Liquidity and Market Quality Around Predictable Trades:

Evidence from Crude Oil ETF Rolls*

Hendrik Bessembinder
University of Utah

Allen Carrion
Lehigh University

Laura Tuttle
Securities and Exchange Commission

Kumar Venkataraman
Southern Methodist University

This Draft: November 2014

Keywords: Predatory trading; Sunshine Trading; Resiliency; ETFs; Commodity trading.

---

*Earlier versions of this paper were titled "Predatory or Sunshine Trading: Evidence From Crude Oil ETF Rolls." For their comments, we thank Amber Anand, Tarun Chordia, Thierry Foucault, Ruslan Goyenko (Central Bank Conference Discussant), Jeff Harris (WFA Discussant), John Hyland, Steve Mann, Lubos Pastor, Ravi Shukla, Juan Ignacio Pena (CNMV discussant), Nela Richardson, Myron Slovin, Elvira Solji, Wing Wah Tham, Selim Topaloglu and seminar participants at Erasmus University, HEC-Paris, University of Notre Dame, University of Washington, Syracuse University, Tinbergen Institute, Texas Christian University, Queens University, Second CNMV International Conference on Securities Markets, the 8th Annual Central Bank Workshop on Market Structure, the US Securities and Exchange Commission, the 2012 Commodity Futures Trading Commission Research Conference, and the 2013 Western Finance Association (WFA) Annual Meeting. We also thank Jeff Harris, Andrei Kirilenko and Jim Moser for their help in accessing and interpreting data, and Stephen Meek for research assistance.

Liquidity and Market Quality Around Predictable Trades

Evidence from Crude Oil ETF Rolls*

Abstract

We extend the theory of strategic trading around a predictable liquidation by considering the role of market resiliency. Our model predicts that strategic trading improves market quality and increases liquidator proceeds when trades' temporary price impacts are quickly reversed. We provide related empirical evidence by studying prices, liquidity, and individual account trading activity around the large and predictable "roll" trades undertaken by the largest ETF tracking crude oil futures prices. The evidence indicates narrower bid-ask spreads, greater order book depth, improved market resiliency, and more trading accounts provide liquidity on roll dates. However, the large volume of trading associated with the roll transactions leads to substantial trade execution costs that average three percent per year. On balance, the theory and evidence supports that strategic traders choose to provide liquidity rather than exploit predictable trades in resilient markets.

I. Introduction

A trader who gains knowledge that another investor will transact a substantial quantity of a security can potentially profit by trading in the same direction as the investor. In the case of brokers who are aware of pending client orders, the practice is known as "front running." More broadly, the practice has been dubbed "predatory trading" by Brunnermeier and Pedersen (2005). Their model implies that the practice degrades market quality, in that it causes prices to temporarily overshoot the longer-term equilibrium, and is harmful in that it causes the investor to realize a less advantageous price.

Admati and Pfleiderer (1991) present an alternative theory of trading around a predictable order. In their "sunshine trading" theory, investors who intend to transact a substantial quantity publicly announce their intention to trade, thereby attracting additional liquidity suppliers as well as natural counterparties to the market. Their model implies that the public announcement of the upcoming trade results in smaller market movement and a more advantageous price to the liquidator.

In this paper, we provide a simple extension of the theory of trading around a predictable order, and present relevant empirical evidence by studying individual account trading, liquidity, and market resiliency around the time of large and predictable monthly trades undertaken by the United States Oil Fund (USO), which is the largest of the ETFs that are designed to provide returns that track crude oil prices.[1] Rather than holding crude oil inventories, which would entail substantial storage costs, USO gains exposure to crude oil prices by holding positions in New York Mercantile Exchange (NYMEX) crude oil futures contracts. Since individual NYMEX contracts periodically expire, the strategy involves regularly "rolling" positions by selling the expiring contract and purchasing contracts with more distant expiration dates. Data on crude oil ETFs' Assets-Under-Management are publicly available, and USO announces on its web site the dates on which it will roll its positions.[2] The magnitude, direction, and timing of USO roll trades are therefore highly predictable.

---

[1] During our sample period, USO accounted for 95% of the assets-under-management in crude oil ETFs. Data on ETF's assets-under-management are obtained from ALPS Fund Services.

[2] USO's investment objective, as well as a calendar schedule of recent and future roll dates, are disseminated on the web site http://www.unitedstatesoilfund.com/.

USO was launched in April 2006, and by early 2009 had more than $4.2 billion under management, equating at prevailing prices to over 90 million barrels of crude oil. Returns to USO investors have lagged crude oil prices, as displayed on Figure 1. Some observers have suggested that predatory trading explains the USO share price record. For example, the Wall Street Journal reported that "Since the fund (USO) is so big, it is unable to switch in and out of contracts….without moving markets and giving speculators an opportunity to make bets on those moves."[3] The article quotes a trader as stating that "It's like taking candy from a baby" and asserts that the "... candy comes out of returns of the investors in the fund."

USO's stated investment objective involves tracking futures settlement prices, which are established during a short two-minute interval at the end of the normal trading day.[4] The magnitude of USO's roll often exceeded NYMEX market volume during the settlement interval, and at times was over fifteen percent of volume on the roll day. Since USO predictably demands a very large quantity of liquidity during a short trading interval, its trades provide an ideal experiment to study the economics of liquidity provision around the execution of large predictable trades.

We employ data on individual orders and trades in crude oil futures made available to us by the Chicago Mercantile Exchange, which owns and operates the NYMEX market. In addition, we use Commodity Futures Trading Commission (CFTC) data that identifies the individual trading accounts associated with each crude oil futures transaction. The former dataset allows us to study posted liquidity in the form of bid and ask quotes, as well as unexecuted displayed depth in the limit order book. The latter dataset allows us to evaluate the strategies used by owners of specific trading accounts around the time of USO's rolls. Our study of individual orders and trades spans the period March 1, 2008 to

[3] "U.S. Oil Fund Finds Itself at the Mercy of Traders", by Gregory Meyer and Carolyn Cui, The Wall Street Journal, March 6, 2009, page C1.

[4] USO's investment objective is stated on the company website http://www.unitedstatesoilfund.com/ as follows: "The investment objective of USO is for the daily changes in percentage terms of its units' net asset value ("NAV") to reflect the daily changes in percentage terms of the spot price of light, sweet crude oil delivered to Cushing, Oklahoma, as measured by the changes in the price of the futures contract for light, sweet crude oil traded on the New York Mercantile Exchange (the "NYMEX") that is the near month contract to expire, except when the near month contract is within two weeks of expiration, in which case it will be measured by the futures contract that is the next month contract to expire (the "Benchmark Oil Futures Contract"), less USO's expenses."

February 28, 2009, and therefore includes twelve monthly rolls.[5] We also study daily crude oil settlement price data for the longer time interval January 1990 through December 2013.

In addition to providing empirical evidence, we provide some new analysis of the economics of strategic trading around a known liquidation. Brunnermeier and Pedersen (2005) rely on the assumption that trades have permanent but not transitory effects on prices, and show that the effects of predatory trading are worst when there is a monopolist predator. We analyze the effects of strategic trading when markets are resilient, in the sense that some or all of the immediate price impact of trades is subsequently reversed. Our analysis reveals that the profit maximizing strategy for a monopolist trader who is aware of a pending liquidation in a resilient market is to sell before (and, for some parameters, after) the period where the liquidator sells, while purchasing during the period that the liquidator sells. In other words, the strategic trader essentially chooses to absorb a portion of the liquidator's order imbalance while it occurs, while offloading the resulting inventory both before and after the liquidation. The effect of the strategic trades is to increase rather than decrease the liquidator's proceeds and to decrease rather than increase the temporary price impact of the liquidating trade if the market is sufficiently resilient. Our analysis shows that a key implication of the Brunnermeier and Pedersen (2005) model, that a profit-maximizing strategic trader will necessarily degrade market quality and reduce liquidator proceeds, does not generalize to a resilient market.

Our empirical analysis reveals several findings. First, the oil futures market is indeed resilient. Using CME order book data, we estimate (a) the permanent and temporary component of trading costs and (b) a resiliency parameter that captures the extent to which temporary price impacts persist beyond the period of the order imbalance. The resulting estimates imply that the temporary price impact of an order imbalance is almost entirely reversed within one minute in the expiring contract and within three minutes

[5] Note that the apparent underperformance in the USO share price as displayed on Figure 1 accelerates after the March 2008 to February 2009 period covered by our main datasets. We show in the Appendix that the main reason for the apparent underperformance is storage costs, rather than predatory trading or other costs of completing the roll trades. Storage costs, in turn, did not soar until crude oil inventories increased in the wake of the financial crisis.

in the second nearest-to-expiration contract on roll days. We also document a reduction in the permanent price impact of order imbalances on ETF roll days versus non-roll days.

Second, we find that several measures indicate improved liquidity on roll versus non-roll days. In particular, quoted and effective bid-ask spreads are narrower on roll days, and the quantity of unexecuted orders in the limit order book at prices near the inside quote is greater on roll days. Further, a larger number of distinct accounts provide liquidity through limit orders on ETF roll days relative to non-roll days. These findings are consistent with Admati and Pfleiderer's (1988, 1991) prediction that a preannouncement strategy by a large liquidity trader increases market size by attracting liquidity providers and natural counterparties.[6]

Third, we analyze the CFTC account-level data, and find little or no evidence that individual trading accounts use strategies that would reasonably be considered predatory. Consistent with the simple framework introduced in our paper, we find significant increased usage of a liquidity provision strategy where strategic traders sell the expiring contract the trading day before the roll and offload the resulting position on and after the roll day, thereby absorbing a portion of the ETF sales during the roll day, while shifting a portion of the selling pressure to the preceding day. Our theoretical analysis implies that this strategy mitigates temporary price impacts in the resilient market and improves prices for the rolling ETF.

Fourth, our analysis of daily settlement prices indicates that USO does pay to execute its roll trades – about 25 basis points on average per roll, or 3% per year, in the form of adverse changes in settlement prices in advance of the rolls. While this estimate indicates a substantial roll cost associated with the ETF's large liquidity demand, it needs to be evaluated in light of the high rate of turnover (1200% annually) implied by the monthly roll strategy.

The findings of our study are relevant for portfolio managers who need to rebalance their portfolios for non-informational reasons. For example, revisions to the names of index component stocks are typically disclosed in advance of the index reconstitution date; however participants such as index funds

[6] In the limit order book model of Foucault, Kadan, and Kandel (2005), an increase in the proportion of liquidity suppliers narrows the bid ask spread as liquidity suppliers compete by submitting more aggressively priced limit orders. The preannouncement strategy can be viewed as an event that increases the proportion of liquidity suppliers.

rebalance their portfolios on the reconstitution date to minimize tracking error. Index reconstitution is associated with higher trading activity and significant abnormal returns prior to the event, including both transitory price pressure and the permanent effects of index membership (see Madhavan (2003) and Chen, Noronha and Singal (2004)). To avoid predatory trading, some index funds and transition portfolio managers choose not to disclose the timing of their trades.[7] Our study shows that predatory trading is unlikely to degrade market quality in the absence of long lived price impacts, implying that the preannouncement of trading intentions is a viable strategy for large non-informed traders in more resilient markets. In contrast, our analysis confirms the Brunnermeier and Pedersen finding that predatory trading can harm liquidators and market quality when trades contain information that permanently alters prices, when markets are not resilient, or when the predatory trades are executed in very close time proximity to the liquidator's trades.

The rest of the paper is organized as follows. Section II discusses the related literature, the structure of the NYMEX market, data sources and summary statistics on USO trading activity on roll days. Section III presents our simple extension of the theory of strategic trading around a known liquidation, while Section IV provides evidence confirming that the NYMEX futures market is indeed resilient. Section V reports on measures of market quality on roll versus non-roll days. Section VI examines account level activity by strategic traders surrounding the roll, while Section VII presents estimates of the effective trading costs paid by USO for their roll trades. Finally, we present in the Appendix an analysis that reconciles USO's share price performance to the level of crude oil prices, as documented on Figure 1.

II. Related Literature, Data Sources and NYMEX market structure

a. Related Literature

We extend the theory of trading around a predictable order, and provide related empirical evidence from an important commodity market, that for NYMEX crude oil futures. The cornerstone theory of

[7] Vanguard Emerging Markets Stock Index Fund and ETF recently adopted the FTSE Emerging Index as the new target index, replacing the MSCI Emerging Markets Index. In describing the adjustments to fund investors, Vanguard states that "To protect the fund from the potential for harmful front-running by traders, the exact timing of the index changes will not be disclosed in advance to investors."

strategic trading when some traders become aware of another trader's need to liquidate a position is provided by Brunnermeier and Pedersen (2005), who introduce the label "predatory trading" to describe the strategies followed. In their model the predators sell ahead of or alongside the liquidating trader, before reversing their positions. The predatory trades damage market quality in that they cause the price to temporarily overshoot its long-term equilibrium. Further, their profits come at the expense of lower proceeds to the liquidating trader. Carlin, Lobo, and Vishwanathan (2007) present a multi-period model in which traders typically provide liquidity to each other. However, in situations where the potential profit from following a predatory strategy is sufficiently large, traders can abandon liquidity provision to follow predatory strategies. Their model therefore predicts episodic periods of illiquidity due to predation.

The model presented by Brunnermeier and Pedersen (2005) assumes that trades have permanent price impacts proportional to the size of the order imbalance. In practice large trades can have both temporary and permanent price impacts. Schoneborn and Schied (2007) show that strategic traders may react to known liquidations by trading in the opposite rather than the same direction as the liquidator when price impacts are temporary. Our model extends theirs in that we also assess the effect of market "resiliency," i.e. the extent to which trades' temporary price impacts spill over to periods subsequent to trade execution, and because we assess whether strategic trading improves or degrades market quality and liquidator proceeds as compared to a scenario where strategic traders are absent. The extension to a resilient market is important in our setting, because although ETF roll trades are unlikely to have permanent price impact, long-lived temporary price impacts could accommodate predatory trading.

Admati and Pfleiderer (1991) consider a trader who needs to complete a large transaction and who is not motivated by private information regarding asset value. They show that a public announcement of the intent to trade, termed "sunshine trading," can attract liquidity suppliers who might not otherwise have been present, and can therefore allow the trader to achieve a more favorable price.

Our empirical analysis of potential price impacts of roll trades in commodity futures is not entirely unprecedented. Stoll and Whaley (2010) and Mou (2011), study commodity trading by index investors.

In contrast to specialized ETFs that focus on a single commodity, index investors seek to generate returns that match the performance of multi-commodity indices, such as the Standard and Poor's-Goldman Sachs Commodity Index (SP-GSCI). To the extent that these index investors rely on futures positions to track the indices, they also generate periodic roll trades.[8] Mou (2011) reports that significant profits can be earned by investors who trade in advance of the dates that the SP-GSCI index begins to track the second- rather than nearest-to-expiration futures contract. In contrast, Stoll and Whaley (2010) find little or no price effects around index shift dates in a broad cross-section of commodity futures prices.

Our study is distinguished from these in that we are able to exploit account-level data on individual trades as well as limit order book updates to assess the relevance of sunshine and predatory trading theories. In addition, we present a model of strategic trading around a known liquidation that extends prior work, estimate trade execution costs and market resiliency for an important commodity market, and explain the apparent underperformance of the largest crude oil ETF.

b. The NYMEX Market

Our empirical analysis focuses on the New York Mercantile Exchange, which trades crude oil and other energy futures contracts. NYMEX prices are widely-used benchmarks for valuing derivative contracts and determining final prices for over-the-counter contracts. Although the NYMEX continues to operate a physical trading floor, the large majority of transactions occur on NYMEX's electronic limit order market, known as Globex. In addition, large traders can negotiate block trades. Though physical delivery is rare, each individual NYMEX crude oil contract calls for delivery of 1,000 barrels of crude oil at Cushing, Oklahoma, during a designated delivery month. Transaction prices reflect prices at which oil is to be delivered in the future, not an amount paid to enter the contract. Trading hours for floor trades are 9:00 AM to 2:30 PM New York time. Globex trading occurs around the clock, except for a 45-minute break from 4:15 PM to 5:00 PM New York time. The weighted average price during the two-minute interval 2:28 to 2:30 PM comprises the contract's "settlement price" for the day, and is used to calculate

[8] In addition, a number of authors have assessed whether index investors and other passive long-only investors have affected the level and/or the volatility of commodity prices. See, for example, Boyd, Harris and Nowak (2011), Buyuksahin and Harris (2011), Irwin and Sanders (2012), and Kilian (2009).

gains and losses on outstanding positions. In particular, long positions receive and short positions pay the change in the settlement price since the prior day (or since the transaction price if entered the same day).

