CHAPTER 7

1-2 PARALLEL-COUNTERFLOW:
SHELL-AND-TUBE EXCHANGERS

INTRODUCTION

The Tubular Element. The fulfillment of many industrial services requires the use of a large number of double pipe hairpins. These consume considerable ground area and also entail a large number of points at which leakage may occur. Where large heat-transfer surfaces are required, they can best be obtained by means of shell-and-tube equipment.

Shell-and-tube equipment involves expanding a tube into a tube sheet and forming a seal which does not leak under reasonable operating conditions. A simple and common example of an expanded tube is shown in Fig. 7.1. A tube hole is drilled in a tube sheet with a slightly greater diameter than the outside diameter of the tube, and two or more grooves are cut in the wall of the hole. The tube is placed inside the tube hole, and a tube roller is inserted into the end of the tube. The roller is a rotating mandril having a slight taper. It is capable of exceeding the elastic limit of the tube metal and transforming it into a semiplastic condition so that it flows into the grooves and forms an extremely tight seal. Tube rolling is a skill, since a tube may be damaged by rolling to paper thinness and leaving a seal with little structural strength.

In some industrial uses it is desirable to install tubes in a tube sheet so that they can be removed easily as shown in Fig. 7.2. The tubes are
actually packed in the tube sheet by means of ferrules using a soft metal packing ring.

**Heat-exchanger Tubes.** Heat-exchanger tubes are also referred to as condenser tubes and should not be confused with steel pipes or other types of pipes which are extruded to iron pipe sizes. The outside diameter of heat exchanger or condenser tubes is the actual outside diameter in inches within a very strict tolerance. Heat-exchanger tubes are available in a variety of metals which include steel, copper, admiralty, Muntz metal, brass, 70-30 copper-nickel, aluminum bronze, aluminum, and the stainless steels. They are obtainable in a number of different wall thicknesses defined by the Birmingham wire gage, which is usually referred to as the BWG or gage of the tube. The sizes of tubes which are generally available are listed in Table 10 of the Appendix of which the 3/4 outside diam-

![Fig. 7.3. Common tube layouts for exchangers.](image)

eter and 1 in. OD are most common in heat-exchanger design. The data of Table 10 have been arranged in a manner which will be most useful in heat-transfer calculations.

**Tube Pitch.** Tube holes cannot be drilled very close together, since too small a width of metal between adjacent tubes structurally weakens the tube sheet. The shortest distance between two adjacent tube holes is the clearance or ligament, and these are now fairly standard. Tubes are laid out on either square or triangular patterns as shown in Fig. 7.3a and b. The advantage of square pitch is that the tubes are accessible for external cleaning and cause a lower pressure drop when fluid flows in the direction indicated in Fig. 7.3a. The tube pitch \( P \) is the shortest center-to-center distance between adjacent tubes. The common pitches for square layouts are 3/4 in. OD on 1-in. square pitch and 1 in. OD on 1 1/4-in. square pitch. For triangular layouts these are 3/4 in. OD on 1 5/8-in. triangular pitch, 3/4 in. OD on 1-in. triangular pitch, and 1-in. OD on 1 1/4-in. triangular pitch. In Fig. 7.3c the square-pitch layout has been rotated 45°, yet it is essentially the same as in Fig. 7.3a. In Fig. 7.3d a mechanically cleanable modification of triangular pitch is
shown. If the tubes are spread wide enough, it is possible to allow the cleaning lanes indicated.

Shells. Shells are fabricated from steel pipe with nominal IPS diameters up to 12 in. as given in Table 11. Above 12 and including 24 in. the actual outside diameter and the nominal pipe diameter are the same. The standard wall thickness for shells with inside diameters from 12 to 24 in. inclusive is \( \frac{3}{8} \) in., which is satisfactory for shell-side operating pressures up to 300 psi. Greater wall thicknesses may be obtained for greater pressures. Shells above 24 in. in diameter are fabricated by rolling steel plate.

Stationary Tube-sheet Exchangers. The simplest type of exchanger is the fixed or stationary tube-sheet exchanger of which the one shown in Fig. 7.4 is an example. The essential parts are a shell (1), equipped with two nozzles and having tube sheets (2) at both ends, which also serve as flanges for the attachment of the two channels (3) and their respective channel covers (4). The tubes are expanded into both tube sheets and are equipped with transverse baffles (5) on the shell side. The calculation of the effective heat-transfer surface is frequently based on the distance between the inside faces of the tube sheets instead of the overall tube length.

Baffles. It is apparent that higher heat-transfer coefficients result when a liquid is maintained in a state of turbulence. To induce turbulence outside the tubes it is customary to employ baffles which cause the liquid to flow through the shell at right angles to the axes of the tubes. This causes considerable turbulence even when a small quantity of liquid flows through the shell. The center-to-center distance between baffles is called the baffle pitch or baffle spacing. Since the baffles may be spaced close together or far apart, the mass velocity is not entirely dependent upon the diameter of the shell. The baffle spacing is usually not greater than a distance equal to the inside diameter of the shell or closer than a distance equal to one-fifth the inside diameter of the shell. The baffles are held securely by means of baffle spacers (6) as shown in Fig. 7.4, which consist of through-bolts screwed into the tube sheet and a number of
smaller lengths of pipe which form shoulders between adjacent baffles. An enlarged detail is shown in Fig. 7.5.

There are several types of baffles which are employed in heat exchangers, but by far the most common are segmental baffles as shown in Fig. 7.6. Segmental baffles are drilled plates with heights which are generally 75 per cent of the inside diameter of the shell. These are known as 25 per cent cut baffles and will be used throughout this book although other fractional baffle cuts are also employed in industry. An excellent review of the influence of the baffle cut on the heat-transfer coefficient has been presented by Donohue.¹ They may be arranged, as shown, for "up-and-down" flow or may be rotated 90° to provide "side-to-side" flow, the latter being desirable when a mixture of liquid and gas flows

through the shell. The baffle pitch and not the 25 per cent cut of the baffles, as shown later, determines the effective velocity of the shell fluid.

Other types of baffles are the disc and doughnut of Fig. 7.7 and the orifice baffle in Fig. 7.8. Although additional types are sometimes employed, they are not of general importance.

Fixed-tube-sheet Exchanger with Integral Channels. Another of several variations of the fixed-tube-sheet exchanger is shown in Fig. 7.9, in which the tube sheets are inserted into the shell, forming channels which are integral parts of the shell. In using stationary tube-sheet exchangers it is often necessary to provide for differential thermal expansion between the tubes and the shell during operation, or thermal stresses will develop across the tube sheet. This can be accomplished by the use of an expansion joint on the shell, of which a number of types of flexible joints are available.

Fixed-tube-sheet 1-2 Exchanger. Exchangers of the type shown in Figs. 7.4 and 7.9 may be considered to operate in counterflow, notwithstanding the fact that the shell fluid flows across the outside of the tubes. From a practical standpoint it is very difficult to obtain a high velocity when one of the fluid flows through all the tubes in a single pass. This can be circumvented, however, by modifying the design so that the tube fluid is carried through fractions of the tubes consecutively. An example of a two-pass fixed-tube-sheet exchanger is shown in Fig. 7.10, in which all the tube fluid flows through the two halves of the tubes successively.
The exchanger in which the shell-side fluid flows in one shell pass and the tube fluid in two or more passes is the 1-2 exchanger. A single channel is employed with a partition to permit the entry and exit of the tube fluid from the same channel. At the opposite end of the exchanger a bonnet is provided to permit the tube fluid to cross from the first to the second pass. As with all fixed-tube-sheet exchangers, the outsides of the tubes are inaccessible for inspection or mechanical cleaning. The insides of the tubes can be cleaned in place by removing only the channel cover and using a rotary cleaner or a wire brush. Expansion problems are extremely critical in 1-2 fixed-tube-sheet exchangers, since both passes, as well as the shell itself, tend to expand differently and cause stress on the stationary tube sheets.

Removable-bundle Exchangers. In Fig. 7.11 is shown a counterpart of the 1-2 exchanger having a tube bundle which is removable from the shell. It consists of a stationary tube sheet, which is clamped between the single channel flange and a shell flange. At the opposite end of the bundle the tubes are expanded into a freely riding floating tube sheet or floating head. A floating-head cover is bolted to the tube sheet, and the entire bundle can be withdrawn from the channel end. The shell is closed by a shell bonnet. The floating head illustrated eliminates the differential expansion problem in most cases and is called a pull-through floating head.
The disadvantage to the use of a pull-through floating head is one of simple geometry. To secure the floating-head cover it is necessary to bolt it to the tube sheet, and the bolt circle requires the use of space where it would be possible to insert a great number of tubes. The bolting not only reduces the number of tubes which might be placed in the tube bundle but also provides an undesirable flow channel between the bundle and the shell. These objections are overcome in the more conventional split-ring floating-head 1-2 exchanger shown in Fig. 7.12. Although it is relatively expensive to manufacture, it does have a great number of mechanical advantages. It differs from the pull-through type by the use of a split-ring assembly at the floating tube sheet and an oversized shell cover which accommodates it. The detail of a split ring is shown in Fig. 7.13. The floating tube sheet is clamped between the floating-head cover and a clamp ring placed in back of the tube sheet which is split in half to permit dismantling. Different manufacturers have different modifications of the design shown here, but they all accomplish the purpose of providing increased surface over the pull-through floating head in the same size shell. Cast channels with nonremovable channel covers are also employed as shown in Fig. 7.12.

Tube-sheet Layouts and Tube Counts. A typical example of the layout of tubes for an exchanger with a split-ring floating head is shown in Fig. 7.14. The actual layout is for a 13¼ in. ID shell with 1 in. OD tubes on 1¾-in. triangular pitch arranged for six tube passes. The partition
arrangement is also shown for the channel and floating-head cover along with the orientation of the passes. Tubes are not usually laid out symmetrically in the tube sheet. Extra entry space is usually allowed in the shell by omitting tubes directly under the inlet nozzle so as to minimize the contraction effect of the fluid entering the shell. When tubes are laid out with minimum space allowances between partitions and adjoining tubes and within a diameter free of obstruction called the outer tube limit, the number of tubes in the layout is the tube count. It is not always possible to have an equal number of tubes in each pass, although in large exchangers the unbalance should not be more than about 5 per cent. In Appendix Table 9 the tube counts for $\frac{3}{4}$ and 1 in. OD tubes are given for one pass shells and one, two, four, six, and eight tube pass arrangements.

**Table 7.1. Tube Count Entry Allowances**

<table>
<thead>
<tr>
<th>Shell ID, in.</th>
<th>Nozzle, in.</th>
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<tbody>
<tr>
<td>Less than 12</td>
<td>3</td>
</tr>
<tr>
<td>12-17$\frac{1}{4}$</td>
<td>4</td>
</tr>
<tr>
<td>19$\frac{3}{4}$-21$\frac{1}{4}$</td>
<td>6</td>
</tr>
<tr>
<td>23$\frac{1}{4}$-29</td>
<td>8</td>
</tr>
<tr>
<td>31-37</td>
<td>10</td>
</tr>
</tbody>
</table>

These tube counts include a free entrance path below the inlet nozzle equal to the cross-sectional area of the nozzles shown in Table 7.1. When a larger inlet nozzle is used, extra entry space can be obtained by flaring the inlet nozzle at its base or removing the tubes which ordinarily lie close to the inlet nozzle.

**Packed Floating Head.** Another modification of the floating-head 1-2 exchanger is the packed floating-head exchanger shown in Fig. 7.15.
This exchanger has an extension on the floating tube sheet which is confined by means of a packing gland. Although entirely satisfactory for shells up to 36 in. ID, the larger packing glands are not recommended for higher pressures or services causing vibration.

*U-bend Exchangers.* The 1-2 exchanger shown in Fig. 7.16 consists of tubes which are bent in the form of a U and rolled into the tube sheet.

![Diagram of packed floating head 1-2 exchanger](Fig. 7.15)

![Diagram of U-bend 1-2 exchanger](Fig. 7.16)

![Diagram of U-bend double-tube-sheet exchanger](Fig. 7.17)

The tubes can expand freely, eliminating the need for a floating tube sheet, floating-head cover, shell flange, and removable shell cover. Baffles may be installed in the conventional manner on square or triangular pitch. The smallest diameter U-bend which can be turned without deforming the outside diameter of the tube at the bend has a diameter of three to four times the outside diameter of the tubing. This means that it will usually be necessary to omit some tubes at the center of the bundle, depending upon the layout.
An interesting modification of the U-bend exchanger is shown in Fig. 7.17. It employs a double stationary tube sheet and is used when the leakage of one fluid stream into the other at the tube roll can cause serious corrosion damage. By using two tube sheets with an air gap between them, either fluid leaking through its adjoining tube sheet will escape to the atmosphere. In this way neither of the streams can contaminate the other as a result of leakage except when a tube itself corrodes. Even tube failure can be prevented by applying a pressure shock test to the tubes periodically.

THE CALCULATION OF SHELL-AND-TUBE EXCHANGERS

Shell-side Film Coefficients. The heat-transfer coefficients outside tube bundles are referred to as shell-side coefficients. When the tube bundle employs baffles directing the shell-side fluid across the tubes from top to bottom or side to side, the heat-transfer coefficient is higher than for undisturbed flow along the axes of the tubes. The higher transfer coefficients result from the increased turbulence. In square pitch, as seen in Fig. 7.18, the velocity of the fluid undergoes continuous fluctuation because of the constrained area between adjacent tubes compared with the flow area between successive rows. In triangular pitch even greater turbulence is encountered because the fluid flowing between adjacent tubes at high velocity impinges directly on the succeeding row. This would indicate that, when the pressure drop and cleanability are of little consequence, triangular pitch is superior for the attainment of high shell-side film coefficients. This is actually the case, and under comparable conditions of flow and tube size the coefficients for triangular pitch are roughly 25 percent greater than for square pitch.

Several factors not treated in preceding chapters influence the rate of heat transfer on the shell side. Suppose the length of a bundle is divided by six baffles. All the fluid travels across the bundle seven times. If ten baffles are installed in the same length of bundle, it would require that the bundle be crossed a total of eleven times, the closer spacing causing the greater turbulence. In addition to the effects of the baffle spacing the shell-side coefficient is also affected by the type of pitch, tube size, clearance, and fluid-flow characteristics. Furthermore, there is no true flow area by which the shell-side mass velocity can be computed,
since the flow area varies across the diameter of the bundle with the different number of tube clearances in each longitudinal row of tubes. The correlation obtained for fluids flowing in tubes is obviously not applicable to fluids flowing over tube bundles with segmental baffles, and this is indeed borne out by experiment. However, in establishing a method of correlation the form of the heat-transfer factor \( f_h \) is \((hD/k) (\mu /k) \mu /\mu_w \) to the power 0.14 vs. \( DG/\mu \) has been retained, in agreement with the suggestion of McAdams, but using fictitious values for the equivalent diameter \( D_e \) and the mass velocity \( G_e \) as discussed below.

