Ownership Structure, Incentives, and Asset Prices\

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Abstract

We develop a dynamic equilibrium model to study the interplay among managerial incentive contracts, the ownership dynamics of large controlling shareholders, and asset prices. Our unified framework integrates an asset pricing model with a dynamic principal-agent model by distinguishing a firm’s large controlling shareholders from small shareholders. Large shareholders play the role of mediators who determine incentive contracts for firm managers, while also influencing asset prices through their dynamic trading decisions. We characterize the equilibrium in which the ownership dynamics of large shareholders, long-term contracts for managers, and asset prices are simultaneously and endogenously determined. Relative to the benchmark owner-manager case, the incorporation of agency conflicts between large shareholders and managers leads to more volatile stock returns, higher expected stock returns, and lower Sharpe ratios. The facilitation of risk-sharing through optimal contracts makes large shareholders’ ownership dynamics effectively insulated from fluctuations in firm-specific parameters. We exploit our unified framework to derive a number of novel empirical implications for the equilibrium relations among block ownership levels, managerial incentives, and stock return characteristics. Our results suggest that block ownership and incentive contracts serve as complementary mechanisms in influencing corporate governance.

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1 Introduction

Traditional theoretical research in financial economics reveals a fundamental dichotomy between its two cornerstones; asset pricing and corporate finance (Gorton, He, and Huang (2014; hereafter GHH)). At a broad level, corporate finance research focuses on how agency conflicts among a firm’s various stakeholders influence firms’ cash flows. The market’s valuation of the cash flows through the underlying pricing kernel is, however, typically exogenously specified. In contrast, traditional asset pricing models usually view corporate cash flows as exogenous and focus on identifying a pricing kernel to price the assumed cash flows. In reality, however, firms’ earnings and their valuation by market participants are simultaneously and endogenously determined.

A nascent literature examines the feedback between the effects of agency conflicts among firms’ stakeholders on cash flows and security prices that represent the values of various claims to the cash flows. One group of extant studies (e.g., Admati, Pfleiderer and Zechner (1994), DeMarzo and Urošević (2006; hereafter DU), and GHH) specifically focusses on the role that asset prices play in facilitating risk-sharing between small and large shareholders of firms and, thereby, the incentives of large shareholders who control firms. These studies abstract away from non-market mechanisms such as incentive contracts that play a central role in mediating the trade-off between risk-sharing and incentives. Another group of studies examines how contracts interact with asset prices (e.g., Ou-Yang (2005)), but abstract away from the distinction between large and small shareholders. The frameworks analyzed by these studies confront the fundamental conceptual problem that stems from assuming that shareholders are competitive price-takers, but are nevertheless able to coordinate with each other to enforce managerial incentive contracts.

We contribute to the literature by developing a dynamic equilibrium model that bridges the gap between the frameworks analyzed by the aforementioned groups of studies. Our unified framework synthesizes an asset pricing model with a dynamic principal-agent model by distinguishing a firm’s large controlling shareholders from small shareholders. Large shareholders play the role of mediators who determine incentive contracts for firm managers, while also influencing asset prices through their dynamic trading decisions. Our perspective is motivated by empirical evidence that the vast majority of firms around the world are controlled by large shareholders (e.g., Holderness (2009))\(^1\).

\(^{1}\)Holderness (2009) shows that 96% of randomly selected Compustat- and CRSP-listed firms have shareholders who own at least 5% of the firm’s common stock (“blockholders”), and that 89% of S&P 500 firms have blockholders.
as well as theoretical research that highlights the importance of large shareholders in influencing corporate governance (e.g., Shleifer and Vishny (1986), Holmstrom and Tirole (1993), Burkart, Gromb, and Panunzi (1997)).

We exploit our framework to provide a full-fledged and dynamic equilibrium description of the interactions among large shareholders’ ownership dynamics, managerial long-term contracts, and stock prices. In the benchmark owner-manager case, which corresponds to the scenario studied by DU, one obtains the counterfactual implication that stock prices are deterministic functions of time. In contrast, the incorporation of agency conflicts between large shareholders and managers results in stochastically evolving stock prices stemming from the provision of incentives to managers. Relative to the benchmark owner-manager scenario, the agency conflicts between large shareholders and managers lead to more volatile stock returns, higher expected stock returns, and lower Sharpe ratios, ceteris paribus.

We derive novel testable implications for the equilibrium relations among the degrees of concentration of firms’ ownership structures, firms’ earnings and stock return characteristics, as well as managerial incentives. In particular, we show that (i) expected stock returns, stock return volatilities, and Sharpe ratios decline with block ownership; (ii) managerial pay-performance sensitivities (PPS) are negatively related to expected returns, return volatilities, and Sharpe ratios; (iii) managerial PPS increase with block ownership; and (iv) expected stock returns and volatilities decline with the productivities of managerial effort, but increase with earnings volatilities. Our analysis highlights the importance of examining the relations among these endogenous variables in an equilibrium framework.

We model an infinite time horizon, continuous-time economy with a representative all-equity firm. The firm’s shareholders comprise of small shareholders, who are competitive price-takers, and a representative large shareholder. The large shareholder hires the firm’s manager who influences the firm’s earnings through his unobservable effort. All agents can dynamically consume over time and also have access to a risk-free money market account. The manager’s savings are observable. As in DU, the large shareholder trades the firm’s stock at discrete dates, but cannot commit to her future trades, while small shareholders trade the stock continuously. The stock price is endogenously determined at each instant of time by the market clearing condition for the firm’s equity.

The large shareholder offers the manager a long-term incentive contract that is contingent on
the firm’s earnings. The two parties can renegotiate their existing contract after each trade by the large shareholder. The large shareholder’s trades influence the terms of the manager’s contracts that affect the manager’s effort and the firm’s output. The firm’s earnings, in turn, influence the stock price and, therefore, the large investor’s trades. In this manner, the large shareholder’s trading dynamics, managerial incentives, and the stock price are simultaneously and endogenously determined. We focus on Strong Markov Perfect Public Equilibria (SMPPE) in which the large shareholder’s consumption and optimal contracting choices in any period depend only on the state vector that comprises of her ownership at the beginning of the period, her holdings in the money market account, and the manager’s promised payoff or expected continuation utility from his long-term contract.

We analytically characterize the equilibrium when the firm’s earnings process is Gaussian and all agents have “ConstantAbsoluteRiskAversion” (CARA) preferences, and compare it with the equilibrium in the benchmark “owner-manager” scenario in which there is no separation between the large shareholder and the manager. As in the owner-manager scenario, the large shareholder’s ownership dynamics are deterministic. Because the earnings process is i.i.d. over time, and there are no wealth effects with CARA preferences, the large shareholder’s ownership dynamics are determined by a sequence of static mean-variance optimization problems. In sharp contrast with the benchmark owner-manager case, however, the equilibrium stock price varies stochastically. The stochastic variation arises due to the random evolution of the manager’s promised payoff process that reflects incentive provision to the manager via his long-term contract.

For a given ownership stake of the large shareholder, the expected excess dollar return of the stock and the dollar stock return volatility are lower than in the benchmark owner-manager scenario. The lower dollar stock return volatility in the agency scenario arises due to the fact that incentive provision entails nontrivial risk-sharing between the large shareholder and the manager. As the stock price is determined by the residual earnings (that is, total earnings net of the managerial compensation payments), the higher volatility of managerial compensation payments effectively lowers the volatility of the stock returns, which also lowers the expected dollar stock return. The agency costs of risk-sharing between the large shareholder and the manager, however, have a negative impact on the stock price relative to the benchmark owner-manager scenario that outweighs the negative effects on the expected dollar return and volatility. Consequently, the expected pro-
portional stock return and stock return volatility—the dollar stock return and return volatility normalized by the stock price—are higher than in the owner-manager scenario.

We exploit our analytical characterization of the equilibrium to derive novel testable implications for the relations among the large shareholder’s ownership stake, managerial incentives, and stock return characteristics. The sensitivity of the manager’s pay to the stock return is inversely related to the expected proportional stock return (hereafter, stock return), stock return volatility, and Sharpe ratio. The effects of the large investor’s ownership on stock returns and managerial incentives, as well as the impact of firm-specific parameters—the mean and volatility of earnings—on the equilibrium are difficult to pin down analytically for general parameter values. We, therefore, investigate these effects numerically by calibrating the model to match key moments.

Our analysis of the calibrated model shows that an increase in block ownership has a negative effect on the expected return, return volatility, and Sharpe ratio of the stock. The large shareholder’s ownership has direct and indirect effects on the stock return. The direct effect stems from the fact that an increase in the large shareholder’s ownership stake reduces the stock’s liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected return of the stock. The indirect effect arises from the fact that an increase in the large shareholder’s ownership stake increases the incentive power of the manager’s compensation contract, thereby lowering the volatility of residual cash flows to shareholders and the expected return of the stock. Because the manager’s pay-performance sensitivity is also negatively related to the expected stock return and volatility, the large shareholder’s ownership stake has a positive effect on the manager’s pay-performance sensitivity. This finding, which suggests that block ownership and explicit contracts complement each other in corporate governance, is consistent with the evidence in Almazan, Hartzell, and Starks (2005) and Kim (2010). We also obtain the intuitive results that the productivity of the manager’s effort has a positive effect on incentives, and a negative effect on the excess stock return and volatility. The earnings volatility has a negative effect on incentives, and a positive effect on the expected stock return and volatility.

The separation of ownership and control, and the facilitation of risk-sharing through the manager’s incentive contract effectively insulates the large investor from variations in firm-specific parameters. Consequently, the dynamics of the large investor’s ownership stake are insensitive to firm-specific parameters—the mean and volatility of earnings—and are primarily determined by
the risk aversions of the large and small shareholders as well as the manager. In contrast, the large shareholder’s ownership dynamics are significantly influenced by firm-specific parameters in the benchmark owner-manager model.

As mentioned earlier, we contribute to the burgeoning literature that studies the interactions between agency conflicts and asset prices. GHHH abstract away from non-marketed managerial incentive contracts and assume that agents do not have access to a savings technology. Ou-Yang (2005) considers a contracting framework with lump-sum compensation at the terminal date as in Holmstrom and Milgrom (1987). We consider a more general contracting problem in which agents can consume inter-temporally and have access to a savings technology. As in studies in the dynamic contracting literature (see the survey in Bolton and Dewatripont (2005)), a recursive characterization of the contracting problem is facilitated by including the manager’s promised payoff as an additional state variable.

Among studies in the vast literature on corporate governance, our study is also related to the ongoing literature initiated by Gompers, Ishii and Metrick (2003) that explores the effects of corporate governance mechanisms on equity prices. Cremers and Nair (2005) show complementary interactions between internal (active shareholder ownership) and external (market for corporate control, that is, takeover vulnerability) governance mechanisms in generating long-term abnormal equity returns. Parigi, Pelizzon, and von Thadden (2015) theoretically and empirically show that the quality of corporate governance, which is endogenously chosen by the firm, correlates positively with CAPM beta and idiosyncratic volatility and negatively with returns on assets. In a broad sense, our study complements the literature by examining how the two important governance mechanisms—large shareholder ownership and optimal contracting—endogenously affect firm stock prices and returns.

Lastly, in the asset pricing literature, there are several studies that examine the effects of large shareholders (e.g., Cvitanic (1997), El Karoui, Peng and Quenez (1997), Cuoco and Cvitanic (1998), Subramanian and Jarrow (2001)). Unlike our study, this literature assumes that the effects of a large investor’s trades on asset prices are exogenous.
2 The Model

We consider an economy with a representative, all-equity firm. The firm’s shareholders are comprised of two groups: “large” shareholders or blockholders who hold a block of shares in the firm and a continuum of dispersed small shareholders who collectively hold the remaining equity stake. Because our focus is on asset pricing implications, we assume that both groups of shareholders are risk averse. Based on prior research on firm ownership (e.g., DeMarzo and Urošević [DU] (2006)), and empirical evidence (e.g., Cella, Ellul, and Giannetti (2013)), we consider different holding periods for the two groups of investors.\(^2\) That is, small shareholders trade shares of the firm continuously, while the large shareholder trades the firm’s shares at a discrete set of dates that are equally spaced for notational convenience. In addition to the firm’s shares, both types of investors can trade a risk-free bond (savings account) continuously.

In reality, major strategic corporate decisions require the approval of corporate boards that are significantly influenced by large shareholders (e.g., see Shleifer and Vishny (1997), Holderness (2003), Tirole (2006)). Cronqvist and Fahlenbrach (2009) document evidence for significant blockholder effects on a number of important corporate decisions such as investment, financial, and executive compensation policies. For simplicity, we ignore strategic behavior among different blockholders—that is, they behave as a monolithic unit in their collective interest—so that we refer to them as a single representative blockholder. The large shareholder hires a risk-averse manager to operate the firm who affects the firm’s output through his costly, unobservable effort. The large shareholder influences the manager’s effort through a long-term incentive contract that is contingent on the firm’s observable output process. The two parties can renegotiate their existing contract after trades by the large shareholder, but commit to the contract until the next trading date. Renegotiations must be weakly Pareto improving to be accepted. We now describe the various elements of the model in detail.