In addition to outright trades that specify a delivery price, the NYMEX offers "Trade-at-Settlement" (TAS) contracts. The futures price for a TAS trade is the current day settlement price (potentially plus or minus a specified margin), and is generally not known at the transaction time. Press accounts indicate that the USO Oil ETF routinely uses TAS trades to complete its rolls.

c. Data Sources

We employ three main datasets. The Commodity Futures Trading Commission (CFTC) provided data on all completed trades in NYMEX crude oil futures from March 1, 2008 to February 28, 2009. The CFTC data includes floor and block trades, as well as trades completed on the Globex electronic market. In addition to trade type, contract, price, and volume, the CFTC data includes an account identification variable for each trade, which allow us to assess the number of unique trading accounts active on a given day and assess trading strategies used by each account. Although the buy and sell account is identified for each trade, the initiator of the trade is not. We use a modified Lee-Ready algorithm to assign trades as buyer- or seller-initiated.[9]

We also obtain for the same time period from the Chicago Mercantile Exchange (owner of the NYMEX) a 5-level deep representation of the limit order book and a record of completed trades for crude oil futures on the electronic GLOBEX market. The CME dataset allows us to construct a continuous record of best bid and ask quotes, as well as the depth of unexecuted orders at and behind the best quotes.[10] Third, we obtain from the United States Energy Information Agency (EIA) a daily record of

[9] Specifically, to sign trades, we compare the transaction price with the contemporaneous quote-midpoint (without a 5-second lag), and consider up to five preceding trades to implement the tick-rule.

[10] In addition to the "outright" book for each contract, the CME maintain a limit order book for calendar spread orders. Each leg of the spread order competes for order flow with the corresponding outright book. Further, the CME data has a finer time resolution (to the 100th of a second) as compared to the CFTC data, for which time stamps are truncated to the second. We use the CME transactions to impute centi-second time stamps for CFTC data transactions through an iterative process of matching unique price-quantity pairings. When transactions cannot be perfectly matched between the two datasets, the latest-possible time stamp is imputed within the CFTC dataset to avoid any possible look-ahead bias of matching trades with LOB information. A limitation is that the CME data does not include floor trades or negotiated block trades.

settlement prices for NYMEX crude oil contracts traded over the longer time interval January 1999 through December 2013.

d. Descriptive Statistics

Our most detailed analysis focuses on the period March 1, 2008 to February 28, 2009, during which we have data on individual crude oil trades. USO's assets-under-management (AUM) grew significantly during this period, from $0.47 billion in March 2008 to $3.92 billion in February 2009. Table 1 reports estimated USO roll activity as a percentage of market volume for each monthly roll date, for the "front" contract and for the second nearest-to-expiration contract. Roll activity is estimated based on USO's assets under management relative to the roll-date settlement price, and shows rapid growth during the sample, with sales of the front contract increasing from 4,445 contracts representing 1% of market volume during the March 2008 roll to 85,055 contracts representing 16% of market volume during the February 2009 roll.[11] Aggregated across the twelve roll dates, net roll activity comprised 5% of roll-day volume in the front contract. Roll-date purchases accounted for an average of 10% of roll-day volume in the more lightly traded second contract.

Table 1 also reports market volume during the two-minute settlement period. Since performance is benchmarked against changes in settlement price, ETFs that hold futures positions generally try to track the settlement price on the roll day. USO's roll volume on average exceeds market volume during the settlement period, indicating that it would be difficult for USO to execute its entire roll by use of regular trades during the settlement interval. Crude oil ETFs can employ TAS contracts to ensure a price that closely matches the benchmark settlement price. To the extent that a TAS counterparty to an ETF trade has a 'natural' offsetting position, the TAS trade allows both the ETF and counterparty to offset positions at low cost.[12] If the TAS counterparty is simply providing liquidity, then compensation takes the form of the difference between roll-day settlement prices and prices for the counterparties' offsetting trades.

[11] Our estimates of USO's roll date volume may differ slightly from the fund's actual volume if a portion of assets under management have not yet been invested, or if some roll activity takes place in OTC markets.

[12] An example would be an oil producer, who might hedge oil price risk with short futures positions, and roll the short position in expiring contracts. ETF's reliance on TAS contracts has been discussed in popular press articles. See for example, Financial Times article, http://ftalphaville.ft.com/2010/07/15/287061/the-end-of-diversification.

III. Extending the Theory of Strategic Trading

We present a simple model that relaxes a key assumption of the Brunnermeier and Pedersen (2005) model, that the price impact of trades is entirely permanent. In particular, we allow trades to have both permanent and temporary price impacts, and for the duration of the temporary price impact to depend on the "resiliency" of the market. Allowing for temporary price impacts is particularly important when assessing strategic trading around predictable orders initiated by non-information-motivated traders.

Brunnermeier and Pedersen (2005) show (their Proposition 3) that competition among predatory traders reduces the adverse effects of their activities, and Admati and Pfleiderer (1991) demonstrate how attracting more counterparties to the market improves outcomes for the liquidator. Since the effects of increased competition are already established and we wish to focus on the effects of allowing for market resiliency, we study the case of a single monopolist trader who is aware of the pending trade. We will refer to a pending liquidation, but the analysis implies equally well to a pending purchase or a pending sale and purchase of related securities, as in a futures roll trade. Our main finding is that the predator, who we refer to by the more benign label "strategic trader", can improve market quality and liquidator proceeds when the market is sufficiently resilient.

a. The Setting

We assume that the investor will liquidate a known quantity, Q_L. A strategic trader is aware of the liquidation, and can trade before, simultaneous with, or after the liquidation, and chooses transaction quantities to maximize profits. Strategic trades sum to zero across periods; that is the strategic trader ultimately does not absorb the liquidator's position. Order imbalances generated by the strategic trader as well as the liquidator are absorbed by the limit order book, as described in the next section.

b. Trade Prices When the Market is Resilient

We assume that the private information conveyed by trades is measured by a permanent price impact parameter, λ, so that security value evolves according to $V_t = V_0 + \lambda Q_t$, where q_i is signed marketable order flow in period i, and $Q_t = \sum_{i=1}^{t} q_i$ is the accumulated order flow since base period 0. Trades also have temporary price impacts. The immediate temporary price impact, γq_i, is proportional to the signed order flow, reflecting that small orders execute at the inside quotes that differ from security value and that larger orders walk up the limit order book. The temporary price impact potentially persists beyond the time of the trade. Specifically, the trade price at time t depends on current and prior order flow according to:

$$P_t = V_t + \gamma A_t = V_0 + \lambda Q_t + \gamma A_t \quad (1)$$

where $A_t = \sum_{j=0}^{t-1} \theta^j q_{t-j}$.

Here, A_t is a weighted sum of orders from time 0 to t, and the parameter θ measures the (inverse) resiliency of the market. If $\theta = 0$ the market is completely resilient, and the temporary price impact of the order at time t has no effect beyond time t. This requires that the limit order book refill instantaneously after an order is executed. If so, the midpoint, $M_t = V_0 + \lambda Q_{t-1} + \gamma\theta A_{t-1}$, equals the security value, and $P_t = V_{t-1} + (\lambda+\gamma)q_t$. If $0 < \theta < 1$ the limit order book takes time to refill, the temporary effect of the time t order flow extends beyond time t, and the midpoint differs from security value as a function of recent order imbalances. If $\theta = 1$ the temporary impact is never reversed, and thus is indistinguishable from permanent impact.

c. Market Prices and Outcomes

Trading occurs during three periods: before, during, and after the investor's liquidation. Let Q_p, Q_d, and Q_a denote net signed order flow (sum of liquidator and strategic trading) during the "pre", "during", and "after" periods, respectively. Then, trade prices during the "pre", "during", and "after" periods are:

$$P_p = V_0 + I_0 Q_p, \text{ where } I_0 = [\lambda + \gamma] \quad (2)$$

$$P_d = V_0 + I_1 Q_p + I_0 Q_d, \text{ where } I_1 = [\lambda + \theta\gamma], \text{ and} \quad (3)$$

$$P_a = V_0 + I_2 Q_p + I_1 Q_d, + I_0 Q_a, \text{ where } I_2 = \left[\lambda + \theta^2 \gamma\right] \quad (4)$$

The parameters I_0, I_1 and I_2 measure the effects on trade prices of same, prior, and second-prior period order imbalances, and depend on market resiliency.

We can describe the strategic trader's order flow by a pair of proportionality parameters ρ_d and ρ_p, defined so that positive values indicate trading in the same direction as the liquidator. Including the requirement that the strategic trader order flow sums to zero across the three periods, the imbalance absorbed by the limit order book (the sum of liquidator and strategic order flow) in each period is:

$$Q_p = -\rho_p Q_L , \quad (5)$$

$$Q_d = -(1 + \rho_d) Q_L , \quad (6)$$

and $Q_a = Q_L(\rho_d + \rho_p)$. (7)

The liquidator's proceeds depend only on the price in the liquidation period:

$$LP = Q_L P_d , \quad (8)$$

while the strategic trader's profits depend on price differences across periods:

$$SP = Q_L \left[\rho_p (P_p - P_a) + \rho_d (P_d - P_a)\right] . \quad (9)$$

Using expressions (5) to (7) in expressions (2) to (4) for prices, straightforward calculus reveals that strategic trader profits are maximized when:

$$\rho_d^* = \frac{\theta - (\lambda/\gamma) - 2}{\theta^2 - 4\theta + 3(\lambda/\gamma) + 6} \quad \text{and}$$

$$\rho_p^* = -\frac{\theta^3 - 3\theta^2 - [(\lambda/\gamma) - 2]\theta + 2(\lambda/\gamma)^2 + 5(\lambda/\gamma) + 2}{\theta^4 - 4\theta^3 + [2(\lambda/\gamma) + 4]\theta^2 + [4(\lambda/\gamma) + 8]\theta - 3(\lambda/\gamma)^2 - 12(\lambda/\gamma) - 12} . \quad (10)$$

Note that the strategic trader's optimal strategy depends only on market resiliency, θ, and on the ratio of permanent to temporary price impacts, λ/γ.

d. Illustration of Model Implications

Though the model provides closed form solutions, its implications are not fully transparent. To illustrate the model's implications, Table 2 provides a set of numerical outcomes. The illustration includes an initial price (V_0) equal to $100, Q_L = 20 units liquidated, and temporary price impact $\gamma = 0.05$, for values of the (inverse) resiliency parameter, θ, ranging from zero to one. Panel A focuses on an informationless liquidation, without a permanent price impact ($\lambda = 0$). Since ETFs rolls are unlikely to have a permanent effect on prices, these results are the most relevant. Given the temporary price impact of .05, the price received by the liquidator in the absence of strategic trading is $99 per unit. For comparison, Panel B illustrates outcomes when the permanent price impact is as large as the temporary impact ($\lambda = \gamma = 0.05$). Here, in the absence of strategic trading the liquidator receives $98 per unit.

Focusing first on Panel A, a notable result is that the profit maximizing strategy for the strategic trader is to purchase during the period when the liquidator sells. The strategic trader's purchases are in the vicinity of one third of the amount liquidated, for all θ. Also for all levels of θ, the strategic trader sells in advance of the known liquidation, a strategy that we will loosely refer to as "front running." Importantly, however, the liquidator's proceeds are in many cases *greater* when the strategic trader is present. In particular, given $\lambda = 0$ the liquidator *benefits* from the activities of the strategic trader when θ is less than about 0.76, i.e. when the market is relatively resilient.

The benefit to the liquidator from being "front run" arises because the strategic trader essentially spreads the price impact of the liquidation into the preceding and subsequent periods. When the market is sufficiently resilient the price impacts of the "front running trades" are largely dissipated by the time of the liquidation, and the liquidator is not harmed

In contrast, the strategic trader harms the liquidator if the market is not sufficiently resilient. The main effect of increasing θ (decreasing market resiliency) that can be observed on Table 2 is that the profit maximizing strategic trader sells a larger quantity in the period prior to the liquidation and a smaller quantity in the period after the liquidation. When the market is not resilient the price impacts of the larger front running trades are large enough to depress the price received by the liquidator. Note also that market quality is degraded, in the sense that the strategic trader causes the price to overshoot its longer term equilibrium by a larger amount, for the same parameters that the liquidator receives reduced proceeds, and vice versa.

Panel B of Table 2 presents parallel results for the hypothetical case where the liquidating trades permanently alter the price. In many dimensions the results on Panel B parallel those on Panel A. In particular, the strategic trader purchases during the liquidation period, and the strategic trades benefit rather than harm the liquidator when the market is sufficiently resilient. However, results in Panel B indicate that the strategic trader always sells a larger amount in the period prior to the liquidation and a smaller amount in the period subsequent to the liquidation when there is a permanent price impact. Further, the range of resiliency parameters for which the liquidator benefits from the activities of the strategic trader is reduced to $\theta < 0.52$ when $\lambda = .05$, as compared to $\theta < 0.76$ when the permanent price impact is zero.

The results in Table 2 illustrate the importance of market resilience in assessing whether a predictable liquidator will benefit or be harmed by the trades of a profit maximizing strategic trader. In particular, there exists a "breakeven" level of market resilience. If the market is more resilient than the breakeven level then the liquidator benefits and the magnitude of temporary price movement is reduced by the presence of the strategic trader, and vice versa.

We calculate breakeven resilience by solving for the value of theta that sets the liquidation price from equation (3) equal to the liquidation price received without strategic trading.[13] On Figure 2 we plot the breakeven level of market resiliency for a range of permanent vs. temporary price impact parameters. As might be anticipated, the breakeven resiliency parameter declines monotonically as the ratio of permanent price impact to temporary price impact increases. For very large permanent price impacts (over 2.7 times the temporary price impact), the liquidator is harmed by strategic trading for any positive resiliency parameter.

[13] The expression for break-even theta is complex, and is presented in the Internet Appendix.

This model presented here is based on a number of simplifying assumptions, including a monopolist strategic trader, linear price impacts, and only three trading periods. Nevertheless it delivers important insights. In particular, it implies that strategic trading will benefit rather than harm a party whose trades can be predicted if permanent price impacts are not too large and if temporary price impacts are rapidly reversed. Thus strategic trading in advance of the liquidation is not a concern for a liquidator such as an ETF or index fund who can credibly signal that their trade is not motivated by information, if the market is resilient.

More broadly, the key issue in assessing whether strategic trading in advance of a predictable trade will be harmful is the proportion of any "front running" trades that persists into the period when the predictable trade is executed. The proportion of the price impact of a time t trade that persists to period t+n can be summarized by an "Impact Persistence" parameter, defined as:

$$IP_n = \frac{I_n}{I_0} = \frac{\lambda + \gamma\theta^n}{\lambda + \gamma\theta} \tag{11}$$

Note that impact persistence depends on permanent price impact, temporary impact, market resilience, as well as elapsed time since the trade. An implication is that strategic traders will be more damaging to the liquidator, ceteris paribus, if they occur very shortly before the liquidator trades.

To illustrate, suppose that market resiliency is $\theta = 0.98$, with time is measured in seconds. Recall that breakeven resiliency for informationless (λ=0) trades when assuming that the frontrunning trade occurs one period prior to the liquidation is 0.76. These parameters imply that the liquidator would be harmed by strategic trades occurring immediately or a few seconds prior to liquidation. On the other hand, the liquidator would not be harmed by strategic trades occurring one minute before liquidation, as only the proportion $0.98^{60} = 0.298$ of the price impact would persist into the liquidation period.

As noted, the USO ETF routinely uses "Trade at Settlement" or "TAS" contracts, which are completed at the day's settlement price, to complete its roll trades. The TAS contract allows USO to capture the settlement price for its trades while effectively outsourcing the actual execution of the roll trades to the TAS contract counterparty. Market participants know the date of the USO rolls due to preannouncement, but in general will not know the exact time at which the TAS counterparties will execute trades to offset their TAS commitment. That is, preannouncement of the date of the roll can attract additional counterparties and liquidity providers, as implied by sunshine trading theories, while ambiguity about the precise timing of the roll trades can mitigate the danger of damaging strategic trades.

