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CHAPTER 12

CONDENSATION OF SINGLE VAPORS

Introduction. A fluid may exist as a gas, vapor, or liquid. The change from liquid to vapor is vaporization, and the change from vapor to liquid is condensation. The quantities of heat involved in the vaporization or condensation of a pound of fluid are identical. For a pure fluid compound at a given pressure the change from liquid to vapor or vapor to liquid occurs at but one temperature which is the saturation or equilibrium temperature. Since vapor-liquid heat-transfer changes usually occur at constant or nearly constant pressure in industry, the vaporization or condensation of a single compound normally occurs isothermally. When a vapor is removed upon formation from further contact with a liquid, the addition of heat to the vapor causes superheat, during which it behaves like a gas. If a mixture of vapors instead of a pure vapor is condensed at constant pressure, the change does not take place isothermally in most instances. The general treatment of vapor mixtures differs in certain respects from single compounds and will be studied in the next chapter with the aid of the phase rule of J. Willard Gibbs.

Condensation occurs at very different rates of heat transfer by either of the two distinct physical mechanisms, which will be discussed presently, dropwise or filmwise condensation. The condensing film coefficient is influenced by the texture of the surface on which condensation occurs and also by whether the condensing surface is mounted vertically or horizontally. In spite of these apparent complications condensation, like streamline flow, lends itself to direct mathematical study.

Dropwise and Filmwise Condensation. When a saturated pure vapor comes into contact with a cold surface such as a tube, it condenses and may form liquid droplets on the surface of the tube. These droplets may not exhibit an affinity for the surface and instead of coating the tube fall from it, leaving bare metal on which successive droplets of condensate may form. When condensation occurs by this mechanism, it is called dropwise condensation. Usually, however, a distinct film may appear as the vapor condenses and coats the tube. Additional vapor is then required to condense into the liquid film rather than form directly on the bare surface. This is film or filmwise condensation. The two mechanisms are distinct and independent of the quantity of vapor condensing per square foot of surface. Filmwise condensation is therefore not a transition from dropwise condensation because of the rapidity at which condensate forms on the tube. Due to the resistance of the condensate film to the heat passing through it the heat-transfer coefficients for dropwise condensation are four to eight times those for filmwise condensation. Steam is the only pure vapor known to condense in a dropwise manner, and special conditions are required for its occurrence. These are described by Drew, Nagle, and Smith and principally result from the presence of dirt on the surface or the use of a contaminant which adheres to the surface. Materials have been identified by Nagle which promote the dropwise condensation of steam although these also introduce an impurity into the steam. Dropwise condensation also occurs when several materials condense simultaneously as a mixture and where the condensate mixture is not miscible, as in the case of a hydrocarbon and steam. However, during various periods in the normal operation of a steam condenser the mechanism may initially be filmwise condensation, shift to dropwise condensation, and at some later time revert to film condensation. Because of the lack of control it is not customary in calculations to take advantage of the high coefficients which have been obtained in dropwise-condensation experiments. This chapter consequently deals with the calculation of condensers for various conditions and is based solely upon film-condensation heat-transfer coefficients.

It is fortunate that the phenomenon of film condensation lends itself to mathematical analysis, and the nature of condensation on a cold surface may be considered one of self-diffusion. The saturation pressure of vapor in the vapor body is greater than the saturation pressure of the cold condensate in contact with the cold surface. This pressure difference provides the potential for driving vapor out of the vapor body at a great rate. Compared with the small resistance to heat transfer by diffusion from the vapor into the condensate, the film of condensate on the cold tube wall contributes the controlling resistance. It is the slowness with which the heat of condensation passes through this film that determines the condensing coefficient. The ultimate form of an equation for the condensing coefficient may be obtained from dimensional analysis where the average condensing coefficient $h$ is a function of the properties of the condensate film, $k, \rho, g, \mu, \Delta T,$ and $\lambda$, the last being the latent heat of vaporization. Nusselt theoretically derived the relationships for the mechanism of film condensation, and the results he obtained are in excellent agreement with experiments.

Process Applications. In chemical industry it is a common practice to separate a liquid mixture by distilling off the compounds which have

\footnotesize{\begin{enumerate}
\item Nagle, W. M., U.S. Patent 1,995,381.
\end{enumerate}}
lower boiling points in the pure condition from those having higher boiling points. In a solution of several compounds each exerts a partial pressure and the most volatile cannot be boiled off from the rest without carrying some of the heavier or higher boiling compounds along with it. The proportion of heavier compounds carried off when a solution starts to boil is less than existed in the original solution before boiling commenced. If the vapor coming off initially is condensed, it has a lower boiling point than the original solution, indicating the increase in the proportion of the more volatile compounds. By successively boiling off

only part of a liquid mixture, condensing the vapor formed, and boiling off only a part of the condensate, it is possible to obtain a nearly pure quantity of the most volatile compound by numerous repetitions of the procedure. Thus the separation by distillation is accomplished by partial vaporization and subsequent condensation.

In distillation it is customary to obtain a number of partial vaporizations and condensations by directly contacting a vapor and a liquid cooling medium in a continuous distilling column. The bubble cap distilling column shown in Fig. 12.1 is representative of modern practice and derives its name from a series of inverted slotted caps which are placed over vapor risers on each plate of the column. Vapor from below a plate enters the risers and is broken into bubbles as it passes through the slotted bottoms of the bubble caps and thence through the layer of liquid maintained by the downcomer on each plate. The feed, which is usually a liquid, is a mixture of more and less volatile compounds and enters the distilling column at the feed plate where the volatile compounds are partially vaporized by the rising vapors as the feed travels across the plate. The remainder of the liquid on the plate is less volatile than the feed and overflows to the plate below through the downcomer. The boiling points of the liquids on each of the lower plates is consequently higher.

