for the power is now written as

\[ w_P = \left[ \left(I - \frac{2E}{R_m}\right)^2 w_I^2 + E^2 w_E^2 + \left(\frac{E^2}{R_m^2}\right)^2 w_{R_m}^2 \right]^{1/2} \]

Inserting the appropriate numerical values gives

\[ w_P = \left[ \left(5 - \frac{1000}{1000}\right)^2 + (25 \times 10^4)(25 \times 10^{-4}) + \left(25 \times \frac{10^4}{10^9}(2500) \right)^2 \right]^{1/2} \]
\[ = [16 + 25 + 6.25]^{1/2}(5) \]
\[ = 34.4 \text{ W} \]

or

\[ \frac{w_P}{P} = \frac{34.4}{2250} = 1.53\% \]

In order of influence on the final uncertainty in the power we have

1. Uncertainty of current determination
2. Uncertainty of voltage measurement
3. Uncertainty of knowledge of internal resistance of voltmeter

There are other conclusions we can draw from this example. The relative influence of the experimental quantities on the overall power determination is noted above. But this listing may be a bit misleading in that it implies that the uncertainty of the meter impedance does not have a large effect on the final uncertainty in the power determination. This results from the fact that \( R_m \gg R \) \((R_m = 10R)\). If the meter impedance were lower, say, 200 \( \Omega \), we would find that it was a dominant factor in the overall uncertainty. For a very high meter impedance there would be little influence, even with a very inaccurate knowledge of the exact value of \( R_m \). Thus, we are led to the simple conclusion that we need not worry too much about the precise value of the internal impedance of the meter as long as it is very large compared with the resistance we are measuring the voltage across. This fact should influence instrument selection for a particular application.

Example 3-4 A certain obstruction-type flowmeter (orifice, venturi, nozzle), shown in the accompanying figure, is used to measure the flow of air at low velocities. The relation describing the flow rate is

\[ m = C \frac{2A}{RT_1} \left( p_1 - p_2 \right)^{1/2} \]  \hauline

where \( C \) is an empirical-discharge coefficient, \( A \) is the flow area, \( p_1 \) and \( p_2 \) are the upstream and downstream pressures, \( T_1 \) is the upstream temperature, and

\[ \frac{p_1}{T_1} \]

Flow

\[ \Delta p \]

\[ 1 \]

\[ 2 \]
$R$ is the gas constant for air. Calculate the percent uncertainty in the mass flow rate for the following conditions:

- $C = 0.92 \pm 0.005$ (from calibration data)
- $p_1 = 25$ psia $\pm 0.5$ psia
- $T_1 = 70^\circ F \pm 2^\circ F$, $T_i = 530^\circ R$
- $\Delta p = p_1 - p_2 = 1.4$ psia $\pm 0.005$ psia (measured directly)
- $A = 1.0$ in$^2 \pm 0.001$ in$^2$

**SOLUTION** In this example the flow rate is a function of several variables, each subject to an uncertainty.

$$\dot{m} = f(C, A, p_1, \Delta p, T_1)$$

Thus, we form the derivatives:

$$\frac{\partial \dot{m}}{\partial C} = A \left(\frac{2g_i p_1}{RT_1} \Delta p\right)^{1/2}$$

$$\frac{\partial \dot{m}}{\partial A} = C \left(\frac{2g_i p_1}{RT_1} \Delta p\right)^{1/2}$$

$$\frac{\partial \dot{m}}{\partial p_1} = 0.5CA \left(\frac{2g_i}{RT_1} \Delta p\right)^{1/2} p_1^{1/2}$$

$$\frac{\partial \dot{m}}{\partial \Delta p} = 0.5CA \left(\frac{2g_i p_1}{RT_1} \Delta p\right)^{1/2} \Delta p^{-1/2}$$

$$\frac{\partial \dot{m}}{\partial T_1} = -0.5CA \left(\frac{2g_i p_1}{R} \Delta p\right)^{1/2} T_1^{-3/2}$$