To summarize, our model confirms the findings of Brunnermeier and Pedersen (2005) that strategic trading will harm the liquidator and degrade market quality if trades have permanent but not temporary price impacts. However, strategic trading, even by a monopolist strategic trader, can benefit the liquidator and improve market quality if some or all of trades' price impacts are temporary and are reversed quickly. The key issue is the proportion of the strategic trades' price impact that persists during the liquidation period. If this proportion is low the liquidator benefits from having price impacts spread over more time periods. However, even in a resilient market strategic trading can harm the liquidator if the strategic trader can time the liquidator's trades and execute in very close proximity to the time of the liquidation. The model implies that traders should be concerned about potential predation mainly when their trades are motivated by private information that will affect long-run security value, or when the market is non-resilient, or when trade timing can be forecast with such precision that the price impact of prior-period trades persists almost fully into the liquidation period.

IV. Estimating the Resiliency of the NYMEX Crude Oil Market

The analysis in the preceding section underscores that the effects of a strategic trader who becomes aware of the trading intentions of a large investor depend in an important way on whether the price impact of trades is permanent or temporary, and in the case of temporary price impacts, on the resiliency of the market. To assess the likely effect of strategic traders for crude oil ETF rolls it is useful to obtain estimates of trades' permanent and temporary price impacts, as well as market resiliency, in the NYMEX crude oil markets. Expression (1) implies that these parameters can be estimated by a geometric lag regression of the form:

$$P_t - P_{t-1} = \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2} + ... + \beta_j q_{t-j} + \varepsilon_t \qquad (12)$$

where P_t is the time t trade price, q_j is the signed order imbalance at time j, and the coefficients are restricted such $\beta_0 = \lambda + \gamma, \beta_1 = \gamma(\theta - 1), \beta_2 = \beta_1\theta, \beta_3 = \beta_1\theta^2, \beta_4 = \beta_1\theta^3, \beta_4 = \beta_1\theta^3$, etc. Note that if θ = 0, i.e. if the market is completely resilient, then expression (12) reduces to a regression of the price change on q_t and q_t - q_{t-1}, a direct extension of the indicator variable model of Huang and Stoll (1997).

We implement equation (12) using a ten-second window to measure the net order imbalance (excess of buyer-initiated over seller-initiated trades, measured in contracts), based on the CME order level data from the period March 1, 2008 to February 28, 2009.[14] The number of lags (j) is set to 30. Table 3, Panel A presents results for the front contract for the full sample as well as separately for USO roll and non-roll days, while Panel B presents analogous results for the second-nearest-to-expiration contract.

As might be anticipated, coefficient estimates based on the pooled sample of roll and non-roll days indicate that the front contract is more liquid than second contract. In particular, the front contract has smaller permanent price impact (4.3 cents versus 7.0 cents), smaller temporary price impact (2.0 cents versus 3.7 cents) and is more resilient (0.597 versus 0.777).

The parameter estimates for roll and non-roll days indicate that the market absorbs the rolls effectively. In particular, the permanent price impact in the front contract is .041 on roll days versus .045 on non-roll days, and the difference is statistically significant.[15] Perhaps more important, the estimated resiliency parameter, θ, for the front month contract is smaller (implying greater resiliency) on roll days as compared to non-roll days (0.442 vs. 0.597). Note that these results are observed despite the fact that the roll brings heightened liquidity demand to the market. Parameter estimates for the second contract

[14] For time periods with multiple trades P_t is measured as the last trade price during the time period. The geometric lag expression (equation 12) is implemented using NYMEX order data by Generalized Method of Moments (GMM), using SAS Proc Model with a Bartlett Kernel set equal to the lag length plus one.

[15] It may be surprising that the estimated permanent price impact on roll days is positive, if ETF rolls comprise non-informed trading. However, while ETF trades are large, they still comprise a minority of trading activity on roll days, and informed traders may be present.

and the estimated temporary price impact for the front contract do not differ significantly across roll and non-roll days.

Recall that the resiliency parameter, θ, measures the proportion of the temporary price impact attributable to current period order imbalance that persists to the next period. The model in the preceding section shows that the resiliency parameter is crucial in assessing the trading patterns that will maximize strategic trader profits. The estimates of θ reported on Table 3 are uniformly less than 0.6 for the front contract and less than 0.8 for the second month contract. Focusing on roll dates in particular, the front month θ estimate of 0.442 implies the estimated proportion of the temporary impact that persists after one minute is $0.442^6 = 0.007$. The corresponding estimate for the second month contract on roll dates is 0.798, which implies that the proportion of temporary price impact that persists after one minute is $0.798^6 = 0.258$, while the proportion that persists after three minutes is $0.798^{18} = 0.017$. Clearly these estimates indicate that the crude oil futures markets are indeed quite resilient.

The numerical illustrations of the theoretical analysis presented in preceding section indicate that the "breakeven" resiliency parameter for informationless, liquidity-motivated trades exceeds 0.75, even under the strong assumption of a monopolist strategic trader. The model and empirical estimates provided here therefore imply that strategic trading around the USO roll will benefit rather than harm market quality unless strategic traders are able to complete same-direction trades within seconds prior to the execution of the USO roll.

V. Additional Evidence Regarding Liquidity on Roll and Non-Roll Days

We provide additional evidence by constructing and comparing across roll and non-roll periods several liquidity measures during the March 2008 to February 2009 period. The USO roll occurs prior to what might be termed the "market roll" i.e. when overall trading activity and open interest moves from the front month contract to second month contract. Since the market roll itself can induce changes in market conditions, we compare trading activity during the roll period to a non-roll period that precedes the USO roll date. Specifically, we define the period extending from seven to three days prior to the USO roll as the non-roll period.

We compare roll period and non-roll period measures for each minute between 9:00 AM and 3:00 PM EST, and report results averaged across minutes. To address non-normality attributable to potential time-of-day effects we implement a non-parametric (Wilcoxon signed) test, which requires less stringent distributional assumptions for tests of difference in location.

Results, reported on Table 4, verify that trading volume per minute (measured based on the CFTC data) is substantially greater on roll days, averaging 777 contracts in the front month and 355 contracts in the second month, compared to 575 contracts in front month and 193 contracts in the second month on non-roll days. Figure 3 also displays average trading volume by minute for roll and non-roll days. Most notable is the spike in trading activity at the time of the daily settlement, particularly on roll days.

We also examine a standardized "trade imbalance" measure for each minute based on the difference between buyer-initiated and seller-initiated trading volume. To account for intraday patterns, we normalize the measure by subtracting the mean and dividing by the standard deviation of imbalance observed during the same minute on both roll and non-roll periods. For the front contract the results show that the net trade imbalance is on average negative on roll days while for the second contract the net trade imbalance is positive on roll days. The trade imbalance results are consistent with increased usage of marketable orders by large traders (USO, or the counterparties to its TAS contracts) to sell the front month contract and buy the second month contract on the roll days.

Importantly, the evidence indicates enhanced liquidity provision on roll days. Quoted bid-ask spreads (the difference between the lowest limit price for unexecuted sell orders and the highest limit price for unexecuted buy orders) on roll days decline from an average of 1.17 basis points to 1.13 basis points in the front contract, and from 1.52 basis points to 1.42 basis points in the second contract. Figure 3, Panel B displays average quoted spreads by minute for roll and non-roll days. The patterns indicate smaller intraday quoted spreads for the majority of minutes on roll days. These declines, while modest, are statistically significant, and must be evaluated in light of the substantial increase in liquidity demand attributable to the USO rolls, which might have been anticipated to widen spreads.

We also assess liquidity supply by computing the "depth" of unexecuted orders in the limit order book. In particular, we determine the total volume of unexecuted sell (ask depth) and buy (bid depth) orders at prices within four ticks of the most competitive prices. Bid depth for the front contract, which is relevant for those seeking to sell, increases from an average of 47.6 contracts on non-roll days to 51.9 contracts on roll days (t-statistic = 10.98), while ask depth (relevant for those seeking to buy) for the second contract increases from an average of 18.5 contracts to 20.5 contracts (t-statistic = 9.07). Figure 4 displays average bid and ask depths by minute for roll and non-roll days and supports increased liquidity provision in both contracts throughout the day.

Next, we examine effective spreads, defined as the excess of the trade price over the bid-ask midpoint for each buyer-initiated trade and the excess of the midpoint over the trade price for each seller-initiated trade. Effective spreads differ from quoted spreads because large trades can execute outside the quotes, and any order may execute inside the quotes due to "hidden" orders in the limit order book. We focus on volume-weighted effective spreads for individual trades for each minute. The results reported in Table 4 indicate modest reductions in effective spreads on roll days, from an average of 2.03 basis points to 1.96 basis points (t-statistic = -3.97) for the front contract, and from 2.42 basis points to 2.29 basis points for the second contract (t-statistic = -4.66).

Finally, we use the CFTC data to assess the number of distinct accounts that supply liquidity on roll and non-roll days. An account is deemed to supply liquidity if more than one buy (sell) limit order posted by the account for the front (second) contract is executed on the corresponding day. We find that an average of 10,470 distinct accounts provide liquidity in the front contract on roll days, compared to 9,698 accounts on non-roll days. For the second contract the number of liquidity-supplying increases from 860 accounts on non-roll days to 1,416 accounts on roll days.

These comparisons of liquidity across roll and non-roll days are generally consistent with the sunshine trading theory of Admati and Pfleiderer (1991), who predict that the announcement of an upcoming trade by a credible uninformed trader will attract additional liquidity suppliers to market. The results are also generally consistent with our modified theory of strategic trading around known liquidations, which predicts that strategic traders will effectively supply liquidity during the liquidation in this resilient market. In particular, despite large liquidity demand on roll days, quoted spreads and effective spreads (which measure costs of trading) decline, while available liquidity on the relevant side of the limit order book and the number of distinct accounts providing liquidity both increase.

VI. Account-level analysis

We next study the trading strategies followed by individual accounts around the time of USO roll trades, to assess if a significant proportion of these accounts follow strategies that might reasonably be considered strategic The CFTC data identifies the trading accounts associated with both the buy and sell side of each transaction. In our main analysis, which we refer to as the "long window," we define the "During" interval as 9 A.M. to 4:15 P.M. New York time on the ETF roll day, while the "Before" interval covers from midnight on Day -3 (three trading days prior) to 9 A.M. on roll day, and the "After" interval is the reopening of trading at 5 P.M. on the roll day through Midnight on Day +3 (three trading days after) the roll day. We also analyze a shorter window where the "During" interval is defined as the two-minute settlement period on the roll day, and the "Before" and "After" intervals are defined as the one hour of trading before and after, respectively, the settlement period. The "During" interval in the long window analysis is the day of the roll, while the "During" interval in the short window analysis is the two-minute period of trading that determines the ETF's benchmark price for the roll day. The short window analysis examines whether strategic traders change behavior in the minutes prior to the settlement period on the day of the USO roll.

Our theoretical analysis focuses on a strategic trader who seeks to profit from the predictable liquidation, and who does not ultimately absorb any portion of the liquidation. Since strategic traders might also transact for reasons unrelated to the roll, we relax this definition for the empirical implementation. Specifically, we identify an account as potentially being a strategic trader if the absolute value of the net change in the account's inventory as a fraction of the account's total activity in the period surrounding the roll is less than 0.25.[16] The ETF's natural counterparties (i.e., accounts that hold or can be induced by price concessions to hold opposite positions as the ETF) are unlikely to be classified as strategic traders, since their inventory change to total trading ratio is likely to exceed the threshold of 0.25.

We categorize each account identified as a possible strategic trader as following one of twelve possible trading strategies, as described in Panel A of Table 5. Those strategic traders whose signed position change in the "During" interval is of the opposite sign as the ETF's order flow are potentially following liquidity provision or sunshine strategies, labeled ST1 to ST5. Those strategic traders whose signed position change in the "During" interval is in the same direction as the ETF order flow are deemed to be following potentially predatory strategies, along the lines described by Brunnermeier and Pedersen (2005), labeled ST8 to ST12. The categories ST6 and ST7 represent accounts without any trading activity in the During interval.

We further classify strategic traders into five sub-strategies within liquidity provision (ST1-ST5) and predatory trading (ST8-ST12) categories based on the account's trading before and after the During interval. Specifically, among sunshine traders, an account that trades in the opposite (same) direction as the ETF in the Before interval and trades in the same (opposite) direction as the ETF in the After interval is placed in category ST1 (ST3). Our objective in identifying the sub-strategies is to assess the relative importance of strategies implied by various theories. Our analysis in Section III above implies that a strategic trader will trade in the same direction as the ETF in the Before interval and in the opposite direction as the ETF in the During interval (ST3, ST4, and ST5). Further, for the modest resiliency parameters estimated in Section IV, the strategic trader will trade in the opposite direction as the ETF in the After interval (ST3).[17]

[16] As an illustration, suppose an account sells 1,000 contracts before, sells 1,000 contracts during, and buys 1,500 contracts after the roll. The absolute value of net change in account's inventory is 500 contracts, while total trading activity is 4,500 contracts. Since the ratio (500/4500) = 0.11, we classify the account as a strategic trader.

[17] As a robustness check, we implement an additional screen where an interval is classified as 'no trade' if the account's trading activity in the interval as a percentage of the account's total trading activity across intervals is lower than the absolute value of 10%. Results are similar to those reported in Table 5.

To assess whether particular strategies are used more frequently around the ETF roll as compared to other periods, we estimate trading activity for each of the sub-strategies on all usable non-roll days. We identify non-roll (or control) day for the long window analysis if the interval three days before to three days after does not overlap with the three days before or after an actual USO roll day. To identify abnormal activity, we compare trading volume associated with a strategy on roll days with the trading volume observed for the same strategy on control days.

Having assigned each strategic account to one of the 12 strategies at each roll period, we aggregate the *strategic volume* across all trading accounts associated with a strategy, where the strategic volume is simply the round trip volume associated with an account surrounding the roll. Note that for each identified strategy there is a complementary strategy involving opposite trading patterns. For example, ST1 and ST12 are complimentary, in that ST1 involves trading against, against, and with the ETF during the three intervals, while ST12 involves the opposite pattern of trading with, with, and against the ETF during the three intervals. Since some strategies might be more common than others for reasons entirely unrelated to the ETF roll, we focus on *normalized strategic volume*, which is the strategic volume in a category less the strategic volume in the complementary strategy.

In Panel B of Table 5 we report regression coefficients obtained when normalized strategic volume in categories ST1 to ST6 for all days (roll and control days) based on the long window is regressed on an indicator variable that equals one for USO roll days.[18] Increases in normalized strategic volume for categories ST1 to ST5 at the roll (positive coefficient on the roll day) would indicate increased usage of liquidity providing strategies, while decreases in normalized strategic volume for categories ST1 to ST5 at the roll would indicate increased use of predatory strategies.

Several results from the long-window analysis are noteworthy. First, the statistically significant intercepts in regressions for ST3 and ST4 for the front contract imply that strategic trading in category ST4 is more prevalent and in ST3 less prevalent relative to complementary strategies on non-roll days.

[18] Note that results for strategies ST7 to ST12 would simply be the opposite of results for ST1 to ST6, since they are the complements of the first six strategies.

Second, we estimate statistically significant positive coefficients on the roll day indicator in regressions for ST3 for both contracts, and significant negative coefficients on the roll day indicator in regressions for ST1for both contracts. ST3 involves trading in the same direction as the preannounced ETF trades prior to the roll, while ST1 involves trading in the opposite direction as the preannounced ETF prior to the roll. The analysis presented in Section III and illustrated in Table 2 indicates that the profit maximizing strategy always involves (for any resiliency parameter) trading in the same direction as the ETF in the period prior to the roll, as in ST3, ST4, and ST5. Further, when trades have some permanent impact, as documented in Table 3, the most profitable strategy based on Table 2 is ST3. These results therefore support a selection by strategic traders of the more profitable liquidity provision strategies.