\(^2\)Brav, Jiang, and Thomas (2008) document that the median holding period of hedge fund activists, who file with the SEC within 10 days of acquiring more than 5% of a publicly traded company’s shares with the intention to influence the management of the company, is about 20 months. Cella, Ellul, and Giannetti (2013) show that institutional investors, on average, rotate almost 13% of their portfolio each quarter, and 52% in each year. They also document that most Compustat- and CRSP-listed firms have institutional investors that tend to hold their positions for between 14 months and 40 months.
2.1 Firm Output Process

We consider a continuous-time model in which uncertainty is generated by a standard Brownian motion \( Z_t \) on a probability space \((\Omega, \mathcal{F}, P)\). The firm’s cumulative output \( X_t \) evolves as follows:

\[
dX_t = \mu(a_t, t)dt + \sigma dZ_t, \tag{1}
\]

where \( a_t \in [0, A] \) with \( 0 < A < \infty \) is the manager’s effort at date \( t \) and the constant \( \sigma > 0 \) is the output volatility. The output process is publicly observable. We denote the (complete and augmented) information filtration generated by the cumulative output process, \( X_t \), by \( \{ \mathcal{F}_t \} \). All instantaneous cash flows net of the manager’s compensation are paid out to the firm’s shareholders.

2.2 Large and Small Shareholders

The firm has a representative large shareholder, \( L \), and a continuum of small, dispersed shareholders, \( S \) (uniformly indexed over the unit interval, that is, \( S \in [0, 1] \)). We normalize the total number of the firm’s shares outstanding to one. As in DU, small shareholders trade continuously, while the large shareholder trades on the discrete set of dates \( \{ t_i; i = 1, 2, \ldots, N \} \) with \( t_1 = 0, t_{N+1} = \infty, \) and \( t_{i+1} - t_i = \Delta. \) A small shareholder \( S \)'s shareholding process, \( \theta^S = \{ \theta^S_t \} \), is an \( \{ \mathcal{F}_t \} \)-adapted process, where \( \theta^S_t \) is the number of shares held by \( S \) at date \( t \). The large shareholder \( L \)'s ownership process, \( \Theta^L = \{ \Theta^L_t \} \), is an \( \{ \mathcal{F}_t \} \)-adapted process, where \( \Theta^L_t \) is \( L \)'s holding at date \( t \). \( L \)'s and \( S \)'s ownership levels are publicly observable. \( L \)'s shareholding process is piecewise constant (that is, it is constant between successive trading dates), and is right continuous with left limits. If \( L \)'s shareholdings just prior to her trade at date \( t_i \) are \( \Theta^L_{t_i^-} \), the difference \( \Theta^L_{t_i} - \Theta^L_{t_i^-} > 0 (< 0) \) represents the number of shares that she purchases (sells) at date \( t_i \). As in DU, \( L \) cannot commit to her trading policy. Due to the market clearing condition for the firm’s shares, the total number of shares collectively held by small shareholders at any date \( t \in [t_i, t_{i+1}) \) is \( \int_0^1 \theta^S_t dS = 1 - \Theta^L_t = 1 - \Theta^L_{t_i} \).

Investors also have access to a risk-free bond that is in perfectly elastic supply so that it pays a continuously compounded constant return \( r > 0. \) The investors are initially endowed with wealth,

\(^3\) As in DU, we assume a finite number of trading dates, \( N \), in our main analysis to facilitate a direct comparison with their results. We can extend our analysis to incorporate an infinite number of trading dates for \( L \) as we describe in Section A2 of the Appendix. Under additional transversality conditions that guarantee the finiteness of the agents' value functions, our main implications are unchanged.

\(^4\) We can alternatively assume that the risk-free bond is in zero net supply, which would endogenously determine
Both groups of investors are risk averse and their preferences are described by the utility function, \( u^j(c^j_t) \) for \( j \in \{L, S\} \), that is twice continuously differentiable, strictly increasing, and strictly concave in the instantaneous consumption rate, \( c^j_t \), at date \( t \).

### 2.3 Preferences and Contracting

The large shareholder hires a manager, \( M \), to operate the firm and offers him a long-term contract. The contract can be renegotiated by both parties at any of \( L \)'s trading dates, \( t_i \), for \( i = 1, 2, \ldots, N \), provided that it is in the interests of both parties to do so. Both parties, however, commit to the terms of the contract over the trading period, \([t_i, t_{i+1})\). The time line of events in any trading period \([t_i, t_{i+1})\) is as follows. At trading date \( t_i \), given her prior shareholdings \( \Theta^L_{t_i} = \Theta^L_{t_i-1} \), \( L \) makes a trading decision \( \Theta^L_t \). \( L \) and \( M \) then renegotiate the terms of their existing contract. In making her trade, \( L \) rationally anticipates its effects on the terms of the renegotiated contract, the manager’s effort, and the firm’s output and stock price. However, \( L \) cannot commit to the terms of the renegotiated contract before she makes her trade. Small shareholders \( S \) competitively and continuously trade shares of the firm—that is, they take the stock price as given—by rationally anticipating \( L \)'s trading and contracting decisions. The stock price, \( P_t \), at any time \( t \) is determined by market clearing for the firm’s shares.

Because any contractual terms that are renegotiated in the future can be rationally incorporated in the original contract, we can, without loss of generality, restrict consideration to long-term contracts that are renegotiation-proof (RP) at each date \( t_i \) for \( i = 1, 2, 3, \ldots, N \) (see also Laffont and Martimort (2002)). A long-term contract is RP at each date \( t_i \) if it is weakly Pareto optimal among the set of continuation contracts that are themselves RP at future trading dates \([t_{i+1}, t_{i+2}, \ldots]\) (see, for example, Wang (2000)).

A long-term contract \( \Pi \) can be formally expressed as \( \Pi \equiv \{c^M, a^M\} \), where we augment the definition of the manager \( M \)'s contract to include the recommended effort process, \( \{a^M\} \), and the manager’s compensation process, \( \{c^M\} \). \( M \)'s effort and compensation processes are \( \mathcal{F}_t \)-adapted stochastic processes. The manager continuously chooses an unobservable effort level \( a^M_t \in [0, A] \), given his contractual compensation, which affects the firm’s output \( X_t \) as shown in (1). He incurs the instantaneous effort cost, \( \Psi(a_t, t) \), that is twice continuously differentiable, increasing and convex the risk-free rate over time without altering our main results.
in the effort level, \( a_t^M \). Throughout, we assume that the upper bound, \( A \), on \( M \)'s effort is large enough that \( M \)'s optimal effort choices take interior values. The manager can also invest in the risk-free bond and the stock, but his savings are observable. Without loss of generality, therefore, we can restrict consideration to "savings proof" contracts in which \( M \)'s contract directly specifies his consumption at each date. Therefore, the process \( c^M \) specified by the contract is \( M \)'s consumption process.

The manager’s total utility function, \( u^M(c^M_t, a^M_t) \), satisfies \( u^M_c \equiv \partial u^M / \partial c > 0 \), \( u^M_{cc} \equiv \partial^2 u^M / \partial c^2 < 0 \), \( u^M_a \equiv \partial u^M / \partial a < 0 \), and \( u^M_{aa} \equiv \partial^2 u^M / \partial a^2 < 0 \). The cost of effort is a monetary cost at the same unit of his consumption. The manager’s continuation value or promised payoff at any date \( t \)—that is, his expected utility from the future consumption and effort under the contract \( \Pi \)—is given by

\[
W_t^M(\Pi) \equiv E_t^{a^M} \left[ \int_t^\infty e^{-\delta^M (\tau - t)} u^M(c^M_\tau, a^M_\tau) d\tau \right],
\]

where \( E_t^{a^M} [\cdot] \) denotes the conditional expectation at time \( t \) with respect to the probability distribution induced by \( M \)'s effort choices \( a^M \), and \( \delta^M \) is \( M \)'s time discount rate.

The contract must be incentive compatible for the manager at each time \( t \). That is, given the stream of future compensation \( c^M \), it is optimal for the manager to choose the effort levels \( a^M \) specified by the contract. By (2),

\[
a^M = \arg \max_{\Pi'=(c^M,a')} W_t^M(\Pi')
\]

\[
= \arg \max_{a'} E_t^{a'} \left[ \int_t^\infty e^{-\delta^M (\tau - t)} u^M(c^M_\tau, a'_\tau) d\tau \right].
\]

Next, the contract must satisfy the manager’s participation constraint at date zero, that is,

\[
W_0^M(\Pi) \geq W_0,
\]

where \( W_0 \) is \( M \)'s reservation utility at date zero. A contract is incentive feasible if it is incentive compatible and satisfies the manager’s participation constraint at date zero.

Finally, we characterize the renegotiation-proofness (RP) constraints. Let \( \Pi_{(t_i, \infty)} \) be the restriction of the long-term contract \( \Pi \) to the period after date \( t_i \), where \( t_i \) is a trading date for \( L \).
The RP constraints can be recursively characterized as follows (e.g., see Wang (2000), Giat and Subramanian (2013)).

**Renegotiation Proofness:**

1. The contract, $\Pi_{[t_N, \infty)}$, over the last trading period $[t_N, \infty]$ is weakly Pareto optimal among all incentive compatible, continuation contracts, $\Pi'_{[t_N, \infty)}$.

2. At any trading date $t_i; i < N$, the contract, $\Pi_{[t_i, \infty)}$, is weakly Pareto optimal among all incentive compatible, continuation contracts, $\Pi'_{[t_i, \infty)}$, that are themselves RP at future trading dates $\{t_{i+1}, ..., t_N\}$.

The key consequence of renegotiation proofness for our analysis is *sequential optimality*, that is, the manager’s contract must be sequentially optimal at each trading date $t_i$ for $i = 1, 2, 3, ..., N$.

To solve for the equilibrium fully, we assume the following for the rest of the paper. First, all the players have constant absolute risk aversion (CARA) preferences over their consumption, and their risk aversion parameters are $\gamma_L$, $\gamma_M$, and $\gamma_S$, respectively. More specifically, their utility functions are given by

$$u^M(c^M_t, a^M_t) = -\frac{1}{\gamma_M} e^{-\gamma_M (c^M_t - \Psi(a^M_t))},$$

$$u^j(c^j_t) = -\frac{1}{\gamma^j} e^{-\gamma^j c^j_t}, \text{ for } j = L, S. \quad (5)$$

Second, the firm’s mean cash flow and the cost of exerting effort are given by

$$\mu(a_t, t) = \mu(a_t) = \mu_0 + \mu_1 a_t; \quad \Psi(a_t, t) = \Psi(a_t) = \frac{1}{2} \psi a_t^2, \quad (6)$$

where the constants $\mu_0$, $\mu_1$, and $\psi$ are positive.

### 2.4 The Objectives of Large and Small Shareholders

$L$ chooses her ownership policy, $\Theta^L$, and consumption policy, $c^L$, and the manager’s incentive feasible RP contract, $\Pi$, to maximize her expected utility, that is, $L$ solves

$$\max_{(\Theta^L, c^L, \Pi)} E^a_0 \left[ \int_0^\infty e^{-\delta t} u^L(c^L_t) d\tau \right], \quad (7)$$
where $\delta^L$ is $L$’s time discount rate, and $E^a_0$ denotes the expectation with respect to the probability measure induced by the effort process $a$ specified by $M$’s contract. The above optimization program is subject to $L$’s budget constraint.

To characterize $L$’s budget constraint, we need to specify $L$’s proceeds from trading at a trading date $t_i$ both on and off-equilibrium. Let $\Theta^L_{t_i^-}$ and $\Theta^L_{t_i^+}$ be $L$’s share ownership before and after trading, where $\Theta^L_{t_i^-}$ and $\Theta^L_{t_i^+}$ could be on or off the equilibrium path. We follow DU and Gorton et al. [GHH] (2014) by assuming that $L$’s proceeds from trading are given by

$$L’s 	ext{ trading proceeds} = [\Theta^L_{t_i^-} - \Theta^L_{t_i^+}] P_{t_i}(\Theta^L_{t_i^+}),$$ (8)

where $P_{t_i}(\Theta^L_{t_i^+})$ is the stock price after trading. We characterize $L$’s budget constraint by the evolution of her money market account balance as follows.

$$dB^L_{t_i} = (rB^L_{t_i} - c^L_{t_i}) dt + \Theta^L (dX_t - c^M_t dt) - P_{t_i}(\Theta)d\Theta$$

$$= (rB^L_{t_i} - c^L_{t_i} + \Theta^L(\mu(a_t) - c^M_t)) dt + \Theta^L \sigma dZ_t - P_{t_i}(\Theta)d\Theta,$$ (9)

where the differential, $P_{t_i}(\Theta)d\Theta$, is nonzero only at $L$’s trading dates, and is given by (8). At time $t \in [t_i, t_{i+1})$ between any two consecutive trading dates of $L$, during which her shareholdings remain the same, the budget constraint reflects a change in her risk-free money market account balance in a time interval $(t, t + dt)$ due to the instantaneous consumption and dividend payment (the change in the firm’s cumulative output net of $M$’s instantaneous compensation payment). When $L$ trades at each trading date $t_i$ (that is, when she changes her shareholdings from $\Theta^L_{t_{i-1}}$ to $\Theta^L_{t_i}$), her money market account balance also changes due to the proceeds from trading shares. Note that the stock price is affected by $L$’s subsequent ownership and contracting decisions, and that $L$ incorporates the effects of her trades on the stock price in making these decisions. We make the following assumption about the observability of $L$’s ownership, contracting, and consumption decisions.