The uncertainty in the mass flow rate may now be calculated by assembling these derivatives in accordance with Eq. (3-2). Designating this assembly as Eq. (c) and then dividing by Eq. (a) gives

$$\frac{w_m}{\dot{m}} = \left[\frac{\left(\frac{w_{m}}{C}\right)^2}{\frac{w_{m}}{A}} + \frac{1}{4} \left(\frac{w_{m}}{p_1}\right)^2 + \frac{1}{4} \left(\frac{w_{m}}{\Delta p}\right)^2 + \frac{1}{4} \left(\frac{w_{m}}{T_1}\right)^2\right]^{1/2}$$

We may now insert the numerical values for the quantities to obtain the percent uncertainty in the mass flow rate.

$$\frac{w_m}{\dot{m}} = \left[\frac{\left(0.005\right)^2}{0.92} + \frac{\left(0.001\right)^2}{1.0} + \frac{1}{4} \left(\frac{0.5}{25}\right)^2 + \frac{1}{4} \left(\frac{0.005}{1.4}\right)^2 + \frac{1}{4} \left(\frac{2}{530}\right)^2\right]^{1/2}$$

$$= \left[29.5 \times 10^{-6} + 1.0 \times 10^{-6} + 1.0 \times 10^{-4} + 3.19 \times 10^{-6} + 3.57 \times 10^{-6}\right]^{1/2}$$

$$= \left[1.373 \times 10^{-4}\right]^{1/2} = 1.172\%$$

The main contribution to uncertainty is the $p_1$ measurement with its basic uncertainty of 2 percent. Thus, to improve the overall situation the accuracy of this measurement should be attacked first. In order of influence on the flow-rate uncertainty, we have

1. Uncertainty in $p_1$ measurement (± 2 percent)
2. Uncertainty in value of $C$
3. Uncertainty in determination of $T_1$
4. Uncertainty in determination of $\Delta p$
5. Uncertainty in determination of $A$

By inspecting Eq. (e) we see that the first two items make practically the whole contribution to uncertainty. The value of the uncertainty analysis in this example is that it shows the investigator how to improve the overall measurement accuracy of this technique. First, obtain a more precise measurement of $p_1$. Then try to obtain a better calibration of the device, i.e., a better value of $C$. In Chap. 7 we shall see how values of the discharge coefficient $C$ are obtained.

3-5 EVALUATION OF UNCERTAINTIES FOR COMPLICATED DATA REDUCTION

We have seen in the preceding discussion and examples how uncertainty analysis can be a useful tool to examine experimental data. In many cases data reduction is a rather complicated affair and is often performed with a computer routine written specifically for the task. A small adaptation of the routine can provide for direct calculation of uncertainties without resorting to an analytical determination of the partial derivatives in Eq. (3-2). We still assume that this equation applies, although it could involve several computational steps. We also assume that we are able to obtain estimates by some means of the uncertainties in the primary measurements, i.e., $w_1, w_2, etc.$

Suppose a set of data is collected in the variables $x_1, x_2, \ldots, x_n$ and a result calculated. At the same time one may perturb the variables by $\Delta x_1, \Delta x_2$, and so on, and calculate new results. We would have

$$R(x_1) = R(x_1, x_2, \ldots, x_n)$$

$$R(x_1 + \Delta x_1) = R(x_1 + \Delta x_1, x_2, \ldots, x_n)$$

$$R(x_2) = R(x_1, x_2, \ldots, x_n)$$

$$R(x_2 + \Delta x_2) = R(x_1, x_2 + \Delta x_2, \ldots, x_n)$$
For small enough values of $\Delta x$ the partial derivatives can be well approximated by
\[
\frac{\partial R}{\partial x_1} \approx \frac{R(x_1 + \Delta x_1) - R(x_1)}{\Delta x_1}, \quad \frac{\partial R}{\partial x_2} \approx \frac{R(x_2 + \Delta x_2) - R(x_2)}{\Delta x_2}
\]
and these values could be inserted in Eq. (3-2) to calculate the uncertainty in the result.