In Panel B of Table 5, we also report results obtained when strategic trading is classified into strategies over a short window. The most notable result is that, for both the front and second contract, we estimate a large positive roll day coefficient for ST1. Collectively, the coefficients suggest that strategic traders are trading in the opposite direction of the ETF both in the hour before and during the two-minute settlement window on roll day and reversing the position in the hour after the settlement period. To the extent that USO’s TAS counterparties offset their TAS positions acquired earlier on the roll day both in the hour before and during the two-minute settlement period, the results suggest that strategic traders identified in the short window act as liquidity providers and absorb the imbalance. Further, strategic traders are less likely to trade in the same direction as the ETF in the hour before the settlement window. Thus ETF’s usage of TAS contracts appear to shift the imbalance from the two-minute settlement period to earlier periods on the roll day.

Estimates reported in Table 3 indicate that order imbalances do have a permanent price impact, even around the roll. A strategic trader who provides liquidity by absorbing order imbalances during the roll must offset the inventory (by trading in the same direction as the roll) either before or after the roll. Given long-lived price impacts it is more profitable to conduct offsetting trades ahead of (as in ST3, ST4 or ST5) than after (as in ST1 or ST2) the roll, and the long window evidence supports that strategic traders indeed use this strategy to provide liquidity at the roll.[19] Importantly, our analysis also indicates that the temporary price impacts associated with the large trades are reduced and the liquidator proceeds enhanced by strategic trading of this type.

On balance, we interpret the individual account evidence as failing to provide strong evidence that traders follow strategies that would be reasonably considered predatory around the time of USO rolls. We believe that our methodology has the power to detect predatory trading if it was prevalent, and therefore view the lack of strong evidence as informative.

VII. USO's Trade Execution Costs

The analysis reported here provides little evidence that strategic traders engage in predatory strategies around USO rolls and instead documents enhanced liquidity on roll days. However, USO demands a large amount of liquidity in its rolls, and the absence of predatory trading does not imply that liquidity suppliers need not be compensated in equilibrium for meeting USO's demand for liquidity.

To asses this issue, we compute three daily time series of futures returns from USO's inception date on April 1, 2006 to December 31, 2013, with each futures return defined as the change since the prior day in the log of the futures price for a contract with a fixed maturity. The first series is comprised of returns to the front month contract, colored blue in Figure 5, computed based on the current and prior day price of the front month contract, for all days including the last day of trading for the expiring contract. The second series is the return to the second nearest contract, colored red in Figure 5, computed based on the current and prior day price of the second month contract, for all days including the last day of trading for the expiring contract. The third, denoted "Benchmark Return" is based on settlement prices of the contracts that comprise USO's benchmark and that reflect its roll strategies, colored green in Figure 5. The Benchmark Return equals the return on the front month contract in the days up to and including the

[19] To expand on this reasoning, note that the ETFs (or the TAS counterparties of ETF) actively sell the front contract and actively buy the second month contract on the roll day. The positive permanent price effect implies that ETF trading would cause front contract prices to decrease and second contract prices to increase on the roll day. For this reason, strategic liquidity providers in front month contracts are better off building a short position (at higher prices) before the roll day. Along similar lines, strategic liquidity providers in the second month contracts are better off building a long position (at lower prices) before the roll day.

USO roll date, and equals the return on the second month contract after USO completes its roll.[20]

Each mean return is negative during the USO period. Annualized by multiplying by 250, the mean Benchmark futures return is -8.79% per year. By comparison, the mean return to the front month contract is -5.63% per year and the mean return to the second month contract is -5.80% per year. The benchmark return is 3.16% per year less than the return to the front contract and is 2.99% less than the return to the second contract. The latter differential is statistically significant (t-statistic = 2.48).[21]

USO's roll trades are executed at the relevant settlement prices due to the use of TAS contracts. Trade execution costs arise in the form of adverse changes in settlement prices, i.e. increases in the price of the second contract and/or decreases in the price of the front contract, attributable to the roll. Such costs can potentially arise due to strategic trading, or alternatively because of imperfectly elastic liquidity supply. It is common to measure trade execution costs by comparing the price for a completed trade to either a pre-trade or post-trade benchmark price. In the case of a futures roll, the relevant price is the spread between the prices of the second month and the front month contracts.

Note that the second month return and the benchmark return are identical in the days after USO completes its roll. The statistically significant differential in the second month futures return relative to benchmark returns therefore reflects a widening of the calendar spread between the second- and the front month contract prices in the days before and during the USO roll, which averages 25 basis points per roll. This comprises an estimate of USO's monthly trading costs associated with the roll, analogous to the comparison of a trade price to a pre-trade benchmark.

Similarly, the excess of the front month return over the benchmark return reflects better performance of the front month contract relative to the second month contract, i.e. narrowing of the calendar spread, in the days after the completion of the USO roll. This differential of 3.16% per year

[20] USO completed its roll on a single day through February 2009, after which it shifted to a four-day roll (see http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aZYy1UXKZRb0). The Benchmark return is equal to the front month contract return on roll days through February 2009. Thereafter, the benchmark return is a weighted average of the return on the front month and second month contracts during the four days of the roll, with the weight on second month return equal to 0, 0.25, 0.50, and 0.75 on the four consecutive days of the roll.

[21] That the former is not significant reflects the greater volatility of the Return 1 series, which in turn reflects in part the high volatility in the settlement price of the expiring contract on the final trading day.

equates to 26 basis points per roll, and analogous to estimating costs by comparing execution prices to post-trade benchmark prices, comprises an additional (though noisier) estimate of the roll cost.

Are trading costs of 25 basis points per month, or about 3% per year, excessive? By comparison, Marshall, Nguyen, and Visaltanachoti (2011, Table 1) report that median effective bid-ask spreads in the NYMEX crude oil market are about 11 basis points. However, each effective bid-ask spread is computed based on a single trade, and USO's ETF rolls are far too large to execute with a single order. As a second point of comparison, Lou, Yan, and Zhang (2013) report that prices of U.S. Treasury securities, which trade in what many consider to be the most liquid financial market in the world, decrease by about nine to 18 basis points in the days ahead of Treasury auctions.

The ETF roll trades we study are large – the monthly roll strategy implies that the ETF turns over its entire portfolio twelve times per year. The individual rolls can be as large (Table 1) as 17% of average daily volume for the front month and 28% for the second month, and often exceed 100% of market volume during the two minute settlement period whose price they seek to match. We view these estimated trade execution costs as moderate in light of the sharp demand for liquidity associated with the monthly roll strategy. Whether the monthly roll strategy itself is superior to alternative methods of generating returns that track crude oil prices, such as holding physical inventory or using longer-dated futures contracts, is left for future research. [22]

Finally, it is worth noting that the trade execution costs we estimate, which average about three percent per year, are not sufficient to explain the underperformance of the USO share price relative to the level of crude oil futures prices, as demonstrated on Figure 1. In the Appendix to this paper we demonstrate that the remaining apparent underperformance can be attributed to crude oil storage costs, which have been high on average in recent years.

[22] In unreported results, we observe that both liquidity (wider bid-ask spreads and lower book depth) and trading activity is lower in the longer dated 3-month, 6-month and 12-month crude oil contracts.

VIII. Conclusions

This study contributes to our understanding of the economic issues related to the execution of large predictable trades, and also provides estimates of the resiliency of the crude oil markets and of trading costs for a large trader in crude oil futures. More specifically, we study trading strategies, liquidity, and price patterns around the time of large and predictable monthly trades undertaken by the US Oil ETF, which is designed to provide returns that track changes in crude oil futures prices.

USO demands a large amount of liquidity. Aggregated across the twelve roll dates in our sample period, net roll activity comprises approximately 5% of roll-day volume in the front contract and ten percent of roll-day volume in the second contract. Further, USO typically seeks to trade at the daily settlement price, which is established in a two-minute period, and their roll trades on average exceed market wide volume during this interval. We view the large and predictable liquidity demand associated with the USO roll to comprise an ideal experiment for assessing the relevance of two theories: predatory and sunshine trading, in a setting where both potentially apply.

In addition to presenting empirical evidence, we develop a simple model that extends prior work to consider the implications of strategic trading in a resilient market. We find that the strategic trader necessarily causes prices to overshoot and reduces liquidator proceeds *only* hold when trades' price impacts are permanent or long-lasting. When markets are resilient, the model predicts that the strategic trader will choose to act as a liquidity provider, absorbing a portion of the liquidator's order imbalance on roll day, while offloading the resulting inventory in periods before or after the roll day. Further, for resiliency parameters in line with those estimated from the data, the liquidator's proceeds are *larger* when the strategic trader is present than when the strategic trader is absent. The extended model is relevant to crude oil ETF rolls, as well as other settings where traders need to make large predictable transactions.

In light of the increasing popularity of ETFs in retail and institutional portfolios, regulators are interested in better understanding the impact of ETF activity on market quality. Leveraged ETFs and ETFs that invest in futures contracts pose special challenges because their replication strategies involve frequent large trades. With respect to USO's roll, we find little evidence that strategic traders engage in predatory trading that impairs price discovery or destabilizes the futures market.

We estimate that USO effectively pays about 25 basis points on average to complete its roll trades, which is not much larger than price impacts observed in other liquid markets, including U.S. treasuries (Lou, Yan, and Zhang, 2013). Observing moderate trading costs despite the ETF's large and concentrated demand for liquidity reflect the resiliency of the crude oil futures market, and the effectiveness of the "sunshine trading" strategy where preannouncement attracts liquidity suppliers, including strategic traders, as well as natural counterparties. Still, accumulated trading costs of 3% per year are substantial. These costs arise from the large amount of liquidity demanded by a monthly roll strategy, and highlight the importance of developing innovative strategies to match benchmark returns.

On balance these results are consistent with the theoretical analysis presented here implying that strategic traders have incentives to trade in a benign manner in a resilient market, and are also consistent with the elements of both the sunshine trading theory of Admati and Pfleiderer (1991) and the predatory trading theory of Brunnermeier and Pedersen. The evidence of improved liquidity and greater resiliency on roll days, as well as the observation that USO announces roll dates on its web site, are strongly consistent with the sunshine trading interpretation. At the same time, the fact that USO completes its roll trades through TAS contracts implies that market participants will not in general know the exact timing of the TAS counterparties offsetting trades. As a consequence it would be difficult for a would-be predator to "front run" the USO roll trades by very short time intervals.

Our model does not imply that predatory trading is never a concern. The key practical issue is the proportion of the price impact of strategic trades that persists into the period when the predictable trades are executed. This proportion will be higher, and strategic trading more likely to be harmful, when trades are motivated by private information, if trades are executed in less resilient markets where price impacts are closer to permanent, or if strategic traders are able to execute their trades in very close time proximity to those of the predictable trades.

Appendix: Explaining Crude Oil ETF Stock Price Performance – The Role of Storage Costs

In this paper, we present evidence that liquidity is improved on USO roll days, that there is little evidence of predatory trading around USO rolls, and that the trade execution costs incurred by USO cannot explain the performance of the USO share price relative to the level of crude oil prices, as documented in Figure 1. What then does explain the divergence?

a. Ex post Return Premia and Storage Costs

We assess this issue by relying on the well-known "cost-of-carry" no-arbitrage relation, stated as:

$$F_t(m) = P_t e^{S_t m}, \qquad \text{(A1)}$$

where P_t denotes the date t spot price, $F_t(m)$ denote the date t futures price for delivery at date t+m, and S_t is the marginal investor's continuously compounded per-period cost of carrying inventory, including forgone interest and other storage costs. Non-interest storage costs include costs of renting storage tanks, insurance, etc., and may at times be offset in part or full by "convenience yields" that reflect the option value of holding inventory. Applying (A1) to futures contracts for delivery at dates t+m and t+n, the per-period cost of carrying inventory can be inferred as:

$$S_t = \frac{\ln(F_t(m)/F_t(n))}{(m-n)} \qquad \text{(A2)}$$

which implies that the marginal cost of carrying inventory is revealed by the slope of the futures term structure. Using expression (A2) with the daily EIA data, we compute the cost of storage implied by the settlement prices of the first and second nearest-to-expiration crude oil contracts for each trading day from January 1, 1999 to December 31, 2013, and report the results on Table A1, for the full sample and for subsamples. For the full sample the mean implied storage cost (multiplied by 250 to convert to an annual equivalent) is 0.49%. In contrast, during the post-USO subsample the mean implied storage cost was 12.38%. A positive term slope, whereby futures prices rise for more distant delivery dates, characterizes what practitioners refer to as a "contango" market. The cost-of-carry relation implies that contango will be observed only when net storage costs for the marginal holder of inventory are positive.[23]

Define an ex post spot return premium as:

$$U_{t+1} = \ln\left[\frac{P_{t+1}}{P_t e^{S_t}}\right]. \quad \text{(A3)}$$

The denominator of expression (A3) is the time t spot price adjusted for the cost of storing oil for one period, so U_{t+1} is interpreted as the return in excess of storage costs (analogous to the return in excess of the interest rate often studied in equity markets). We construct a daily time series of spot prices implied by expression (A1), relying on the nearest-to-expiration futures price and the previously computed daily storage cost estimates. From this series we compute the time series of realizations of U_t. Table A1 reports mean outcomes, annualized by multiplying by 250.

For the full sample, the spot return premium for crude oil is 5.46% per year. During the 1990s, the spot return premium was 4.81% per year. In the period before USO was launched, January 2000 through April 9, 2006, the spot return premium surged to 22.80% per year, potentially whetting investor demand for products linked to crude oil prices. In contrast, since USO's April 10, 2006 launch, the mean spot return premium was -7.64% per year. While none of these means are statistically significant (reflecting the high variability of price changes), the accumulated effect is nevertheless economically important. The negative spot return premium for the period when USO was active implies that the appreciation of spot oil prices during the sample period was considerably *less* than sufficient to compensate for the marginal cost of carrying inventory.

[23] Pirrong (2011) documents that the collapse in crude oil prices during the recent financial crisis was accompanied by large increases in physical crude oil inventories and in the marginal cost of carrying inventory. To see that the positive term slope represents marginal storage costs, recognize that S_t also represents the pre-storage-cost daily return to a strategy of purchasing crude oil for delivery at date t+n at price $F_t(n)$ and simultaneously selling the same oil for delivery at date t+m at price $F_t(m)$. Positive arbitrage profits are available if oil can be stored from date t+n to date t+m for a per-period cost less than S_t. Of course, the no-arbitrage condition applies to the *marginal* holder of inventory. Those who can store a commodity for lower cost can earn profits. Anecdotal accounts (e.g. http://blogs.reuters.com/great-debate/2010/07/22/contango-and-the-real-cost-of-carry/) indicate entry by non-traditional firms (e.g. hedge funds) into the oil storage business in recent periods.

Applying expression (A1) at dates t and t+1 and denoting $\Delta S = S_{t+1} - S_t$, the one-period return to a long position in a given futures contract can be expressed as:

$$\ln\left[\frac{F_{t+1}(m-1)}{F_t(m)}\right] = U_t + (m-1)\Delta S \quad , \tag{A4}$$

while, by comparison, the continuously compounded growth in the spot price can be written as:

$$\ln\left[\frac{P_{t+1}}{P_t}\right] = U_t + S_t \quad . \tag{A5}$$

Comparing expressions (A4) and (A5) yields several insights. First, for a given cost of carry ($\Delta S = 0$), the rate of appreciation in the spot price exceeds that of the futures price by S_t, the cost of carrying inventory. Stated alternately, spot price appreciation will exceed changes in prices of individual futures contracts in contango markets, and vice versa in "backwardated" markets (where the implied cost of carry is negative, presumably due to large "convenience yields"). As noted, the marginal cost of carry was large and significant (12.4% per year) during the USO sample period, implying underperformance of long futures positions relative to spot price changes. Second, the futures return does depend on ΔS. Third, both futures and spot returns are equally affected by U_t, the ex post premium in the spot price. Finally, the cost-of-carry itself has no direct implication for futures returns, as S_t does not appear in expression (A4).[24]

This final insight contrasts with what appears to be a rather wide-spread myth among both academics and the financial press that a roll trade generates an immediate gain or loss attributable to the divergence of the near-to-delivery futures price from that of the more distant delivery price.[25] The

[24] However, the expression does not rule out covariation between the cost of carry and futures returns, which has in fact been documented in a number of commodity markets. See, for example, Liu and Tang (2010), Szymanowska, De Roon, Nijman, and Van den Goorbergh (2014), and the papers referenced therein. The data reported in Table A1 are consistent with negative covariation, in that futures returns are positive and the cost of storage negative during the January 2000 to April 2006 period, while futures returns are negative and the cost of storage positive during the April 2006 to December 2013 period.