**Assumption 1** $L$’s ownership process, $\Theta^L$, money market account balance, $B^L$, consumption process, $c^L$, and the manager’s contract, $\Pi$, are publicly observable.
As mentioned earlier, $L$ cannot commit to her ownership policy. Further, the manager’s contract must be sequentially optimal for $L$ at each trading date $t_i$. In addition, as in GHH, we restrict consideration to Strong Markov Perfect Public Equilibria (SMPPE) by imposing the stronger restriction that $L$’s ownership, contracting, and consumption decisions must be sequentially optimal at each trading date $t_i$ both on and off the equilibrium path. In particular, the stock price, $P_t(\Theta)$, in (9) is the stock price at $L$’s possibly off-equilibrium ownership choice, $\Theta$, incorporating the fact that $L$’s consumption policy and continuation of $M$’s contract $\Pi$ are sequentially optimal given the ownership level, $\Theta$. Consequently, $L$’s ownership, contracting and consumption decisions must solve

$$
\max_{(\Theta^L_{[t_i,\infty)}, c^L_{[t_i,\infty)}, \Pi_{[t_i,\infty)})} E^{a}_{t_i} \left[ \int_{t_i}^{\infty} e^{-\delta^L(t-t_i)} u^L(c^L_t) d\tau \right].
$$

(10)

In the above, $(\Theta^L_{[t_i,\infty)}, c^L_{[t_i,\infty)}, \Pi_{[t_i,\infty)})$ denotes the restriction of the vector of processes, $(\Theta^L, c^L, \Pi)$, to the interval $[t_i, \infty)$ with the understanding that $L$’s decisions must be sequentially optimal on or off the equilibrium path, that is, for any past history.

Each small shareholder, $S$, chooses its consumption and ownership policies to maximize its expected utility taking the stock price process as given, and rationally anticipating $L$’s trading and contract choices, that is, $S$ solves

$$
\max_{(\theta^S,c^S)} E^{a}_{0} \left[ \int_{0}^{\infty} e^{-\delta^S\tau} u^S(c^S_\tau) d\tau \right],
$$

(11)

where $\delta^S$ is $S$’s time discount rate. $S$’s money market balance evolves according to

$$
dB^S_t = (rB^S_t - c^S_t) dt + \theta^S_t (dX_t - c^M_t dt) - P_t d\theta^S_t.
$$

(12)

As $S$ changes its shareholdings continuously, its total wealth process, $Y^S_t = B^S_t + \theta^S_t P_t$, evolves as

$$
dY^S_t = (rY^S_t - c^S_t) dt + \theta^S_t (dX_t - c^M_t dt) + dP_t - rP_t dt.
$$

(13)

In the above, we explicitly indicate the fact that $S$, unlike $L$, takes the stock price process $P$ as given in making its trading decisions.
2.5 Equilibrium Characterization

An equilibrium of the model is described by the vector of processes

\[
\{ (\Theta^L, B^L, c^L); (\theta^S, Y^S, c^S); \Pi^*; P^* \}
\]  

(14)

where \((\Theta^L, B^L, c^L)\) is the vector of processes representing \(L\)'s share ownership, money market account balance and consumption, respectively; \((\theta^S, Y^S, c^S)\) is the vector of processes representing a small shareholder \(S\)'s share ownership, total wealth and consumption; \(\Pi^*\) is the manager’s contract; and \(P^*\) is the stock price process.

The processes must satisfy the following equilibrium conditions.

1. \(L\)'s share ownership, money market account balance and consumption, \((\Theta^L, B^L, c^L)\), as well as the manager’s incentive feasible RP contract, \(\Pi^*\), solve (10) subject to the budget constraint (9). In particular, \(L\)'s decisions are sequentially optimal at each trading date \(t_i\) on or off the equilibrium path.

2. Each small shareholder \(S\)'s ownership, total wealth and consumption, \((\theta^S, Y^S, c^S)\), solve (11) subject to the budget constraint (13).

3. The stock price process \(P^*\) clears the market at each time \(t\), that is, \(\int_0^1 \theta_t^S dS = 1 - \Theta_L^t\). In particular, the stock price process must clear the market on or off the equilibrium path.

3 A Benchmark Model: Owner-Manager

We first analyze a benchmark model in which \(L\) directly runs the firm by exerting costly effort herself, that is, there is no agency problem between \(L\) and \(M\). We then turn to the main analysis of the full model with the agency problem.

As discussed in Section 2.3, \(L\)'s consumption and ownership decisions must be sequentially optimal given any past history. Suppose that \(L\)'s ownership policy in the firm is given by \(\Theta^L = \{\Theta^L_i; i = 1, \ldots, N\}\). Adapting (10) to the case where \(L\) also makes effort choices, her problem is to
solve

\[
\max_{(\Theta^L_{t_i,\infty},c^L_{t_i,\infty},a^L_{t_i,\infty})} E^{aL}_{t_i} \left[ \int_{t_i}^{\infty} e^{-\delta^L(\tau-t_i)} u^L(c^L_{\tau},a^L_{\tau}) d\tau \right]
= E^{aL}_{t_i} \left[ -\int_{t_i}^{\infty} \frac{1}{\gamma^L} e^{-\delta^L(\tau-t_i)} - \gamma^L(c^L_{\tau} - \Psi(a^L_{\tau})) d\tau \right],
\]  
\tag{15}

subject to the budget constraint

\[
\begin{align*}
\frac{dB^L_t}{dt} &= (rB^L_t - c^L_t)dt + \Theta^L_t dX_t - P_t(\Theta^L_t) d\Theta^L \\
&= (rB^L_t - c^L_t + \Theta^L_t \mu(a^L_t)) dt + \Theta^L_t \sigma dZ_t - P_t(\Theta^L_t) d\Theta^L.
\end{align*}
\tag{16}
\]

In (15), \(E^{aL}_{t_i}\) signifies the expectation with respect to the probability distribution induced by \(L\)'s effort choice process.

A small shareholder \(S\) chooses its ownership and consumption policies rationally anticipating \(L\)'s ownership, effort and consumption decisions to solve

\[
\max_{(\Theta^S_{t,\infty},c^S_{t,\infty})} E^{aL}_{t} \left[ \int_{t}^{\infty} e^{-\delta^S(\tau-t)} u^S(c^S_{\tau}) d\tau \right]
= E^{aL}_{t} \left[ -\int_{t}^{\infty} \frac{1}{\gamma^S} e^{-\delta^S(\tau-t)} - \gamma^S c^S_{\tau} d\tau \right],
\]  
\tag{17}

subject to the budget constraint

\[
\begin{align*}
\frac{dY^S_t}{dt} &= (rY^S_t - c^S_t)dt + \Theta^S_t dX_t + dP_t - rP_t dt \\
&= (rY^S_t - c^S_t)dt + \Theta^S_t (\mu(a_t) dt + dP_t - rP_t dt) + \Theta^S_t \sigma dZ_t.
\end{align*}
\tag{18}
\]

As in GHH, we consider Strong Markov Public Perfect Equilibria (SMPPE) where the public state vector is \((t, B^L_t, X_t)\). The following proposition describes \(L\)'s optimal consumption and effort choices. We provide detailed proofs of all results in Appendix A.

**Proposition 1 (Large Shareholder’s Optimal Policies for Given Ownership Level)**

Let \(\Theta\) be any (on- or off-equilibrium) level of \(L\)'s ownership at time \(t \in [t_i, t_{i+1})\). \(L\)'s value function at time \(t\) has the form of

\[
W^L_t = -\frac{1}{\gamma^L r} e^{-\frac{\delta^L - t + \frac{\delta^L - \gamma^L}{\gamma^L r}}{\gamma^L r} (rB^L_t + G(t, \Theta))}.
\tag{19}
\]
L’s optimal effort and consumption policies are

\[ a_t^L = \frac{\mu_1}{\psi} \Theta, \]  

(20)

\[ c_t^L = r(B_t^L + G(t, \Theta)) + \frac{\delta^L - r}{\gamma L \rho} + \Psi(a_t^L). \]  

(21)

In the above, the time-deterministic certainty-equivalent payoff, \( G(t, \Theta) \), satisfies the recursion

\[ G(t, \Theta) = \phi_i(t)V(\Theta) + (1 - r\phi_i(t))G(t_{i+1}, \Theta), \]  

(22)

where \( \phi_i(t) = \frac{1}{r} \left( 1 - e^{-r(t_{i+1} - t)} \right) \),

(23)

\[ \lim_{t \to \infty} G(t, \Theta) = 0. \]  

(24)

\( V(\Theta) \) is the net benefit flow to \( L \) from holding \( \Theta \) that is given by

\[ V(\Theta) = \Theta \mu(a_t^L) - \Psi(a_t^L) - \frac{1}{2} \gamma L \rho \Theta^2 \sigma^2 = \mu_0 \Theta + \frac{1}{2} \left[ \frac{\mu_1^2}{\psi} - \gamma L r \sigma^2 \right] \Theta^2. \]  

(25)

By (20), we note that, not surprisingly, \( L \) exerts greater effort when she holds a larger block of shares in the firm, and when the productivity of effort is higher or the unit cost of effort is lower. \( L \)’s net benefit flow from holding \( \Theta \) captures the trade-off between expected payoff, which is reduced by the effort cost solely borne by \( L \) in the owner-manager case, and the cost of the risk that she bears from her ownership stake in the firm. The function \( G \), defined by (22), is \( L \)’s certainty equivalent payoff from holding the firm’s shares that is useful to obtain \( L \)’s optimal trading policy.

The following proposition describes a small shareholder \( S \)’s value function as well as optimal consumption and trading policies.

**Proposition 2 (Small Shareholder’s Optimal Policies for Given Large Shareholder Ownership)**

\( S \)’s value function at time \( t \in [t_i, t_{i+1}) \), given \( L \)’s ownership level \( \Theta \), is

\[ W_t^S = -\frac{1}{\gamma^S r} e^{-\gamma^S \left[ r(Y_t^S + J(t, \Theta)) + \frac{\delta^S - r}{\gamma^S r} \right]}. \]  

(26)
$S$’s optimal consumption and ownership policies are
\begin{align}
    c_t^S &= r(Y_t^S + J(t, \Theta)) = \frac{\delta^S - r}{\gamma^S}, \\
    \theta_t^S &= \frac{\mu_R(\Theta)}{\gamma^S r \sigma_R^2(\Theta)},
\end{align}
\quad(27, 28)

Its certainty equivalent payoff satisfies the recursion
\[ J(t, \Theta) = \phi_i(t)k(\Theta) + J(t_{i+1}, \Theta); \lim_{t \to \infty} J(t, \Theta) = 0, \quad(29) \]
where $\phi_i(t)$ is defined in (23), and $\mu_R(\Theta)$ and $\sigma_R(\Theta)$ are defined in the next proposition that characterizes the equilibrium stock price.

**Proposition 3 (Stock Price for Given Large Shareholder Ownership)**

The stock price, given $L$’s ownership level $\Theta$, is
\[ P(t, \Theta) = \phi_i(t)k(\Theta) + (1-r\phi_i(t))P(t_{i+1}, \Theta); \lim_{t \to \infty} P(t, \Theta) = 0, \quad(30) \]
and
\[ k(\Theta) = \mu(a_t^L) - \gamma^S r(1-\Theta)\sigma^2 = \mu_0 + \frac{\mu^2}{\psi_0} \Theta - \gamma^S r(1-\Theta)\sigma^2. \quad(31) \]

The excess dollar return for holding a share of the firm’s stock within the time interval $(t, t + dt)$ is defined as
\[ dR_t \equiv dX_t + dP_t - rP_t dt = \mu_R(\Theta)dt + \sigma_R(\Theta)dZ_t, \quad(32) \]
where the expectation and volatility of the excess dollar return of the stock are
\[ \mu_R(\Theta) = (1-\Theta) \gamma^S r \sigma^2; \quad \sigma_R(\Theta) = \sigma. \quad(33) \]

The Sharpe ratio is, by definition,
\[ \Sigma_R(\Theta) = \frac{\mu_R(\Theta)}{\sigma_R(\Theta)} = (1-\Theta) \gamma^S r \sigma. \quad(34) \]

The following corollary summarizes how $L$’s ownership level is related to the stock return characteristics.
Corollary 1 (Large Shareholder Ownership and Stock Returns)

The expected excess dollar return of the stock and the Sharpe ratio decline with the large shareholder’s ownership.