Figure 28 in the Appendix is a correlation of industrial data which gives satisfactory results for the hydrocarbons, organic compounds, water, aqueous solutions, and gases when the bundle employs baffles with acceptable clearances between baffles and tubes and between baffles and shell.

It is not the mean curve through the data but a safe curve such that the deviation of the test points from the curve ranges from 0 to approximately 20 per cent high. Inasmuch as the line expressing the equation possesses a curvature, it cannot be evaluated in the simple form of Eq. (3.42), since the proportionality constant and the exponent of the Reynolds number actually vary. For values of Re from 2000 to 1,000,000, however, the data are closely represented by the equation

\[
\frac{h_e D_e}{k} = 0.36 \left( \frac{D_e G_e}{\mu} \right)^{0.55} \left( \frac{c \mu}{k} \right)^{1.5} \left( \frac{\mu}{\mu_w} \right)^{0.14}
\]

where \( h_e, D_e \) and \( G_e \) are as defined below. Calculations using Fig. 28 agree very well with the methods of Colburn and Short and the test data of Breidenbach and O'Connell on a number of commercial heat exchangers. It will be observed in Fig. 28 that there is no discontinuity at a Reynolds number of 2100 such as occurs for fluids in tubes. The different equivalent diameters used in the correlation of shell and tube data preclude comparison between fluids flowing in tubes and across tubes on the basis of the Reynolds number alone. All the data in Fig. 28 refer to turbulent flow.

**Shell-side Mass Velocity.** The linear and mass velocities of the fluid change continuously across the bundle, since the width of the shell and the number of tubes vary from zero at the top and bottom to maxima at the center of the shell. The width of the flow area in the correlation

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2. For mechanical details and standards see Standards of the Tubular Exchanger Manufacturers' Association, New York (1949).
represented by Fig. 28 was taken at the hypothetical tube row possessing the maximum flow area and corresponding to the center of the shell. The length of the flow area was taken equal to the baffle spacing $B$. The tube pitch is the sum of the tube diameter and the clearance $C'$. If the inside diameter of the shell is divided by the tube pitch, it gives a fictitious, but not necessarily integral, number of tubes which may be assumed to exist at the center of the shell. Actually in most layouts there is no row of tubes through the center but instead two equal maximum rows on either side of it having fewer tubes than computed for the center. These deviations are neglected. For each tube or fraction there are considered to be $C' \times 1$ in$^2$ of crossflow area per inch of baffle space. The shell-side or bundle crossflow area $a_s$ is then given by

$$a_s = \frac{\text{ID} \times C'B}{P_T \times 144} \text{ ft}^2$$  (7.1)

and as before, the mass velocity is

$$G_s = \frac{W}{a_s} \text{ lb/(hr)(ft}^2)$$  (7.2)

**Shell-side Equivalent Diameter.** By definition, the hydraulic radius corresponds to the area of a circle equivalent to the area of a noncircular flow channel and consequently in a plane at right angles to the direction of flow. The hydraulic radius employed for correlating shell-side coefficients for bundles having baffles is not the true hydraulic radius. The direction of flow in the shell is partly along and partly at right angles to the long axes of the tubes of the bundle. The flow area at right angles to the long axes is variable from tube row to tube row. A hydraulic radius based upon the flow area across any one row could not distinguish between square and triangular pitch. In order to obtain a simple correlation combining both the size and closeness of the tubes and their type of pitch, excellent agreement is obtained if the hydraulic radius is calculated along instead of across the long axes of the tubes. The equivalent diameter for the shell is then taken as four times the hydraulic radius obtained for the pattern as laid out on the tube sheet. Referring to Fig. 7.19, where the crosshatch covers the free area$^1$ for square pitch

$$D_e = \frac{4 \times \text{free area}}{\text{wetted perimeter}} \text{ ft}$$  (7.3)

or

$$d_e = \frac{4 \times (P_T^4 - \pi d_0^2/4)}{\pi d_0} \text{ in.}$$  (7.4)

$^1$ The expression free area is used to avoid confusion with the free-flow area, an actual entity in the hydraulic radius.
where \( P_r \) is the tube pitch in inches and \( d_o \), the tube outside diameter in inches. For triangular pitch as shown in Fig. 7.19, the wetted perimeter of the element corresponds to half a tube.

\[
d_s = \frac{4 \times (\frac{3}{2} P_r \times 0.8 P_r - \frac{3}{2} \pi d_o^2/4)}{\frac{3}{2} \pi d_o} \text{ in.} \quad (7.5)
\]

The equivalent diameters for the common arrangements are included in Fig. 28.

It would appear that this method of evaluating the hydraulic radius and equivalent diameter does not distinguish between the relative percentage of right-angle flow to axial flow, and this is correct. It is possible, using the same shell, to have equal mass velocities, equivalent diameters, and Reynolds numbers using a large quantity of fluid and a large baffle pitch or a small quantity of fluid and a small baffle pitch, although the proportions of right-angle flow to axial flow differ. Apparently, where the range of baffle pitch is restricted between the inside diameter and one-fifth the inside diameter of the shell, the significance of the error is not too great to permit correlation.

**Example 7.1.** Compute the shell-side equivalent diameter for a \( \frac{3}{4} \) in. OD tube on 1-in. square pitch. From Eq. (7.4)

\[
d_s = \frac{4(1^2 - 3.14 \times 0.75^2/4)}{3.14 \times 0.75} = 0.95 \text{ in.}
\]

\[
P_r = \frac{0.95}{12} = 0.079 \text{ it}
\]

The True Temperature Difference \( \Delta t \) in a 1-2 Exchanger. A typical plot of temperature vs. length for an exchanger having one shell pass and two tube passes is shown in Fig. 7.20 for the nozzle arrangement indicated. Relative to the shell fluid, one tube pass is in countercflow and the other in parallel flow. Greater temperature differences have
been found, in Chap. 5, to result when the process streams are in counterflow and lesser differences for parallel flow. The 1-2 exchanger is a combination of both, and the LMTD for counterflow or parallel flow alone cannot be the true temperature difference for a parallel flow-counterflow arrangement. Instead it is necessary to develop a new equation for calculation of the effective or true temperature difference \( \Delta t \) to replace the counterflow LMTD. The method employed here is a modification of the derivation of Underwood\(^1\) and is presented in the final form proposed by Nagle\(^4\) and Bowman, Mueller, and Nagle.\(^3\)

The temperature of the shell fluid may undergo either of two variations as it proceeds from the inlet to outlet, crossing the tube bundle several times in its progress: (1) So much turbulence is induced that the shell fluid is completely mixed at any length \( X \) from the inlet nozzle, or (2) so little turbulence is induced that there is a selective temperature atmosphere about the tubes of each tube pass individually. The baffles and turbulent nature of the flow components across the bundle appear to eliminate (2) so that (1) is taken as the first of the assumptions for the derivation of the true temperature difference in a 1-2 exchanger. The assumptions are

1. The shell fluid temperature is an average isothermal temperature at any cross section.
2. There is an equal amount of heating surface in each pass.
3. The overall coefficient of heat transfer is constant.
4. The rate of flow of each fluid is constant.
5. The specific heat of each fluid is constant.
6. There are no phase changes of evaporation or condensation in a part of the exchanger.
7. Heat losses are negligible.

The overall heat balance, where \( \Delta t \) is the true temperature difference, is

\[
Q = U A \Delta t = WC(T_1 - T_2) = wc(t_2 - t_1) \quad (7.6)
\]

From which

$$
\Delta t = \left( \frac{T_1 - T_2}{UA/\bar{W}C} \right)_{true} = \left( \frac{t_2 - t_1}{tA/\bar{w}c} \right)_{true}
$$

(7.7)

In Fig. 7.20a let $T$ be the temperature of the shell fluid at any cross section of the shell $L = X$ between $L = 0$ and $L = L$. Let $t'$ and $t''$ represent temperatures in the first and second tube passes, respectively, and at the same cross section as $T$. Let $a''$ be the external surface per foot of length. In the incremental surface $dA = a''dL$ the shell temperature changes by $-dT$. Over the area $dA$

$$
-WC \, dT = U \frac{dA}{2} \left( T' - t' \right) + U \frac{dA}{2} \left( T - t'' \right)
$$

(7.8)

$$
-WC \, dT = t' \, dA \left( T - \frac{t' + t''}{2} \right)
$$

(7.9)

$$
- \int \frac{U \, dA}{WC} = \int \frac{dT}{T - \left( t' + t'' \right)/2}
$$

(7.10)

But in this equation $T$, $t'$, and $t''$ are dependent variables. The heat balance from $L = X$ to the hot-fluid inlet is

$$
WC(T - T_2) = wc(t'' - t')
$$

(7.11)

and the heat balance per pass

$$
wc \, dt' = U \frac{dA}{2} \left( T' - t' \right)
$$

(7.12)

$$
wc \, dt'' = -U \frac{dA}{2} \left( T - t'' \right)
$$

(7.13)

Dividing Eq. (7.13) by Eq. (7.12),

$$
\frac{dt''}{dt'} = -\frac{T - t''}{T' - t'}
$$

(7.14)

To eliminate $t''$ and $dt''$ from Eq. (7.11) and (7.13)

$$
t'' = \frac{WC}{wc} \left( T_2 - T \right) + t'
$$

(7.15)

Differentiating Eq. (7.15), with the hot fluid inlet $T_1$ constant,

$$
dt'' = -\frac{WC}{wc} \, dT + dt'
$$

(7.16)

Substituting in Eq. (7.14) and rearranging,

$$
\frac{WC \, dT}{wc \, dt'} = 1 + \frac{T - t'}{T - t'' - (WC/wc)(T_2 - T)}
$$

(7.17)
The number of variables in Eq. (7.15) has been reduced from three \((T, t^i, t^m)\) to two \((T\) and \(t^i)\). For a solution it is necessary to eliminate either \(T\) or \(t^i\). Simplifying by the use of parameters as in the case of the double pipe exchanger let

\[
R = \frac{T_1 - T_2}{t_1 - t_2} = \frac{wc}{WC} \quad \text{and} \quad S = \frac{t_2 - t_1}{t_1' - t_2}
\]

Rearranging Eq. (7.8),

\[
WC \frac{dT}{dA} + \frac{U}{2} (T - t^i) + \frac{U}{2} (T - t^m) = 0 \tag{7.18}
\]

Simplifying and substituting \(WC = wc/R\),

\[
\frac{dT}{dA} + \frac{URT}{wc} - \frac{UR}{2wc} (t^i + t^m) = 0 \tag{7.19}
\]

Differentiating with respect to \(A\),

\[
\frac{d^2T}{dA^2} + \frac{URT}{wc} \frac{dT}{dA} - \frac{UR}{2wc} \left( \frac{dt^i}{dA} + \frac{dt^m}{dA} \right) = 0 \tag{7.20}
\]

Substituting Eqs. (7.12) and (7.13),

\[
\frac{d^2T}{dA^2} + \frac{URT}{wc} \frac{dT}{dA} - \frac{U^2R}{(2wc)^2} (t^m - t^i) = 0 \tag{7.21}
\]

Since the heat change is sensible, a direct proportionality exists between the percentage of the temperature rise (or fall) and \(Q\).

\[
\frac{T_1 - T_2}{t_1 - t_2} = \frac{t^i - t^m}{t_1 - t_2} \tag{7.22}
\]

or

\[
t^i = t^m = \frac{T_1 - T_2}{R} \tag{7.23}
\]

\[
\frac{d^2T}{dA^2} + \frac{URT}{wc} \frac{dT}{dA} - \frac{U^2T}{(2wc)^2} = - \frac{U^2T}{(2wc)^2} \tag{7.24}
\]

Differentiating again with respect to \(A\),

\[
\frac{d^3T}{dA^3} + \frac{URT}{wc} \frac{d^2T}{dA^2} - \frac{U^2dT}{(2wc)^2} dA = 0 \tag{7.25}
\]

The solution of this equation will be found in any standard differential-equations text. The equation is

\[
T = K_1 + K_2 e^{-\left(\frac{UA}{2wc}(R + \sqrt{R^2 + 1})\right)} + K_3 e^{-\left(\frac{UA}{2wc}(R - \sqrt{R^2 + 1})\right)} \tag{7.26}
\]

When \(T = T_1\), \(A\) will have increased from 0 to \(A\), and from the solution
of Eq. (7.24) $K_1 = T_2$ so that Eq. (7.26) becomes

$$-K_2e^{-\frac{UA}{2wc}(R+\sqrt{R^2+1})} = K_2e^{-\frac{UA}{2wc}(R-\sqrt{R^2+1})}$$ (7.27)

Taking logarithms of both sides and simplifying,

$$\frac{UA}{wc} = \frac{1}{\sqrt{R^2+1}} \ln \left( -\frac{K_1}{K_2} \right)$$ (7.28)

Differentiate Eq. (7.26):

$$\frac{dT}{dA} = -K_2 \frac{U}{2wc} (R + \sqrt{R^2+1})e^{-\frac{UA}{2wc}(R+\sqrt{R^2+1})}$$

$$-K_2 \frac{U}{2wc} (R - \sqrt{R^2+1})e^{-\frac{UA}{2wc}(R-\sqrt{R^2+1})}$$ (7.29)

Substituting the value of $dT/dA$ from Eq. (7.19) and since at $A = 0$, $t' = t_1$, $t'' = t_2$, and $T = T_1$, $t' + t'' = t_1 + t_2$.

$$R(t_1 + t_2) - 2RT_1 = -K_2(R + \sqrt{R^2+1})$$

$$-K_3(R - \sqrt{R^2+1})$$ (7.30)

From Eq. (7.26) at $A = 0$ and $T = T_1$ and $K_1 = T_2$,

$$T_1 - T_2 = K_2 + K_3$$ (7.31)

Multiplying both sides of Eq. (7.31) by $(R + \sqrt{R^2+1})$,

$$(R + \sqrt{R^2+1})(T_1 - T_2) = K_2(R + \sqrt{R^2+1})$$

$$+ K_3(R + \sqrt{R^2+1})$$ (7.32)

Adding Eqs. (7.31) and (7.32) and solving for $K_3$,

$$K_3 = \frac{R(t_1 + t_2) + (T_1 - T_2)(R + \sqrt{R^2+1}) - 2RT_1}{2 \sqrt{R^2+1}}$$ (7.33)

Returning to Eq. (7.31),

$$-K_3 = K_2 - (T_1 - T_2) =$$

$$\frac{(R + \sqrt{R^2+1})(T_1 - T_2) - 2\sqrt{R^2+1}(T_1 - T_2) - 2RT_1 + R(t_1 + t_2)}{2 \sqrt{R^2+1}}$$ (7.34)