To vaporize a portion of the feed, vapor from below the feed plate must exchange heat with the liquid on the feed plate, thereby driving the more volatile compounds to the plate above the feed. By supplying heat at the bottom of the column where the increased concentration of the least volatile compounds represents the highest boiling temperature in the system, a thermal gradient is established plate by plate between the bottom of the column and the top. Heat supplied at the bottom by vaporization in a reboiler is transmitted to the top of the column plate-by-plate due to the temperature differences corresponding to the differences in boiling points between plates. Continuous distillation requires the presence of liquid at all times on the plates, so that vapors of the less volatile compounds in the feed may be condensed and carried downward.

To accomplish this, some volatile liquid from the condenser, which represents one plate above the top plate and which is therefore colder, is introduced onto the top plate and flows downward in the column. The volatile liquid which is poured back into the column from the condenser is the reflux. The quantity of volatile components removed from the system at the top and having the same composition as the reflux is called the distillate or overhead product. The heavier compounds removed at the bottom are variously called waste or residue or, if they are of value, bottoms product. The quantitative aspects of the heat balance are treated in Chap. 14.

It is the condensing temperature in the condenser which determines the operating pressure of the distilling column, since the saturation temperature of a vapor varies with its pressure. The overhead product must condense in the condenser at a temperature sufficiently high so that its latent heat can be removed by cooling water. The size of the condenser is dependent upon the difference between the condensing temperature and the range of the cooling water. If the condensing temperature is very close to the cooling water range at atmospheric pressure, the distillation pressure must be elevated to permit the attainment of a larger ΔT.

In the power industry the term surface condenser is reserved for tubular equipment which condenses steam from the exhaust of turbines and engines. Since a turbine is primarily designed to obtain mechanical work
from heat, the maximum conversion is obtained in the turbine by maintaining a low turbine-discharge temperature. If the turbine were to discharge to the atmosphere, the lowest attainable steam temperature would be 212°F, but if the steam were to discharge into a condenser under vacuum, it would be possible to operate at discharge temperatures of 75°F and lower and to convert the enthalpy difference from 212 to 75°F into useful work.

Condensation on Surfaces—Nusselt’s Theory. In condensation on a vertical surface a film of condensate is formed as shown in Fig. 12.2 and further condensation and heat transfer to the surface occurs by conduction through the film which is assumed to be in laminar flow downward.

The thickness of this film greatly influences the rate of condensation, since the heat accompanying the removal of vapors from the vapor phase encounters the condensate film as a resistance which may be quite large. The thickness of the film is a function of the velocity of drainage which varies with the deviation of the surface from a vertical position. For a vertical surface the thickness of the film cumulatively increases from top to bottom. For this reason the condensing coefficient for a vapor condensing on a vertical surface decreases from top to bottom, and for the attainment of a large condensing coefficient the height of the surface should not be very great. The velocity of drainage for equal quantities of condensate is also a function of the viscosity of the condensate; the lower the viscosity the thinner the film. For all liquids the viscosity decreases as the temperature increases, and the condensing coefficient consequently increases with the condensate temperature. The derivations given in this chapter through Eq. (12.34) are those of Nusselt.1

The following assumptions are involved:
1. The heat delivered by the vapor is latent heat only.
2. The drainage of the condensate film from the surface is by laminar flow only, and the heat is transferred through the film by conduction.
3. The thickness of the film at any point is a function of the mean velocity of flow and of the amount of condensate passing at that point.
4. The velocity of the individual layers of the film is a function of the relation between frictional shearing force and the weight of the film (see Chap. 3).

5. The quantity of condensate is proportional to the quantity of heat transferred, which is in turn related to the thickness of the film and the temperature difference between the vapor and the surface.
6. The condensate film is so thin that the temperature gradient through it is linear.
7. The physical properties of the condensate are taken at the mean film temperature.
8. The surface is assumed to be relatively smooth and clean.
9. The temperature of the surface of the solid is constant.
10. The curvature of the film is neglected.

Condensation. Vertical Surfaces. In Fig. 12.2 the rate at which heat passes from the vapor through the liquid condensate film and into the cooling surface per unit area is given by

\[ \frac{Q}{A} = \frac{k(t' - t)}{y} = \lambda W' = h(t' - t) \]  \hspace{1cm} (12.1)

where \( \lambda \) is the latent heat of vaporization, \( W' \) the pounds of condensate formed per hour per square foot, and \( y' \) is the thickness of the condensate film at the generalized point in the figure whose coordinates are \( x', y' \). The other symbols have their conventional meaning.

The rate at which the vapor condenses is then given by

\[ W' = \frac{k(t' - t)}{\lambda y'} \]  \hspace{1cm} (12.2)

The liquid flows downward over the vertical surface with a velocity \( u \) varying from zero at the tube-film interface and increasing outward to the condensate-vapor interface. The velocity also increases vertically as the condensate flows downward.