[25] For example, Mou (2011) claims (page 13) that the excess of the front month futures price over the more distant-delivery futures price "is the amount of gain (or loss) per unit of the commodity when rolling futures forward" and asserts (page 2) that futures investors earn a "return called 'roll yield', which refers to the difference between log price of the maturing contract they roll from and the deferred contract they roll into." Similarly, the Wall Street Journal recently claimed ("Winning by Waiting in Commodities: Investors Cash In With a Commodities Trading Strategy," July 15, 2014) that "A fund manager buys a futures contract for delivery next month. Right

confusion seems to arise from a misconception that traders pay or receive the futures price when they transact in futures contracts. In practice, cash flows to or from margin accounts depend only on same-contract price changes. With regard to roll trades in particular, if the roll is completed on date t at the settlement prices, then the date t cash flow depends on the change from date t-1 to date t in the settlement price for the expiring contract, while the date t+1 cash flow depends on the change from date t to date t+1 in the settlement price for the second-nearest-to-expiration contract.

We summarize this analysis as follows. The mean annualized return (daily mean x250) to the USO ETF during the April 10, 2006 to July 31, 2012 period was -8.42%, while the mean annualized rate of change in implied spot prices during the same period was 4.74% (Table A1). This gives rise to the perception, reinforced by Figure 1, that the USO ETF performed very poorly. The perceived underperformance has been attributed to predatory trading or to the effect of "contango" on futures prices. We show that neither of these explanations is particularly relevant.

The most important factor is storage costs, which for the marginal investor averaged 12.38% per year during this period. The actual post-storage-cost return to the marginal investor holding spot crude oil was therefore -7.64% per year. Crude oil futures markets as a whole performed somewhat better – a long position rolled so as to always remain invested in the second (front) month contract earned -5.80% (-5.63%) per year. The USO benchmark, which reflected the price impacts of the actual USO roll strategies, performed worse, delivering -8.79% per year. We attribute the underperformance of the USO benchmark relative to hypothetical long futures strategies invested in the front or second month contract to trade execution costs that average about 25 basis points per roll. Finally, the actual USO ETF return of -8.42% per year slightly exceeds the benchmark futures return of -8.79%, reflecting the net effect of interest on cash balances versus management fees. We conclude that USO's stock performance can be well explained by the combination of crude oil storage costs and moderately large trade execution costs.

before it expires, the investor sells the contract, buys a cheaper one for delivery at a later date and pockets the difference."

References

Admati, A.R., and P. Pfleiderer, 1988, A Theory of Intraday Patterns: Volume and Price Variability, *Review of Financial Studies*, Vol 1 (1), 3-40.

Admati, A.R., and P. Pfleiderer, 1991, Sunshine trading and financial market equilibrium, *Review of Financial Studies*, Vol 4 (3), 443-481.

Boyd, N.E., Harris, J.H., and A. Nowak, 2011, The role of speculators during times of financial distress, *The Journal of Alternative Investments*, Summer 2011, 1-16.

Brunnermeier, M., and L.H. Pedersen, 2005, Predatory trading, *Journal of Finance*, Vol 60 (4), 1825-1863.

Buyuksahin, B., and J.H. Harris, 2011, Do speculators drive crude oil prices?, *The Energy Journal*, 32, 167-202.

Carlin, B., Lobo, M.S. and S. Viswanathan, 2007, Episodic liquidity crises: Cooperative and predatory trading, *Journal of Finance*, 62 (5), 2235-2270.

Erb, C. and C. Harvey, 2006, The strategic and tactical value of commodity futures, *Financial Analysts Journal*, 62, 69-97.

Foucault, T., O. Kadan, and E. Kandel, 2005, Limit order book as a market for liquidity, *Review of Financial Studies*, Vol 18 (4), 1171-1217.

Honghui, C., Noronha, G., and V. Singal, 2004, The Price Response to S&P 500 Index Additions and Deletions: Evidence of Asymmetry and a New Explanation, *Journal of Finance*, Vol. 59(4), 1901-1929.

Huang, R., and H. Stoll, 1997, The components of the bid-ask spread: a general approach, *Review of Financial Studies*, 10, 995-1034.

Irwin, S., and D. Sanders, 2012, Testing the Masters Hypothesis in commodity futures markets, *Energy Economics*, 34, 256-269.

Kilian, L. 2009, Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market, *American Economic Review* 99, 1053-1069.

Lou, D., H. Yan, and J. Zhang, 2013, Anticipated and repeated shocks in liquid markets, Review of Financial Studies, forthcoming.

Madhavan, A., 2003, The Russell Reconstitution Effect, Financial Analyst Journal, Vol. 59(4), 51-64.

Manaster, S., and S.C. Mann, 1996, Life in the Pits: Competitive Market Making and Inventory Control, *Review of Financial Studies*, Vol. 9(3), 953-975.

Marshall, B.R., Nguyen, N.H., and N. Visaltanachoti, 2012, Commodity liquidity measurement and transaction costs, *Review of Financial Studies*, Vol. 25 (2), 599-638.

Mou, Y., 2011, Limits to Arbitrage and Commodity Index Investment: Front-running the Goldman Roll, working paper, Columbia University.

Pirrong, C., 2011, An evaluation of the performance of oil price benchmarks during the financial crisis, working paper, University of Houston.

Schoneborn, T., and A. Schied, 2009, Liquidity in the face of adversity: Stealth vs. sunshine trading, predatory trading vs. liquidity provision, working paper, Technical University Berlin.

Stoll, H., and R.E. Whaley, 2010, Commodity Index Investing and Commodity Future Prices, *Journal of Applied Finance*, Issue 1, 1-40.

Szymanowska, M., F. De Roon, T. Nijman, and R. Van den Goorbergh, 2014, An Anatomy of Commodity Futures Risk Premia, *Journal of Finance*, 69, 453-482.

Table 1. USO and Market Trading Activity on Roll Days

Reported are trading volumes (in contracts) in the NYMEX Crude Oil futures market for the full trading day, and during the two-minute settlement period, on USO roll days from March 2008 to February 2009. Also reported are the estimated sell and buy volumes attributable to USO's roll trading on these days. We rely on the Commodity Futures Trading Commission (CFTC) dataset to calculate the market-wide trading volumes for the full day and settlement periods. The CFTC dataset includes all completed trades in NYMEX crude oil futures, including floor and block trades, as well as trades completed on the GLOBEX electronic market. USO roll trading volume is estimated on the basis of USO's Total Net Assets (TNA) on the roll date relative to front and second month settlement prices on the roll day. Settlement prices are obtained from the Energy Information Agency (EIA). TNA values were provided by ALPS on behalf of USO. Roll dates are two weeks prior to the expiration of the nearest-delivery contract. A calendar schedule of USO's recent and future roll dates is available on the website: hhtp://www.unitedstatesoilfund.com/.

	Front Contract on Roll Date				Second Contract on Roll Date			
Roll date	ETF Selling Activity (contracts)	Market Trading Volume (contracts)	ETF %	Market Trading Volume During Settlement	ETF Buying Activity (contracts)	Market Trading Volume (contracts)	ETF %	Market Trading Volume During Settlement
3/5/08	4,445	414,308	1%	16,756	4,480	205,827	2%	10,449
4/8/08	5,524	307,800	2%	16,338	5,557	165,544	3%	15,775
5/6/08	4,995	331,913	2%	11,933	5,015	129,110	4%	6,632
6/6/08	8,582	508,749	2%	18,139	8,572	231,984	4%	11,112
7/8/08	7,205	382,404	2%	15,378	7,169	154,453	5%	13,299
8/6/08	6,289	307,994	2%	16,189	6,296	140,471	4%	13,489
9/8/08	11,960	317,923	4%	18,581	11,929	142,644	8%	14,791
10/7/08	9,116	342,917	3%	21,235	9,254	193,234	5%	15,414
11/6/08	13,025	292,018	4%	6,756	12,877	87,869	15%	3,578
12/5/08	23,723	327,140	7%	27,508	22,552	157,572	14%	22,765
1/6/09	55,688	331,307	17%	9,145	50,919	183,802	28%	7,659
2/6/09	85,055	518,382	16%	32,674	74,033	318,960	23%	29,187
Sum	235,607	4,382,855	5%	210,632	218,653	2,111,470	10%	164,150

Table 2. Strategic Trading Around a Known Liquidation – Numerical Outcomes

Reported are th outcomes when the Strategic Trader chooses trade quantities to maximize profits. The initial price is $100 and the liquidation amount is 20 units. The temporary price impact parameter is $\gamma = .05$. In Panel A the permanent price impact parameter is zero, while in Panel B the permanent price impact parameter is λ=0.05.

Table 1: Numerical Illustration of Closed Form Solutions

	Panel A: Lambda = 0, Gamma = 0.05							Panel B: Lambda = 0.05, Gamma = 0.05						
	Liquidation Price w/o Strategic Trading = 99.							Liquidation Price w/o Strategic Trading = 98.						
	Optimal Strategic Trading as proportion of liquidation			Strategic Profit	Prices with Strategic Trading			Optimal Strategic Trading as proportion of liquidation			Strategic Profit	Prices with Strategic Trading		
Theta	Pre	During	After		Pre	During	After	Pre	During	After		Pre	During	After
0.00	0.167	-0.333	0.167	3.33	99.833	99.333	99.833	0.333	-0.333	0.000	6.67	99.333	98.333	99.000
0.02	0.172	-0.334	0.162	3.31	99.828	99.331	99.824	0.337	-0.334	-0.003	6.71	99.326	98.324	98.990
0.04	0.178	-0.336	0.158	3.29	99.822	99.328	99.815	0.341	-0.335	-0.006	6.76	99.318	98.315	98.979
0.06	0.183	-0.337	0.153	3.27	99.817	99.326	99.806	0.345	-0.335	-0.009	6.81	99.311	98.306	98.968
0.08	0.189	-0.338	0.149	3.26	99.811	99.323	99.797	0.348	-0.336	-0.012	6.86	99.303	98.296	98.957
0.10	0.194	-0.339	0.144	3.24	99.806	99.319	99.788	0.352	-0.337	-0.016	6.91	99.295	98.286	98.946
0.12	0.200	-0.340	0.140	3.23	99.800	99.316	99.778	0.356	-0.337	-0.019	6.96	99.287	98.276	98.934
0.14	0.206	-0.341	0.135	3.22	99.794	99.312	99.769	0.360	-0.338	-0.022	7.01	99.279	98.265	98.922
0.16	0.211	-0.342	0.130	3.21	99.789	99.308	99.759	0.364	-0.339	-0.026	7.07	99.271	98.255	98.911
0.18	0.217	-0.343	0.126	3.20	99.783	99.304	99.749	0.368	-0.339	-0.029	7.13	99.263	98.244	98.898
0.20	0.223	-0.344	0.121	3.19	99.777	99.299	99.739	0.373	-0.340	-0.033	7.19	99.255	98.232	98.886
0.22	0.229	-0.344	0.116	3.19	99.771	99.294	99.729	0.377	-0.340	-0.036	7.25	99.246	98.221	98.873
0.24	0.234	-0.345	0.111	3.19	99.766	99.289	99.719	0.381	-0.341	-0.040	7.32	99.238	98.209	98.860
0.26	0.240	-0.346	0.106	3.19	99.760	99.284	99.708	0.386	-0.341	-0.044	7.38	99.229	98.197	98.847
0.28	0.246	-0.347	0.101	3.19	99.754	99.278	99.697	0.390	-0.342	-0.048	7.45	99.220	98.184	98.833
0.30	0.252	-0.348	0.095	3.19	99.748	99.272	99.686	0.394	-0.342	-0.052	7.52	99.211	98.172	98.819
0.32	0.259	-0.348	0.090	3.20	99.741	99.266	99.675	0.399	-0.343	-0.056	7.6	99.202	98.158	98.805
0.34	0.265	-0.349	0.084	3.20	99.735	99.259	99.664	0.404	-0.343	-0.061	7.67	99.192	98.145	98.791
0.36	0.271	-0.350	0.079	3.21	99.729	99.252	99.652	0.409	-0.343	-0.065	7.75	99.183	98.131	98.776
0.38	0.278	-0.350	0.073	3.23	99.722	99.245	99.640	0.413	-0.344	-0.070	7.84	99.173	98.117	98.761
0.40	0.284	-0.351	0.067	3.24	99.716	99.237	99.628	0.418	-0.344	-0.075	7.92	99.163	98.102	98.745
0.42	0.291	-0.351	0.061	3.26	99.709	99.229	99.616	0.424	-0.344	-0.079	8.01	99.153	98.087	98.729
0.44	0.298	-0.352	0.054	3.28	99.702	99.221	99.603	0.429	-0.344	-0.084	8.1	99.143	98.071	98.713
0.46	0.305	-0.352	0.048	3.30	99.695	99.212	99.590	0.434	-0.345	-0.090	8.2	99.132	98.055	98.696
0.48	0.312	-0.353	0.041	3.33	99.688	99.203	99.577	0.440	-0.345	-0.095	8.3	99.121	98.039	98.679
0.50	0.319	-0.353	0.034	3.36	99.681	99.193	99.563	0.445	-0.345	-0.100	8.4	99.110	98.022	98.661
0.52	0.327	-0.353	0.026	3.40	99.673	99.183	99.549	0.451	-0.345	-0.106	8.51	99.098	98.004	98.643
0.54	0.335	-0.353	0.019	3.43	99.665	99.173	99.535	0.457	-0.345	-0.112	8.62	99.086	97.986	98.625
0.56	0.343	-0.353	0.011	3.47	99.657	99.162	99.520	0.463	-0.345	-0.118	8.74	99.074	97.968	98.606
0.58	0.351	-0.354	0.002	3.52	99.649	99.150	99.504	0.469	-0.345	-0.124	8.86	99.062	97.949	98.586
0.60	0.360	-0.354	-0.006	3.57	99.640	99.138	99.489	0.475	-0.345	-0.131	8.99	99.049	97.929	98.566
0.62	0.369	-0.353	-0.015	3.63	99.631	99.125	99.473	0.482	-0.345	-0.137	9.12	99.036	97.909	98.546
0.64	0.378	-0.353	-0.025	3.69	99.622	99.111	99.456	0.489	-0.345	-0.144	9.26	99.022	97.887	98.525
0.66	0.387	-0.353	-0.034	3.76	99.613	99.097	99.439	0.496	-0.344	-0.151	9.4	99.008	97.866	98.503
0.68	0.397	-0.353	-0.045	3.83	99.603	99.082	99.421	0.503	-0.344	-0.159	9.55	98.994	97.843	98.480
0.70	0.408	-0.352	-0.056	3.91	99.592	99.067	99.402	0.511	-0.344	-0.167	9.71	98.979	97.820	98.457
0.72	0.419	-0.352	-0.067	4.00	99.581	99.050	99.383	0.518	-0.343	-0.175	9.88	98.963	97.795	98.433
0.74	0.430	-0.351	-0.079	4.10	99.570	99.033	99.363	0.526	-0.343	-0.183	10.05	98.947	97.770	98.409
0.76	0.442	-0.351	-0.092	4.20	99.558	99.014	99.343	0.535	-0.343	-0.192	10.23	98.931	97.744	98.384
0.78	0.455	-0.350	-0.105	4.32	99.545	98.995	99.321	0.543	-0.342	-0.201	10.42	98.914	97.717	98.357
0.80	0.469	-0.349	-0.120	4.45	99.531	98.974	99.299	0.552	-0.342	-0.211	10.62	98.896	97.689	98.330
0.82	0.483	-0.348	-0.135	4.58	99.517	98.952	99.276	0.561	-0.341	-0.220	10.83	98.877	97.660	98.303
0.84	0.498	-0.347	-0.151	4.74	99.502	98.929	99.251	0.571	-0.340	-0.231	11.05	98.858	97.630	98.274
0.86	0.514	-0.345	-0.168	4.90	99.486	98.904	99.225	0.581	-0.340	-0.242	11.29	98.837	97.598	98.244
0.88	0.531	-0.344	-0.187	5.09	99.469	98.877	99.199	0.592	-0.339	-0.253	11.53	98.816	97.565	98.213
0.90	0.549	-0.343	-0.207	5.29	99.451	98.848	99.170	0.603	-0.338	-0.265	11.79	98.794	97.531	98.181
0.92	0.569	-0.341	-0.228	5.51	99.431	98.817	99.140	0.614	-0.337	-0.277	12.07	98.771	97.495	98.147
0.94	0.591	-0.339	-0.251	5.76	99.409	98.784	99.108	0.627	-0.336	-0.290	12.36	98.747	97.457	98.113
0.96	0.614	-0.337	-0.276	6.03	99.386	98.748	99.075	0.639	-0.335	-0.304	12.66	98.722	97.418	98.077
0.98	0.639	-0.335	-0.304	6.33	99.361	98.709	99.039	0.653	-0.334	-0.318	12.99	98.695	97.377	98.039
1.00	0.667	-0.333	-0.333	6.67	99.333	98.667	99.000	0.667	-0.333	-0.333	13.33	98.667	97.333	98.000