Since $R = (T_1 - T_2)/(t_1 - t_1)$,

$$-\frac{K_3}{K_2} = \frac{(R - \sqrt{R^2+1})(t_1 - t_2) - (T_1 - T_1) - (T_1 - t_2)}{(R + \sqrt{R^2+1})(t_1 - t_2) - (T_1 - t_1) - (T_1 - t_2)}$$ (7.35)
Dividing by \( T_1 - t_1 \) and substituting \( S = (t_2 - t_1)/(T_1 - t_1) \) and
\[ 1 - S = (T_1 - t_2)/(T_1 - t_1), \]
\[
- \frac{K_2}{K_3} = \frac{2 - S(R + 1 - \sqrt{R^2 + 1})}{2 - S(R + 1 + \sqrt{R^2 + 1})}
\]
(7.36)

Substituting in Eq. (7.28),
\[
\left( \frac{U/A}{wc} \right)_{\text{true}} = \frac{1}{\sqrt{R^2 + 1}} \ln \frac{2 - S(R + 1 - \sqrt{R^2 + 1})}{2 - S(R + 1 + \sqrt{R^2 + 1})}
\]
(7.37)

Equation (7.37) is the relationship for the true temperature difference for 1-2 parallel flow-counterflow. How does this compare with the LMTD for counterflow employing the same process temperatures? For counterflow
\[
Q = wc(t_2 - t_1) = UA \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln (T_1 - t_2)/(T_2 - t_1)}
\]
(7.38)

from which
\[
\left( \frac{U/A}{wc} \right)_{\text{counterflow}} = \frac{t_2 - t_1}{(T_1 - t_2) - (T_2 - t_1)} = \frac{\ln (1 - S)/(1 - RS)}{R - 1}
\]
(7.39)

The ratio of the true temperature difference to the LMTD is
\[
\frac{t_2 - t_1}{(U/A/wc)_{\text{true}}} = \frac{\left( (U/A/wc)_{\text{counterflow}} \right)}{(U/A/wc)_{\text{true}}}
\]
(7.40)

Calling the fractional ratio of the true temperature difference to the LMTD \( F_T \),
\[
F_T = \frac{\sqrt{R^2 + 1} \ln (1 - S)/(1 - RS)}{(R - 1) \ln \frac{2 - S(R + 1 - \sqrt{R^2 + 1})}{2 - S(R + 1 + \sqrt{R^2 + 1})}}
\]
(7.41)

The Fourier equation for a 1-2 exchanger can now be written:
\[
Q = UA \Delta t = UAF_T(\text{LMTD})
\]
(7.42)

To reduce the necessity of solving Eq. (7.37) or (7.41), correction factors \( F_T \) for the LMTD have been plotted in Fig. 18 in the Appendix as a function of \( S \) with \( R \) as the parameter. When a value of \( S \) and \( R \) is close to the vertical portion of a curve, it is difficult to read the figure and \( F_T \) should be computed from Eq. (7.41) directly. When an exchanger has one shell pass and four, six, eight, or more even-numbered tube passes such as a 1-4, 1-6, or 1-8 exchanger, Eq. (7.10) becomes for a 1-4 exchanger
\[
- \int U dA = \int \frac{dT}{T - (t^i + t^o + t^\text{int} + t^\text{ext})/4}
\]
for a 1-6 exchanger

\[ - \int \frac{U dA}{WE} = \int \frac{dT}{T - (T_1 + T_2 + T_3 + T_4 + T_5 + T_6)} \]

It can be shown that the values of \( F_T \) for a 1-2 and 1-8 exchanger are less than 2 per cent apart in the extreme case and generally considerably less. It is therefore customary to describe any exchanger having one shell pass and two or more even-numbered tube passes in parallel flow—counterflow as a 1-2 exchanger and to use the value of \( F_T \) obtained from Eq. (7.41). The reason \( F_T \) will be less than 1.0 is naturally due to the fact that the tube passes in parallel with the shell fluid do not contribute so effective a temperature difference as those in counterflow with it.

There is an important limitation to the use of Fig. 18. Although any exchanger having a value of \( F_T \) above zero will theoretically operate, it is not practically true. The failure to fulfill in practice all the assumptions employed in the derivation, assumptions 1, 3, and 7 in particular, may cause serious discrepancies in the calculation of \( \Delta t \). As a result of the discrepancies if the actual value of \( t_i \) in Fig. 7.20a at the end of the parallel pass is required to approach \( T_2 \) more closely than the derived value of \( t_i \), it may impose a violation of the rule of parallel flow; namely, the outlet of the one stream \( t_i \) may not attain the outlet of the other, \( T_2 \), without infinite surface. Accordingly it is not advisable or practical to use a 1-2 exchanger whenever the correction factor \( F_T \) is computed to be less than 0.75. Instead some other arrangement is required which more closely resembles counterflow.

The temperature relationship for the case where the orientation of the shell nozzles has been reversed is shown in Fig. 7.21 for the same inlet and outlet temperatures plotted in Fig. 7.20. Underwood\(^1\) has shown that the values of \( F_T \) for both are identical.\(^2\) Since a 1-2 exchanger is a combination of counterflow and parallel-flow passes, it may be expected that the outlet of one process stream cannot approach the inlet of the

\(^1\) Underwood, op. cit.
\(^2\) The values of \( t_i \), however, differ for both cases.
other very closely. In fact it is customary in parallel flow-counterflow equipment to call \( T_2 - t_2 \) the approach, and if \( t_2 > T_2 \), then \( t_2 - T_2 \) is called the temperature cross.

It is useful to investigate several typical process temperatures and to note the influence of different approaches and crosses upon the value of \( F_T \).

**Fig. 7.22. Influence of approach temperature on \( F_T \) with fluids having equal ranges in a 1-2 exchanger.**

**Fig. 7.23. Influence of approach temperature on \( F_T \) with fluids having unequal ranges in a 1-2 exchanger.**

\( F_T \). For a given service the reduction of \( F_T \) below unity in Eq. (7.42) is compensated for by increasing the surface. Thus if the process temperatures are fixed it may be advisable to employ a parallel flow-counterflow exchanger as against a counterflow exchanger, since it increases the cost of the equipment beyond the value of its mechanical advantages. In Fig. 7.22 two pairs of fluids each with equal ranges of 100 and 50°F
are studied. The operating temperatures of the cold fluid are fixed, while the hot-fluid temperatures are variable thereby changing the approach in each case. Note the conditions under which \( F_T \) shrinks rapidly, particularly the approach at the practical minimum \( F_T = 0.75 \) and the influence of the relationship between \( T_2 \) and \( t_2 \). The calculation of several points is demonstrated.

**Example 7.2. Calculation of \( F_T \) for Fluids with Equal Ranges**

<table>
<thead>
<tr>
<th>Point:</th>
<th>(a) 50° approach</th>
<th>(b) Zero approach</th>
<th>(c) 20° cross</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 )</td>
<td>350 200 (t_1)</td>
<td>300 200 (t_1)</td>
<td>280 200 (t_2)</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>250 100 (t_2)</td>
<td>200 100 (t_2)</td>
<td>180 100 (t_2)</td>
</tr>
<tr>
<td>( T_1 - T_2 )</td>
<td>100 100</td>
<td>100 100</td>
<td>100 100</td>
</tr>
<tr>
<td>( R ) = ( \frac{T_1 - T_2}{T_1 - t_1} )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( S ) = ( \frac{t_2 - t_1}{T_2 - T_1} )</td>
<td>0.40</td>
<td>0.50</td>
<td>0.555</td>
</tr>
<tr>
<td>( F_T ) = 0.925 (Fig. 18)</td>
<td>( F_T = 0.80 )</td>
<td>( F_T = 0.64 )</td>
<td></td>
</tr>
</tbody>
</table>

In Fig. 7.23 are shown the results of the calculation when one fluid has a range five times as great as the other.

**Shell-side Pressure Drop.** The pressure drop through the shell of an exchanger is proportional to the number of times the fluid crosses the bundle between baffles. It is also proportional to the distance across the bundle each time it is crossed. Using a modification of Eq. (3.44) a correlation has been obtained using the product of the distance across the bundle, taken as the inside diameter of the shell in feet \( D_s \) and the number of times the bundle is crossed \( N + 1 \), where \( N \) is the number of baffles. If \( L \) is the tube length in feet,

Number of crosses, \( N + 1 = \text{tube length, in.}/\text{baffle space, in.} = 12 \times \frac{L}{B} \) \( (7.43) \)

If the tube length is 160” and the baffles are spaces 18 in. apart, there will be 11 crosses or 10 baffles. There should always be an odd number of crosses if both shell nozzles are on opposite sides of the shell and an even number if both shell nozzles are on the same side of the shell. With close baffle spacings at convenient intervals such as 6 in. and under, one baffle may be omitted if the number of crosses is not an integer. The equivalent diameter used for calculating the pressure drop is the same as for heat transfer, the additional friction of the shell itself being neglected. The isothermal equation for the pressure drop of a fluid being heated or cooled and including entrance and exit losses is

\[
\Delta P_s = \frac{fG_i^2D_s(N + 1)}{2g_pD_s\phi_t} = \frac{fG_i^2D_s(N + 1)}{5.22 \times 10^{10}D_s\phi_t} \text{ psf} \quad (7.44)
\]
where \( s \) is the specific gravity of the fluid. Equation (7.44) gives the pressure drop in pounds per square foot. The common engineering unit is pounds per square inch. To permit the direct solution for \( \Delta P_t \) in psi dimensional shell-side friction factors, square foot per square inch, have been plotted in Fig. 29. To obtain the pressure drop in consistent units by Eq. (7.44) multiply \( f \) in Fig. 29 by 144.

**Tube-side Pressure Drop.** Equation (3.44) may be used to obtain the pressure drop in tubes, but it applies principally to an isothermal fluid. Sieder and Tate have correlated friction factors for fluids being heated or cooled in tubes. They are plotted in dimensional form in Fig. 26 and are used in the equation

\[
\Delta P_t = \frac{5.22 \times 10^{16} D_s \phi_t \lambda}{f \phi t} \text{ psi}
\]

where \( \phi_t \) is the number of tube passes, \( L \) the tube length, and \( Ln \) is the total length of path in feet. The deviations are not given, but the curve has been accepted by the Tubular Exchanger Manufacturers Association. In flowing from one pass into the next at the channel and floating head the fluid changes direction abruptly by 180°, although the flow area provided in the channel and floating-head cover should not be less than the combined flow area of all the tubes in a single pass. The change of direction introduces an additional pressure drop \( \Delta P_r \), called the return loss and accounted for by allowing four velocity heads per pass. The velocity head \( \frac{V^2}{2g} \) has been plotted in Fig. 27 against the mass velocity for a fluid with a specific gravity of 1, and the return losses for any fluid will be

\[
\Delta P_r = \frac{4n V^2}{s 2g} \text{ psi}
\]

where \( V = \text{velocity, fps} \)

\( s = \text{specific gravity} \)

\( g' = \text{acceleration of gravity, ft/sec}^2 \)

The total tube-side pressure drop \( \Delta P_T \) will be

\[
\Delta P_T = \Delta P_t + \Delta P_r \text{ psi}
\]

**The Analysis of Performance in an Existing 1-2 Exchanger.** When all the pertinent equations are used to calculate the suitability of an existing exchanger for given process conditions, it is known as rating an exchanger. There are three significant points in determining the suitability of an existing exchanger for a new service.

1. What clean coefficient \( U_c \) can be "performed" by the two fluids as the result of their flow and individual film coefficients \( h_w \) and \( h_o \)?
2. From the heat balance \( Q = WC(T_1 - T_2) = wc(t_2 - t_1) \), known
surface \( A_s \), and the true temperature difference for the process tempera-
tures a value of the design or dirty coefficient, \( U_c \) is obtained. \( U_c \) must
exceed \( U_d \) sufficiently so that the dirt factor, which is a measure of the
excess surface, will permit operation of the exchanger for a reasonable
period of service.

3. The allowable pressure drops for the two streams may not be
exceeded.

When these are fulfilled, an existing exchanger is suitable for the process
conditions for which it has been rated. In starting a calculation the first
point which arises is to determine whether the hot or cold fluid should be
placed in the shell. There is no fast rule. One stream may be large
and the other small, and the baffle spacing may be such that in one
instance the shell-side flow area \( a_s \) will be larger. Fortunately any selec-
tion can be checked by switching the two streams and seeing which
arrangement gives the larger value of \( U_c \) without exceeding the allowable
pressure drop. Particularly in preparation for later methods there is
some advantage, however, in starting calculations with the tube side,
and it may be well to establish the habit. The detailed steps in the
rating of an exchanger are outlined below. The subscripts \( s \) and \( t \) are
used to distinguish between the shell and tubes, and for the outline the
hot fluid has been assumed to be in the shell. By always placing the
hot fluid on the left the usual method of computing the LMTD may be
retained.

The Calculation of an Existing 1-2 Exchanger. Process conditions
required:

- Hot fluid: \( T_1, T_2, W, c, s, \mu, k, R_s, \Delta P \)
- Cold fluid: \( t_1, t_2, w, c, s, \mu, k, R_s, \Delta P \)

For the exchanger the following data must be known:

<table>
<thead>
<tr>
<th>Shell side</th>
<th>Tube side</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>Number and length</td>
</tr>
<tr>
<td>Baffle space</td>
<td>OD, BWG, and pitch</td>
</tr>
<tr>
<td>Passes</td>
<td>Passes</td>
</tr>
</tbody>
</table>

(1) Heat balance, \( Q = WC(T_1 - T_2) = wc(t_2 - t_1) \)
(2) True temperature difference \( \Delta t \):

\[
\text{LMTD}, \quad R = \frac{T_1 - T_2}{t_2 - t_1}, \quad S = \frac{t_2 - t_1}{T_1 - T_2} \quad (5.14)
\]

\[
\Delta t = \text{LMTD} \times F_T \quad (F_T \text{ from Fig. 18}) \quad (7.42)
\]
(3) Caloric temperature $T_e$ and $t_e$.

\[ T_e = T_a + \frac{h_e/a}{h_{oa}/\phi_a + h_e/\phi_e} \quad (5.28), \quad (5.29) \]

(5') Mass vol. $G_t = W/a_t$, lb/(hr)(ft\(^2\))

\[ G_t = \frac{N_t a_t^f/144n}{\text{No. of tubes} \times \text{flow area/tube}} \quad \text{Eq. (7.48)} \]

(6) Mass vol. $G_t = w/a_t$, lb/(hr)(ft\(^2\))

\[ G_t = \frac{N_t a_t^f/144n}{\text{No. of passes}} \quad \text{Eq. (7.48)} \]

(8') Obtain $D$, from Fig. 28 or computer from Eq. (7.4).