Consider a small cube of unit depth \( dz = 1 \), defined by \( dx \ dy \) in the moving condensate film of Fig. 12.2. On the side near the cold vertical surface there is a tangential force acting upward and tending to support the cube. On the side away from the cold vertical surface there is a tangential force acting downward due to the more rapid movement of the liquid downward as the distance from the surface is increased. If the resultant force upward through the cube is designated by \( \tau \), then the respective forces are \( \tau - \frac{dr}{dy} \) and \( \tau + \frac{dr}{dy} \). The differential tangential force must be offset by gravity acting down.

\[ \rho \frac{dr}{dy} \, dy \, dz = \left( \tau - \frac{dr}{dy} \frac{dy}{2} \right) - \left( \tau + \frac{dr}{dy} \frac{dy}{2} \right) = -\rho \frac{dr}{dy} \]  \hspace{1cm} (12.3)

On unit area, \( dx \, dz = 1 \)

\[ \rho = -\frac{dr}{dy} \]
From the basic definition of viscosity in Chap. 3 the tangential stress is defined by Eq. (3.3), using (lb-force)/(hr)/(ft²) as the dimensions for the viscosity. Since it is customary to use the dimensions (lb-mass)/(ft)(hr) for the dimensions, Eq. (3.3) becomes

\[ \tau = \frac{\mu}{g} \frac{d u}{dy} \]
\[ \frac{d \tau}{dy} = \frac{\mu}{g} \frac{d^2 u}{dy^2} = -\rho \]  
(12.4)
(12.5)

Take \( \rho/\mu \) as constant.

\[ \frac{d^2 u}{dy^2} = -\frac{\rho g}{\mu} \]  
(12.6)
\[ u = -\frac{\rho g y^2}{2\mu} + C_1 y + C_2 \]  
(12.7)

The constants \( C_1 \) and \( C_2 \) must now be evaluated. Since the liquid adheres to the wall, \( u \) must equal zero at \( y = 0 \), making \( C_2 \) equal zero.

![Fig. 12.3. Vertical condensate film flow.](image)

At the outer boundary of the film (condensate-vapor interface) there is no tangential stress and from \( \tau = \mu \frac{du}{dy} \) when \( y = y' \),

\[ \left( \frac{d u}{dy} \right)_{y=y'} = 0 = -\frac{\rho g y'}{\mu} + C_1 \quad C_1 = \frac{\rho g y'}{\mu} \]  
(12.8)

At a distance \( x \) from the top of the condensing surface the average velocity downward \( \bar{u} \) is given by

\[ \bar{u} = \frac{1}{y'} \int_0^{y'} u \, dy = \frac{\rho g}{3\mu} y' \]  
(12.9)

When the value of \( x \) from the top of a vertical wall is taken to be unity as shown in Fig. 12.3a, the quantity of downward flow across a horizontal plane of area \( 1y' \) of the condensate is

\[ 1y' \bar{u} \]

At \( x + dx \) there is a gain in the amount of downward flow of condensate as shown in Fig. 12.3b. Using the value for \( \bar{u} \) from Eq. (12.9), multiplying by \( \rho g y' \), and differentiating with respect to \( x \) to obtain the increase from \( x \) to \( x + dx \),

\[ d(\rho g y') = d \left( \frac{\rho g y'}{2\mu} \right) \frac{dy'}{\mu} \]  
(12.10)

And this increase must come from condensation out of the vapor and into the condensate film

\[ d(\rho g y') = W' \frac{dx}{x} \]

where \( W' \) is the condensate flow out of the vapor and "normal" to the falling condensate layer per unit area as in Fig. 12.3b. From Eq. (12.2), however, \( W' \) was defined in terms of the heat transfer as

\[ W' = \frac{k(t' - t)}{\lambda y'} \]

Substituting for \( W' \) in Eq. (12.10) the value from Eq. (12.2),

\[ \frac{k}{\lambda y'} (t' - t) \frac{dx}{\mu} = \frac{\rho g y'}{\mu} \frac{dy'}{\mu} \]  
(12.11)
\[ (t' - t) \frac{dx}{x} = \frac{\rho g y'}{k \mu} \frac{dy'}{y'} \]  
(12.12)

For a limited range set \( t' - t \), \( \rho \), \( \lambda \), \( \mu \), and \( k \) constant and integrate.

When \( y' = 0 \), \( x = 0 \)

\[ y' = \int_0^{y'} u \, dy = \frac{\rho g y'}{3\mu} \quad \frac{dy'}{3} \]  
(12.13)
\[ y' = \left[ \frac{4\mu k}{\rho^2 \lambda} (t' - t) x \right]^{3/4} \]  
(12.14)

The heat-transfer coefficient across the condensate layer at the distance \( x \) from the origin per unit of interfacial area is given from Eq. (12.1) by

\[ h_x = \frac{Q}{t' - t} = \frac{k}{y'} \]  
(12.15)

Substitute \( y' \) from Eq. (12.11):

\[ h_x = \left[ \frac{k^3 p \lambda}{4 \mu} \right]^{1/3} \frac{1}{x^{1/3}} \]  
(12.16)
The total heat through the condensate layer from 0 to \( x \) is \( Q_x \):

\[
Q_x = \int_0^x I_x(t' - t) \, dx = \int_0^x \left[ -\frac{k_f p \rho g}{\mu} \right]^{1/4} (t' - t) \, dx
\]

\[
= \frac{4^{1/4}}{3} \left( \frac{k_f p \rho g}{\mu} \right)^{1/4} [(t' - t)x]^{1/4} \tag{12.17}
\]

If the average coefficient between the two points is \( \bar{h} \):

\[
\bar{h} = \frac{(Q_x)_{vortal}}{(t' - t)L} = \frac{4^{1/4}}{3} \left( \frac{k_f p \rho g}{\mu} \right)^{1/4} \frac{[(t' - t)L]^{1/4}}{(t' - t)L}
\]

\[
\bar{h} = 0.943 \left( \frac{k_f p \rho g}{\mu \Delta t_f} \right)^{1/4} \tag{12.18}
\]

where \( k_f, \rho, \) and \( \mu \) are evaluated at the film temperature \( t_f \) and where the film temperature is

\[
t_f = \frac{1}{2}(t' + t) = \frac{1}{2}(T_s + t_o) \tag{12.19}
\]

and

\[
\Delta t_f = (t_f - t_o) = t' - t \tag{12.19}
\]

In the above, as in the derivation which follows, the stress caused by the passage of the saturated vapor over the condensate-vapor interface has been neglected. It can be included, although its is not of practical consequence. The variation of the film thickness and local heat-transfer coefficient are shown in Fig. 12.4. The shapes of the curves follow the thickness and consequently the resistances of the condensate film.