At this point we must again alert the reader to the ways uncertainties or errors of instruments are normally specified. Suppose a pressure gage is available and the manufacturer states that it is accurate within $\pm 1.0$ percent. This statement normally refers to percent of full scale. So a gage with a range of 0 to 100 kPa would have an uncertainty of $\pm 10$ percent when reading a pressure of only 10 kPa. Of course, this means that the uncertainty in the calculated result, either as an absolute value or percentage, can vary widely depending on the range of operation of instruments used to make the primary measurements. The above procedure can be used to advantage in complicated data-reduction schemes.

A very full description of this technique and many other considerations of uncertainty analysis are given by Moffat [7].

**Example 3-5** Calculate the uncertainty of the wire resistance in Example 3-1 using the technique of this section.

**Solution** In Example 3-1 we have already calculated the nominal resistance as 6.24 $\Omega$. We now perturb the three variables $R_0$, $\alpha$, and $T$ by small amounts to evaluate the partial derivatives. We shall take
\[
\Delta R_0 = 0.01, \quad \Delta \alpha = 1 \times 10^{-5}, \quad \Delta T = 0.1
\]
Then
\[
R(R_0 + \Delta R_0) = (6.01)(1 + (0.004)(30 - 20)) = 6.2504
\]
and the derivative is approximated as
\[
\frac{\partial R}{\partial R_0} \approx \frac{R(R_0 + \Delta R_0) - R}{\Delta R_0} = \frac{6.2504 - 6.24}{0.01} = 104
\]
or the same result as in Example 3-1. Similarly,
\[
R(\alpha + \Delta \alpha) = (6.0)(1 + (0.00401)(30 - 20)) = 6.2406
\]
\[
\frac{\partial R}{\partial \alpha} \approx \frac{R(\alpha + \Delta \alpha) - R}{\Delta \alpha} = \frac{6.2406 - 6.24}{1 \times 10^{-5}} = 60
\]

\[
R(T + \Delta T) = (6)[1 + (0.004)(30.1 - 20)] = 6.2424
\]
\[
\frac{\partial R}{\partial T} \approx \frac{R(T + \Delta T) - R}{\Delta T} = \frac{6.2424 - 6.24}{0.1} = 0.24
\]
All the derivatives are the same as in Example 3-1 so the uncertainty in $R$ would be the same, or 0.0305 $\Omega$.

### 3-6 STATISTICAL ANALYSIS OF EXPERIMENTAL DATA

We shall not be able to give an extensive presentation of the methods of statistical analysis of experimental data; we may only indicate some of the more important methods currently employed. First, it is important to define some pertinent terms.

When a set of readings of an instrument is taken, the individual readings will vary somewhat from each other, and the experimenter is usually concerned with the mean of all the readings. If each reading is denoted by $x_i$ and there are $n$ readings, the arithmetic mean is given by
\[
x_m = \frac{1}{n} \sum_{i=1}^{n} x_i
\]
(3.3)

The deviation $d_i$ for each reading is defined by
\[
d_i = x_i - x_m
\]
(3.4)

We may note that the average of the deviations of all the readings is zero since
\[
d_m = \frac{1}{n} \sum_{i=1}^{n} d_i = \frac{1}{n} \sum_{i=1}^{n} (x_i - x_m) = x_m - x_m = 0
\]
(3.5)
The average of the absolute values of the deviations is given by
\[
|d_m| = \frac{1}{n} \sum_{i=1}^{n} |d_i| = \frac{1}{n} \sum_{i=1}^{n} |x_i - x_m|
\]
(3.6)

Note that this quantity is not necessarily zero. The standard deviation or root mean square deviation is defined by
\[
\sigma = \left[ \frac{1}{n} \sum_{i=1}^{n} (x_i - x_m)^2 \right]^{1/2}
\]
(3.7)