Obtain $\mu$ at $T_e$, lb/(ft)(hr) = cp $\times$ 2.42

\[ Re = \frac{DgT_e}{\mu} \quad \text{Eq. (7.5)} \]

(7') Obtain $f$ from Fig. 28.

(7') At $T_e$ obtain $a_e$, Btu/(lb)(°F) and $k_e$, Btu/(hr)(ft\(^2\))(°F/ft).

Compute $f = \left(\frac{cp}{k}\right)^{0.14}$

(9') $h_e = fD \left\{ \frac{cp}{k} \right\}^{0.14} \phi_e$ \quad [Eq. (6.156)]

(10') Tube-wall temp. $t_e$

\[ t_e = t_a + \frac{h_e/\phi_e}{h_{oa}/\phi_a + h_e/\phi_e} \quad (T_e - t_o) \quad \text{Eq. (5.31)} \]

(11') Obtain $\mu_w$ and $\phi_w = (\mu/\mu_w)^{0.14}$.

(12) Corrected coefficient, $h_w = \frac{h_e/\phi_w}{\phi_w}$

\[ h_w = \frac{h_e/\phi_w}{\phi_w} \quad \text{Eq. (6.36)} \]

(13) Clean overall coefficient $U_e$:

\[ U_e = h_{oa} + h_e \quad \text{(6.38)} \]

(14) Design overall coefficient $U_D$: Obtain external surface/in ft $a''$ from Appendix Table 10.

Heat-transfer surface, $A = a''L/N_b$, ft\(^2\)

\[ U_D = \frac{Q}{A \Delta t} \quad \text{Btu/(hr)(ft\(^2\))(°F)} \]

(15) Dirt factor $R_d$:

\[ R_d = \frac{U_e - U_D}{(hr)(ft\(^2\))(°F)/\text{Btu}} \quad \text{(6.13)} \]

If $R_d$ equals or exceeds the required dirt factor, proceed under the pressure drop.

\[ 1 \text{ The use of caloric temperatures is in partial contradiction of the derivation for the 1-2 parallel flow counterflow temperature difference in which $T'$ was assumed constant. The use of caloric temperatures presumes that a linear variation of $U$ with $t$ can be accounted for as the product of $U_{\text{calor},\Delta t}$ where $\Delta t$ is the true parallel flow-counterflow temperature difference when $U$ is constant.} \]

\[ 2 \text{ A convenient graph of } k(\mu/k)^{0.14} \text{ vs. } \mu \text{ for petroleum fractions is given in Fig. 16.} \]
Pressure Drop

(1') For $Re_s$ in (5') obtain $f$, ft$^2$/in.$^9$  
\[ f = \frac{C_{fL}}{5.22 \times 10^{14}/\phi} \]  
[Fig. 29']

(2') No. of crosses, $N + 1 = 12L/B$  
\[ \Delta P_s = \frac{\zeta D_s(N + 1)}{5.22 \times 10^{14}D_s\phi} \text{ psi} \]  
[Eq. (7.43)]

(3') $\Delta P_s = \frac{D_s(N + 1)}{5.22 \times 10^{14}D_s\phi} \text{ psi}$  
\[ \Delta P_r = \frac{4n V^2}{2q} \frac{62.5}{144} \text{ psi} \]  
[Eq. (7.44)]

\[ \Delta P = \Delta P_t + \Delta P_r \text{ psi} \]  
[Eq. (7.47)]

Example 7.3. Calculation of a Kerosene–Crude Oil Exchanger. 43,800 lb/hr of a 42°API kerosene leaves the bottom of a distilling column at 390°F and will be cooled to 200°F by 149,000 lb/hr of 34°API Mid-continent crude coming from storage at 100°F and heated to 170°F. A 10 psi pressure drop is permissible on both streams, and in accordance with Table 12, a combined dirt factor of 0.003 should be provided. Available for this service is a 21½ in. ID exchanger having 158 1 in. OD, 13 BWG tubes 160° long and laid out on 1½-in. square pitch. The bundle is arranged for four passes, and baffles are spaced 5 in. apart.

Will the exchanger be suitable; i.e., what is the dirt factor?

Solution:

Exchanger:

**Shell side**  
ID = 21½ in.  
Baffle space = 5 in.  
Passes = 1

**Tube side**  
Number and length = 158, 160°  
OD, BWG, pitch = 1 in., 13 BWG, 1½-in. square  
Passes = 4

(1) Heat balance:

Kerosene,  
\[ Q = 43,800 \times 0.605(390 - 200) = 5,100,000 \text{ Btu/hr} \]

Mid-continent crude,  
\[ Q = 149,000 \times 0.49(170 - 100) = 5,100,000 \text{ Btu/hr} \]

(2) $\Delta t$:

<table>
<thead>
<tr>
<th>Hot Fluid</th>
<th>Cold Fluid</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>390</td>
<td>Higher Temp</td>
<td>170</td>
</tr>
<tr>
<td>200</td>
<td>Lower Temp</td>
<td>100</td>
</tr>
</tbody>
</table>
| 190       | Differences | 70    | 120   | $(\Delta t_2 - \Delta t_3)$

\[ (T_t - T_s) \quad (t_2 - t_1) \]

\[ LMTD = 152.5°F \]  
\[ R = \frac{189.5}{390 - 100} = 2.71 \]

\[ S = \frac{70}{300 - 100} = 0.241 \]

\[ F = 0.905 \]

\[ \Delta t = 0.905 \times 132.5 = 138°F \]  
[Fig. 18]  
(7.42)
\( T_a \) and \( t_c \):

\[
\frac{\Delta s}{\Delta t} = 0.455
\]

\( K_s = 0.20 \) (crude oil controlling)

\( F_r = 0.42 \)

\( T_c = 200 + 0.42 \times 190 = 280^\circ F \)  
(5.28)

\( t_i = 100 + 0.42 \times 70 = 129^\circ F \)  
(5.29)

Since the flow area of both the shell and tube sides will be nearly equal, assume the larger stream to flow in the tubes and start calculation on the tube side.

\( \text{Hot fluid: shell side, kerosene} \)

\( \text{(4') Flow area, } a_s = 1D \times C^2D/144P_T \)  
[Eq. (7.1)]

\[
= 21.25 \times 0.25 \times 5/144 \times 1.25
= 0.1475 \text{ ft}^2
\]

\( \text{(5') Mass vel, } G_s = W/a_s \)  
[Eq. (7.2)]

\[
= 42,800/0.1475 = 297,000 \text{ lb/(hr)}(\text{ft}^2)
\]

\( \text{(6') } R_s = D_s G_s / \mu \)  
[Eq. (7.3)]

\[
\text{At } T_s = 280^\circ F, \mu = 0.40 \times 2.42
= 0.97 \text{ lb/(ft)}(\text{hr})  
\text{ [Fig. 14]}
\]

\( D_s = 0.99/12 = 0.0825 \text{ ft} \)  
[Fig. 28]

\( R_s = 0.0825 \times 297,000/0.97 = 25,300 \)

\( \text{(7') } J_k = 93 \)  
[Fig. 28]

\( \text{(8') At } T_s = 280^\circ F, c = 0.59 \text{ Btu/(lb)}(\text{°F}) \)  
[Fig. 4]

\( k = 0.0765 \text{ Btu/(hr)}(\text{ft})(\text{°F/ft}) \)  
[Fig. 1]

\( (c_{mu}/k)^{1/4} = (0.59 \times 0.97/0.0765)^{1/4} = 1.95 \)

\( \text{(9') } h_s = J_k \left( \frac{k}{k_s} \right)^{1/4} \phi_s \)  
[Eq. (6.16b)]

\[
\frac{h_s}{\phi_s} = 93 \times 0.0765 \times 1.95 = 169
\]

\( \text{(10') Tube-wall temperature:} \)

\[
t_w = t_c + \frac{h_s/\phi_s}{h_s/\phi_s + h_s/\phi_s} (T' - t_c)
= 129 + \frac{169}{109 + 169} (280 - 129)
= 221^\circ F
\]

\( \text{(11') At } t_w = 221^\circ F, \mu_w = 0.50 \times 2.42
= 1.36 \text{ lb/(ft)}(\text{hr}) \)  
[Fig. 14]

\( \phi_s = (\mu/\mu_w)^{0.14} = (0.97/1.36)^{0.14}
= 0.96 \)  
[Fig. 24 insert]

\( \text{(12') Corrected coefficient, } h_s = \frac{h_s}{\phi_s} \)  
[Eq. (6.36)]

\[
= 169 \times 0.96 = 162 \text{ Btu/(hr)}(\text{ft})(\text{°F})
\]

\( \text{Cold fluid: tube side, crude oil} \)

\( \text{(4') Flow area, } a_t = 0.515 \text{ in.}^2 \)  
[Table 10]

\( a_t = No_t^2/144 \)  
[Eq. (7.48)]

\[
= 158 \times 0.515/144 \times 1 = 0.141 \text{ ft}^2
\]

\( \text{(5') Mass vel, } G_t = w/a_t \)

\[
= 149,000/0.141 = 1,060,000 \text{ lb/hr}(\text{ft}^2)
\]

\( \text{(6') } R_t = D G_t / \mu \)

\[
\text{At } t_c = 129^\circ F, \mu = 3.6 \times 2.42
= 8.7 \text{ lb/(ft)}(\text{hr})  
\text{ [Fig. 14]}
\]

\( D = 0.81/12 = 0.0675 \text{ ft} \)  
[Table 10]

\( R_t = 0.0675 \times 1,060,000/8.7 = 8,220 \)

\( \text{(7') } L/D = 16/0.0675 = 237 \)

\( J_L = 31 \)  
[Fig. 28]

\( \text{(8') At } T_t = 129^\circ F, c = 0.49 \text{ Btu/(lb)}(\text{°F}) \)  
[Fig. 4]

\( k = 0.077 \text{ Btu/(hr)}(\text{ft})(\text{°F/ft}) \)  
[Fig. 1]

\( (c_{mu}/k)^{1/4} = (0.49 \times 8.7/0.077)^{1/4} = 3.81 \)

\( \text{(9') } h_t = J_L \left( \frac{k}{D} \right)^{1/4} \phi_t \)  
[Eq. (6.16a)]

\[
\frac{h_t}{\phi_t} = 31 \times 0.077 \times 0.0675 \times 3.81 = 135
\]

\( \text{(10') } h_{tot} = \frac{h_t}{\phi_t} \times \frac{1D}{OD} = 135 \times \frac{1.0}{109}
= 1.11 \)  
[Eq. (6.5)]

\( \text{(11') At } t_w = 221^\circ F, \mu_w = 1.5 \times 2.42
= 3.63 \text{ lb/(ft)}(\text{hr}) \)  
[Fig. 14]

\( \phi_t = (\mu/\mu_w)^{0.14} = (8.7/3.63)^{0.14}
= 1.11 \)  
[Fig. 24 insert]

\( \text{(12') Corrected coefficient, } h_t = \frac{h_t}{\phi_t} \)  
[Eq. (6.37)]

\[
= 109 \times 1.11 = 121 \text{ Btu/(hr)}(\text{ft})(\text{°F})
\]
Clean overall coefficient \( U_C \):

\[
U_C = \frac{h_{sh0}}{h_a + h_s} = \frac{121 \times 162}{121 + 162} = 69.3 \text{ Btu/(hr)(ft)\(^{3}\)(F)} \tag{5.38}
\]

Design overall coefficient \( U_D \):

\[
\alpha'' = 0.2618 \text{ ft}^2/\text{ln ft} \quad \text{(Table 10)}
\]

Total surface, \( A = 158 \times 10'0' \times 0.2618 = 662 \text{ ft}^2 \)

\[
U_D = \frac{Q}{A \Delta T} = \frac{5,100,000}{662 \times 138} = 55.8 \text{ Btu/(hr)(ft)\(^{3}\)(F)}
\]

Dirt factor \( R_d \):

\[
R_d = \frac{U_C - U_D}{U_C U_D} = \frac{69.3 - 55.8}{69.3 \times 55.8} = 0.00348 \quad \text{hr)(ft)\(^{3}\)(F)/Btu} \tag{6.13}
\]

<table>
<thead>
<tr>
<th>162</th>
<th>Outside</th>
<th>121</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_C )</td>
<td>69.3</td>
<td></td>
</tr>
<tr>
<td>( U_D )</td>
<td>55.8</td>
<td></td>
</tr>
<tr>
<td>( R_d ) Calculated</td>
<td>0.00348</td>
<td></td>
</tr>
<tr>
<td>( R_d ) Required</td>
<td>0.00300</td>
<td></td>
</tr>
</tbody>
</table>

Pressure Drop

(1) For \( Re_0 = 25,300 \),

\[
f = 0.00175 \text{ ft}^2/\text{in}.* \quad \text{[Fig. 28]}
\]

\[
s = 0.75 \quad \text{[Fig. 6]}
\]

\[
D_s = 21.25/12 = 1.77 \text{ ft} \quad \text{(2) No. of rows, } N + 1 = \frac{12L}{H}
\]

\[
= 12 \times 1.5 = 30
\]

\[
\Delta P_s = \frac{fG^2L_1}{5.22 \times 10^{10}D_{sh} s} \quad \text{[Eq. (7.43)]}
\]

\[
= 0.000285 \times 1,060,000^4 \times 16 \times 4
\]

\[
= 5.22 \times 10^{10} \times 0.0675 \times 0.83 \times 1.11
\]

\[
= 0.3 \text{ psi}
\]

\[
\Delta P_s = \frac{fG^2D_s(N + 1)}{5.22 \times 10^{10}D_{sh} s} \quad \text{[Eq. (7.44)]}
\]

\[
= 0.00175 \times 297,000^4 \times 1.77 \times 30
\]

\[
= 5.22 \times 10^{10} \times 0.0675 \times 0.73 \times 0.96
\]

\[
= 3.5 \text{ psi}
\]

Allowable \( \Delta P_s = 10.0 \) psi

(3) \( \Delta P_r = \frac{4nV^2}{2g} \quad \text{[Fig. 27]}
\]

\[
\Delta P_r = \frac{4 \times 4}{0.83 \times 0.15} = 2.9 \text{ psi}
\]

It is seen that a dirt factor of 0.00348 will be obtained although only 0.003 will be required to provide reasonable maintenance period. The pressure drops have not been exceeded and the exchanger will be satisfactory for the service.
Exchangers Using Water. Cooling operations using water in tubular equipment are very common. Despite its abundance, the heat-transfer characteristics of water separate it from all other fluids. It is corrosive to steel, particularly when the tube-wall temperature is high and dissolved air is present, and many industrial plants use nonferrous tubes exclusively for heat-transfer services involving water. The commonest nonferrous tubes are admiralty, red brass, and copper, although in certain localities there is a preference for Muntz metal, aluminum bronze, and aluminum. Since shells are usually fabricated of steel, water is best handled in the tubes. When water flows in the tubes, there is no serious problem of corrosion of the channel or floating-head cover, since these parts are often made of cast iron or cast steel. Castings are relatively passive to water, and large corrosion allowances above structural requirements can be provided inexpensively by making the castings heavier. Tube sheets may be made of heavy steel plate with a corrosion allowance of about \( \frac{1}{8} \) in. above the required structural thickness or fabricated of naval brass or aluminum without a corrosion allowance.