**Inclined Surfaces.** Consider a cube making the angle \( \alpha \) as shown in Fig. 12.5. The gravity component acting in a plane parallel to the surface is \( \rho \sin \alpha \), and Eq. (12.3) becomes

\[
\rho \sin \alpha \, dy \, dz \, dx = \left( \tau - \frac{d \tau}{dy} \frac{dy}{2} \right) - \left( \tau + \frac{d \tau}{dy} \frac{dy}{2} \right) = -d \tau \tag{12.20}
\]

On unit area \( dx \, dz = 1 \)

\[
\rho \sin \alpha = -\frac{d \tau}{dy} \tag{12.21}
\]

Equation (12.6) becomes

\[
\frac{d^2 u}{dy^2} = -\frac{\rho \sin \alpha}{\mu} \tag{12.22}
\]

Equation (12.7) becomes

\[
u = -\frac{\rho \sin \alpha}{2 \mu} y^2 + C_1 y + C_2 \tag{12.23}
\]

At the start of condensation on the tube where \( y = 0 \) and there is no velocity along the tube, \( u = 0 \) and \( C_2 = 0 \)

\[
C_1 = \frac{\rho \sin \alpha}{\mu} \tag{12.24}
\]

and Eq. (12.9) becomes

\[
\bar{h} = \frac{3}{2} \frac{\rho \sin \alpha}{\mu \Delta t_f} \tag{12.25}
\]

**Horizontal Tubular Surfaces.** Consider a cube of unit length at radius \( r \) making an angle \( \alpha \) with the vertical as seen in Fig. 12.6. The mass flow of vapor into the condensate film through the area \( r \, dr \) and with a film
thickness \(y'\) is given by the conduction equation.

\[
W' = \frac{k(k' - l)r\,d\alpha}{\lambda y'}.
\]

The condensation must give rise to an increase in the inclined falling film. For a differential amount of condensation at the increase through the condensing area \(r\,d\alpha\) is \(d(\rho y')\) and for Eq. (12.10)

\[
d(\rho y') = \frac{\rho \eta}{3\mu} d(y'^* \sin \alpha) = W'\,d\alpha
\]

Substituting for \(W'\), Eq. (12.11) becomes

\[
\frac{k(k' - l)r\,d\alpha}{\lambda y'} = \frac{\rho \eta}{3\mu} d(y'^* \sin \alpha)
\]

\[
3\mu k(k' - l)r\,d\alpha = y'd(y'^* \sin \alpha)
\]

Let

\[
m = \frac{3\mu k(k' - l)r}{\rho \eta}
\]

\[
md\alpha = y'd(y'^* \sin \alpha)
\]

Differentiating

\[
m\,d\alpha = y'(3y'^* \sin \alpha \,dy'^* + y'^* \cos \alpha \,d\alpha)
\]

\[
= 3y'^* \sin \alpha \,dy'^* + y'^* \cos \alpha \,d\alpha
\]

In Eq. (12.26) the term \(3y'^* \,dy'^*\) appears, but \(d(y'^*) = 4y'^* \,dy'^*\) and

\[
3y'^* \,dy'^* = \frac{3}{4} y'^* dy'^*
\]

Rearranging Eq. (12.26) and substituting,

\[
d\alpha = \frac{3}{4m} \sin \alpha \,dy'^* + \frac{y'^*}{m} \cos \alpha \,d\alpha
\]

Let \(y'/m = \psi\)

\[
d\alpha = \frac{3}{4} \sin \alpha \,dz + z \cos \alpha \,d\alpha
\]

\[
3 \frac{\sin \alpha}{d\alpha} \frac{dz}{\sin \alpha} + z \cos \alpha - 1 = 0
\]

Equation (12.28) is a linear differential equation whose solution is

\[
z = \frac{1}{\sin^{3\alpha}} \left(\frac{4}{3} \int \sin^{3\alpha} \alpha \,d\alpha + C_2\right)
\]

When \(\alpha = 0, C_2 = 0,\)

\[
z = \frac{4}{3} \frac{1}{\sin^{3\alpha}} \int \sin^{3\alpha} \alpha \,d\alpha
\]

The value of this integral for different values of \(\alpha\) may be determined by graphical methods. From the substitution in Eq. (12.27)

\[
y' = \psi m^{\frac{1}{4}} = \psi \left[\frac{3\mu k(k' - l)r}{\rho \eta}\right]^{\frac{1}{4}}
\]

As shown in Eq. (12.15), \(h_x = k/y'\).