L’s shareholdings affect the risk premium for the stock through its effect on the supply of shares. As L holds a larger block of the company’s stock, the stock’s liquidity available to small shareholders is lower, thereby increasing the current stock price due to the lower supply of shares and, thus, lowering the expected stock return as well as the Sharpe ratio.

We now discuss L’s optimal ownership choice at each trading date \( t_i \). Recall that L’s ownership process is piecewise constant, that is, her holdings in the firm are constant over a trading period, \([t_i, t_{i+1})\). L’s ownership decision for the trading period \([t_i, t_{i+1})\) is made at \( t_i^- \) when she holds \( \Theta_{t_{i-1}}^L \) shares in the firm and her money market balance is \( B_{t_i}^L \). By (19), L’s value function once she chooses the new equity stake \( \Theta \) is

\[
W_{t_i}^L = -\frac{1}{\gamma^{t_i}} e^{-\gamma^{t_i} \left[ r(B_{t_i}^L + G(t_i, \Theta)) + \frac{\mu_{t_i} - r}{\gamma^{t_i}} \right]}, \tag{35}
\]

so that L’s optimal ownership choice maximizes \( B_{t_i}^L + G(t_i, \Theta) \). As noted earlier, her money market account balance changes from \( B_{t_i}^L \) to \( B_{t_{i+1}}^L \) by the proceeds from trading shares at \( t_i \) as below:

\[
B_{t_{i+1}}^L = B_{t_i}^L - P(t_i, \Theta)(\Theta - \Theta_{t_{i-1}}^L). \tag{36}
\]

L’s optimal ownership choice for the period \([t_i, t_{i+1}), \Theta_{t_{i+1}}^L \), given her current equity stake, \( \Theta_{t_i}^L = \Theta_{t_{i+1}}^L \), thus solves

\[
G(t_i^-, \Theta_{t_{i-1}}^L) = \max_{\Theta} (\Theta_{t_{i-1}}^L - \Theta) P(t_i, \Theta) + G(t_i, \Theta),
\]

\[
= \max_{\Theta} (\Theta_{t_{i-1}}^L - \Theta) P(t_i, \Theta) + \phi_i(t_i) V(\Theta) + (1 - r \phi_i(t_i)) G(t_{i+1}, \Theta),
\]

\[
\Rightarrow \quad \text{FOC} : \quad (\Theta_{t_{i-1}}^L - \Theta) \frac{\partial P(t_i, \Theta)}{\partial \Theta} + \phi_i(t_i) \left[ V'(\Theta) - k(\Theta) \right] = 0, \tag{37}
\]

where \( \phi_i(t_i) \) is given by (23). Note that the second and third lines follow from (22), (30), and \( \frac{\partial G(t_{i+1}, \Theta)}{\partial \Theta} = P(t_{i+1}, \Theta_{t_{i+1}}^L) \), which we obtain by applying the envelope theorem. The following proposition characterizes L’s optimal ownership path.
Proposition 4 (Large Shareholder’s Optimal Ownership Path)

L’s optimal ownership stake in the firm evolves according to the following recursive form:

\[
\Theta^L_{t_i}(\Theta^L_{t_{i-1}}) = \frac{\eta_{1i} \Theta^L_{t_{i-1}} + \phi_i(t_i) \gamma_S r \sigma^2}{\eta_{1i} + \phi_i(t_i)(\gamma_S + \gamma_L) r \sigma^2},
\]

(38)

where the coefficient \(\eta_{1i}\) evolves as

\[
\eta_{1i} = \phi_i(t_i) \nu + (1 - r \phi_i(t_i)) \frac{\eta^2_{1i+1}}{\eta_{1i+1} + \phi_{i+1}(t_{i+1})(\gamma_S + \gamma_L) r \sigma^2},
\]

(39)

and \(\nu = \frac{\mu^2}{\sigma} + \gamma_S r \sigma^2\).

By (38), we see that L’s ownership path is a deterministic function of time. It then follows from Proposition 3 that the equilibrium stock price is also a deterministic function of time. Because L’s ownership level and effort are constant between successive trading dates, the firm’s expected output and S’s cost of risk from its ownership stake are also constant between L’s successive trading dates. The stock price is determined by the net present value of the expected earnings net of the cost of risk and is, therefore, a deterministic function of time. As we show in the next section, however, the equilibrium stock price is stochastic when we introduce the agency conflict between L and M. As shown above, the equilibrium is similar to that obtained by DU, that is, the stock price and ownership path are deterministic functions of time.

The following corollary describes the steady state equilibrium, which corresponds to the case where the number of trades by L goes to \(\infty\), that is, \(N \to \infty\).

Corollary 2 (Steady State Equilibrium)

In the steady state, L’s ownership stake is

\[
\bar{\Theta}^L = \frac{\gamma_S}{\gamma_S + \gamma_L}.
\]

(40)

The steady state equilibrium stock price is

\[
\bar{P} = \frac{1}{r} k(\bar{\Theta}^L),
\]

(41)

and the optimal policies of L are given by Proposition 1 with L’s ownership level equal to \(\bar{\Theta}^L\).
In the steady state, L’s ownership level equals the competitive equilibrium level that depends only on the risk aversions of L and S. By Propositions 3 and 4, L’s ownership level and the stock price converge deterministically to their competitive equilibrium values.

4 Contracting

We now examine the main model that incorporates the agency conflict between L and M. As in the owner-manager case, we consider SMPPE and derive the equilibrium using backward induction.

4.1 Optimal Contracting

As noted earlier, we consider renegotiation-proof contracting along the lines of Wang (2000) and Giat and Subramanian (2013). The incorporation of the possibility of renegotiation is consistent with the assumption that L cannot commit to her future trading policy at the outset. The contract specifies the instantaneous compensation payoff and recommended effort level that maximize L’s expected utility (7) subject to the incentive compatibility and participation constraints for the manager as specified in (3) and (4).

Following the dynamic contracting literature (see the survey by Cvitanić and Zhang (2013)), the contracting problem can be characterized recursively if we take the manager’s continuation value or promised payoff, $W_t^M$, defined in (2) as an additional state variable whose value evolves according to the following stochastic process

$$dW_t^M = \left(\delta^M W_t^M - u^M(c_t^M, q_t^M)\right) dt + \chi_t^M \sigma dZ_t.$$  (42)

In the above, $\chi_t^M$ is a $\mathcal{F}_t$-adapted process that represents the sensitivity of the manager’s continuation value to the exogenous shock in the firm’s output and plays a key role in the provision of incentives.

Under technical conditions that are satisfied under CARA preferences for the principal and agent, the incentive compatibility constraint for the manager can be replaced by the following local
incentive compatibility constraint (e.g., see Williams (2009)):
\[
\chi_t^M = -\frac{u''_M(c_t^M, a_t^M)}{\mu'(a_t^M)} = \frac{\Psi'(a_t^M)}{\mu'(a_t^M)}H(c_t^M, a_t^M) > 0, \tag{43}
\]
where \( H(c_t^M, a_t^M) = e^{-\gamma M(c_t^M - \Psi(a_t^M))}. \) The manager’s contract is publicly observable so that we can assume (without loss of generality) that his promised payoff, \( W^M, \) is publicly observable.

Assumption 2 \( W^M \) is public observable.

As we now show, a SMPPE is completely characterized by the state vector, \((t, B_t^L, W_t^M, X_t)\).

Proposition 5 (Large Shareholder’s Optimal Policies for Given Ownership Level)
Let \( \Theta \) be any (on- or off-equilibrium) level of \( L \)'s ownership at \( t \in [t_i, t_{i+1}) \). Suppose that \( M \)'s promised payoff at date \( t \) is \( W_t^M \). \( L \)'s value function at time \( t \) has the form of
\[
W_t^L = -\frac{1}{\gamma L} e^{-\gamma L [r(B_t^L + G^*(t, \Theta)) + \frac{\Theta}{\gamma M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma L r}]} . \tag{44}
\]
\( L \)'s optimal consumption and \( M \)'s optimal effort and compensation are given by
\[
c_t^L = r(B_t^L + G^*(t, \Theta)) + \frac{\Theta}{\gamma M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma L r}, \\
a_t^M = \alpha, \\
c_t^M = -\frac{1}{\gamma M} \ln(\beta) - \frac{1}{\gamma M} \ln(-W_t^M) + \Psi(a_t^M), \tag{45}
\]
where the contractual parameters \( \alpha \) and \( \beta \) are determined by
\[
\beta = \gamma^M r \left[ \gamma^M \alpha (\psi \alpha - \mu_1) + 1 \right], \\
\left( 1 + \frac{\gamma L}{\gamma M} \Theta \right) \left( \frac{\psi}{\mu_1} \alpha \beta \sigma \right)^2 - (\gamma L r \Theta \sigma) \left( \frac{\psi}{\mu_1} \alpha \beta \sigma \right) - \left( r - \frac{\beta}{\gamma M} \right) = 0. \tag{46}
\]
In (44), the time-deterministic certainty-equivalent payoff \( G^* \) satisfies the recursion,
\[
G^*(t, \Theta) = \phi(t)V^*(\Theta) + (1 - r \phi(t))G^*(t_{i+1}, \Theta); \lim_{t \to \infty} G^*(t, \Theta) = 0, \tag{48}
\]
where $\phi_i(t)$ is defined in (23), and the net benefit flow to $L$ from holding $\Theta$ is

$$V^*(\Theta) = \Theta \left( \mu(\alpha) - \left( -\frac{1}{\gamma_M} \ln(\beta) - \frac{1}{\gamma_{Mr}} \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right) \right) - \frac{1}{2} \gamma^2 \Theta^2 \left( \sigma - \frac{\sigma_W}{\gamma_{Mr}} \right)^2,$$

(49)

with

$$\mu_W \equiv \delta^M - \frac{\beta}{\gamma_M}; \quad \sigma_W \equiv \frac{\psi}{\mu_1} \alpha \beta \sigma.$$

(50)

In the above, both $L$’s and $M$’s consumption processes are linear in the log of $M$’s promised utility process $W^M_t$. We note that $M$’s effort level $\alpha$ and the fixed component $\beta$ in $M$’s compensation are constant within the time period $[t_i, t_{i+1})$ and depend nonlinearly on $L$’s ownership level, $\Theta$, during the period. The evolution of the manager’s promised payoff process, (42), is described by the following corollary.

**Corollary 3 (Manager’s Promised Payoff Process)**

$M$’s promised utility process $W^M_t$ follows a geometric Brownian motion,

$$d \ln(-W^M_t) = \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) dt - \sigma_W dZ_t,$$

(51)

where $\mu_W, \sigma_W$ are defined in (50).

The manager’s dollar-dollar pay-performance sensitivity (PPS), $PPS_X$, is the dollar change in $M$’s pay for a dollar change in the firm’s cumulative output $X_t$. From (45) and (51), we have

$$dc^M_t = -\frac{1}{\gamma_M} d \ln(-W^M_t) = -\frac{1}{\gamma_M} \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) dt + \frac{\sigma_W}{\gamma_M} dZ_t.$$

(52)

By neglecting the drift terms, we obtain

$$PPS_X = \frac{dc^M_t}{dX_t} = \frac{\sigma_W}{\gamma_M} \frac{dZ_t}{\sigma dZ_t} = \frac{\sigma_W}{\gamma_M \sigma} \left( \psi \left( \frac{\mu_1}{\alpha \beta} \right) \right),$$

(53)

where the last equality follows from the definition of $\sigma_W$ in (50). The PPS is positively associated with the volatility term $\sigma_W$ of the manager’s promised payoff process. As the manager’s contract terms $\alpha$ and $\beta$ depend on $L$’s ownership level, $\Theta$, $M$’s incentives with respect to the firm’s cash flows depend on $L$’s ownership level. The relation between $M$’s incentives and $L$’s ownership level is
difficult to pin down analytically because \( M \)'s optimal effort, \( \alpha \), satisfies a sixth degree polynomial equation (see (46) and (47)) whose solutions cannot be characterized analytically. We thus calibrate the model in the next section and numerically analyze the relation between \( L \)'s ownership level and \( M \)'s incentives.

4.2 Large Shareholder Ownership, and Stock Prices and Returns

We now derive the stock price for a given level of \( L \)'s ownership by analyzing a small shareholder \( S \)'s portfolio choice problem. Note that, as small shareholders are homogeneous, each \( S \)'s problem is identical. \( S \) makes his portfolio choice by taking the stock price as given, but rationally anticipating \( L \)'s ownership stake and her decision on the manager’s optimal contract. The following proposition characterizes the stock market equilibrium derived from \( S \)'s optimal portfolio and consumption problem and the stock market clearing condition.