Table 3. Regression Estimates of Permanent and Temporary Price Impact and the Resiliency of the Market

Reported are estimates of the permanent price impact (λ), the temporary price impact (γ) and the resiliency of the market (θ) in the NYMEX crude oil markets for the full sample, and separately on USO roll and non-roll days. The analysis relies on trades and limit order book data from Chicago Mercantile Exchange's Datamine database on GLOBEX electronic market. USO's roll dates are identified based on their publicly stated roll strategy. Non-roll days are defined as Days [-5,-2] before the roll day. We estimate these parameters with geometric lag regressions of the form:

$$P_t - P_{t-1} = \beta_0 q_t + \beta_1 q_{t-1} + \beta_2 q_{t-2} + ... + \beta_j q_{t-j} + \varepsilon_t$$

where P_t is the time t trade price, q_j is the signed order imbalance at time j, and the coefficients are restricted such that

$$\beta_0 = \lambda + \gamma, \beta_1 = \gamma(\theta - 1), \beta_2 = \beta_1\theta, \beta_3 = \beta_1\theta^2, \beta_4 = \beta_1\theta^3, \beta_4 = \beta_1\theta^3 \text{, etc.}$$

Results are based on ten-second time periods and j = 30 lags. For time periods with multiple trades, P_t is measured as the last trade price and Q_j is measured as net trade imbalance during the period. The geometric lag expression (12) is estimated by Generalized Method of Moments (GMM), using SAS Proc Model with a Bartlett Kernel set equal to the lag length plus one.

	Observations	Lambda (λ)	Gamma (γ)	Theta (θ)	R^2
Panel A: Front Contract					
Full sample	545,124	0.043	0.020	0.597	33.78%
Non-Roll Days		0.045	0.019	0.596	
Roll Days		0.041	0.018	0.442	
Difference		-0.004 **	-0.002	-0.154 ***	
p-value		(0.01)	(0.24)	(0.00)	
Panel B: Second Contract					
Full sample	544,505	0.070	0.037	0.777	11.13%
Non-Roll Days	`	0.067	0.037	0.788	
Roll Days		0.078	0.033	0.798	
Difference		0.011	-0.004	0.010	
p-value		(0.21)	(0.66)	(0.88)	

Table 4. Average Market Quality Measures on USO Roll and Non-roll days

Reported are market quality measures on USO roll days and non-roll days in the NYMEX Oil Futures market. USO's roll dates are identified based on their publicly stated roll strategy during the period March 1, 2008 to February 28, 2009. Non-roll days are defined as Days [-7,-3] before the roll day. Market quality is calculated each minute of the day and then averaged across roll and non-roll days. We rely on Commodity Futures Trading Commission (CFTC) data for calculating trading volume and number of liquidity providing accounts. Trading volume includes all completed trades in NYMEX crude oil futures, including floor and block trades, as well as trades completed on GLOBEX. The spread, depth and imbalance measures are based on the Chicago Mercantile Exchange's Datamine database for the GLOBEX electronic market. Trade imbalance is the signed difference between buyer and seller initiated volume standardized by subtracting the mean and dividing by the standard deviation of imbalance during the same minute (across roll and non-roll days). Quoted bid-ask spread (in basis points) is the difference between the lowest limit price for unexecuted sell orders and the highest limit price for unexecuted buy orders. Depth is the total volume of unexecuted sell (ask depth) and buy (bid depth) orders at prices within four ticks of the most competitive prices. Effective spread (in basis points) for a buyer (seller) initiated trade is twice the excess of trade price (quote midpoint) over the quote midpoint (trade price). Reported are Wilcoxon signed rank t-statistics and p-values with the null hypothesis of zero difference in median.

	Roll Period		Non-Roll Period			Wilcoxon Signed Rank	
	Mean	Median	Mean	Median	Difference:	T-Stat	P-Value
Front Month Crude Oil Futures Contract							
Trading Volume per Minute (contracts)	777.0	716.4	574.9	544.9	202.04	13.75	<.0001
Standardized Trade Imbalance	-0.023	-0.023	0.014	0.015	-0.04	-2.23	0.0129
Quoted Spread	1.13	1.12	1.17	1.15	-0.04	-5.08	<.0001
Effective Spread	1.96	1.83	2.03	1.97	-0.08	-3.97	<.0001
Near-inside Bid Depth (contracts)	51.9	51.0	47.6	46.9	4.33	10.98	<.0001
Near-inside Ask Depth (contracts)	49.2	48.2	45.2	44.8	3.97	11.38	<.0001
Liquidity Supplying accounts (N)	10,470	10,541	9,698	9,787	772.00	0.82	0.413
Second Month Crude Oil Futures Contract							
Trading Volume per Minute (contracts)	354.6	283.9	193.2	169.3	161.36	15.20	<.0001
Standardized Trade Imbalance	0.0020	0.0122	0.0099	0.0082	-0.01	-0.52	0.3015
Effective Spread	2.29	2.06	2.42	2.29	-0.14	-4.66	<.0001
Quoted Spread	1.42	1.39	1.52	1.48	-0.10	-6.99	<.0001
Near-inside Bid Depth (contracts)	23.9	23.3	21.6	21.4	2.25	9.80	<.0001
Near-inside Ask Depth (contracts)	20.5	19.9	18.5	18.1	1.96	9.07	<.0001
Liquidity Supplying accounts (N)	1,416	1,198	860	835	556.00	2.98	0.0028

Table 5: Strategic Trading Surrounding the USO Roll

Reported in Panel A are patterns associated with twelve strategic trading strategies associated with the USO roll. To be identified as a strategic trader, the absolute value of net change in the (non-ETF) account's inventory to the account's total activity surrounding the roll must be less than 0.25. The "During" period is defined as between 9 A.M. and 5 P.M. EST on the USO roll day, the "Before" period is defined from Midnight on Day -3 (three trading days prior) to 9 A.M. on roll day, and the "After" period is defined from 5 P.M. on roll day to Midnight on Day +3 (three trading days after) relative to the roll day. A strategic trader whose signed position change on roll day moves *against* USO's inventory change is deemed a liquidity provider (Strategies ST1 to ST5) while a strategic trader whose signed position change on roll day moves *with* USO's inventory change is deemed a predatory trader (Strategies ST8 to ST12). Categories ST6 and ST7 correspond to trading patterns with no trading activity on the roll day. Strategic traders are further classified into one of five sub-strategies within liquidity provision (ST1-ST5) and predatory trading (ST8-ST12) based on the account's change in net positions in the Before and After period. Also identified below is the complementary strategy where strategic traders pursue an opposite trading pattern surrounding the USO roll. Panel A reports the direction of USO activity and those for each strategy for expiring (front) contract and next-to-expiring (second) contract on USO roll days.

Panel A: Direction of ETF and Strategic Trading Surrounding the USO Roll

Strategy	Trading Pattern (relative to ETF)			Front month			Second month			Complement strategy
	Before	During	After	Before	During	After	Before	During	After	
ETF*				none	sell	none	none	buy	none	
ST 1	against	against	with	buy	buy	sell	sell	sell	buy	ST 12
ST 2	none	against	with	none	buy	sell	none	sell	buy	ST 11
ST 3	with	against	against	sell	buy	buy	buy	sell	sell	ST 10
ST 4	with	against	none	sell	buy	none	buy	sell	none	ST 9
ST 5	with	against	with	sell	buy	sell	buy	sell	buy	ST 8
ST 6	against	none	with	buy	none	sell	sell	none	buy	ST 7
ST 7	with	none	against	sell	none	buy	buy	none	sell	ST 6
ST 8	against	with	against	buy	sell	buy	sell	buy	sell	ST 5
ST 9	against	with	none	buy	sell	none	sell	buy	none	ST 4
ST 10	against	with	with	buy	sell	sell	sell	buy	buy	ST 3
ST 11	none	with	against	none	sell	buy	none	buy	sell	ST 2
ST 12	with	with	against	sell	sell	buy	buy	buy	sell	ST 1

Table 5, Panel B presents regressions coefficients of normalized strategic volume for all days (USO roll and Control days) on an USO roll day indicator variable. The analysis relies on the CFTC dataset. The *strategic volume* is aggregated across all (non-ETF) trading accounts associated with a specific strategy, identified in Panel A, where strategic volume is simply the round trip volume associated with an account surrounding the roll. We define *Normalized strategic volume* as the strategic volume in a strategy less strategic volume in complementary strategy. Such normalization accounts for abnormal trading volume and liquidity associated with a roll day and allows comparison of trading activity across USO roll and non-roll days. We calculate Normalized strategic volume for each strategy on every roll day and every usable non-roll day. A usable non-roll day (Control Day 0) is defined as a day when Days [-3, +3] relative to Control Day 0 does not have any overlap with [-3,+3] relative to USO roll day. The roll-day indicator variable takes the value of one for an USO roll day, and equals zero otherwise.

	Long Window						Short Window					
Panel B: Normalized Strategic	**ST1**	**ST2**	**ST3**	**ST4**	**ST5**	**ST6**	**ST1**	**ST2**	**ST3**	**ST4**	**ST5**	**ST6**
Front Month Contract												
Intercept	-306	-52	-851	368	3	-244	60	29	10	-84	-25	5
t(Intercept)	(-0.86)	(-0.77)	(-2.14)	(4.09)	(0.01)	(-1.44)	(1.70)	(2.31)	(0.38)	(-1.79)	(-0.51)	(0.40)
Roll_day	-2187	200	3178	219	9	-807	720	-4	-196	104	544	85
t(Roll_day)	(-2.06)	(0.98)	(2.66)	(0.81)	(0.01)	(-1.60)	(4.49)	(-0.07)	(-1.60)	(0.49)	(2.43)	(1.41)
Second Month Contract												
Intercept	-89	30	-396	-79	102	-83	70	54	70	146	173	0
t(Intercept)	(-0.42)	(0.43)	(-1.52)	(-0.87)	(0.59)	(-0.59)	(2.35)	(4.89)	(2.86)	(5.53)	(5.00)	(-0.06)
Roll_day	-1453	179	2028	67	2	56	736	43	-287	-6	13	0
t(Roll_day)	(-2.30)	(0.88)	(2.60)	(0.25)	(0.00)	(0.13)	(5.39)	(0.86)	(-2.58)	(-0.05)	(0.08)	(0.00)

Table A1: Understanding the USO ETF Stock Price Performance

The table reports the performance of various futures benchmarks, based on daily data from January 1990 to December 2013. All data except USO ETF share prices are obtained from the United States Energy Information Agency (EIA). Each mean has been annualized by multiplying by 250. The cost of storage is the futures term slope implied by the settlement prices of the first and second month crude oil futures contracts. The spot price return is the change in the implied (by the nearest futures price and the cost of carry relation) spot price, while the ex-post Spot Premium is the excess of the spot return over the cost of storage. The nearest futures contract return is based on price changes in the nearest-to-expiration contract, for all days including the last day of trading for the expiring contract. The second nearest futures contract return is based on price changes in the second-nearest-to-expiration contract, for all days including the last day of trading for the expiring contract. The Futures "Benchmark Return" series is based on the settlement price changes that track the USO roll. The divergence of benchmark returns from returns based on alternative roll dates provide estimates of the cost of executing trades at USO benchmark prices.

Spot Returns, USO Returns, and Futures Benchmarks

	ETF Period 4/10/06 to 12/31/13		2000s, pre ETF 1/1/00 to 4/9/06		1990s 1/1/90 to 12/31/99		Full Sample 1/1/90 to 12/31/13	
Days	1946		1564		2510		6020	
Variable	**Mean (x250)**	**T-stat**	**Mean (x250)**	**T-stat**	**Mean (x250)**	**T-stat**	**Mean (x250)**	**T-stat**
Spot Price Return: S + U	4.74%	0.33	15.11%	0.94	1.18%	0.09	5.95%	0.70
Cost of Storage S	12.38%	22.22	-7.69%	-12.04	-3.63%	-6.46	0.49%	1.39
Expost Spot Premium U	-7.64%	-0.53	22.80%	1.41	4.81%	0.35	5.46%	0.64
Return, Nearest Futures	-5.63%	-0.43	26.02%	1.79	2.41%	0.21	5.95%	0.79
Return, Second Nearest	-5.80%	-0.47	25.37%	1.86	5.24%	0.52	6.90%	1.02
Futures Benchmark Return	-8.79%	-0.70	20.80%	1.49	4.84%	0.45	4.59%	0.64
Nearest - Benchmark	3.16%	1.02	5.22%	2.17	-2.43%	-0.90	1.36%	0.83
Second - Benchmark	2.99%	2.48	4.57%	2.82	0.40%	0.15	2.31%	1.88
USO ETF Return	-8.42%	-0.69						

Figure 1: United States Oil Fund (USO) Share Price and Front Month NYMEX Crude Oil Price

The figure presents daily USO share prices and front month NYMEX crude oil prices over the period April 12, 2006 to December 31, 2013. The USO share price is obtained from Bloomberg while the NYMEX crude oil price is obtained from the United States Energy Information Agency.

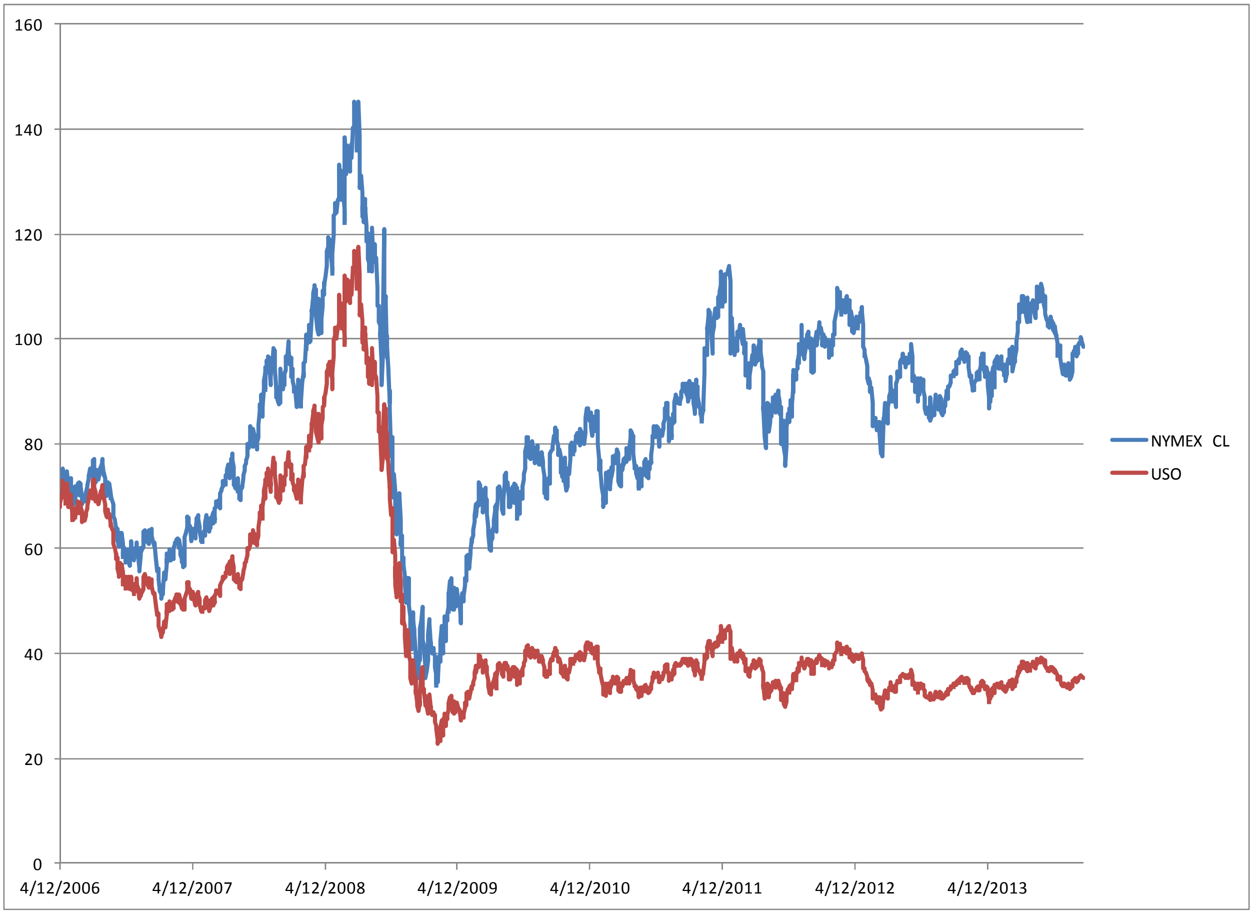


Figure 2: Breakeven Resilience

The figure plots the market resilience parameter that provides the liquidator with the same revenue with and without strategic trading for variations in the ratio of permanent price impact (λ) to temporary price impact(γ). A smaller resiliency parameter (a more resilient market) than the breakeven implies larger liquidator revenues with strategic trading than without, and vice versa. The shaded area below the curve indicates the region where strategic trading increases the liquidator's revenues.