The thickness of the film actually decreases slightly as \(\alpha\) increases from 0 to 5°, and then it increases steadily and breaks into drops. The local-heat-transfer coefficient at any point is then

\[
h_x = \frac{k}{y'} = \frac{1}{\psi} \left[\frac{3\mu k(k' - l)r}{\rho \eta}\right]^{\frac{1}{4}}
\]

The average heat-transfer coefficient \(h\) of the segment between angles \(\alpha_1\) and \(\alpha_2\) is

\[
h_s \int_{\alpha_1}^{\alpha_2} = \frac{k}{m^{\frac{1}{4}}(\alpha_2 - \alpha_1)} \int_{\alpha_1}^{\alpha_2} d\alpha
\]

Employing graphical methods as before, where \(D_1\) is the outside diameter of the tube, the average heat-transfer coefficients are found to be

\[
h_s \int_{0^\circ}^{100^\circ} = 0.860 \left(\frac{k\rho \eta}{\mu D_1 \Delta T_f}\right)^{\frac{1}{4}}
\]

\[
h_s \int_{100^\circ}^{180^\circ} = 0.589 \left(\frac{k\rho \eta}{\mu D_1 \Delta T_f}\right)^{\frac{1}{4}}
\]

From 0 to 180° which is one-half the tube, the other half being symmetrical,

\[
h = 0.725 \left(\frac{k\rho \eta}{\mu D_1 \Delta T_f}\right)^{\frac{1}{4}}
\]

The variation of the film thickness and heat-transfer coefficient for steam on a horizontal tube is shown in Fig. 13.2. As in the preceding case it is governed by the resistance of the condensate film to conduction.

**Development of Equations for Calculations.** McAdams\(^1\) found from the correlation of the data of several investigations that observed condensing coefficients for steam on vertical tubes were 75 per cent greater than the theoretical coefficients calculated by Eq. (12.18). The values calculated from Eq. (12.18) agree, however, for a condensate in streamline flow with the values calculated from Eq. (6.1) for ordinary streamline flow.

When a liquid descends vertically on the outside of a tube, it is certainly
in streamline flow at the top of the tube, where the accumulation of condensate is small. If a relatively large amount of vapor is condensed on the tube, it is possible that at some point below the top the film will change to turbulent flow. This may be estimated from the diameter and length of the tube, the viscosity of the condensate, and the quantity being condensed. Referring to the tube as shown in Fig. 12.8, the crosshatched area outside the tube represents condensate film as seen at any point looking down. This is similar to the flow in the annulus of a double pipe exchanger except that the outer surface of the tube is not formed by a concentric pipe. In the case of the double pipe exchanger the equivalent diameter was taken as four times the hydraulic radius.

Then

\[ D_s = 4r_h = 4 \times \frac{\text{free flow area}}{\text{wetted perimeter}} \]

and

\[ Re = \frac{D_s \dot{G}}{\mu} \]

For vertical tubes let

- \( A_f \) = cross-sectional area (shaded)
- \( P = \) wetted perimeter per tube
- \( D_s = 4 \times A_f / P \)

Letting the loading per tube be \( w' = W / N_t \), where \( N_t \) is the number of tubes,

\[ G = \frac{w'}{A_f} \quad \text{lb/(hr)(ft)}^2 \]

\[ Re = \frac{D_s \dot{G}}{\mu} = \left( \frac{4A_f / P}{w'/A_f} \right) / \mu = 4w'/\mu P \quad (12.35) \]

Calling the condensate loading per linear foot \( G' \),

\[ G' = \frac{w'}{P}, \quad \text{lb/(hr)(lin ft)} \quad (12.36) \]

Eq. (12.35) becomes

\[ Re = \frac{4G'}{\mu} \]

The total heat load is given by \( Q = \lambda w' \).

\[ \dot{q} = \frac{Q}{A \Delta t_f} = \frac{\lambda w'}{PL \Delta t_f} = \frac{\lambda}{L} \frac{G'}{\Delta t_f} \quad (12.37) \]

Substituting in Eq. (12.18),

\[ \dot{q} = 0.943 \left( \frac{k^3 \rho \mu \dot{g}}{\mu_f \dot{g}} \right)^{14} \quad (12.38) \]

Multiplying the right term by \((4\mu/4\mu)^{14}\),

\[ \dot{q} = 0.943 \left( \frac{4k^3 \rho \mu \dot{g}}{4 \mu_f \dot{g}} \right)^{14} \]

\[ \dot{q} \left( \frac{\mu^3}{k^3 \rho \mu} \right)^{15} = 1.47 \left( \frac{4G'}{\mu_f} \right)^{14} \quad (12.39) \]

For horizontal tubes Eq. (12.39) becomes

\[ \dot{q} \left( \frac{\mu^3}{k^3 \rho \mu} \right)^{15} = 1.51 \left( \frac{4G''}{\mu_f} \right)^{14} \quad (12.40) \]

where the loading for a single horizontal tube is

\[ G'' = \frac{W}{LN_t} \quad (12.41) \]
Using the corresponding tables as given by either Eq. (12.36) or (12.41) as the case may be, Eqs. (12.39) and (12.40) may both be represented by

\[ k \left( \frac{\mu_f}{k_f \rho_f} \right)^{15} = 1.5 \left( \frac{dG}{d\mu_r} \right)^{15} = 1.5 \left( \frac{\mu_0}{\mu_r} \right)^{15} \] (12.42)

Equations (12.39) and (12.40) were obtained for condensation on single tubes. In a vertical-tube bundle the presence of one or more tubes does not alter the assumptions on which the derivation was predicated. However, on horizontal tubes in tube bundles it has been found that the splashing of the condensate as it drips over successive rows of tubes causes \( G'' \) to be more nearly inversely proportional to \( N_r^{15} \) rather than \( N_r \) so that it is preferable to use a fictitious value for horizontal tubes

\[ G'' = \frac{W}{LN_r^{15}} \text{ lb/(hr)(lin ft)} \] (12.43)

Figure 12.9 is a line chart of solutions of Eq. (12.42) prepared for convenience. Its use requires that the film be in streamline flow corresponding to an average Reynolds number of about 1800 to 2100 for the flow gradient assumed by the condensate. For steam at atmospheric pressure Eq. (12.42) reduces to the equations given by McAdams.\(^1\) For horizontal tubes

\[ k = \frac{3100}{D_r^{15} \Delta T^{15}} \] (12.44a)

and for vertical tubes

\[ k = \frac{4000}{L^{15} \Delta T^{15}} \] (12.44b)

where \( \Delta T \) ranges from 10 to 150°F.