**Proposition 6 (Stock Price for Given Large Shareholder Ownership)**

Given any (on- or off-equilibrium) level of \( L \)'s ownership, \( \Theta \), and \( M \)'s promised utility, \( W_t^M \), at time \( t \in [t_i, t_{i+1}) \), the stock price is given by

\[
P^*(t, \Theta, W_t^M) = \Lambda^*(t, \Theta) + \frac{1}{\gamma M_r} \ln(-W_t^M),
\]

where

\[
\Lambda^*(t, \Theta) = \phi_i(t)k^*(\Theta) + (1 - r\phi_i(t))\Lambda^*(t_{i+1}, \Theta) \quad \text{lim}_{t \to \infty} \Lambda^*(t, \Theta) = 0,
\]

\[
k^*(\Theta) = \mu(\alpha) - \left(-\frac{1}{\gamma M} \ln \beta - \frac{1}{\gamma M_r} \left(\mu W - \frac{1}{2} \sigma_W^2\right) + \Psi(\alpha)\right) - \gamma^S r (1 - \Theta) \left(\sigma - \frac{\sigma_W}{\gamma M_r}\right)^2.
\]

The excess dollar return for holding a share of the firm’s stock within the time interval \( (t, t + dt) \) is

\[
dR_t^* = dP_t^* + dX_t - c_t^M dt - rP_t^* dt = \mu^*_R(\Theta) dt + \sigma^*_R(\Theta) dZ_t,
\]

where

\[
\mu^*_R(\Theta) = (1 - \Theta) \gamma^S r \left(\sigma - \frac{\sigma_W}{\gamma M_r}\right)^2 ; \quad \sigma^*_R(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma M_r}\right).
\]
The Sharpe ratio is 
\[ \Sigma^*_R (\Theta) = \frac{\mu^*_R(\Theta)}{\sigma^*_R(\Theta)} = (1 - \Theta) \gamma^S r \left( \sigma - \frac{\sigma W}{\gamma M r} \right). \] (59)

From (51) and (54), we see that the stock price is stochastic as it depends on the manager’s promised payoff that evolves stochastically. Consequently, in sharp contrast with the benchmark owner-manager case (and with DU), the presence of the agency conflict between \( L \) and \( M \) makes the stock price evolve stochastically (see Proposition 3). As one can easily see from (58), there is a positive relation between the stock return characteristics, 
\[ \mu^*_R(\Theta) = (1 - \Theta) \gamma^S r \sigma^*_R(\Theta)^2, \] (60)

which arises because the collective demand for the stock by small shareholders with CARA preferences is competitively determined by the expected stock returns adjusted for their risk premium, \( \mu^*_R/\left(\gamma^S r \sigma^*_R \right)^2 \), which must equal the supply of the shares, \( 1 - \Theta \), to ensure market clearing. The following corollary compares the stock return characteristics with the benchmark owner-manager scenario.

**Corollary 4 (Stock Returns in Agency and Benchmark Models)**

For a given level of \( L \)’s ownership, \( \Theta \), the expected excess dollar return, dollar return volatility and Sharpe ratio of the stock are lower in the agency model than in the benchmark owner-manager model.

The possibility of risk-sharing between the large shareholder and the manager through the manager’s incentive contract makes the manager’s compensation payments volatile. As shareholders receive the dividend streams net of the manager’s compensation payments, the sharing of risk with the manager makes the residual cash flows less volatile and, thereby, lowers the stock return volatility. The expected excess return of the stock and its Sharpe ratio are also lower than in the benchmark scenario due to the agency costs of risk-sharing between \( L \) and \( M \).

Note from (60) that, in contrast to the owner-manager case, \( L \)’s ownership level, \( \Theta \), has both direct and indirect influence on the expected dollar return, \( \mu^*_R \), and volatility, \( \sigma^*_R \). Her ownership stake reduces the stock’s liquidity available to small shareholders, thereby increasing the current stock price by lowering the supply of shares and, thus, lowering the expected dollar return, as shown
by the term \((1 - \Theta)\). By Proposition 5 and Corollary 3, however, her ownership also affects the mean and volatility of the manager’s promised payoff process and compensation payments that, in turn, influence the stock returns. The net effects are difficult to pin down analytically for general parameter values so that we explore the relations by numerically analyzing the calibrated model in the next section.

As shown by Propositions 5 and 6, the manager’s optimal compensation payment \(c_t^M\) and the equilibrium stock price \(P_t^*\) are negatively and positively related to the state variable, \(\ln(-W_t^M)\), respectively, which suggests a negative relation between CEO pay and the contemporaneous stock price. This negative relation appears because the current stock price reflects the costs of compensating the manager that are shared by all shareholders.

Combining (58) with (52), we obtain the sensitivity of managerial pay to the dollar return:

\[
PPS_R = \frac{dc_t^M}{dR_t^*} = \frac{(\sigma_W/\gamma^M)dZ_t}{\sigma^R_t(\Theta)dZ_t} = \frac{r}{\gamma^M r(\sigma/\sigma_W) - 1}.
\]

By (58), (59) and (61), the expected excess dollar return, return volatility and Sharpe ratio of the stock decrease with the quantity \(\sigma_W\), but the managerial pay-stock return sensitivity increases. We immediately obtain the following corollary.

**Corollary 5 (Pay-Performance Sensitivity and Stock Return)**

The sensitivity of managerial pay to the dollar return is negatively correlated with the expected excess dollar return, return volatility and Sharpe ratio of the stock.

### 4.3 Large Shareholder’s Optimal Ownership Policy

\(L\)’s optimal ownership choice at each trading date \(t_i\) maximizes \(L\)’s value function (44). Similar to the owner-manager case, \(L\)’s optimal trading policy at \(t_i^-\) solves

\[
\Theta_{L_t^*} = \arg \max_{\Theta} \left[ B_{t_i^-}^L + (\Theta_{t_i^-} - \Theta) P^*(t_i, \Theta, W_{t_i}^M) + G^*(t_i, \Theta) + \frac{\Theta}{\gamma^M r} \ln(-W_{t_i}^M) \right],
\]

By (58), (59) and (61), the expected excess dollar return, return volatility and Sharpe ratio of the stock decrease with the quantity \(\sigma_W\), but the managerial pay-stock return sensitivity increases. We immediately obtain the following corollary.

**Proposition 7 (Large Shareholder’s Ownership Path)**

\(L\)’s optimal ownership level, \(\Theta_{L_t^*}\), in period \([t_i, t_{i+1})\) satisfies the following equation:
$$\text{FOC: } (\Theta_{t_i} - \Theta_{t_i}^*) \frac{\partial \Lambda^*(t_i, \Theta_{t_i}^*)}{\partial \Theta} + \phi_i(t_i) [\gamma^S - (\gamma^L + \gamma^S)\Theta_{t_i}^*] r\sigma_R^2(\Theta_{t_i}^*) = 0,$$

where $\Lambda^*$ and $\sigma_R^*$ are given by (55) and (58).

In the above, we see two forces that determine $L$’s optimal ownership choices. The first force, which also exists in the owner-manager case, is $L$’s risk-sharing with $S$ as their risk aversions directly affect $L$’s ownership choice. The other is $L$’s costly risk-sharing with $M$ through his optimal contract, which is captured by the firm’s stock return volatility that is affected by $L$’s equity stake. In the steady state, which corresponds to the case where $N \to \infty$, the latter force disappears as described in the following corollary.

**Corollary 6 (Steady State Equilibrium)**

In the steady state, $L$’s ownership stake is

$$\tilde{\Theta}^L = \frac{\gamma^S}{\gamma^S + \gamma^L}.$$  

(64)

$L$’s optimal consumption and contract choices are given by Proposition 5 with $L$’s ownership level equal to $\tilde{\Theta}^L$. The stock price is given by

$$\tilde{P}^*(t, \tilde{\Theta}^L, W_t^M) = \frac{1}{r} k^*(\tilde{\Theta}^L) + \frac{1}{\gamma^M r} \ln(-W_t^M).$$

(65)

Comparing (64) with (40), we see that, in the steady state, $L$’s ownership level is the same as in the benchmark case with no agency conflicts, which is the ownership level in the competitive equilibrium where $L$ takes the stock price as given in making her ownership choice. Agency conflicts, therefore, affect the convergence of $L$’s ownership level to the competitive equilibrium level. However, as is apparent from (65), the stock price evolves stochastically in the steady state equilibrium because of the stochastic evolution of the manager’s promised payoff process. As $L$’s steady-state ownership level is the same as in the benchmark owner-manager scenario, we can compare other equilibrium variables in the steady state.

**Corollary 7 (Comparison of Steady State Equilibria)**

In the steady state, the manager’s effort, the expected excess dollar return and dollar return volatility are lower than in the benchmark owner-manager scenario.
As emphasized above, the optimal incentive provision to the manager plays a significant role of risk-sharing, which effectively lowers the firm’s dollar return volatility.

5 Numerical Analysis

We calibrate the model and numerically analyze it to obtain further implications that are difficult to derive analytically.

5.1 Baseline Parametrization

We determine the baseline parameters for our analysis by setting some of the parameters directly using guidance from previous studies and calibrating the remaining parameters to match salient output variables in the steady state equilibrium that we are described in the previous section. Table 1 reports the baseline parameters of the model. We set the annual risk-free rate \( r \) to 1.94\%, and each economic agent’s time discount rate \( \delta^j \) to 0.0404 so that the annual discount factor \( e^{-\delta^j} = 0.96 \) for \( j = L, M, S \) (Guvenen (2009)). The time interval between the large shareholder’s successive trading dates corresponds to a quarter. We set the unit cost of effort \( \psi \) to 1 as this parameter appears together with the productivity of effort, \( \mu_1 \).

We calibrate the remaining parameters to match selected variables in the steady state equilibrium of the model. The parameters that we calibrate include the coefficients of absolute risk aversion for the manager and for large and small shareholders; the parameters, \( \mu_0 \) and \( \mu_1 \), that determine the impact of the manager’s effort on the mean output; the output volatility, \( \sigma \); and the manager’s mean promised payoff in the steady state, \( \ln(-\tilde{W}^M) \).

We match the following empirical moments whose values are reported in Table 2: (i) the median block ownership in firms that have been public for at least five years as reported in Foley and Greenwood (2010); (ii) the mean volatility of stock returns; (iii) the mean Sharpe ratio; (iv) the mean market to book ratio; (v) the mean dividend-price ratio; (vi) the mean sensitivity of CEO pay to shareholder value; and (vii) the mean ratio of CEO pay to market value. The stock-related moments (volatility, Sharp ratio, and dividend-price ratio) are the values used in Guvenen (2009).

The model counterpart of \( L \)’s ownership is given by (64). We define the model-predicted market-to-book ratio (\( \tilde{MB}^* \)) in the steady-state using the fact that the firm’s stock price represents its
total market value as the number of shares outstanding is normalized to one. The market value of equity ($\tilde{MV}^*$) is thus computed by the mean stock price in the steady-state equilibrium, which is given by

$$\tilde{MV}^* = \frac{1}{r}k^*(\bar{\Theta}^L) + \frac{1}{\gamma M_r}\ln(-\tilde{W}^M).$$

(66)

Our proxy for the book value of equity ($\tilde{BV}^*$) is the stock price of the hypothetical firm in the absence of the manager’s human capital inputs (e.g., Ou-Yang (2005), Jung and Subramanian (2014)).

$$\tilde{BV}^* = \frac{1}{r}\hat{k}^*(\bar{\Theta}^L)$$

where

$$\hat{k}^*(\bar{\Theta}^L) = \mu_0 - (1 - \bar{\Theta}^L)\gamma S r \sigma^2.$$

(67)

By matching their ratios to the mean book-to-market ratio from data, we can pin down the parameters, $\mu_0$ and $\mu_1$, that determine the mean output of the firm. To match the observed stock return volatility (the standard deviation of the dollar return on the stock normalized by the stock price), we divide the dollar return volatility $\sigma^*_{R}$ in (58) by the steady-state equilibrium mean stock price ($\tilde{MV}^*$).

Table 2 reports the actual and model-predicted values of the moments. We see that the baseline model matches the moments reasonably well. According to the baseline parameter values in Table 1, the manager is more risk-averse than the large shareholder who is, in turn, more risk-averse than small shareholders. This result conforms to the intuition that outside shareholders are well-diversified relative to blockholders who are, in turn, better diversified than managers who have significant human capital invested in their firms. We should note, however, that the calibration exercise is only intended to provide a reasonable set of baseline parameter values for our numerical analysis.