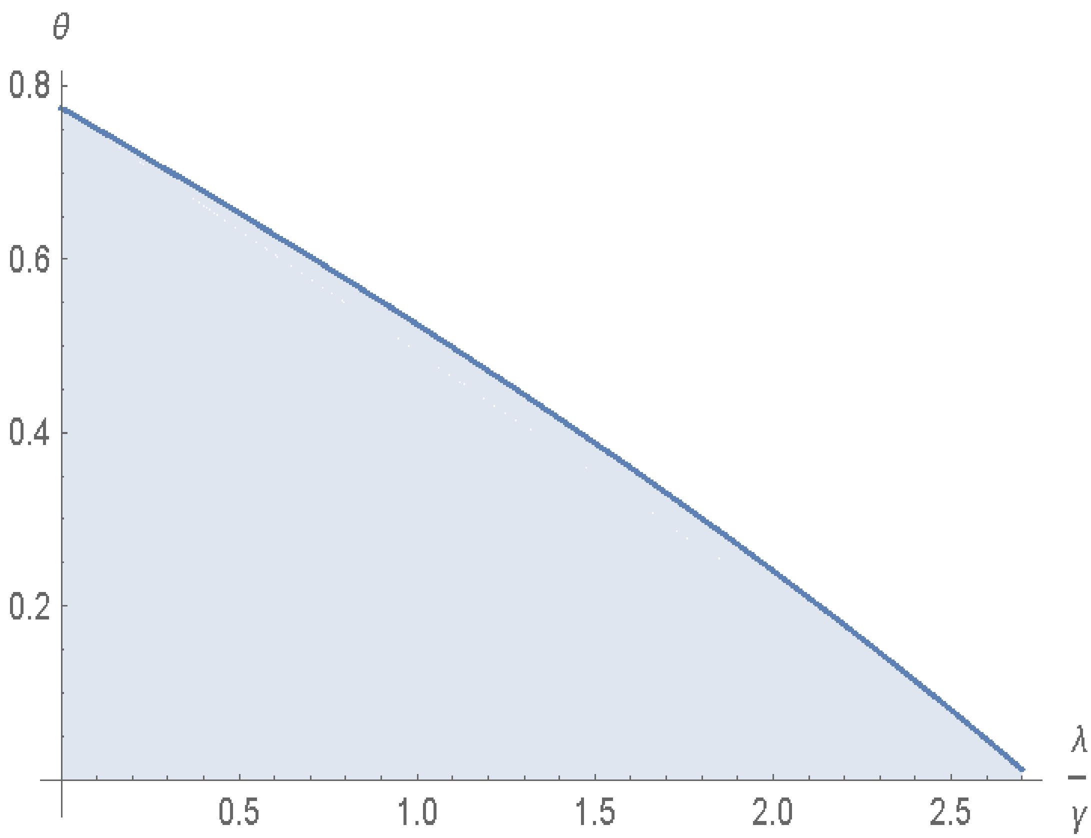
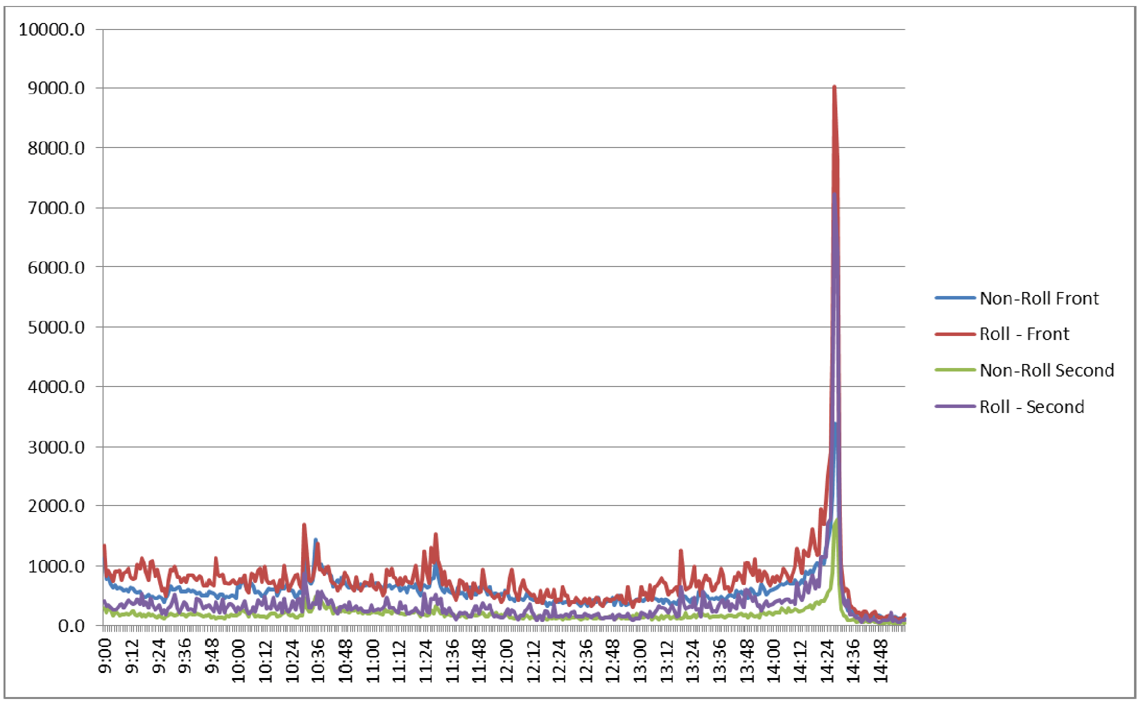


Figure 3: Trading Volume and Trade Imbalance on USO Roll and Non-roll days.

Reported are intra-day patterns in trading volume and quoted spread on USO roll days and non-roll days in the NYMEX Oil Futures market. We rely on Commodity Futures Trading Commission (CFTC) data for trading volume and Chicago Mercantile Exchange's Datamine database for quoted spread measures. USO's roll dates are identified based on their publicly stated roll strategy. Panel A reports on trading volume on roll days during the period March 1, 2008 to February 28, 2009. Non-roll days are defined as Days [-7,-3] before the roll day. Market quality measures are calculated each minute of the day and then averaged across USO roll and non-roll days. Trading volume includes all completed trades in NYMEX crude oil futures, including floor and block trades, as well as trades completed on GLOBEX. Quoted bid-ask spread (in basis points) is the difference between the lowest limit price for unexecuted sell orders and the highest limit price for unexecuted buy orders.

Panel A: Trading volume on roll and non-roll days



Panel B: Front month contract – quoted and effective spread

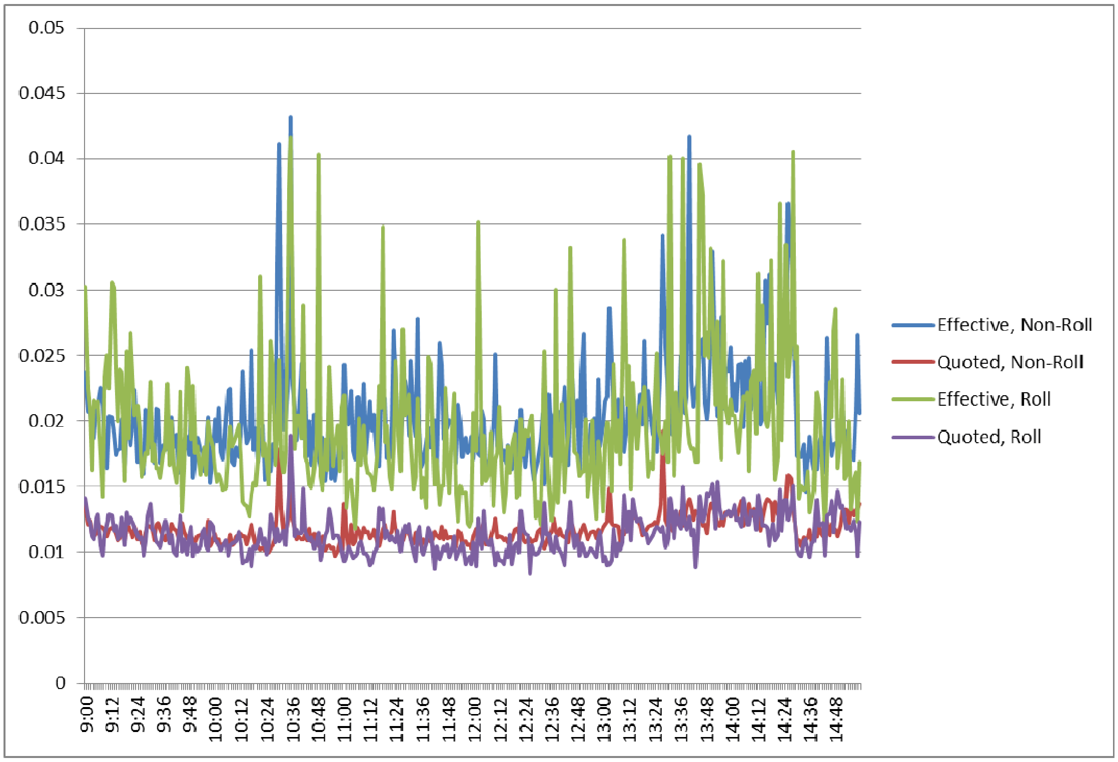


Figure 4: Intraday Limit Order Book Depth on USO Roll and Non-roll Days

Reported are limit order book depth on USO roll days and non-roll days in the NYMEX Oil Futures market. We rely on Chicago Mercantile Exchange's Datamine database on the GLOBEX electronic market for depth measures. USO's roll dates are identified based on their publicly stated roll strategy Non-roll days are defined as days [-7,-3] before the roll day. Market quality is calculated each minute of the day and then averaged across roll and non-roll days. Depth is the total volume of unexecuted sell (ask depth) and buy (bid depth) orders at prices within four ticks of the most competitive prices.

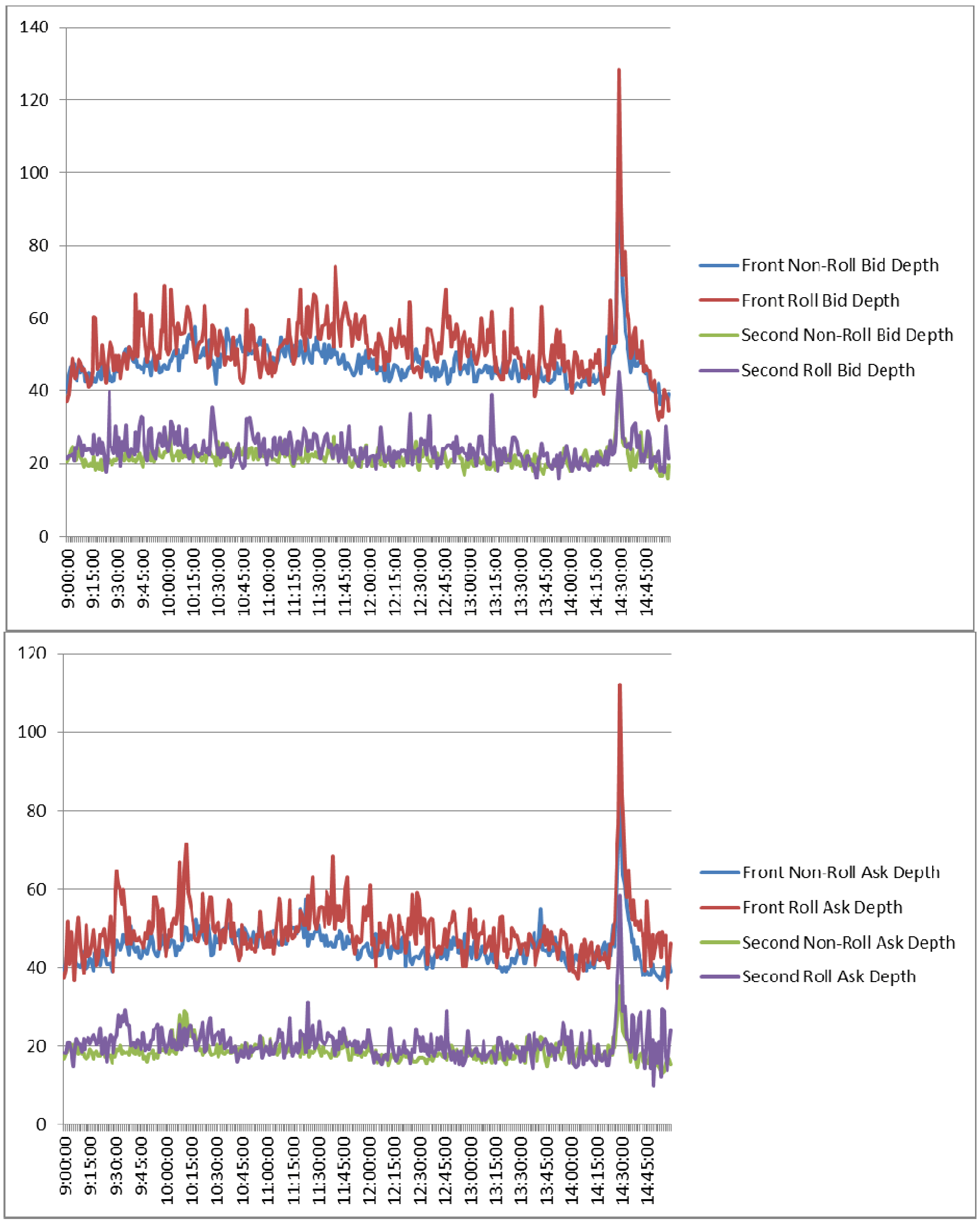
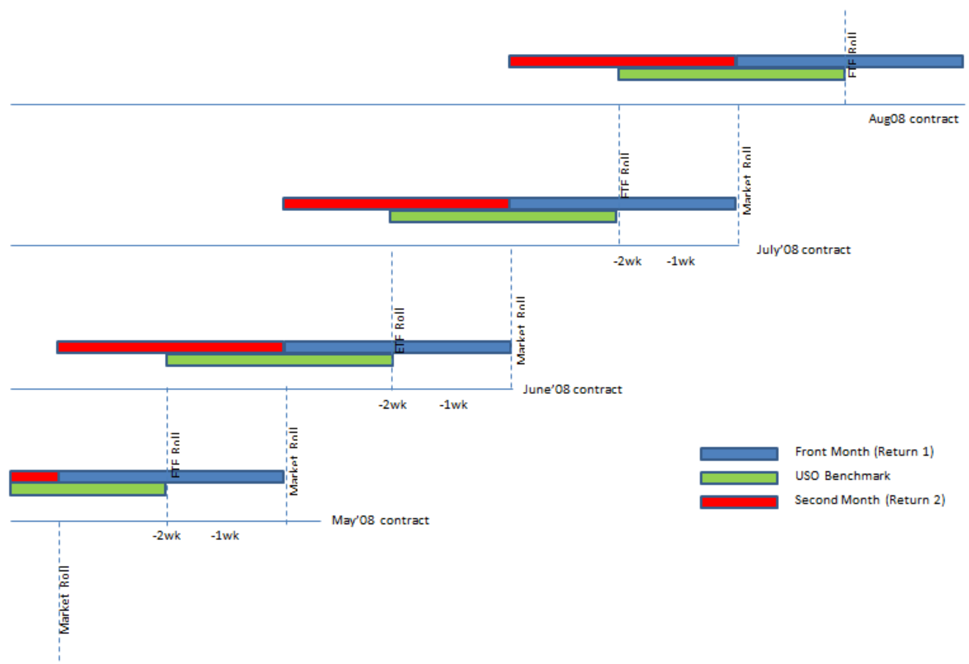


Figure 5: Identifying Front Month contract, Second Month contract, and the USO Roll Strategy

Trading activity in the futures market shifts from nearest-to-expiration (or front) contract to next-nearest-to-expiration (or second) contract a few days before the expiration of the front contract. We refer to the shift in overall trading activity as the Market Roll. USO's roll dates are identified based on their publicly stated investment objective, by which the fund tracks the price of the front NYMEX contract until two weeks before expiration, after which the fund tracks the second contract price. During the March 2008 to February 2009 sample period, USO's roll trades occurred on a single day.