It is frequently desirable to apply Eqs. (12.39), (12.40), and (12.42) to the calculation of condensers which are modifications of the 1-2 exchanger with condensation in the shell. Such condensers have baffled tube bundles. The baffles do not affect the condensing film coefficients in horizontal condensers, since the coefficients are independent of the vapor mass velocity, but they do influence the accumulation of condensate on the tubes of vertical condensers. Moreover, in condensers with multi-pass tubes the tube-wall temperature is different at every point in each pass, whereas the surface temperature was assumed constant in the derivations. A correction for the latter cannot be accounted for in the calculations except by the treatment of small surface increments of each pass individually. The error introduced by using the mean tube-wall temperature as being effective over all the surface is apparently too small to justify the lengthier calculation. Since the baffle holes are ordinarily

about \(\frac{1}{4}\) in. greater in diameter than the tube outside diameter, the baffles in vertical condensers prevent the condensate film from reaching a thickness greater than \(\frac{1}{6}\) in. before impinging on the baffle. This, however, is a favorable limitation except at high tube loadings where the condensate film might otherwise grow sufficiently to change to turbulent flow.

**Comparison between Horizontal and Vertical Condensation.** The value of the condensing film coefficient for a given quantity of vapor on a given surface is significantly affected by the position of the condenser. In a vertical tube about 60 per cent of the vapor condenses in the upper half of the tube. By combining Eqs. (12.18) and (12.34) the ratio of the theoretical horizontal and vertical condensing coefficients is given by

\[
K_t = \frac{0.725}{(0.943)(L/D)^{0.4}}
\]

For a \(\frac{3}{4}\) in. OD tube 160\" long, the horizontal coefficient should be 3.07 times as great as the vertical coefficient, provided that the condensate film is in streamline flow throughout. Ordinarily, however, the advantage is not quite so great due to the other modifications which prevail such as a transition of the vertical film into turbulent flow.

For the condensation of exhaust steam from turbines with vacuum discharge the condenser surface is usually very great, from 10,000 to 60,000 ft\(^2\) per shell, and the economies are such that tubes up to 26 ft long are employed. These large condensers are designed with overall transfer coefficients as high as 800 Btu/(hr)(ft\(^2\))(\(^\circ\)F) as treated later. Condensers for such services are universally installed in a horizontal position to facilitate the distribution of the vapor and the removal of the condensate.

When a condenser is employed on a distillation column, several specific factors must be taken into consideration. A typical arrangement of such a condenser is shown in Fig. 12.10, in which the reflux is returned to the column by gravity. The condensate leg 1-2 of height \(z_0\), must supply sufficient hydrostatic head to return the condensate to the column through the seal. In Fig. 12.11 a vertical condenser is employed for the same service, but it is not well suited for gravity return of the condensate, since it must be elevated considerably above the column, which in many cases is very tall by itself. Maintenance and structural support for the vertical condenser may be costly and considerably more difficult. On the other hand if it is desired not only to condense the overhead vapor but also to subcool the condensate, the vertical condenser is admirably suited. **Subcooler** is the operation of cooling the condensate below its saturation temperature, and this is done very frequently when the overhead product is a volatile liquid to be sent to storage. By subcooling it is possible to avoid large evaporation losses during initial storage. The combination of condensation and subcooling in a single unit eliminates the need for a separate overhead product cooler, as shown in Fig. 12.1.

**Condensation inside Tubes. Horizontal Condensers.** The equations developed so far give excellent results when applied to condensation outside tubes, although the deviations in commercial condensers have not been reported except in isolated instances. Often, however, the condensate is corrosive, or it is desired to recover the latent heat from the vapor by using it to preheat the feed to a column. In such cases it may be preferable to condense the vapor in the tubes rather than on the tubes wherein the original derivation is no longer applicable. Within the tubes of a single-pass horizontal condenser each tube condenses an equal amount of vapor and there is no change in the coefficient due to the splashing of condensate from one tube row to another. As the condensate flows along the inside bottoms of the tubes, however, it builds up a thicker condensate resistance film than that anticipated in the derivation. Little is available of a theoretical nature to permit a rational analysis, but it has been found that the film coefficient may be safely computed by Eq. (12.40) when \(G''\) is replaced by the fictitious loading

\[
G'' = \frac{W}{0.5LN_i} \quad \text{lb/}(\text{hr})(\text{lin ft})
\]

Equation (12.45) is especially useful when condensation occurs in the inner pipe of a double pipe exchanger. For condensation in the tubes of a condenser with multipass tubes it is preferable to compute the average film coefficient for each pass. The condensate formed in the first pass is carried through the second pass by one or more of the lower tubes in the pass, which may flow full of condensate and therefore expose no surface.
12.4. 24,000 lb/hr of nearly pure methyl ethyl ketone vapor at 2 psig (boiling point 180°F) is to be condensed and cooled to 100°F by water from 85 to 120°F. Pressure drops of 2.0 psig for the vapor and 10.0 for the water are permissible.

Available for the service is a 25 in. ID 1-2 horizontal condenser with 468 2/3 in. OD, 16 BWG tubes, 160" long arranged for four passes on 15 1/4-in. triangular pitch. Baffles are spaced 25" apart.