Table 3 shows key output variables in the steady state in the agency contracting and owner-manager cases that are computed at the baseline parameter values in Table 1. As discussed in Section 4, we notice that the steady state ownership stakes of the large shareholder are identical in the two scenarios. The effort, expected dollar return, dollar return volatility, and Sharpe ratio of the stock are all lower in the contracting case. Although earnings and dividends are lower in the contracting case due to agency conflicts, the stock price is also lower than in the benchmark owner-manager case. The difference in the stock price relative to the owner-manager case outweighs the differences in expected dollar returns and in dividends so that the expected percentage stock
return and volatility as well as the dividend-price ratio are higher in the agency contracting case.

5.2 Large Shareholder Ownership, Stock Returns, and Incentives

Figure 1 shows the effects of varying the large shareholder’s ownership level on the steady-state equilibrium expected percentage stock return ($\hat{\mu}_R$), stock return volatility ($\hat{\sigma}_R$) and Sharpe ratio ($\hat{\mu}_R/\hat{\sigma}_R$) as well as the two PPS measures; the sensitivities of managerial pay to earnings ($PPS_X$) and stock returns ($PPS_R$), respectively. The expected percentage return, return volatility and Sharpe ratio of the stock all decline with the large shareholder’s ownership level. As discussed in the previous section, the large shareholder’s ownership has direct and indirect effects on the stock returns. The direct effect stems from the fact that her ownership stake reduces the stock’s liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected stock return. The indirect effect stems from the effects of the large shareholder’s ownership stake on the manager’s incentive compensation and, therefore, the volatility of the firm’s residual earnings and stock return volatility.

The large shareholder’s ownership stake has a positive impact on the two PPS measures suggesting that block ownership and incentive contracting are complementary mechanisms in effecting corporate governance. This finding is consistent with the evidence in Almazan, Hartzell and Starks (2005) and Kim (2010) that the sensitivity of CEO compensation to firm performance is positively associated with the level of outside block ownership.

5.3 Productivity, Risk, Stock Returns, and Incentives

Figure 2 shows the effects of the productivity of effort, $\mu_1$, on the expected stock return, volatility, Sharpe Ratio, as well as the manager’s PPS measures. The expected return, return volatility and Sharpe ratio of the stock decline with the productivity of effort, while the PPS measures increase. An increase in the effort productivity increases the power of incentives that can be provided to the manager and, therefore, the manager’s effort. Consequently, the stock price increases, thereby lowering the expected return. The return volatility declines because of the increase in the power of incentives to the manager so that the volatility of the residual cash flows to shareholders declines. The Sharpe ratio barely varies due to the offsetting effects.

Figure 3 shows the effects of the output volatility, $\sigma$. The expected stock return, return volatility,
and Sharpe ratio all increase sharply, while the PPS measures decline. An increase in the output volatility increases costs of risk-sharing, thereby lowering the power of incentives and lowering the manager’s effort. Consequently, the stock price decreases so that the expected return of the stock increases. The volatility of the stock also increases because of the decline in the incentive power of the manager’s contract.

5.4 Productivity, Risk, and Large Shareholder Ownership Dynamics

Figures 4 and 5 show the effects of productivity and risk on the dynamics of the large shareholder’s ownership stake in the agency contracting case and the benchmark owner-manager case. We set L’s initial ownership stake to 60% that is the median ownership of blockholders for IPO firms as reported in Foley and Greenwood (2010). Both parameters have a significant impact on the large shareholder’s optimal ownership choices in the benchmark scenario, but the ownership choices are largely insensitive to the parameters in the agency contracting scenario. Indeed, the separation of ownership and control, and the facilitation of risk-sharing through contracting with the manager, effectively insulates the large shareholder from fluctuations in firm-specific parameters. Consequently, the large shareholder’s ownership choices are largely determined by the risk aversions of the large and small shareholders as well as the manager. In other words, in the contracting case, risk-sharing between the large and small shareholders that determines the stock price is effectively separated from the contracting problem that determines the manager’s effort and the firm’s earnings. In sharp contrast with the benchmark owner-manager scenario, the large shareholder’s ownership stake plays less of an incentive role so that it is largely determined by risk-sharing with small shareholders.

5.5 Empirical Implications

The results of the analytical and quantitative analyses of the model can be interpreted as empirical implications for the relations among block ownership, stock return characteristics, and managerial contracts. We summarize them below as a guide for future empirical research.

1. Ceteris paribus, managerial pay-performance sensitivities are negatively associated with expected stock returns, return volatilities, and Sharpe ratios.
2. *Ceteris paribus*, block ownership (or large shareholder ownership) is, negatively associated with expected stock returns, volatilities and Sharpe ratios.

3. *Ceteris paribus*, block ownership (or large shareholder ownership) is positively related to managerial incentives.

4. *Ceteris paribus*, block ownership dynamics are more sensitive to firm-specific characteristics such as firm productivities and cash flow volatilities in owner-managed firms than firms characterized by significant separation between ownership and control.

6 Conclusions

We present a simple tractable dynamic framework that embeds a standard managerial moral hazard problem into an asset pricing model with a large shareholder. Using a simple CARA-normal continuous-time framework, we fully characterize the endogenous determination of a representative firm’s ownership structure, managerial compensation, and stock return characteristics and examine their equilibrium interactions. We compare the equilibrium solutions in the owner-manager and agency contracting cases and then discuss the impact of optimal contracting on the firm’s equity prices and the large shareholder’s ownership dynamics.

We show that, in contrast to the benchmark owner-manager case, the incorporation of agency conflicts between large shareholders and managers results in a stochastically evolving stock price. The separation between ownership and control as well as optimal contracting lead to less volatile dollar stock returns, lower expected dollar stock returns, and lower Sharpe ratios, *ceteris paribus*. The negative impact of the agency costs of risk-sharing on the expected stock price, however, causes percentage stock returns and stock return volatilities to be higher relative to the owner-manager scenario. The facilitation of risk-sharing through optimal contracts effectively insulates the ownership stakes of large shareholders from firm-specific parameters such as the mean and volatility of earnings, and largely determined by the risk aversions of large and small shareholders. We also obtain novel testable implications for the equilibrium relations among block ownership stakes, expected excess dollar returns and return volatilities, and managerial incentives. In particular, we highlight the importance of examining the relations among these endogenous variables in an...
equilibrium framework.

There are several directions in which the relatively simple structure in this paper can be extended. First, we consider a representative firm in the current paper so that there are only two assets available to investors: the risk-free bond and the representative firm’s stock. We intend to extend this simple model to a general framework of multiple firms in a subsequent paper that would allow us to present a CAPM with optimal managerial contracting and cross-sectional implications for block ownership, managerial incentives, and stock return characteristics (expected returns, volatilities, and stock betas). More importantly, these theoretical predictions should be supported by empirical asset pricing tests.

Second, we adopt a simple CARA-normal setting in this paper. This enables us to obtain a closed-form solution for the optimal contract, equilibrium asset price and ownership structure. There are two theoretical limitations to the generalization of the framework. As Sannikov (2008) presents numerical solutions to the contracting problems with different non-CARA preferences, it would be difficult to find a closed-form solution even to the optimal contracting problem. In addition, under different preferences and cumulative payoff structures, it would not be plausible to guess and verify the specific form of the equilibrium price. More work would be necessary to obtain the general form of the price process in such a generalization.

Lastly, as in Admati et. al (1994) and DeMarzo and Urošević (2006), we do not consider an endogenous distinction between the large shareholder and small shareholders. It would be interesting to further explore an ex-ante identical investor’s choice to be a large shareholder in a particular firm in equilibrium, possibly by taking into account a fee structure that the large shareholder charges small shareholders for the extra risk she bears.

References


A1 Proofs

A1.1 Proofs for Section 3: The Owner-Manager Case

Consider the last period \( t \in [t_i, t_{i+1}) \) with \( i = N \) and \( t_{N+1} = \infty \) during which \( L \)'s shareholdings are held constant: \( \Theta_L^t = \Theta \). We first determine \( L \)'s optimal effort and consumption choices given her ownership stake \( \Theta \) that maximize the expected utility,

\[
W^L_t = \max_{c_t^L, a_t^L} E_t^{a_t^L} \left[ \int_t^\infty e^{-\delta^L(\tau-t)}u^L(c^L_{\tau}, a^L_{\tau})d\tau \right] = E_t^{a_t^L} \left[ -\frac{1}{\gamma^L} \int_t^\infty e^{-\delta^L(\tau-t)-\gamma^L(c^L_{\tau}-\Psi(a^L_{\tau}))}d\tau \right],
\]  

(A1)

subject to the budget constraint,

\[
\frac{dB^L_t}{dt} = (rB^L_t - c^L_t)dt + \Theta dX_t = [rB^L_t - c^L_t + \Theta \mu(a^L_t)] dt + \Theta \sigma dZ_t.
\]  

(A2)

Let \( F(t, B^L_t) \) be the format of \( L \)'s value function \( (W^L_t) \) representing her expected utility from the optimal choices, given the evolution of her risk-free account \( B^L_t \). With CARA preferences, we
conjecture that
\[ W_t^L = F(t, B_t^L) = -\frac{1}{\gamma^L_r} e^{-\gamma^L_r \left[ r(B_t^L + G(t, \Theta)) + \frac{\delta^L - r}{\gamma^L_r} \right]}. \]  

L’s Hamilton-Jacobi-Bellman (HJB) equation is then given by
\[ F_t + \max_{c^L, a^L} \left[ F_B \left( rB_t^L - c^L + \Theta \mu(a^L) \right) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 + u^L(c^L, a^L) - \delta^L F \right] = 0. \]  

The first order conditions (FOCs) for \((c^L, a^L)\) are then
\[ -F_B + u^L(c^L, a^L) = 0, \]
\[ F_B \Theta \mu'(a^L) + u^L_a(c^L, a^L) = 0. \]

By the equations above and the derivative of \(F\), we obtain the following optimal policies
\[ c_t^L = r(B_t^L + G(t, \Theta)) + \frac{\delta^L - r}{\gamma^L_r} + \Psi(a_t^L), \]
\[ a_t^L = \frac{\mu_1}{\psi} \Theta, \]

where the second equation follows from the time-invariant mean and volatility terms in the firm’s cash flow process, \(\mu(a) = \mu_0 + \mu_1 a\), and \(\Psi(a) = \frac{1}{2} \psi a^2\). Substituting the optimal policies and the derivatives of \(F\) into the HJB equation (A4) yields
\[ G_t(t, \Theta) = rG(t, \Theta) - V(\Theta), \]

where \(V(\Theta)\) is the net benefit flow to \(L\) from holding \(\Theta\) that is given by
\[ V(\Theta) = \Theta \mu(a_t^L) - \Psi(a_t^L) - \frac{1}{2} \gamma^L r \Theta^2 \sigma^2 = \Theta \mu_0 + \frac{1}{2} \Theta^2 \left[ \frac{\mu_1^2}{\psi} - \gamma^L r \sigma^2 \right]. \]

By integrating (A8) over the period, we obtain
\[ G(t, \Theta) = \phi_1(t) V(\Theta) + (1 - r \phi_1(t)) G(t_{N+1}, \Theta) = \phi_1(t) V(\Theta), \]

where \(\phi_i(t) \equiv \frac{1}{i} \left( 1 - e^{-r(t_{i+1}-t)} \right)\) and the second equation follows from the assumption on the terminal payoff to \(L\), \(G(t_{N+1}) = \lim_{t \rightarrow \infty} G(t, \Theta) = 0\).