Internet Appendix: Breakeven Resiliency Calculation

This appendix details the calculation of breakeven resiliency described in Section III and presented in Figure 2. Breakeven resiliency is the value of the resiliency parameter θ for which the liquidator's revenues will be the same with or without the presence of a monopolistic profit-maximizing strategic trader following the strategy described in Section III. The liquidator's revenues will be increased when resiliency is lower than this threshold, and decreased when it is higher.

The liquidator's revenues are given by expression (8), reproduced here:

$$LP = Q_L P_d \quad \text{(IA1)}$$

where Q_L is the quantity liquidated and P_d is the price during the liquidation period. The price during the liquidation period is given by expression (3), reproduced here:

$$P_D = V_0 + I_1 Q_p + I_0 Q_d \quad \text{(IA2)}$$

where V_0 is the initial value of the security, Q_p is the net signed order flow in the "pre" period, Q_d is the net signed order flow in the "during" period, and I_0 and I_1 are transformations of the parameters of the price impact function that are described below. Without the presence of the strategic trader, $Q_p = 0$ and $Q_d = -Q_L$, giving:

$$P_{DN} = V_0 - I_0 Q_L \quad \text{(IA3)}$$

With the strategic trader, $Q_p = -\rho_p^* Q_L$ and $Q_d = -(1 + \rho_d^*) Q_L$, giving:

$$P_{DS} = V_0 - I_1 \rho_p^* Q_L - I_0 (1 + \rho_d^*) Q_L \quad \text{(IA4)}$$

where ρ_p^* and ρ_d^* are proportionality parameters that specify the strategic trader's profit maximizing order flow in the "pre" and "during" periods relative to the liquidation quantity. Setting $P_{DN} = P_{DS}$ yields:

$$V_0 - I_0 Q_L = V_0 - I_1 \rho_p^* Q_L - I_0 (1 + \rho_d^*) Q_L \quad \text{(IA5)}$$

Which simplifies to:

$$I_1 \rho_p^* + I_0 \rho_d^* = 0 \quad \text{(IA6)}$$

I_1, ρ_p^*, and ρ_d^* are all functions of the resiliency parameter θ. Further, ρ_p^* and ρ_d^* depend on $I_0$and I_2, and I_2 is also a function of θ. We use the definitions of I_0, I_1, and I_2 from expressions (2)-(4), and make the substitution $S = \frac{\lambda}{\gamma}$ to yield:

$$I_0 = \lambda + \gamma = \gamma(S+1) \tag{IA7}$$

$$I_1 = \lambda + \theta\gamma = \gamma(S+\theta) \tag{IA8}$$

$$I_2 = \lambda + \theta^2\gamma = \gamma(S+\theta^2) \tag{IA9}$$

We also repeat the solutions for ρ_d^* and ρ_p^* from expression (10), and use the same substitution as above to give:

$$\rho_d^* = \frac{\theta-(\lambda/\gamma)-2}{\theta^2-4\theta+3(\lambda/\gamma)+6} = \frac{\theta-S-2}{\theta^2-4\theta+3S+6} \tag{IA10}$$

$$\rho_p^* = -\frac{\theta^3-3\theta^2-[(\lambda/\gamma)-2]\theta+2(\lambda/\gamma)^2+5(\lambda/\gamma)+2}{\theta^4-4\theta^3+[2(\lambda/\gamma)+4]\theta^2+[4(\lambda/\gamma)+8]\theta-3(\lambda/\gamma)^2-12(\lambda/\gamma)-12}$$

$$= -\frac{\theta^3-3\theta^2-(S-2)\theta+2S^2+5S+2}{\theta^4-4\theta^3+(2S+4)\theta^2+(4S+8)\theta-3S^2-12S-12} \tag{IA11}$$

Substituting expressions (IA7) – (IA11) into expression (IA6) and simplifying yields the following condition for breakeven θ:

$$\frac{\theta^4-4\theta^3+(S^2-S+4)\theta^2+(2S^2+10S+4)\theta+S^3-6S-4}{-\theta^4+4\theta^3-(2S+4)\theta^2-(4S+8)\theta+3S^2+12S+12} = 0 \tag{IA12}$$

This expression has four roots:

$$
\begin{aligned}
&\Bigg\{\Bigg\{\theta \to 1 - \frac{1}{2}\sqrt{\Bigg(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + \left(2^{1/3}\left(4 + 5s + s^2\right)^2\right)\Big/} \\
&\quad \Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297216 + 2350080\,s + \right.} \\
&\qquad 8028288\,s^2 + 15370560\,s^3 + 17792784\,s^4 + 12346992\,s^5 + 4526928\,s^6 + \\
&\qquad 366768\,s^7 - 265680\,s^8 - 48816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) + \frac{1}{3 \times 2^{1/3}} \\
&\quad \Big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297216 + 2350080\right.} \\
&\qquad s + 8028288\,s^2 + 15370560\,s^3 + 17792784\,s^4 + 12346992\,s^5 + 4526928 \\
&\qquad s^6 + 366768\,s^7 - 265680\,s^8 - 48816\,s^9 + 5616\,s^{10} - 432\,s^{11})\Big)^{1/3}\Bigg) - \\
&\frac{1}{2}\sqrt{\Bigg(4 + s - s^2 + \frac{1}{3}\left(-4 + s - s^2\right) - \left(2^{1/3}\left(4 + 5s + s^2\right)^2\right)\Big/} \\
&\quad \Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297216 + 2350080\,s + \right.} \\
&\qquad 8028288\,s^2 + 15370560\,s^3 + 17792784\,s^4 + 12346992\,s^5 + 4526928\,s^6 + \\
&\qquad 366768\,s^7 - 265680\,s^8 - 48816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) - \frac{1}{3 \times 2^{1/3}} \\
&\quad \Big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297216 + 2350080\right.} \\
&\qquad s + 8028288\,s^2 + 15370560\,s^3 + 17792784\,s^4 + 12346992\,s^5 + 4526928 \\
&\qquad s^6 + 366768\,s^7 - 265680\,s^8 - 48816\,s^9 + 5616\,s^{10} - 432\,s^{11})\Big)^{1/3} - \\
&\quad \left(64 - 16\left(4 - s + s^2\right) - 16\left(2 + 5s + s^2\right)\right)\Big/\Bigg(4\sqrt{\Bigg(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + } \\
&\qquad \left(2^{1/3}\left(4 + 5s + s^2\right)^2\right)\Big/\Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + \\
&\qquad\quad 2\,s^6 + \sqrt{\left(297216 + 2350080\,s + 8028288\,s^2 + 15370560\,s^3 + \right.} \\
&\qquad\quad 17792784\,s^4 + 12346992\,s^5 + 4526928\,s^6 + 366768\,s^7 - 265680 \\
&\qquad\quad s^8 - 48816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) + \frac{1}{3 \times 2^{1/3}}\Big(560 + 2208\,s + \\
&\qquad 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297216 + 2350080\,s + \right.} \\
&\qquad\quad 8028288\,s^2 + 15370560\,s^3 + 17792784\,s^4 + 12346992\,s^5 + 4526928 \\
&\qquad\quad s^6 + 366768\,s^7 - 265680\,s^8 - 48816\,s^9 + 5616\,s^{10} - 432\,s^{11})\Big)^{1/3}\Bigg)\Bigg)\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{\theta \to 1 - \frac{1}{2}\sqrt{\Big(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + \left(2^{1/3}\left(4 + 5\,s + s^2\right)^2\right)\Big/} \\
&\quad \Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297\,216 + 2\,350\,080\,s + \right.} \\
&\qquad 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928\,s^6 + \\
&\qquad 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11}))^{1/3}\Big) + \\
&\quad \frac{1}{3\times 2^{1/3}}\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297\,216 + \right.} \\
&\qquad 2\,350\,080\,s + 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + \\
&\qquad 4\,526\,928\,s^6 + 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11}))^{1/3}\Big) + \\
&\frac{1}{2}\sqrt{\Big(4 + s - s^2 + \frac{1}{3}\left(-4 + s - s^2\right) - \left(2^{1/3}\left(4 + 5\,s + s^2\right)^2\right)\Big/} \\
&\quad \Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297\,216 + 2\,350\,080\,s + \right.} \\
&\qquad 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928\,s^6 + \\
&\qquad 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11}))^{1/3}\Big) - \\
&\quad \frac{1}{3\times 2^{1/3}}\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297\,216 + \right.} \\
&\qquad 2\,350\,080\,s + 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + \\
&\qquad 4\,526\,928\,s^6 + 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11}))^{1/3} - \\
&\quad \left(64 - 16\left(4 - s + s^2\right) - 16\left(2 + 5\,s + s^2\right)\right)\Big/\Big(4\sqrt{\Big(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + } \\
&\qquad \left(2^{1/3}\left(4 + 5\,s + s^2\right)^2\right)\Big/\Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + \\
&\qquad\quad 2\,s^6 + \sqrt{\left(297\,216 + 2\,350\,080\,s + 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + \right.} \\
&\qquad\qquad 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928\,s^6 + 366\,768\,s^7 - 265\,680 \\
&\qquad\qquad s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11}))^{1/3}\Big) + \frac{1}{3\times 2^{1/3}}\big(560 + 2208\,s + \\
&\qquad 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{\left(297\,216 + 2\,350\,080\,s + \right.} \\
&\qquad\quad 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928 \\
&\qquad\quad s^6 + 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11}))^{1/3}\Big)\Big)\Big)\Big\},
\end{aligned}
$$

$$\Big\{\theta \to 1 + \frac{1}{2}\sqrt{\Big(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + \left(2^{1/3}\left(4 + 5s + s^2\right)^2\right)\Big/}$$

$$\Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{}\,(297\,216 + 2\,350\,080\,s +$$

$$8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928\,s^6 +$$

$$366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) +$$

$$\frac{1}{3\times 2^{1/3}}\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{}\,(297\,216 +$$

$$2\,350\,080\,s + 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 +$$

$$4\,526\,928\,s^6 + 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) -$$

$$\frac{1}{2}\sqrt{\Big(4 + s - s^2 + \frac{1}{3}\left(-4 + s - s^2\right) - \left(2^{1/3}\left(4 + 5s + s^2\right)^2\right)\Big/}$$

$$\Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{}\,(297\,216 + 2\,350\,080\,s +$$

$$8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928\,s^6 +$$

$$366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) -$$

$$\frac{1}{3\times 2^{1/3}}\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{}\,(297\,216 +$$

$$2\,350\,080\,s + 8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 +$$

$$4\,526\,928\,s^6 + 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3} +$$

$$\left(64 - 16\left(4 - s + s^2\right) - 16\left(2 + 5s + s^2\right)\right)\Big/\Big(4\sqrt{\Big(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) +}$$

$$\left(2^{1/3}\left(4 + 5s + s^2\right)^2\right)\Big/\Big(3\,\big(560 + 2208\,s + 3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 +$$

$$2\,s^6 + \sqrt{}\,(297\,216 + 2\,350\,080\,s + 8\,028\,288\,s^2 + 15\,370\,560\,s^3 +$$

$$17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928\,s^6 + 366\,768\,s^7 - 265\,680$$

$$s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big) + \frac{1}{3\times 2^{1/3}}\big(560 + 2208\,s +$$

$$3180\,s^2 + 1894\,s^3 + 282\,s^4 - 78\,s^5 + 2\,s^6 + \sqrt{}\,(297\,216 + 2\,350\,080\,s +$$

$$8\,028\,288\,s^2 + 15\,370\,560\,s^3 + 17\,792\,784\,s^4 + 12\,346\,992\,s^5 + 4\,526\,928$$

$$s^6 + 366\,768\,s^7 - 265\,680\,s^8 - 48\,816\,s^9 + 5616\,s^{10} - 432\,s^{11})\big)^{1/3}\Big)\Big)\Big)\Big\},$$

$$\Big\{\theta \to 1 + \frac{1}{2}\sqrt{\Big(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + \left(2^{1/3}\left(4 + 5 s + s^2\right)^2\right) \Big/ \Big(3\left(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{\left(297216 + 2350080 s + 8028288 s^2 + 15370560 s^3 + 17792784 s^4 + 12346992 s^5 + 4526928 s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}\right)}\right)^{1/3}\Big) + \frac{1}{3 \times 2^{1/3}}\left(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{\left(297216 + 2350080 s + 8028288 s^2 + 15370560 s^3 + 17792784 s^4 + 12346992 s^5 + 4526928 s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}\right)}\right)^{1/3}\Big)} +$$

$$\frac{1}{2}\sqrt{\Big(4 + s - s^2 + \frac{1}{3}\left(-4 + s - s^2\right) - \left(2^{1/3}\left(4 + 5 s + s^2\right)^2\right) \Big/ \Big(3\left(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{\left(297216 + 2350080 s + 8028288 s^2 + 15370560 s^3 + 17792784 s^4 + 12346992 s^5 + 4526928 s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}\right)}\right)^{1/3}\Big) - \frac{1}{3 \times 2^{1/3}}\left(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{\left(297216 + 2350080 s + 8028288 s^2 + 15370560 s^3 + 17792784 s^4 + 12346992 s^5 + 4526928 s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}\right)}\right)^{1/3} + \left(64 - 16\left(4 - s + s^2\right) - 16\left(2 + 5 s + s^2\right)\right) \Big/ \Big(4\sqrt{\Big(s - s^2 + \frac{1}{3}\left(4 - s + s^2\right) + \left(2^{1/3}\left(4 + 5 s + s^2\right)^2\right) \Big/ \Big(3\left(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{\left(297216 + 2350080 s + 8028288 s^2 + 15370560 s^3 + 17792784 s^4 + 12346992 s^5 + 4526928 s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}\right)}\right)^{1/3}\Big) + \frac{1}{3 \times 2^{1/3}}\left(560 + 2208 s + 3180 s^2 + 1894 s^3 + 282 s^4 - 78 s^5 + 2 s^6 + \sqrt{\left(297216 + 2350080 s + 8028288 s^2 + 15370560 s^3 + 17792784 s^4 + 12346992 s^5 + 4526928 s^6 + 366768 s^7 - 265680 s^8 - 48816 s^9 + 5616 s^{10} - 432 s^{11}\right)}\right)^{1/3}\Big)}\Big)\Big)}\Big\}$$

(IA13)

Only the second of the four roots is an economically sensible solution for breakeven θ. This solution is graphed against $\frac{\lambda}{\gamma}$ in Figure 2. The other roots are imaginary or imply values of θ outside of its theoretical range of [0,1]. We have validated this analytical solution by implementing a numerical solution that yields the same values of breakeven θ for a large range of values of $\frac{\lambda}{\gamma}$.