(a) What is the true temperature difference?
(b) What are the dirt factor and the pressure drops?

12.5. For the data of Prob. 12.4 calculate the true temperature difference and size of a horizontal 1-2 condenser-subcooler required to fulfill the conditions using 2/3 in. OD, 16 BWG tubes 120" long on 15 1/4-in. triangular pitch with a dirt factor of 0.003.

12.6. 50,000 lb/hr of ethyl acetate at 35 psig (boiling point 245°F) enters a horizontal 1-2 desuperheater-condenser at 300°F and leaves at 218°F. Cooling is effected by water from 85 to 120°F.

Available for the service is a 27 in. ID 1-2 condenser containing 432 2/3 in. OD, 16 BWG tubes 120" long arranged for four passes on 1-in. square pitch. Baffles are spaced 24 in. apart.

Calculate the true temperature difference and the dirt factor and pressure drops.

12.7. 57,000 lb/hr of nearly pure benzene enters the shell of a vertical 1-2 condenser at 5 psig and 220°F. The condensing range is from 177 to 170°F, at which temperature it goes to storage. Cooling water is used between 90 and 120°F.

Available for the service is a 31 in. ID condenser containing 650 2/3 in. OD, 16 BWG tubes 160" long on four-pass layout using 1-in. triangular pitch. Baffles are spaced 18 in. apart.

Calculate the true temperature difference, the dirt factor, and the pressure drops.

12.8. 50,000 lb/hr of a mixture of light hydrocarbons, principally propane, enters a 1-2 horizontal condenser at the initial condensing temperature of 135°F at 275 psig. The condensing range is from 135 to 115°F, at which temperature 49,000 lb/hr is condensed by cooling water from 90 to 110. The remainder of the condensation occurs with refrigerated water.

Available for the service is a 37 in. ID 1-2 horizontal condenser with 1100 2/3 in. OD, 16 BWG tubes 160" long arranged for four passes on 15 1/4-in. triangular pitch. Baffles are spaced 36 in. apart.

What is the dirt factor, and what are the pressure drops?

**NOMENCLATURE FOR CHAPTER 12**

- **A** Heat-transfer surface, ft²
- **A<sub>c</sub>, A<sub>k</sub>, A<sub>k</sub>** Heat-transfer surface for condensation, desuperheating, and subcooling, respectively, ft²
- **A<sub>c</sub>** Cross-sectional area of a film, ft²
- **A<sub>c,s</sub>** Total clean heat-transfer surface, ft²
- **a** Flow area, ft²
- **a<sup>c</sup>** External surface per linear foot, ft²
- **a<sub>s</sub>** Submerged cross section of shell, ft²
- **a<sub>t</sub>** Flow area per tube, in.²
- **a<sub>0</sub>, a<sub>1</sub>** Constants
- **B** Baffle spacing, in.
- **C** Specific heat of hot fluid in derivations, Btu/(lb)(°F)
- **C<sub>0</sub>, C<sub>1</sub>, C<sub>2</sub>** Cleanliness, temperature, and loading factors, dimensionless
- **C<sub>t</sub>** Tube factor, dimensionless
- **C<sub>r</sub>** Resistance of condenser film, tube metal, and water, respectively, (hr)(°F)/(°F/ft)
- **C<sub>r</sub>** Radius of tube, ft
- **C<sub>r</sub>** Hydraulic radius, r<sub>s</sub> = flow area/wetted perimeter, ft
- **C<sub>r</sub>** Reynolds number, dimensionless
- **C<sub>n</sub>** Specific entropy, dimensionless
- **C<sub>n</sub>** Temperature in general, inlet and outlet of hot fluid, respectively, °F

**CONDENSATION OF SINGLE VAPORS**

- Clearance between tubes, in.
- Constants
- Specific heat of cold fluid, Btu/(lb)(°F)
- Inside diameter of tube, ft
- Outside diameter of tube, ft
- Equivalent diameter for heat transfer and pressure drop, ft
- Inside diameter of tubes, in.
- Outside diameter of tubes, in.
- Friction factor, ft/psig
- Mass velocity, lb/(hr)(ft²)
- Condensate loading for vertical tubes, lb/(hr)(ft²)
- Condensate loading for horizontal tubes, lb/(hr)(ft²)
- Circulating water rate, gpm
- Acceleration of gravity, ft/psig²
- Acceleration of gravity, ft/sec²
- h, h<sub>s</sub>, h<sub>s</sub>, h<sub>s</sub> Heat-transfer coefficient in general, for inside fluid, and for outside fluid, respectively, Btu/(hr)(°F)
- Value of h when referred to the tube OD, Btu/(hr)(°F)
- Condensing film coefficient at a distance x from the top of the tube, Btu/(hr)(°F)
- Condensing film coefficient at angle α<sub>c</sub>, Btu/(hr)(°F)
- k Average value of the condensing film coefficient between two points, Btu/(hr)(°F)
- j<sub>w</sub> Factor for heat transfer, dimensionless
- Btu/(hr)(°F/ft²)
- Tube length, ft
- Tube length exposed to condensation, ft
- Log mean temperature difference, °F
- A Constant
- N Number of shell-side baffles
- n Number of tube passes
- Number of tubes effective for condensation
- P Perimeter, ft
- P<sub>r</sub> Tube pitch, in.
- D Pressure drop in general, psi
- D<sub>r</sub>, D<sub>p</sub>, D<sub>r</sub> Total, tube, return pressure drop, respectively, psi
- D<sub>r</sub> Pressure head of condensate, psi
- Q Heat flow, Btu/hr
- Q<sub>r</sub> Heat flow at distance x from top of tube, Btu/hr
- Q<sub>r</sub>, Q<sub>s</sub>, q<sub>r</sub> Heat flow for condensation, desuperheating and subcooling respectively, Btu/hr
- R<sub>r</sub> Combined dirt factor, (hr)(°F/°F)/Btu
- R<sub>r</sub>, R<sub>r</sub>, R<sub>r</sub> Resistance of condensate film, tube metal, and water, respectively, (hr)(°F)/(°F/ft)
- r Radius of tube, ft
- r<sub>s</sub> Hydraulic radius, r<sub>s</sub> = flow area/wetted perimeter, ft
- Re Reynolds number, dimensionless
- s Specific gravity, dimensionless
- T<sub>s</sub>, T<sub>c</sub>, T<sub>j</sub> Temperature in general, inlet and outlet of hot fluid, respectively, °F
PROCESS HEAT TRANSFER