When water travels slowly through a tube, dirt and slime resulting from microorganic action adhere to the tubes which would be carried away if there were greater turbulence. As a standard practice, the use of cooling water at velocities less than 3 fps should be avoided, although in certain localities minimum velocities as high as 4 fps are required for continued operation. Still another factor of considerable importance is the deposition of mineral scale. When water of average mineral and air content is brought to a temperature in excess of 120°F, it is found that tube action becomes excessive, and for this reason an outlet water temperature above 120°F should be avoided.

Cooling water is rarely abundant or without cost. One of the serious problems facing the chemical and power industries today results from the gradual deficiency of surface and subsurface water in areas of industrial concentration. This can be partially overcome through the use of cooling towers (Chap. 17), which reuse the cooling water and reduce the requirement to only 2 per cent of the amount of water required in once-through use. River water may provide part of the solution to a deficiency of ground water, but it is costly and presupposes the proximity of a river. River water must usually be strained by moving screens and pumped considerable distances, and in some localities the water from rivers servicing congested industrial areas requires cooling in cooling towers before it can be used.

Many sizable municipalities have legislated against the use of public water supplies for large cooling purposes other than for make-up in cooling towers or spray-pond systems. Where available, municipal water
may average about 1 cent per 1000 gal., although it has the advantage of being generally available at from 30 to 60 psi pressure which is adequate for most process needs including the pressure drops in heat exchangers. Where a cooling tower is used, the cost of the water is determined by the cost of fresh water, pumping power, fan power, and write-off on the original investment.

The shell-side heat-transfer curve (Fig. 28) correlates very well for the flow of water across tube bundles. The high thermal conductivity of water results in relatively high film coefficients compared with organic fluids. The use of the tube-side curve (Fig. 24), however, gives coefficients which are generally high. In its place the data of Eagle and Ferguson for water alone are given in Fig. 25 and are recommended whenever water flows in tubes. Since this graph deals only with water, it has been possible to plot film coefficients vs. velocity in feet per second with temperature as the parameter. The data have been plotted with the 3/4 in., 16 BWG tube as the base, and the correction factor obtained from the insert in Fig. 25 should be applied when any other inside diameter is used.

In a water-to-water exchanger with individual film coefficients ranging from 500 to 1500 for both the shell and tube, the selection of the required dirt factor merits serious judgment. As an example, if film coefficients of 1000 are obtained on the shell and tube sides, the combined resistance is 0.002, or \( U_c = 500 \). If a fouling factor of 0.004 is required, the fouling factor becomes the controlling resistance. When the fouling factor is 0.004, \( U_f \) must be less than 1/0.004 or 250. Whenever high coefficients exist on both sides of the exchanger, the use of an unnecessarily large fouling factor should be avoided.

The following heat-recovery problem occurs in powerhouses. Although it involves a moderate-size exchanger, the heat recovery is equivalent to nearly 1500 lb/hr of steam, which represents a sizable economy in the course of a year.

**Example 7.4. Calculation of a Distilled-water-Raw-water Exchanger.** 175,000 lb/hr of distilled water enters an exchanger at 93°F and leaves at 85°F. The heat will be transferred to 280,000 lb/hr of raw water coming from supply at 73°F and leaving the exchanger at 80°F. A 10 psi pressure drop may be expected on both streams while providing a fouling factor of 0.0005 for distilled water and 0.0015 for raw water when the tube velocity exceeds 6 fps.

Available for this service is a 15½ in. ID exchanger having 160 ¾ in. OD, 18 BWG tubes 16 ft long and laid out on 1½-in. triangular pitch. The bundle is arranged for two passes, and baffles are spaced 12 in. apart.

Will the exchanger be suitable?

---

Solution:

Exchanger:

**Shell side**
- ID = 15\(\frac{3}{4}\) in.
- Number and length = 160, 16\(\frac{0}{0}\)
- Baffle space = 12 in.
- Passes = 1

**Tube side**
- OD, BWG, pitch = \(\frac{3}{4}\) in., 18 BWG, 1\(\frac{3}{4}\) in. tri.
- Passes = 2

(1) Heat balance:

- Distilled water, \(Q = 178,000 \times 1(93 - 85) = 1,400,000\) Btu/hr
- Raw water, \(Q = 280,000 \times 1(80 - 75) = 1,400,000\) Btu/hr

(2) **\(\Delta t\)**:

<table>
<thead>
<tr>
<th>Hot Fluid</th>
<th>Cold Fluid</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
<td>Higher Temp</td>
<td>80</td>
</tr>
<tr>
<td>85</td>
<td>Lower Temp</td>
<td>75</td>
</tr>
<tr>
<td>8</td>
<td>Differences</td>
<td>5</td>
</tr>
</tbody>
</table>

LMTD = 11.4\(^{\circ}\)F

\[
K = \frac{8}{5} = 1.6 \quad S = \frac{5}{93 - 75} = 0.278
\]

\[
P_T = 0.945
\]

\[
\Delta t = 0.945 \times 11.4 = 10.75^{\circ}\text{F}
\]

(7.42)

(3) **\(T_r\) and \(t\)**:

The average temperatures \(T_r\) and \(t\) of 89 and 77.5\(^{\circ}\)F will be satisfactory for the short ranges and \(\phi_r\) and \(\phi_t\) taken as 1.0. Try hot fluid in shell as a trial, since it is the smaller of the two.

**Hot fluid: shell side, distilled water**

\[(4') a_r = 1D \times C^rB/144P_T \quad \text{[Eq. (7.1)]} \]

\[= 15.25 \times 0.1875 \times 12/144 \times 0.9375 = 0.254 \text{ ft}^2 \]

\[(5') G_r = \frac{w}{\alpha_r} \quad \text{[Eq. (7.2)]} \]

\[= 175,000/0.254 = 690,000 \text{ lb/(hr)}(\text{ft}^2) \]

\[(6') At T_r = 89^{\circ}\text{F}, \mu = 0.81 \times 2.42 = 1.96 \text{ lb}/(\text{ft})(\text{hr}) \quad \text{[Fig. 14]} \]

\[D_r = 0.55/12 = 0.0458 \text{ ft} \quad \text{[Fig. 28]} \]

\[Re_r = \frac{D_rG_r}{\mu} \quad \text{[Eq. (7.3)]} \]

\[= 0.0458 \times 690,000/1.96 = 16,200 \]

\[(7') \phi_r = 73 \quad \text{[Fig. 28]} \]

\[(8') At T_r = 89^{\circ}\text{F}, \epsilon = 1.0 \text{ Btu}/(\text{hr})(\text{ft})^2^{\circ}\text{F} \]

\[k = 0.36 \text{ Btu}/(\text{hr})(\text{ft})^{(\circ}\text{F}/\text{ft}) \quad \text{[Table 4]} \]

\[\epsilon u/k = (1.0 \times 1.96/0.36)^{\frac{1}{2}} = 1.76 \]

**Cold fluid: tube side, raw water**

\[(4') a_t = 0.334 \text{ in.}^2 \quad \text{[Table 10]} \]

\[= \frac{N_aP_t}{144} \quad \text{[Eq. (7.48)]} \]

\[= 160 \times 0.334/144 \times 2 = 0.186 \text{ ft}^2 \]

\[(5') G_t = \frac{w}{\alpha_t} \]

\[= 280,000/0.186 = 1,505,000 \text{ lb/(hr)}(\text{ft}^2) \]

\[
V_{el} = G_t/3600
\]

\[= 1,505,000/3600 \times 62.5 = 5.70 \text{ fps} \]

\[(6') At t_r = 77.5^{\circ}\text{F}, \mu = 0.92 \times 2.42 = 2.23 \text{ lb}/(\text{ft})(\text{hr}) \quad \text{[Fig. 14]} \]

\[D = 0.65/12 = 0.054 \text{ ft} \quad \text{(Re}_t\text{ is for pressure drop only)} \quad \text{[Table 10]} \]

\[Re_t = D\epsilon u/\mu \text{[Eq. (28)]} \]

\[= 0.054 \times 1,505,000/2.23 = 36,400 \]
Hot fluid: shell side, distilled water

\[ h_c = \frac{k}{\delta_c} \times 1 \text{ [Eq. (6.15b)]} \]
\[ = 73 \times 0.36 \times 1.76/0.0458 = 1010 \]

Cold fluid: tube side, raw water

\[ h_i = 1350 \times 0.99 = 1335 \text{ [Fig. 25]} \]
\[ h_{iv} = h_i \times \text{ID/OD} = 1335 \times 0.65/0.75 = 1155 \text{ [Eq. (6.3)]} \]

(13) Clean overall coefficient \( U_C \):

\[ U_C = \frac{h_c h_w}{h_c + h_w} = \frac{1155 \times 1010}{1155 + 1010} = 537 \text{ Btu/hr}(\text{ft}^2)(^\circ\text{F}) \] (6.38)

When both film coefficients are high the thermal resistance of the tube metal is not necessarily insignificant as assumed in the derivation of Eq. (6.38). For a steel 18 BWG tube \( R_w = 0.00017 \) and for copper \( R_w = 0.000017 \).

(14) Design overall coefficient \( U_D \):

\[ \frac{Q}{A \Delta T} = \frac{1,400,000}{502 \times 10.75} = 259 \] (5.3)

(15) Dirt factor \( R_d \):

\[ R_d = \frac{U_C - U_D}{U_C \times U_D} = \frac{537 - 259}{537 \times 259} = 0.0020 \text{ [hr] (ft}^2)(^\circ\text{F})/\text{Btu} \] (6.13)

**Summary**

<table>
<thead>
<tr>
<th>1010</th>
<th>( h ) outside</th>
<th>1155</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_C )</td>
<td>537</td>
<td></td>
</tr>
<tr>
<td>( U_D )</td>
<td>259</td>
<td></td>
</tr>
</tbody>
</table>

- \( R_D \) Calculated: 0.0020
- \( R_D \) Required: 0.0020

**Pressure Drop**

(1) For \( Re = 16,200, f = 0.0019 \text{ ft}^3/\text{in.}^2 \text{ ft} \) [Fig. 29]

(2) For \( Re = 38,400, f = 0.00019 \text{ ft}^3/\text{in.}^2 \) [Fig. 26]

\( \Delta P = \frac{\rho Q \Delta T}{5.22 \times 10^{12}} \) [Eq. (7.45)]

\[ = \frac{0.00019 \times 1,605,000 \times 10 \times 2}{5.22 \times 10^{12} \times 0.064 \times 1.0 \times 1.0} \approx 4.9 \text{ psi} \]
\(\Delta P_t = \frac{kP_D(N + 1)}{5.22 \times 10^{14}D_{s0} \phi_i} \) [Eq. (7.44)]

\[\Delta P_t = \frac{0.0019 \times 690,000 \times 1.27 \times 16}{5.22 \times 10^{14} \times 0.0468 \times 1.0 \times 1.0} = 7.7 \text{ psi}\]

Allowable \(\Delta P_t = 10.0 \text{ psi}\)

\(\Delta P_r = \frac{(4n/\pi)(V^4/2g^2)}{\frac{1}{4} \times 2 \times 0.23} = 2.5 \text{ psi}\) [Eq. (7.46)]

\(\Delta P_T = P_t + P_r = 4.9 + 2.5 = 7.6 \text{ psi}\)

Allowable \(\Delta P_T = 10.0 \text{ psi}\)

It is seen that the overall coefficient for this problem is five times that of the oil-to-oil exchanger of Example 7.3, the principal difference being due to the excellent thermal properties of water. The exchanger is satisfactory for the service.

**Optimum Outlet-water Temperature.** In using water as the cooling medium for a given duty it is possible to circulate a large quantity with a small temperature range or a small quantity with a large temperature range. The temperature range of the water naturally affects the LMTD. If a large quantity is used, \(t_2\) will be farther from \(T_1\) and less surface is required as a result of the larger LMTD. Although this will reduce the original investment and fixed charges, since depreciation and maintenance will also be smaller, the operating cost will be increased owing to the greater quantity of water. It is apparent that there must be an optimum between the two conditions: much water and small surface or little water and large surface.

In the following it is assumed that the line pressure on the water is sufficient to overcome the pressure drop in the exchanger and that the cost of the water is related only to the amount used. It is also assumed that the cooler operates in true countercflow so that \(\Delta t = L\text{M}T\text{D}\). If the approach is small or there is a temperature cross, the derivation below requires an estimate of \(P_T\) by which the LMTD is multiplied.

The total annual cost of the exchanger to the plant will be the sum of the annual cost of water and the fixed charges, which include maintenance and depreciation.

If \(C_T\) is the total annual cost,

\[C_T = (\text{water cost/\text{lb}})(\text{\text{lb/\text{hr}}})(\text{annual \text{hr}})\]

\[+ \text{(annual fixed charges/ft}^2)(\text{ft}^2)\]

\[Q = wc(t_2 - t_1) = UA(L\text{M}T\text{D}) \tag{7.49}\]

Substituting the heat-balance terms in Eq. (7.49), where \(w = Q/[c(t_2 - t_1)]\) and the surface \(A = Q/U(L\text{M}T\text{D})\)

\[C_T = \frac{Q\theta C_w}{c(t_2 - t_1)} + \frac{C_T Q}{U(L\text{M}T\text{D})}\]
where $\theta = \text{annual operating hours}$

$C_w = \text{water cost/lb}$

$C_r = \text{annual fixed charges/ft}^2$

Assuming $U$ is constant

$$\text{LMTD} = \frac{A_t - A_{t1}}{\ln \frac{A_t}{A_{t1}}}$$

Keeping all factors constant except the water-outlet temperature and consequently $A_t$,

$$C_r = \frac{Q \theta C_w}{c(t_2 - t_1)} + \frac{C_r Q}{U \left[ T_1 - t_2 - \frac{A_{t1}}{T_1 - t_2} \right]}$$  \hspace{1cm} (7.50)$$

The optimum condition will occur when the total annual cost is a minimum, thus when $dC_r/dt_2 = 0$.

Differentiating and equating the respective parts,

$$\frac{U \theta C_w}{C_r C} \left( \frac{t_1 - t_2 - \frac{A_{t1}}{t_2 - t_1}}{t_1 - t_2} \right)^2 = \ln \frac{T_1 - t_2}{A_{t1}} - \left[ 1 - \frac{1}{(T_1 - t_2)/A_{t1}} \right]$$  \hspace{1cm} (7.51)$$

Equation (7.51) has been plotted by Colburn and is reproduced in Fig. 7.24.