We now turn to each small shareholder \(S\)’s problem and determine the equilibrium stock price from the market clearing condition. Let \(\hat{\Theta}\) and \(\hat{a}_t^L\) denote \(L\)’s trading policy and effort choice anticipated by \(S\). \(S\) maximizes its expected utility by solving the optimal consumption and portfolio problem, given the stock price process and \(L\)’s policies anticipated. More specifically, \(S\)’s problem is
\[ W_t^S = \max_{c_t^L, \Theta_t^S} E_t^S \left[ \int_t^\infty e^{-\delta^S (\tau-t)} u^S(c_t^L) d\tau \right] = E_t^S \left[ -\frac{1}{\gamma^S} \int_t^\infty e^{-\delta^S (\tau-t) - \gamma^S c_t^S} d\tau \right], \]
subject to the budget constraint that captures the evolution of $S$’s stock and risk-free bond balances. Specifically, $S$’s bond balance evolves according to

$$dB_t^S = (rB_t^S - c_t^S)dt + \theta_t^S dX_t - P_t d\theta_t^S.$$  (A12)

$S$’s total wealth $Y_{t,t}$ from his portfolio holdings is $Y_t^S = B_t^S + \theta_t^S P_t$ and evolves according to

$$dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S (dX_t + dP_t - rP_t dt).$$  (A13)

We denote the last term in (A13) by

$$dR_t = dX_t + dP_t - rP_t dt$$  (A14)

that represents the excess dollar return from holding a share of the firm’s stock within the time interval $(t, t + dt)$. As will be shown below, this process is linear in the firm’s underlying cash flow stochastic process,

$$dR_t = \mu_R(t, \hat{\Theta}) dt + \sigma_R(t, \hat{\Theta}) dZ_t.$$  (A15)

Let $Q(t, Y_t^S)$ be the format of $S$’s value function ($W_t^S$) that depends on the evolution of his total wealth $Y_t^S$. With CARA preferences, we conjecture and verify the following exponential form as in $L$’s problem above,

$$W_t^S = Q(t, Y_t^S) = -\frac{1}{\gamma^S r} e^{-\gamma^S \left[ r(Y_t^S + J(t, \hat{\Theta})) + \frac{\delta^S - r}{\gamma^S r} \right]}.$$  (A16)

The HJB equation is then

$$Q_t + \max_{c_t, \theta_t^S} \left[ Q_Y \left( rY_t^S - c_t^S + \theta_t^S \mu_R(t, \hat{\Theta}) \right) + \frac{1}{2} Q_{YY} \left( \theta_t^S \sigma_R(t, \hat{\Theta}) \right)^2 + u^S(c_t^S) - \delta^S Q \right] = 0,$$  (A17)

whose first order conditions (FOCs) with respect to $c_t^S$ and $\theta_t^S$ are

$$Q_Y = e^{-\gamma^S c_t^S} ; \quad \theta_t^S = -\frac{Q_Y \mu_R(t, \hat{\Theta})}{Q_{YY} \sigma_R^2(t, \hat{\Theta})}.$$  (A18)

Using the derivatives of $Q$ (that is, $Q_Y = -\gamma^S r Q$ and $Q_{YY} = (\gamma^S r)^2 Q$), the FOCs above imply the following optimal choices of $S$:

$$c_t^S = r(Y_t^S + J(t, \hat{\Theta})) + \frac{\delta^S - r}{\gamma^S r};$$  (A19)

$$\theta_t^S = \frac{\mu_R(t, \hat{\Theta})}{\gamma^S r \sigma_R^2(t, \hat{\Theta})}.$$  (A20)
In equilibrium, $S$ rationally anticipate $L$’s optimal policies ($\hat{\Theta} = \Theta$ and $\hat{a}_L^t = a_L^t$) and the stock market clears ($\int S \theta^S dS = 1 - \Theta$) so that
\[
\mu_R(t, \Theta) = \gamma^S r (1 - \Theta) \sigma^2_R(t, \Theta).
\] (A21)

By plugging the optimal policies and the derivatives of $Q$ into the HJB equation (A17),
\[
J_t(t, \Theta) = r J(t, \Theta) - \frac{1}{2} \mu^2_R(t, \Theta) - \frac{1}{2} \sigma^2_R(t, \Theta).
\] (A22)

One can easily verify that the equilibrium stock price is simply a time-deterministic function that depends on $L$’s ownership as below:
\[
P(t, \Theta) = \Lambda(t, \Theta); \quad d\Lambda_t - r \Lambda_t dt = -k(\Theta) dt,
\] (A23)

where the time deterministic function, $\Lambda(t, \Theta)$, and, therefore, the stock price can be shown to follow the recursive format of
\[
\Lambda(t, \Theta) = \phi_1(t) k(\Theta) + (1 - r \phi_1(t)) \Lambda(t - i, \Theta) = \phi_i(t) k(\Theta),
\] (A24)

where $\phi_i(t) = \frac{1}{r} (1 - e^{-r(t+i-t)})$, and the second equation follows from the fact that the terminal price, $\Lambda(t_{N+1}, \Theta) = \lim_{t \to \infty} \Lambda(t, \Theta) = 0$. The expected excess dollar return of the stock and stock return volatility are then derived as
\[
\mu_R(t, \Theta) = \mu(a_L^t) - k(\Theta); \quad \sigma_R(t, \Theta) = \sigma,
\] (A25)

and, together with (A21),
\[
k(\Theta) = \mu(a_L^t) - \mu_R(t, \Theta) = \mu_0 + \frac{\mu^2}{\psi} \Theta - \gamma^S r (1 - \Theta) \sigma^2.
\] (A26)

By (A23), (A24), and (A26), note that the stock price is linear in $L$’s equity stake $\Theta$:
\[
P(t, \Theta) = \phi_N(t) k(\Theta) = \eta_{0N} + \eta_{1N} \Theta,
\] (A27)

where $\eta_{0N} \equiv \phi_N(t)(\mu_0 - \gamma^S r \sigma^2)$ and $\eta_{1N} \equiv \phi_N(t) \left( \frac{\mu^2}{\psi} + \gamma^S r \sigma^2 \right)$ where $\phi_N(t) = \frac{1}{t}$ because $t_{N+1} = \infty$.

We now consider $L$’s optimal ownership choice at date $t_N$, more exactly, $t_N^-$. By (A3), $L$’s value function at $t_N$ right after she chooses the ownership level of $\Theta$ is
\[
W_{t_N}^L = -\frac{1}{\gamma^{t_N}} e^{-\gamma^L \left[ r (B_{t_N}^L + G(t_N, \Theta)) + \frac{t_{N+1} - t_N}{\gamma^{t_{N+1}}} \right]}.
\] (A28)

We denote $L$’s risk-free account balance and shareholdings at $t_N^-$ by $B_{t_N}^L$ and $\Theta_{t_N}^L = \Theta_{t_{N-1}}^L$, respec-
tively. L’s trading decision is made in a way that maximizes $B_N^L + G(t_N, \Theta)$ in the value function above. The proceeds from trading shares is given by $(\Theta_{t_{n-1}} - \Theta)P(t_N, \Theta)$. Then the optimal choice of $\Theta_{t_{n-1}}^L$ is obtained by:

$$G(t_N, \Theta_{t_{n-1}}^L) = \max_{\Theta} (\Theta_{t_{n-1}}^L - \Theta)P(t_N, \Theta) + G(t_N, \Theta)$$

FOC: $-P(t_N, \Theta) + (\Theta_{t_{n-1}}^L - \Theta)\frac{\partial P(t_N, \Theta)}{\partial \Theta} + \frac{\partial G(t_N, \Theta)}{\partial \Theta} = 0$

$\Rightarrow -\phi_N(t_N)k(\Theta) + (\Theta_{t_{n-1}}^L - \Theta)\frac{\partial \phi_N(t_N)k(\Theta)}{\partial \Theta} + \frac{\partial \phi_N(t_N)V(\Theta)}{\partial \Theta} = 0$

$\Rightarrow \phi_N(t_N)\left[-k(\Theta) - (\Theta - \Theta_{t_{n-1}}^L)k'(\Theta) + V'(\Theta)\right] = 0$

$\Rightarrow \Theta_{t_{n-1}}^L = \frac{\nu \Theta_{t_{n-1}} + \gamma^S r \sigma^2}{\nu + (\gamma^L + \gamma^S) r \sigma^2}$, \hspace{1cm} (A29)

where $\nu = \frac{\mu^2}{\sigma^2} + \gamma^S r \sigma^2 > 0$. Accordingly, L’s optimal trading policy, $\Theta_{t_{n}}^L(\Theta_{t_{n-1}}^L)$, is determined as a function of $\Theta_{t_{n-1}}^L$ at $t_{n}$. The stock price must be continuous before and after the trading date $t_N$:

$$P(t_N, \Theta_{t_{n-1}}^L) = P(t_N, \Theta_{t_{n}}^L(\Theta_{t_{n-1}}^L)) = \eta_0 + \eta_1 \Theta_{t_{n}}^L(\Theta_{t_{n-1}}^L).$$ \hspace{1cm} (A30)

We now work on the problems for the earlier years in general and provide the proofs of Propositions 1 to 4 for the owner-manager benchmark case.

**Proof of Proposition 1:** Consider any period $t \in [t_i, t_{i+1})$ for $i < N$ during which L’s shareholdings are held constant: $\Theta_t = \Theta$. We first determine L’s optimal effort and consumption choices given her ownership stake $\Theta$ that maximize the following expected utility

$$W_t^L = \max_{c_t^L, a_t^L} E_{a_t^L} \left[ \int_t^\infty e^{-\delta^L(t-t)}u^L(c_t^L, a_t^L)dt \right] = E_{a_t^L} \left[ -\frac{1}{\gamma^L} \int_t^\infty e^{-\delta^L(t-t)}-\gamma^L(c_t^L-a_t^L)dt \right],$$ \hspace{1cm} (A31)

subject to the budget constraint

$$dB_t^L = (rB_t^L - c_t^L)dt + \Theta dX_t = [rB_t^L - c_t^L + \Theta \mu(a_t^L)] dt + \Theta \sigma dZ_t.$$ \hspace{1cm} (A32)

Let $F(t, B_t^L)$ be the format of L’s value function ($W_t^L$) representing the expected utility from her optimal policy, given the evolution of her risk-free account $B_t^L$. With CARA preferences, we conjecture that

$$W_t^L = F(t, B_t^L) = -\frac{1}{\gamma^L} e^{-\gamma^L \left[r(B_t^L + G(t, \Theta)) + \frac{\delta^L - \gamma^L}{\gamma^L} \right]}.$$ \hspace{1cm} (A33)

The Hamilton-Jacobi-Bellman (HJB) equation is then given by

$$F_t + \max_{c_t^L, a_t^L} \left[ FB(rB_t^L - c_t^L + \Theta \mu(a_t^L)) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 + u^L(c_t^L, a_t^L) - \delta^L F \right] = 0.$$ \hspace{1cm} (A34)
The first order conditions (FOCs) for \((c^L, a^L)\) are then
\[
-F_B + u^L_c(c^L, a^L) = 0,
F_B \Theta \mu'(a^L) + u^L_a(c^L, a^L) = 0.
\] (A35)

By the equations above and the guessed format of \(F\) (in particular, \(F_B = -\gamma^L r F\)), we obtain the following optimal policies
\[
c^L_t = r(B^L_t + G(t, \Theta)) + \frac{\delta^L - r}{\gamma^L r} + \Psi(a^L_t),
\] (A36)
\[
a^L_t = \frac{\mu^1}{\psi} \Theta,
\] (A37)
where the second equation follows from the time-invariant mean and volatility terms in the firm’s cash flow process, \(\mu(a) = \mu_0 + \mu_1 a\), and \(\Psi(a) = \frac{1}{2} \psi a^2\). Substituting the optimal policies and the derivatives of \(F\) into the HJB equation (A34) yields
\[
G^L_t(t, \Theta) = r G^L_t(t, \Theta) - V(\Theta)
\] (A38)
where \(V(\Theta)\) is the net benefit flow to \(L\) from holding \(\Theta\) that is given by
\[
V(\Theta) = \Theta \mu(a^L_t) - \Psi(a^L_t) - \frac{1}{2} \gamma^L r \Theta^2 \sigma^2 = \Theta \mu_0 + \frac{1}{2} \Theta^2 \left[ \frac{\mu^2_1}{\psi} - \gamma^L r \sigma^2 \right].
\] (A39)

By integrating (A38) over the period, we obtain
\[
G(t, \Theta) = \phi_i(t) V(\Theta) + (1 - r\phi_i(t)) G(t^+_i, \Theta),
\] (A40)
where \(\phi_i(t) = \frac{1}{r} \left(1 - e^{-r(t_i+1-t)}\right)\).

Proof of Propositions 2 and 3: We now turn to each small shareholder \(S\)’s portfolio problem and determine the equilibrium stock price from the market clearing condition. Let \(\hat{\Theta}\) and \(\hat{a}^L_t\) denote \(L\)’s trading policy and effort choice anticipated by \(S\). \(S\) maximizes its expected utility by solving the optimal consumption and portfolio problem, given the stock price process and \(S\)’s policies anticipated. More specifically, \(S\)’s problem is specified by
\[
W^S_i = \max_{c^S_t, \Theta^S_i} E^S_t \left[ \int_t^\infty e^{-\delta^S (\tau-t)} u^S(c^S_{\tau}) d\tau \right] = E^S_t \left[ -\frac{1}{\gamma^S} \int_t^\infty e^{-\delta^S (\tau-t) - \gamma^S c^S_{\tau}} d\tau \right],
\] (A41)
subject to the budget constraint that captures the evolution of \(S\)’s stock and risk-free bond balances. Specifically, \(S\)’s bond balance evolves according to
\[
 dB^S_t = (r B^S_t - c^S_t) dt + \theta^S_t dX_t - P_i d\theta^S_i,
\] (A42)
S’s total wealth \( Y_{S,t} \) from its portfolio holdings is \( Y_{S,t} = B_{S,t} + \theta_{S,t} P_t \) and evolves according to
\[
dY_{S,t} = (rY_{S,t} - c_{S,t})dt + \theta_{S,t}(dX_t + dP_t - rP_t dt) = (rY_{S,t} - c_{S,t})dt + \theta_{S,t}(\mu_R(t, \Theta) dt + \sigma_R(t, \Theta) dZ_t),
\] (A43)
where we guess that the excess dollar return of the stock, \( dR_t = dX_t + dP_t - rP_t dt \), is linear in the underlying stochastic process, and verify it later.