\[ T_s \] Average temperature of hot fluid, °F
\[ T_c \] Caloric temperature of hot fluid, °F
\[ T_s \] Steam temperature, °F
\[ t, t_i, t_o \] Cold-fluid temperature in general, inlet, and outlet, respectively, °F
\[ t_c \] Average temperature of cold fluid, °F
\[ t_r \] Caloric temperature of cold fluid, °F
\[ t_t, t_w \] Temperature of film and tube wall respectively, °F
\[ t'_e \] Temperature at outer surface of condensate film, °F
\[ \Delta t \] True temperature difference in \( Q = U \Delta t \), °F
\[ \Delta t_e, (\Delta t)_o, (\Delta t)_a \] True or fictitious temperature difference for condensation, desuperheating, and subcooling, respectively, °F
\[ \Delta t_f \] \( (T_h - T_c)/2 \), °F
\[ U, U_c, U_D \] Overall coefficient of heat transfer, clean coefficient, and design coefficient, respectively, Btu/(hr)(ft²)(°F)
\[ U_n, U_o, U_s \] Clean overall coefficient for condensation, desuperheating, and subcooling respectively, Btu/(hr)(ft²)(°F)
\[ u \] Film velocity along axis of tube, ft/hr
\[ \bar{u} \] Average film velocity, ft/hr
\[ V \] Velocity, fps
\[ W \] Weight flow in general, weight flow of hot fluid, lb/hr
\[ W' \] Rate of condensation, lb/(hr)(ft²)
\[ w \] Weight flow of cold fluid, lb/hr
\[ w' \] Rate of condensation per tube \( W/N_t \), lb/(hr)(tube)
\[ x, x' \] Length of tube, ft
\[ z, z' \] Distance from top of tube at which change from streamline to turbulent flow occurs, ft
\[ y, y' \] Width of film, ft
\[ z \] Distance, ft; height of a hydrostatic leg, ft; a synthetic function
\[ \alpha \] Angle of tube, deg
\[ \lambda \] Latent heat of condensation or vaporization, Btu/lb
\[ \mu \] Viscosity, centipoises \( \times 2.42 = \text{lb}/(\text{ft})(\text{hr}) \)
\[ \mu' \] Viscosity, centipoises
\[ \mu'' \] Viscosity at tube-wall temperature, centipoises \( \times 2.42 = \text{lb}/(\text{ft})(\text{hr}) \)
\[ \rho \] Density, lb/ft³
\[ \tau \] Tangential stress, lb/ft²
\[ \phi \] Viscosity ratio, \( (\mu/\mu_0)^{1/3} \), dimensionless
\[ \psi \] A function

Subscripts (except otherwise noted)

\[ c \] Condensate, condensing
\[ f \] Film or film temperature
\[ l \] Liquid
\[ s \] Shell side
\[ v \] Vapor
\[ t \] Tube side

CHAPTER 13.

CONDENSATION OF MIXED VAPORS

Introduction. In the preceding chapter it was assumed that the condensing vapor consisted of a pure or substantially pure compound which condensed isothermally. If the vapor was mixed with another compound having a slightly different boiling point, the mixture condensed over a small condensing range. It was further assumed that, where there was a condensing range, the latent heat of condensation was transferred to the cooling medium uniformly over the entire condensing range.

However, consider a mixture of two diverse fluids having a condensing range of 100°F. In order to effect the first 10 per cent reduction in the vapor temperature it may be necessary to remove 50 per cent of the total heat load from the vapor mixture, since the less volatile component condenses more rapidly as the vapor temperature is reduced. Previously, in using the LMTD for condensing vapors in counterflow it was assumed that during the first 10 per cent reduction in the vapor temperature only 10 per cent of the heat was removed. The substitution of the logarithmic mean for the true temperature difference based on assumed uniform condensing characteristics for the vapor mixture may lead to a conservative or an unsafe value of \( \Delta t \) and the selection of the wrong condenser. The problems imposed by the condensation of a vapor mixture do not end there. Depending upon the nature of the mixture, the average condensing coefficient \( \bar{k} \) may not remain constant throughout the condensing range and may vary greatly with the composition of the vapor mixture as the less volatile components are condensed. The latter is particularly true of a mixture consisting of a vapor and a noncondensible gas, such as steam and air.

The Phase Rule. The different types of vapor mixtures may be studied qualitatively through the use of the phase rule of J. Willard Gibbs. Phases are defined as homogeneous amounts of matter in the solid, liquid, or gaseous form which are distinguishable from each other by the presence of an interface between any two phases. A vapor is the coexistence of a gas and a liquid and, like any boiling liquid and its vapor, consists of two phases.

\(^1\)The treatment of the phase rule is simplified here for the particular applications in this book.

\(^2\) In nonthermodynamic texts these are frequently referred to as states.