Example 7.5. Calculation of the Optimum Outlet-water Temperature. A viscous fluid is cooled from 175 to 150°F with water available at 85°F. What is the optimum outlet water temperature?

$$175 - x = A_{t1}$$

$$150 - 85 = A_{t1} = 65$$

It will first be necessary to assume a value of $U$. Since the material is viscous, assume $U = 15$. To evaluate the group $U \theta C_w/C_r C$:

$\theta = 8000$ operating hours annually

$C_w$ computed at $0.01/1000$ gal. = $0.01/8300$, dollars/lb

For annual charges assume 20 per cent repair and maintenance and 10 per cent depreciation. At a unit cost of $4 per square foot the annual fixed charge is

$$4 \times 0.30 = 1.20$$

The specific heat of water is taken as 1.0.

$$\frac{U \theta C_w}{C_r C} = \frac{15 \times 8000 \left( 0.01 \right)}{1.20 \times 1.0 \left( 8300 \right)} = 0.1205$$

$$\frac{T_1 - T_2}{A_{t1}} = \frac{175 - 150}{150 - 85} = 0.39$$

From Figure 7.24,

$$\frac{A_{t2}}{A_{t1}} = 0.96$$

$$A_{t2} = T_1 - t_2 - 0.96 \times 65 = 62.3°F$$

$$t_2 = 175 - 62.3 = 112.7°F$$
When the value of $U$ is high or there is a large hot-fluid range, the optimum outlet-water temperature may be considerably above the upper limit of 120°F. This is not completely correct, since the maintenance cost will probably rise considerably above 20 per cent of the initial cost when the temperature rises above 120°F. Usually information is not available on the increase in maintenance cost with increased water-outlet temperature, since such data entail not only destructive tests but records kept over a long period of time.

**Solution Exchangers.** One of the commonest classes of exchangers embraces the cooling or heating of solutions for which there is a paucity of physical data. This is understandable, since property vs. temperature plots are required not only for each combination of solute and solvent but for different concentrations as well. Some of the data available in the literature and other studies permit the formulation of rules for estimating the heat-transfer properties of solutions when the rules are used with considerable caution. They are given as follows:
Thermal conductivity:

Solutions of organic liquids: use the weighted conductivity.

Solutions of organic liquids and water: use 0.9 times the weighted conductivity.

Solutions of salts and water circulated through the shell: use 0.9 times the conductivity of water up to concentrations of 30 per cent.

Solutions of salts and water circulating through the tubes and not exceeding 30 per cent: use Fig. 24 with a conductivity of 0.8 that of water.

Colloidal dispersions: use 0.9 times the conductivity of the dispersion liquid.

Emulsions: use 0.9 times the conductivity of the liquid surrounding the droplets.

Specific heat:

Organic solutions: use the weighted specific heat.

Organic solutions in water: use the weighted specific heat.

Fusible salts in water: use the weighted specific heat where the specific heat of the salt is for the crystalline state.

Viscosity:

Organic liquids in organics: use the reciprocal of the sum of the terms, (weight fraction/viscosity) for each component.

Organic liquids in water: use the reciprocal of the sum of the terms, (weight fraction/viscosity) for each component.

Salts in water where the concentration does not exceed 30 per cent and where it is known that a syrup-type of solution does not result: use a viscosity twice that of water. A solution of sodium hydroxide in water under even very low concentrations should be considered syrupy and cannot be estimated.

Wherever laboratory tests are available or data can be obtained, they will be preferable to any of the foregoing rules. The following demonstrates the solution of a problem involving an aqueous solution:

Example 7.6. Calculation of a Phosphate Solution Cooler. 20,160 lb/hr of a 30% K$_2$PO$_4$ solution, specific gravity at 120°F = 1.30, is to be cooled from 150 to 90°F using well water from 68 to 90°F. Pressure drops of 10 psi are allowable on both streams, and a total dirt factor of 0.002 is required.

Available for this service is a 10.02-in. ID 1-2 exchanger having 52 3/4 in. OD, 16 HWC tubes 16'0" long laid out on 1-in. square pitch. The bundle is arranged for two passes, and the baffles are spaced 2 in. apart.

Will the exchanger be suitable?
Solution:

Exchanger:

Shell side

ID = 10.02 in.  Number and length = 52, 160'’
Baffle space = 2 in.  OD, BWG, pitch = 3/4 in., 16 BWG, 1 in. square
Passes = 1

Tube side

OD, BWG, pitch = 3/4 in., 16 BWG, 1 in. square
Passes = 2

(3) Heat balance:
Specific heat of phosphate solution = 0.3 \times 0.19 + 0.7 \times 1 = 0.737 Btu/(lb)(^\circ F)
30% K,\text{PO}_4 solution, Q = 20,100 \times 0.737(150 - 90) = 913,000 Btu/hr
Water, Q = 41,600 \times 1.0(90 - 68) = 915,000 Btu/hr

(2) \Delta t:

<table>
<thead>
<tr>
<th>Hot Fluid</th>
<th>Cold Fluid</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>Higher Temp</td>
<td>90</td>
</tr>
<tr>
<td>90</td>
<td>Lower Temp</td>
<td>68</td>
</tr>
<tr>
<td>60</td>
<td>Differences</td>
<td>22</td>
</tr>
</tbody>
</table>

\text{LMTD} = 37.9^\circ F \quad (5.14)

\begin{align*}
R &= \frac{60}{22} = 2.73 \\
S &= \frac{22}{150 - 68} = 0.268 \\
F_T &= 0.81 \\
\Delta t &= 0.81 \times 37.9 = 30.7^\circ F \quad (7.42)
\end{align*}

(5) \text{T}_s \text{ and } \text{t}_s: \text{The average temperatures } \text{T}_s \text{ and } \text{t}_s \text{ of 120 and 79^\circ F will be satisfactory.}

\text{Hot fluid: shell side, phosphate solution}

(4') \text{a}_s = \frac{10.02 \times 0.25 \times 2}{144 \times 1} = 0.0347 \text{ ft}^2

(6') \text{G}_s = \frac{w}{\text{a}_s} = 41,600/0.0347 = 1,200,000 \text{ lb/}(\text{hr})(\text{ft}^2)

\text{Cold fluid: tube side, water}

(4) \text{a}_t = 0.302 \text{ in.}^2 \quad \text{Table 10}

\text{G}_t = \frac{w}{\text{a}_t} = 41,600/0.0545 = 762,000 \text{ lb/}(\text{hr})(\text{ft}^2)

\text{Y} = \frac{\text{G}_t}{3600} = 762,000/3600 \times 62.5 = 3.40 \text{ iph}

(6) \text{At } \text{t}_s = 79^\circ F, \mu = 0.91 \times 2.42

\text{At } \text{t}_s = 79^\circ F, \mu = 0.91 \times 2.42

\text{D} = \frac{0.56}{12} - 0.0517 \text{ ft} \quad \text{Table 10}

(9') \text{h}_s = \frac{k}{D_e} \left(\frac{c_p}{k}\right)^{0.25} \times 1 \quad \text{Eq. (6.15b)}

\text{h}_t = \frac{0.33 \times 1.88 \times 0.079}{558 \text{ Btu/}(\text{hr})(\text{ft}^2)(^\circ F)}

\text{a}_s \text{ and } \phi = 1

\text{(4') a}_s = \frac{10.02 \times 0.25 \times 2}{144 \times 1} = 0.0347 \text{ ft}^2

\text{a}_t = \frac{144}{302/144 \times 2} = 0.0545 \text{ ft}^2

\text{G}_s = \frac{w}{\text{a}_s} = 41,600/0.0347 = 1,200,000 \text{ lb/}(\text{hr})(\text{ft}^2)

\text{G}_t = \frac{w}{\text{a}_t} = 41,600/0.0545 = 762,000 \text{ lb/}(\text{hr})(\text{ft}^2)

\text{Y} = \frac{\text{G}_t}{3600} = 762,000/3600 \times 62.5 = 3.40 \text{ iph}
(13) Clean overall coefficient $U_c$:

$$U_c = \frac{h_a h_e}{h_a + h_e} = \frac{662 \times 558}{662 + 558} = 303 \text{ Btu/(hr)(ft²)(°F)}$$  \hspace{1cm} (6.38)

(14) Design overall coefficient $U_d$:

$$U_d = \frac{Q}{A\Delta T} = \frac{915,000}{163 \times 30.7} = 183 \text{ Btu/(hr)(ft²)(°F)}$$

(15) Dirt factor $R_d$:

$$R_d = \frac{U_c - U_d}{U_c U_d} = \frac{303 - 183}{303 \times 183} = 0.00216$$

Summary

<table>
<thead>
<tr>
<th>558</th>
<th>$h_{\text{outside}}$</th>
<th>662</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_c$</td>
<td>303</td>
<td></td>
</tr>
<tr>
<td>$U_d$</td>
<td>183</td>
<td></td>
</tr>
</tbody>
</table>

$R_d$ Calculated 0.00216

$R_d$ Required 0.0020

Pressure Drop

(1') For $Re = 15,750, f = 0.0019 \text{ ft}^3/\text{in} \cdot \text{lb}$ \hspace{1cm} [Fig. 26]

(2') No. of passes, $N + 1 = 12L/R$ \hspace{1cm} [Eq. (7.44)]

$$= 12 \times 1\frac{5}{9} = 96$$

$D_s = 10.02/12 = 0.833 \text{ ft}$

(3') $\Delta P_s = \frac{fG_1D_s(N + 1)}{5.22 \times 10^{10} \rho g_s}$ \hspace{1cm} [Eq. (7.44)]

$$= \frac{0.0019 \times 578,000 \times 0.833 \times 96}{5.22 \times 10^{10} \times 0.079 \times 1.30 \times 1.0} = 9.5 \text{ psi}$$

Allowable $\Delta P_s = 10.0 \text{ psi}$

$$\Delta P_T = \frac{4 \times V^2}{2g}$$

$$= \frac{4 \times 0.08}{2} \times 0.08 = 0.7 \text{ psi}$$

$\Delta P_T = \Delta P_s + \Delta P_r$ \hspace{1cm} [Eq. (7.47)]

Allowable $\Delta P_T = 10.0 \text{ psi}$

The exchanger is satisfactory for the service.

Steam as a Heating Medium. Thus far none of the heat-transfer services studied has employed steam although it is by far the commonest...
heating medium. Steam as a heating medium introduces several difficulties: (1) Hot steam condensate is fairly corrosive, and care must be exercised to prevent condensate from accumulating within an exchanger where continuous contact with metal will cause damage. (2) The condensate line must be connected with discretion. Suppose exhaust steam at 5 psig and 228°F is used to heat a cold fluid entering at a temperature of 100°F. The tube wall will be at a temperature between the two but nearer that of the steam, say 180°F, which corresponds to a saturation pressure of only 7.5 psia for the condensate at the tube wall. Although the steam entered at 5 psig, the pressure on the steam side may drop locally to a pressure below that of the atmosphere, so that the condensate will not run out of the heater. Instead it will remain and build up in the exchanger until it blocks off all the surface available for heat transfer. Without surface, the steam will not continue to condense and will retain its inlet pressure long enough to blow out some or all of the accumulated condensate so as to reexpose surface, depending upon the design. The heating operation will become cyclical and to overcome this difficulty and attain uniform flow, it may be necessary to employ a trap or suction for which piping arrangements will be discussed in Chap. 21.

The heat-transfer coefficients associated with the condensation of steam are very high compared with any which have been studied so far. It is customary to adopt a conventional and conservative value for the film coefficient, since it is never the controlling film, rather than obtain one by calculation. In this book in all heating services employing relatively air-free steam a value of 1500 Btu/(hr)(ft²)(°F) will be used for the condensation of steam without regard to its location. Thus \( h_i = h_o = h_w = 1500 \).

It is advantageous in heating to connect the steam to the tubes of the heater rather than the shell. In this way, since the condensate may be corrosive, the action can be confined to the tube side alone, whereas if the steam is introduced into the shell, both may be damaged. When steam flows through the tubes of a 1-2 exchanger, there is no need for more than two tube passes. Since steam is an isothermally condensing fluid, the true temperature difference \( \Delta t \) and the LMTD are identical.

When using superheated steam as a heating medium, except in desuperheaters, it is customary to disregard the temperature range of desuperheating and consider all the heat to be delivered at the saturation temperature corresponding to the operating pressure. A more intensive analysis of the condensation of steam will be undertaken in the chapters dealing with condensation.
Pressure Drop for Steam. When steam is employed in two passes on the tube side, the allowable pressure drop should be very small, less than 1.0 psi, particularly if there is a gravity return of condensate to the boiler. In a gravity-return system, condensate flows back to the boiler because of the difference in static head between a vertical column of steam and a vertical column of condensate. The pressure drop including entrance and end losses through an exchanger can be calculated by taking one-half of the pressure drop for steam as calculated in the usual manner by Eq. (7.45) for the inlet vapor conditions. The mass velocity is calculated from the inlet steam rate and the flow area of the first pass (which need not be equal to that of the second pass). The Reynolds number is based on the mass velocity and the viscosity of steam as found in Fig. 15. The specific gravity used with Eq. (7.45) is the density of the steam obtained from Table 7 for the inlet pressure divided by the density of water taken as 62.5 lb/ft³.

Quite apparently this calculation is an approximation. It is conservative inasmuch as the pressure drop per foot of length decreases successively with the square of the mass velocity while the approximation assumes a value more nearly the mean of the inlet and outlet.

The Optimum Use of Exhaust and Process Steam. Some plants obtain power from noncondensing turbines or engines. In such places there may be an abundance of exhaust steam at low pressures from 5 to 25 psig which is considered a by-product of the power cycles in the plant. While there are arbitrary aspects to the method of estimating the cost of exhaust steam, it will be anywhere from one-quarter to one-eighth of the cost of process or live steam. Although it possesses a high latent heat, exhaust steam is of limited process value, since the saturation temperature is usually about 215 to 230°F. If a liquid is to be heated to 250 or 275°F, it is necessary to use process steam at 100 to 200 psi developed at the powerhouse specially for process purposes.

When a fluid is to be heated to a temperature close to or above that of exhaust steam, all the heating can be done in a single shell using only process steam. As an alternative the heat load can be divided into two shells, one utilizing as much exhaust steam as possible and the other using as little process steam as possible. This leads to an optimum: If the outlet temperature of the cold fluid in the first exchanger is made to approach the exhaust steam temperature too closely, a small Δt and large first heater will result. On the other hand, if the approach is not close, the operating cost of the higher process steam requirement in the second heater increases so that the initial cost of two shells may not be justified.

In the following analysis it is assumed that the pressure drop, pumping
cost, and overall coefficient are identical in a single and double heater arrangement. It is assumed also that the fixed charges per square foot of surface is constant, although this too is not strictly true. The cost equation is taken as the sum of steam and fixed charges and because steam condenses isothermally, \( \Delta t = \text{LMTD} \).

\[
C_T = wc(t - t_1)\theta C_S + A_1C_T + wc(t_2 - t)\theta C_F + A_2C_T \tag{7.52}
\]

where \( C_T \) = total annual cost, dollars
\( C_F \) = annual fixed charges, dollars/ft²
\( C_S \) = cost of exhaust steam, dollars/Btu
\( T_S \) = cost of process steam, dollars/Btu
\( T_F \) = temperature of exhaust steam, °F
\( T_P \) = temperature of process steam, °F
\( t = \text{intermediate temperature between shells} \)
\( \theta = \text{total annual operating hours} \)

\[
A_1 = \frac{Q_1}{U \Delta t_1} = \frac{wc}{U} \ln \frac{T_S - t_1}{T_S - t} \quad \text{and} \quad A_2 = \frac{Q_2}{U \Delta t_2} = \frac{wc}{U} \ln \frac{T_P - t}{T_P - t_2}
\]

Substituting, differentiating Eq. (7.52) with respect to \( t \), and setting equal to zero

\[
(T_P - t)(T_S - t) = \frac{C_T(T_F - T_P)}{(C_F - C_S)U\theta} \tag{7.53}
\]

**Example 7.7. The Optimum Use of Exhaust and Process Steam.** Exhaust steam at 5 psig (\( \approx 228^\circ \text{F} \)) and process steam at 85 psig (\( \approx 228^\circ \text{F} \)) are available to heat a liquid from 150 to 250°F. Exhaust steam is available at 5 cents per per 1000 lb, and process steam at 30 cents per 1000 lb. From experience an overall rate of 50 Btu/(hr)(ft²)(°F) may be expected. The assumption may be checked later. Use annual fixed charges of \$1.20 per square foot, 8000 annual hours, latent heats of 960.1 Btu/lb for exhaust, and 888.8 Btu/lb for process steam.

**Solution:**

\[
(328 - t)(228 - t) = \frac{1.20(328 - 228)}{(0.30/1000 \times 888.8 - 0.05/1000 \times 960.1 \times 50 \times 8000)} \tag{7.53}
\]

\[
t = 218^\circ \text{F}
\]

**1-2 Exchangers without Baffles.** Not all 1-2 exchangers have 25 per cent cut segmental baffles. When it is desired that a fluid pass through the shell with an extremely small pressure drop, it is possible to depart from the use of segmental baffles and use only support plates. These will usually be half-circle, 50 per cent cut plates which provide rigidity and prevent the tubes from sagging. Successive support plates overlap at the shell diameter so that the entire bundle can be supported by two
half circles which support one or two rows of tubes in common. These may be spaced farther apart than the outside diameter of the shell, but when they are employed, the shell fluid is considered to flow along the axis instead of across the tubes. When the shell fluid flows along the tubes or the baffles are cut more than 25 per cent, Fig. 28 no longer applies. The flow is then analogous to the annulus of a double pipe exchanger and can be treated in a similar manner, using an equivalent diameter based on the distribution of flow area and the wetted perimeter for the entire shell. The calculation of the shell-side pressure drop will also be similar to that for an annulus.

Example 7.8. Calculation of a Sugar-solution Heater without Baffles. 200,000 lb/hr of a 20 per cent sugar solution (\(s = 1.08\)) is to be heated from 100 to 122°F using steam at 5 psi pressure.

Available for this service is a 12 in. ID 1-2 exchanger without baffles having 76 \(\frac{3}{4}\) in. OD, 16 BWG tubes 16'0" long laid out on a 1-in. square pitch. The bundle is arranged for two passes.

Can the exchanger provide a 0.003 dirt factor without exceeding a 10.0 psi solution pressure drop?

Solution:

Exchanger:

<table>
<thead>
<tr>
<th>Shell side</th>
<th>Tube side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D = 12 in.</td>
<td>Number and length = 76, 16'0&quot;</td>
</tr>
<tr>
<td>Baffle space = half circles</td>
<td>OD, BWG, pitch = (\frac{3}{4}) in., 16 BWG, 1 in. square</td>
</tr>
<tr>
<td>Passes = 1</td>
<td>Passes = 2</td>
</tr>
</tbody>
</table>

1) Heat balance:
Specific heat of 20 per cent sugar at 111°F = 0.2 \(\times\) 0.30 + 0.8 \(\times\) 1

\[= 0.86 \text{ Btu/(lb)(°F)}\]

Sugar solution, \(Q = 200,000 \times 0.86(122 - 100) = 3,790,000 \text{ Btu/hr}\)
Steam \(Q = 3950 \times 960.1 = 3,790,000 \text{ Btu/hr}\) (Table 7)

2) \(\Delta t\):

<table>
<thead>
<tr>
<th>Hot Fluid</th>
<th>Cold Fluid</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>228 Higher Temp</td>
<td>122</td>
<td>106</td>
</tr>
<tr>
<td>228 Lower Temp</td>
<td>100</td>
<td>128</td>
</tr>
<tr>
<td>0 Differences</td>
<td>22</td>
<td>22</td>
</tr>
</tbody>
</table>

When \(P = 0, \Delta t = \text{LMTD} = 116.5°F.\) (5.14)

3) \(T_s\) and \(t_s\): The steam coefficient will be very great compared with that for the sugar solution, and the tube wall will be considerably nearer 228°F than the caloric temperature of the fluid. Obtain \(F_s\) from \(U_s\) and \(U_s'.\) Failure to correct for wall effects, however, will keep the heater calculation on the safe side. Use 111°F as the average, \(t_s.\)
Hot fluid: tube side, steam

(4) \( a_l = 0.302 \text{ in}^3 \) \( \text{[Table 10]} \)
\[
\begin{align*}
\text{a}_l & = \frac{N \cdot a_l}{144n} \quad \text{[Eq. (7.48)]} \\
& = 76 \times 0.302 / 144 \times 2 = 0.0797 \text{ ft}^2
\end{align*}
\]

(5) \( G_s \) (for pressure drop only) = \( W/a_s \)
\[
\begin{align*}
& = 3950 / 0.0797 = 49,500 \text{ lb/(hr)(ft}^2
\end{align*}
\]

(6) \( \Delta T_s = 228 \text{ F} \)
\[
\begin{align*}
\rho_{steam} & = 0.0128 \times 2.42 \\
& = 0.031 \text{ lb/(ft)(hr)} \quad \text{[Fig. 15]}
\end{align*}
\]

\( D = 0.62 / 12 = 0.0517 \text{ ft} \) \( \text{[Table 10]} \)
\[
\begin{align*}
\text{Re}_t = DG_t / \mu & \quad \text{[Eq. (3.5)]} \\
& = 0.0517 \times 40,500 / 0.031 = 82,500
\end{align*}
\]
\( \text{Re}_t \) is for pressure drop.

Cold fluid: shell side, sugar solution

(4') \( a_s = (\text{area of shell}) - (\text{area of tubes}) \)
\[
\begin{align*}
& = \frac{4 \pi}{(12^2/4 - 76 \times \pi)} \\
& = 0.65 \text{ ft}^2
\end{align*}
\]

(5') \( G_s = \nu/a_s \)
\[
\begin{align*}
& = 200,000 / 0.55 \\
& = 364,000 \text{ lb/(hr)(ft}^2
\end{align*}
\]

(6') \( \Delta T_s = 111 \text{ F}, \mu = 2 \text{ poise} \)
\[
\begin{align*}
D_s & = 4a_s/(\text{wetted perimeter}) \quad \text{[Eq. (6.3)]} \\
& = 4 \times 0.55 / (76 \times \pi \times 0.75 / 12) = 0.148 \text{ ft}
\end{align*}
\]

\( \text{Re}_s = D_s G_s / \mu \)
\[
\begin{align*}
& = 0.148 \times 364,000 / 3.14 = 17,100
\end{align*}
\]

(7') From Fig. 24 (tube-side data)
\[
\begin{align*}
\Delta T_s & = 210 \text{ F}, \mu_s = 2 \text{ poise} \\
& = 0.51 \times 2.42 = 1.26 \text{ lb/(ft)(hr)} \\
(8') h_s & = \frac{k}{D_s} \left(\frac{\mu_s^2}{\nu^2}\right)^{1/4} \quad \text{[Eq. (6.15b)]} \\
& = 0.333 \text{ Btu/(hr)(ft}^2)(\text{F/F}) \\
(9') \phi_s & = \frac{h_s}{\nu} \quad \text{[Eq. (6.9)]} \\
& = 61.5 \times 0.333 \times 2.0 / 0.148 = 278
\end{align*}
\]

(11') At \( \Delta T_s = 210 \text{ F}, \mu_s = 2 \text{ poise} \)
\[
\begin{align*}
& = 0.51 \times 2.42 = 1.26 \text{ lb/(ft)(hr)} \\
\phi_s & = \frac{(\mu_s^2)}{\nu^2} \quad \text{[Fig. 14]} \\
& = (9.14 / 1.26)^{1.4} = 1.12
\end{align*}
\]

(12') Corrected coefficient, \( h_s = \phi_s \frac{k}{\nu} \quad \text{[Eq. (6.9)]} \\
= 278 \times 1.12 = 311 \text{ Btu/(hr)(ft}^2)(\text{F/F})
\]

Condensation of steam:
\( h_{ce} = 1500 \text{ Btu/(hr)(ft}^2)(\text{F/F}) \)

(10) \( t_o = t_s + \frac{h_{ce}}{h_{ce} + h_o} \) \( \text{(5.31a)} \)
\[
\begin{align*}
& = 111 + \frac{1500}{1500 + 278} \times 228 - 111 \\
& = 210 \text{ F}
\end{align*}
\]

Clean overall coefficient \( U_c \):
\[
U_c = \frac{h_o h_{ce}}{h_o + h_{ce}} = \frac{1500 \times 311}{1500 + 311} = 257 \text{ Btu/(hr)(ft}^2)(\text{F/F})
\]

Design overall coefficient \( U_D \):
\[
\begin{align*}
\Delta T_s & = 0.1963 \text{ ft}^3/\text{lb ft} \\
A & = 76 \times 160'' \times 0.1963 = 238 \text{ ft}^3 \\
U_D & = \frac{Q}{A \Delta T} = \frac{3,790,000}{238 \times 116.5} = 137 \text{ Btu/(hr)(ft}^2)(\text{F/F})
\end{align*}
\]

Dirt factor \( R_d \):
\[
R_d = \frac{U_c - U_D}{U_c U_D} = \frac{257 - 137}{257 \times 137} = 0.0034 \text{ (hr)(ft}^2)(\text{F/F})/\text{Btu}
\]

* Note \( h_o \) in the numerator.
Pressure Drop

(1) Specific vol of steam from Table 7:
\[ s = \frac{1}{20.0 \text{ ft}^3/\text{lb}} \]
\[ s = \frac{1}{20.0 \text{ ft}^3/\text{lb}} = 0.0500 \]

\[ Re_s = 82,500, f = 0.000155 \text{ ft}^{-1} \text{in.}^{-1} \]

[Fig. 26]

\[ \Delta P_r = \frac{1}{2} \times \frac{fG_s^2L_m}{5.22 \times 10^{10}D_s \phi_s} \]

\[ = \frac{1}{2} \times \frac{0.000155 \times 49,500^4 \times 16 \times 2}{5.22 \times 10^{10} \times 0.0517 \times 0.0008 \times 1.0} \]

\[ = 2.8 \text{ psi} \]

This is a relatively high pressure drop for steam with a gravity condensate return. The exchanger is satisfactory.

\[ (1') \quad D_s = 4 \times \text{flow area/side frictional wetted perimeter} \ [\text{Eq. (6.4)}] \]

\[ = 4 \times 0.55/(76 \times 3.14 \times 0.75/12 + 2.14 \times 13/12) = 0.122 \text{ ft} \]

\[ Re_s = D_s \phi_s \mu \]  

[Eq. (7.3)]

\[ = 0.122 \times 364,000/3.14 = 14,100 \]

\[ f(\text{from Fig. 20 for tube side}) = 0.00025 \text{ ft}^{-1} \text{in.}^{-1} \]

\[ \Delta P_r = \frac{f G_s^2 L_m}{5.22 \times 10^{10} D_s \phi_s} \]

[Eq. (7.45)]

\[ = 0.00025 \times 364,000^4 \times 16 \times 1 - 5.22 \times 10^{10} \times 0.122 \times 1.08 \times 1.12 \]

\[ = 0.07 \text{ psi} \]

\[ (2') \quad \Delta P_r = \frac{f G_s^2 L_m}{5.22 \times 10^{10} D_s \phi_s} \]

Heat Recovery in a 1-2 Exchanger. When an exchanger is clean, the hot-fluid outlet temperature is lower than the process outlet temperature and the cold-fluid outlet temperature is higher than the process outlet temperature. For counterflow it was possible to obtain the value of \( T_2 \) and \( t_2 \) for a clean exchanger from Eq. (5.18), starting with

\[ wc(t_2 - t_1) = UA \times \text{LMTD} \]

For a 1-2 exchanger the outlet temperatures can be obtained starting with the expression \( wc(t_2 - t_1) = T/AF_r \times \text{LMTD} \), where the LMTD is defined in terms of parameters \( R \) and \( S \) by Eq. (7.39) and \( F_r \) is defined by Eq. (7.44).

Recognizing that \( F_r \) can be eliminated when \( UA/wc \) in Eq. (7.37) is plotted against \( S \), Ten Broeck\(^1\) developed the graph shown in Fig. 7.25. In an existing 1-2 exchanger \( A \) and \( wc \) are known. \( U \) can be computed

from the flow quantities and temperatures, and \( R \) can be evaluated from \( w_c/W_C \). This permits \( S \) to be read directly from the graph. Since 
\[
S = \frac{t_2 - t_1}{T_1 - t_1}
\]
and \( T_1 \) and \( t_1 \) are known, it is then possible to obtain \( t_2 \) and from the heat balance \( w_c(t_2 - t_1) = W_C(T_1 - T_2) \). The line designated threshold represents the initial points at which a temperature cross occurs. Values on this line correspond to \( T_2 = t_2 \).

![Graph](image)

**Fig. 7.25.** Ten Broeck chart for determining \( t_2 \) when \( T_1 \) and \( t_1 \) are known in a 1-2 exchanger. ([Industrial & Engineering Chemistry.])

**Example 7.9.** Outlet Temperatures for a Clean 1-2 Exchanger. In Example 7.3, kerosene crude oil exchanger, what will the outlet temperatures be when the exchanger is freshly placed in service?

**Solution:**

\[
\begin{align*}
U_c &= 60.3 \\
A &= 662 \\
\nu &= 149,000 \\
\frac{W}{C} &= 43,800 \\
c &= 0.49 \\
R &= \frac{w_c}{W_C} = \frac{149,000 \times 0.49}{43,800 \times 0.60} = 2.78
\end{align*}
\]

From Figure 7.25,

\[
S = \frac{t_2 - t_1}{T_1 - t_1} = 0.265
\]

\[
t_2 = t_1 + 0.265(T_1 - t_1) = 100 + 0.265(390 - 100) = 177^\circ F
\]

\[
T_2 = T_1 - R(t_1 - t_1) = 390 - 2.78(177 - 100) = 176^\circ F
\]

**The Efficiency of an Exchanger.** In the design of many types of apparatus it is frequently desirable to establish a standard of maximum performance. The efficiency is then defined as the fractional performance of an apparatus delivering less than the standard. Dodge\(^1\) gives the definition of the efficiency of an exchanger as the ratio of the quantity

of heat removed from a fluid to the maximum which might have been removed. Using the usual nomenclature,

$$
\epsilon = \frac{\frac{\alpha c(t_2 - t_1)}{\alpha c(t_1 - t_i)}}{\frac{t_2 - t_1}{T_1 - t_i}} = \frac{t_2 - t_1}{T_1 - t_i}
$$

(7.54)

which is identical with the temperature group \( S \) and presumed that \( t_2 = T_1 \). Depending upon whether the hot or cold terminal approaches zero, the efficiency may also be expressed by

$$
\epsilon = \frac{WC(T_1 - T_2)}{WC(T_1 - t_i)}
$$

(7.55)

Although there is merit to this definition from the standpoint of thermodynamics there is a lack of realism in an efficiency definition which involves a terminal difference and a temperature difference of zero. It is the same as defining the efficiency as the ratio of the heat transferred by a real exchanger to an exchanger with infinite surface.

In process heat transfer there is another definition which is useful. The process temperatures are capable of providing a maximum temperature difference if arranged in counterflow. There appears to be some value in regarding the efficiency of an exchanger as the ratio of the temperature difference attained by any other exchanger to that for true counterflow. This is identical with \( F_r \), which proportionately influences the surface requirements. It will be seen in the next and later chapters that other flow arrangements besides 1-2 parallel flow–counterflow can be attained in tubular equipment and by which the value of \( F_r \) may be increased for given process temperatures. These obviously entail flow patterns which approach true counterflow more closely than the 1-2 exchanger.

PROBLEMS

7.1. A 1-2 exchanger is to be used for heating 50,000 lb/hr of methyl ethyl ketone from 100 to 200°F using hot amyl alcohol available at 250°F. (a) What minimum quantity of amyl alcohol is required to deliver the desired heat load in a 1-2 exchanger? (b) If the amyl alcohol is available at 275°F, how does this affect the total required quantity?

7.2. A 1-2 exchanger has one shell and two tube passes. The passes do not have equal surfaces. Instead \( X \) per cent of the tubes are in the first pass and \((1 - X)\) per cent are in the second, but if the tube-side film coefficient is not controlling, the assumption of constant \( U \) is justifiable. (a) Develop an expression for the true temperature difference when \( X \) per cent of the tubes are in the colder of the two tube passes. (b) What is the true temperature difference when the hot fluid is cooled from 485 to 225°F by a noncontrolling cooling medium in the tubes which is heated from 100 to 150°F when 60 per cent of the tubes are in the colder tube pass and (c) when 40 per cent of the tubes are in the colder pass? How do these compare with the 1-2 true temperature difference with equal surfaces in each pass?
7.3. A double pipe exchanger has been designed for the nozzle arrangement shown in Fig. 7.26. If the hot stream is cooled from 275 to 205°F while the cold stream enters at 125°F and is heated to \(t_2 = 190°F\), what is the true temperature difference? (Hint. Establish an equation for the temperature difference with the nozzle arrangement shown and sufficient to allow a numerical trial-and-error solution.) How does this compare with the LMTD for counterflow?

7.4. 43,800 lb/hr of 42°API kerosene between 390 and 200°F is used to heat 149,000 lb/hr of 34°API Mid-continent crude from 100 to 170°F in a 662-ft² exchanger (Example 7.3). The heat transfer coefficient is 69.3 Btu/(hr)(ft²)(°F). When the 1-2 exchanger is clean, what outlet temperatures will be obtained? Calculate the outlet temperatures directly from Eq. How does the total heat load compare with that which could be delivered by a true counterflow exchanger assuming that the same \(U\) could be obtained?

7.5. It is necessary on a new installation to preheat 149,000 lb/hr of 34°API crude oil from 170°F to a temperature of 285°F, corresponding to that of the feed plate of a fractionating tower. There is a utility 33°API gas oil line running near the tower at 530°F of relatively unlimited quantity. Because the pumping cost for cold gas oil is prohibitive, the temperature of the gas oil from the heat exchanger, returning to the line, should not be less than 300°F.

Available on the site is a 25-in. ID 1-2 exchanger containing 252 tubes 1 in. OD, 13 BWG, 160° long arranged on a six-pass 1/4-in. triangular pitch layout. The shell baffles are spaced at 5-in. centers. A pumping head of 10 psi is allowable on the gas oil line and 15 psi on the feed line. Will the exchanger be acceptable if cleaned, and if so, what will the fouling factor be? For the gas oil the viscosities are 0.4 centipoise at 530°F and 0.7 centipoise at 300°F. For the crude oil the viscosities are 0.9 centipoise at 285°F and 2.1 centipoise at 170°F. (Interpolate by plotting \(F^°\) vs. centipoise on logarithmic paper.)

7.6. 96,000 lb/hr of 35°API absorption oil in being cooled from 400 to 200°F is used to heat 35°API distillate from 100 to 200°F. Available for the service is a 29-in. ID 1-2 exchanger having 333 tubes 1 in. OD, 14 BWG, 160° long on 1/4-in. triangular pitch. Baffles are spaced 10 in. apart, and the bundle is arranged for four tube passes. What arrangement gives the more nearly balanced pressure drop, and what is the dirt factor? The viscosity of the absorption oil is 2.6 centipoise at 100°F and 1.15 centipoise at 210°F. (Plot on logarithmic paper \(F^°\) vs. viscosity in centipoise, and extrapolate as a straight line.) The viscosity of the distillate is 3.1 centipoise at 100°F and 1.3 centipoise at 210°F.

7.7. 43,200 lb/hr of 35°API distillate is cooled from 250 to 120°F using cooling water from 85 to 120°F. Available for the service is a 19/4-in. ID 1-2 exchanger having 204 tubes 3/4 in. OD, 16 BWG, 160° long on 1-in. square pitch. Baffles are spaced 5 in. apart, and the bundle is arranged for four passes. What arrangement gives the more nearly balanced pressure drop, and what is the dirt factor? What is the optimum outlet-water temperature? (Viscosities of the distillate are given in Prob. 7.6.)

7.8. 75,000 lb/hr of ethylene glycol is heated from 100 to 200°F using steam at 250°F. Available for the service is a 17/4 in. ID 1-2 exchanger having 224 tubes
\( \frac{1}{4} \) in. OD, 14 BWG, 16\( \frac{3}{4} \)" long on \( \frac{1}{4} \) in. triangular pitch. Baffles are spaced 7 in. apart, and there are two tube passes to accommodate the steam. What are the pressure drops, and what is the dirt factor?

7.8. 100,000 lb/hr of 20 per cent potassium iodide solution is to be heated from 80 to 200°F using steam at 15 psig. Available for the service is a 10 in. ID 1-2 exchanger without baffles having 50 tubes \( \frac{3}{4} \) in. OD, 16 BWG, 16\( \frac{3}{4} \)" long arranged for two passes on \( \frac{1}{4} \) in. triangular pitch. What are the pressure drops and the dirt factor?

7.10. 78,359 lb/hr of isobutane (118°API) is cooled from 203 to 180°F by heating butane (111.6°API) from 154 to 177°F. Available for the service is a 17\( \frac{3}{4} \) in. ID 1-2 exchanger having 173 tubes \( \frac{3}{4} \) in. OD, 14 BWG, 12\( \frac{3}{4} \)" long on 1-in. triangular pitch. Baffles are spaced 8 in. apart, and the bundle is arranged for four passes. What are the pressure drops and the dirt factor?

7.11. A 1-2 exchanger recovers heat from 10,000 lb/hr of boiler blowdown at 135 psig by heating raw water from 70 to 96°F. Raw water flows in the tubes. Available for the service is a 10.02 in. ID 1-2 exchanger having 52 tubes \( \frac{3}{4} \) in. OD, 16 BWG, 8\( \frac{3}{4} \)" long. Baffles are spaced 2 in. apart, and the bundle is arranged for two tube passes. What are the pressure drops and fouling factors?

7.12. 60,000 lb/hr of a 25% NaCl solution is cooled from 150 to 100°F using water with an inlet temperature of 80°F. What outlet water temperature may be used? Available for the service is a 21\( \frac{3}{4} \) in. ID 1-2 exchanger having 302 tubes \( \frac{3}{4} \) in. OD, 14 BWG, 16\( \frac{3}{4} \)" long. Baffles are spaced 5 in. apart, and the bundle is arranged for two tube passes. What are the pressure drops and fouling factor?

NOMENCLATURE FOR CHAPTER 7

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Heat-transfer surface, ft²</td>
</tr>
<tr>
<td>a</td>
<td>Flow area, ft²</td>
</tr>
<tr>
<td>a&quot;</td>
<td>External surface per linear foot, ft</td>
</tr>
<tr>
<td>B</td>
<td>Baffle spacing, in.</td>
</tr>
<tr>
<td>C</td>
<td>Specific heat of hot fluid, in derivations, Btu/(lb)(°F)</td>
</tr>
<tr>
<td>C'</td>
<td>Clearance between tubes, in.</td>
</tr>
<tr>
<td>c</td>
<td>Specific heat of fluid, Btu/(lb)(°F)</td>
</tr>
<tr>
<td>Cs</td>
<td>Cost of exhaust steam, dollars/Btu</td>
</tr>
<tr>
<td>Cr</td>
<td>Annual fixed charges, dollars/ft²</td>
</tr>
<tr>
<td>Cp</td>
<td>Cost of process steam, dollars/Btu</td>
</tr>
<tr>
<td>CT</td>
<td>Total annual cost, dollars/year</td>
</tr>
<tr>
<td>Cw</td>
<td>Cost of water, dollars/lb</td>
</tr>
<tr>
<td>D</td>
<td>Inside diameter of tubes, ft</td>
</tr>
<tr>
<td>d</td>
<td>Outside diameter of tubes, in.</td>
</tr>
<tr>
<td>Dc, Dy</td>
<td>Equivalent diameter for heat transfer and pressure drop, ft</td>
</tr>
<tr>
<td>da, da'</td>
<td>Equivalent diameter for heat transfer and pressure drop, in.</td>
</tr>
<tr>
<td>Ds</td>
<td>Inside diameter of shell, ft</td>
</tr>
<tr>
<td>e</td>
<td>Efficiency, dimensionless</td>
</tr>
<tr>
<td>Fc</td>
<td>Caloric fraction, dimensionless</td>
</tr>
<tr>
<td>Fr</td>
<td>Temperature difference factor, ( \Delta T = F_T \times LMTD ), dimensionless</td>
</tr>
<tr>
<td>f</td>
<td>Friction factor, dimensionless; for ( \Delta P ) in psi, ft²/in²</td>
</tr>
<tr>
<td>G</td>
<td>Mass velocity, lb/(hr)(ft²)</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of gravity, ft/hr²</td>
</tr>
<tr>
<td>g'/g''</td>
<td>Acceleration of gravity, ft/sec²</td>
</tr>
<tr>
<td>h, h', h''</td>
<td>Heat-transfer coefficient in general, for inside fluid, and for outside fluid, respectively, Btu/(hr)(ft²)(°F)</td>
</tr>
</tbody>
</table>
**Value of** $h_i$ when referred to the tube outside diameter, Btu/(hr)·(ft²)(°F)

**ID** Inside diameter, in.

**$j_h$** Factor for heat transfer, dimensionless

**$K_c$** Caloric constant, dimensionless

**$K_1, K_2$** Numerical constants

**$k$** Thermal conductivity, Btu/(hr)(ft²)(°F/ft)

**$L$** Tube length, ft

**LMTD** Log mean temperature difference, °F

**$N$** Number of shell-side baffles

**$N_t$** Number of tubes

**$n$** Number of tube passes

**$p_T$** Tube pitch, in.

**$\Delta P_r, \Delta P_i, \Delta P_r$** Total, tube side and return pressure drop, respectively, psi

**$Q$** Heat flow, Btu/hr

**$T$** Temperature group, $(T_1 - T_2)/(t_2 - t_1)$, dimensionless

**$R_i, R_o, R_e$** Combined, inside and outside dirt factors, respectively, (hr)(ft³)(°F)/Btu

**$Re_i, Re_o$** Reynolds number for heat transfer and pressure drop, dimensionless

**$S$** Temperature group, $(t_2 - t_1)/(T_1 - t_1)$, dimensionless

**$g$** Specific gravity

**$T_1, T_2, T_3$** Temperature in general, inlet and outlet of hot fluid, °F

**$T_E, T_P$** Saturation temperatures of exhaust and pressure steam, °F

**$T_o$** Average temperature of hot fluid, °F

**$T_c$** Caloric temperature of hot fluid, °F

**$t_1, t_2, t_3$** Temperature in general or outlet of the first of two heaters, inlet and outlet of cold fluid, °F

**$t_1^{st}, t_2^{nd}$** Temperatures in first and second passes, °F

**$t_1$, $t_2$** Average temperature of cold fluid, °F

**$t_c$** Temperature at end of first pass, °F

**$t_e$** Caloric temperature of cold fluid, °F

**$t_w$** Tube wall temperature, °F

**$\Delta t$** True temperature difference in $Q = U_i A \Delta t$, °F

**$\Delta t_i, \Delta t_o$** Temperature differences at the cold and hot terminals, °F

**$U, U_c, U_D$** Overall coefficient of heat transfer, clean coefficient, design coefficient, Btu/(hr)(ft²)(°F)

**$V$** Velocity, fps

**$v$** Specific volume, ft³/lb

**$W$** Weight flow in general, weight flow of hot fluid, lb/hr

**$w$** Weight flow of cold fluid, lb/hr

**$X$** Length, ft

**$\varepsilon$** Height, ft

**$\phi$** The viscosity ratio $(\mu/\mu_o)^{0.74}$

**$\mu$** Viscosity, centipoises × 2.42 = lb/(ft·hr)

**$\mu_o$** Viscosity at tube-wall temperature, centipoises × 2.42 = lb/(ft·hr)

**$\rho$** Density, lb/ft³

Subscripts (except as noted above)

$s$ Shell

$t$ Tubes