Let \( Q(t, Y_{S,t}) \) be the format of \( S \)'s value function \( (W_{S,t}) \) that depends on the evolution of its total wealth \( Y_{S,t} \). With CARA preferences, we conjecture and verify the following exponential form as in \( L \)'s problem above,
\[
W_{S,t} = Q(t, Y_{S,t}) = -\frac{1}{\gamma_f} e^{-\gamma_f [r(Y_{S,t} + J(t, \Theta)) + \delta_{S,t} - rY_{S,t}]}.
\] (A44)
The HJB equation is then
\[
Q_t + \max_{c_{S,t}, \theta_{S,t}} \left[ Q_Y \left( rY_{S,t} - c_{S,t} + \theta_{S,t} \mu_R(t, \Theta) \right) + \frac{1}{2} Q_{YY} \left( \theta_{S,t} \sigma_R(t, \Theta) \right)^2 + u_s(c_{S,t}) - \delta_s Q \right] = 0,
\] (A45)
whose first order conditions (FOCs) with respect to \( c_{S,t} \) and \( \theta_{S,t} \) are
\[
Q_Y = e^{-\gamma_f c_{S,t}}, \quad \theta_{S,t} = -\frac{Q_{YY} \mu_R(t, \Theta)}{Q_{YY} \sigma_R^2(t, \Theta)}.
\] (A46)
Using the guessed format of \( Q \) (that is, the derivatives of \( Q \), \( Q_Y = -\gamma_f rQ \) and \( Q_{YY} = (\gamma_f r)^2 Q \)), the FOCs above imply the following optimal choices of \( S \):
\[
c_{S,t} = r(Y_{S,t} + J(t, \Theta)) + \frac{\delta_{S,t} - r}{\gamma_f};
\] (A47)
\[
\theta_{S,t} = \frac{\mu_R(t, \Theta)}{\gamma_f r \sigma_R^2(t, \Theta)}.
\] (A48)
In equilibrium, \( S \) rationally anticipate \( L \)'s optimal policies \( (\hat{\Theta} = \Theta \) and \( \hat{a}_L^t = a_L^t \) and the stock market clears \( (\int_S \theta_{S,t}^2 dS = 1 - \Theta) \) so that
\[
\mu_R(t, \Theta) = \gamma_f r(1 - \Theta) \sigma_R^2(t, \Theta).
\] (A49)
By plugging the optimal policies and the derivatives of \( Q \) into the HJB equation (A45),
\[
J_t(t, \Theta) = rJ(t, \Theta) - \frac{1}{2} \frac{\mu_R^2(t, \Theta)}{\gamma_f r \sigma_R^2(t, \Theta)}.
\] (A50)
As in the problem for the terminal period \( (i = N) \), we show that the equilibrium stock price is a time-deterministic function that depends on \( L \)'s ownership as below:
\[
P(t, \Theta) = \Lambda(t, \Theta); \quad d\Lambda_t - r\Lambda_t dt = -k(\Theta) dt,
\] (A51)
where the time deterministic function, \( \Lambda(t, \Theta) \), and, therefore, the stock price follows the recursive format of
\[
\Lambda(t, \Theta) = \phi_i(t)k(\Theta) + (1 - r\phi_i(t))\Lambda(t_{i+1}, \Theta),
\]
(A52)
where \( \phi_i(t) = \frac{1}{r} \left(1 - e^{-r(t_{i+1} - t)}\right) \). Then, the expected excess dollar return of the stock and stock return volatility are then derived as
\[
\mu_R(t, \Theta) = \mu(a^L_t) - k(\Theta); \quad \sigma_R(t, \Theta) = \sigma,
\]
(A53)
which, together with (A49), implies
\[
k(\Theta) = \mu(a^L_t) - \mu_R(t, \Theta) = \mu_0 + \frac{\mu^2_1}{\psi} \Theta - \gamma^S r(1 - \Theta)\sigma^2.
\]
(A54)

**Proof of Proposition 4:** We now consider \( L \)'s optimal ownership choice at date \( t_i \), more exactly, \( t_i^- \). By (A33), \( L \)'s value function at \( t_i^- \) right after she chooses the ownership level of \( \Theta \) is
\[
W^L_{t_i} = -\frac{1}{\gamma^L} e^{-\gamma^L \left[ r(B^L_{t_i} + G(t, \Theta)) + \frac{\mu^2_1}{\psi} \Theta \right]}.
\]
(A55)
By considering the proceeds from trading shares, \( L \)'s chooses a new equity stake at \( t_i^- \), given her current holdings \( \Theta^L_{t_{i-1}} \), as below:
\[
G(t_i^-, \Theta^L_{t_{i-1}}) = \max_{\Theta} (\Theta^L_{t_{i-1}} - \Theta)P(t_i, \Theta) + G(t_i, \Theta)
\]
\[
= \max_{\Theta} (\Theta^L_{t_{i-1}} - \Theta)P(t_i, \Theta) + \phi_i(t_i)V(\Theta) + (1 - r\phi_i(t_i))G(t_{i+1}^-, \Theta),
\]
(A56)
where the second line follows from (A40). Its FOC is derived as
\[
\text{FOC : } (\Theta^L_{t_{i-1}} - \Theta) \frac{\partial P(t_i, \Theta)}{\partial \Theta} - P(t_i, \Theta) + \phi_i V'(\Theta) + (1 - r\phi_i(t_i)) \frac{\partial G(t_{i+1}^-, \Theta)}{\partial \Theta} = 0,
\]
(A57)
where the last term is, due to the envelope theorem,
\[
\frac{\partial G(t_{i+1}^-, \Theta)}{\partial \Theta} = P(t_{i+1}, \Theta^L_{t_{i+1}}).
\]
(A58)
By (A51) and (A52), the stock price process is given by
\[
P(t_i, \Theta) = \phi_i(t_i)k(\Theta) + (1 - r\phi_i(t_i))P(t_{i+1}, \Theta^L_{t_{i+1}}),
\]
(A59)
where we use the continuity in the stock price before and after the trading date \( t_i^- \) \( P(t_{i+1}^-, \Theta) = P(t_{i+1}, \Theta^L_{t_{i+1}}) \) we rewrite the FOC as
\[
\text{FOC : } (\Theta_{t+1} - \Theta) \frac{\partial P(t, \Theta)}{\partial \Theta} + \phi_i(t_i) [V'(\Theta) - k(\Theta)] = 0. \tag{A60}
\]

We now show that the equilibrium stock price is linear in the large shareholder’s current holdings \(\Theta\):

\[
P(t, \Theta) = \eta_{0i} + \eta_{1i} \Theta, \tag{A61}
\]

which holds at \(t_N\) as shown in (A30). We then assume that (A61) holds at \(t_{i+1}\) and verify that it holds at \(t_i\) as well. If (A61) holds at \(t_i\), \(\frac{\partial P(t_i, \Theta)}{\partial \Theta} = \eta_{1i}\), which, in turn, determines \(L\)’s optimal equity stake from the FOC (A60) as a function of her prior holdings, \(\Theta_{t_i} = (\Theta_{t_i-1} - \Theta)\):

\[
\Theta_{t_i}^L(\Theta_{t_i-1}^L) = \frac{\eta_{1i} \Theta_{t_i-1}^L + \phi_i(t_i) \gamma S r \sigma^2}{\eta_{1i} + \phi_i(t_i)(\gamma S + \gamma L)r \sigma^2}. \tag{A62}
\]

We obtain the recursive relations for the price coefficients by plugging (A54), (A61), and (A62) into the stock price process (A59):

\[
\eta_{0i} + \eta_{1i} \Theta_{t_i}^L = \phi_i(t_i)((\mu_0 - \gamma S r \sigma^2) + \nu \Theta_{t_i}^L) + (1 - r \phi_i(t_i))(\eta_{0i+1} + \eta_{1i+1} \Theta_{t_i+1}^L(\Theta_{t_i}^L)),
\]

where \(\nu = \frac{\mu_0^2}{\sigma^2} + \gamma S r \sigma^2\), so that

\[
\begin{align*}
\eta_{0i} & = \phi_i(t_i) + (1 - r \phi_i(t_i)) \frac{\eta_{0i+1}}{\eta_{0i+1} + \phi_i(t_i+1)(\gamma S + \gamma L)r \sigma^2} \\
\eta_{1i} & = \phi_i(t_i)((\mu_0 - \gamma S r \sigma^2) + (1 - r \phi_i(t_i)) \left[ \eta_{0i+1} + \frac{\eta_{1i+1} \phi_i(t_i)(\gamma S r \sigma^2)}{\eta_{0i+1} + \phi_i(t_i+1)(\gamma S + \gamma L)r \sigma^2} \right].
\end{align*}
\]

\(Q.E.D.\)

\text{Proof of Corollary 2:} It is clear from the above problem and the definitions of \(V(\Theta)\) and \(k(\Theta)\) that, in the steady-state equilibrium, \(L\)’s optimal ownership level \(\Theta^L\) solves \(V'(\Theta) - k(\Theta) = 0 \Rightarrow (\gamma S - (\gamma L + \gamma S)\bar{\Theta})r \sigma^2 = 0\), which yields \(\Theta^L = \gamma S / (\gamma L + \gamma S)\). \(Q.E.D.\)

\section*{A1.2 Proofs for Section 4: The Contracting Case}

The steps of proving the results of our main analysis (agency contracting case) is similar to what we have shown above for the owner-manager benchmark case. We start by solving the problem during the last trading period of \(L\). We then solve the problem recursively for the earlier periods. For expositional convenience, we only show the proofs for the results for earlier periods \([t_i, t_{i+1})\) with \(i < N\) in general.

\text{Proofs of Proposition 5:} Suppose that \(L\) holds an equity stake of \(\Theta\) in the firm during the period \(t \in [t_i, t_{i+1})\). \(L\)’s optimal consumption and managerial contracting problem is now subject
to $M$’s incentive compatibility (IC) constraint:

$$
(\text{IC}) : \quad dW_t^M = (\delta^M W_t^M - u^M (c_t^M, a_t^M)) \, dt - \frac{u_a^M (c_t^M, a_t^M)}{\mu'(a_t^M)} \, \sigma dZ_t
= \left( \delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M) \right) \, dt + \frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M) \sigma dZ_t,
$$

(A63)

where $H(c_t^M, a_t^M) = e^{-\gamma^M (c_t^M - \frac{1}{2} \psi (a_t^M)^2)}$. Given $M$’s incentive contract, $L$’s risk-free bond account evolves according to

$$
dB_t^L = (r B_t^L - c_t^L) \, dt + \Theta (dX_t - c_t^M) \, dt = (r B_t^L - c_t^L + \Theta (\mu(a_t^M) - c_t^M)) \, dt + \Theta \sigma dZ_t.
$$

(A64)

We conjecture and verify that $L$’s value function has the following exponential format of $'$s risk-free bond account

$$
W_t^L = F(t, B_t^L, W_t^M) = -\frac{1}{\gamma^L} e^{-\gamma^L \left[ r(B_t^L + G^L(t, \Theta)) + \frac{\Theta}{\gamma^M} \ln(-W_t^M) + \frac{r}{\gamma^L} \right]}.
$$

(A65)

The Hamilton-Jacobi-Bellman (HJB) equation is then given by

$$
F_t + \max_{c_t^L, c_t^M, a_t^M} \left[ F_B (r B_t^L - c_t^L + \Theta (\mu(a_t^M) - c_t^M)) + F_W (\delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M)) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 + F_{BW} \Theta \sigma \left( \frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M) \right) + \frac{1}{2} F_{WW} \left( \frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M) \right)^2 + u^L(c_t^L) - \delta^L F \right] = 0.
$$

(A66)

The FOCs for $(c_t^L, c_t^M, a_t^M)$ are then

$$
c_t^L : \quad -F_B + u_{c_t^L}^L (c_t^L) = 0,
$$

$$
c_t^M : \quad F_B \Theta + F_W H + F_{BW} \Theta \gamma^M \left( \frac{\psi}{\mu_1} a_t^M H \right) \sigma^2 + F_{WW} \gamma^M \left( \frac{\psi}{\mu_1} a_t^M H \right)^2 \sigma^2 = 0,
$$

$$
a_t^M : \quad F_B \Theta \mu_1 + F_W (\psi a_t^M H) + F_{BW} \Theta (1 + \gamma^M \psi (a_t^M)^2) \left( \frac{\psi}{\mu_1} H \right) \sigma^2 + F_{WW} (1 + \gamma^M \psi (a_t^M)^2) \left( \frac{\psi}{\mu_1} H \right)^2 a_t^M \sigma^2 = 0.
$$

(A67)

Using the FOCs and the derivatives of $F$, we obtain the following optimal policies

$$
c_t^L = r(B_t^L + G^L(t, \Theta)) + \frac{\Theta}{\gamma^M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L} \Theta,
$$

$$
a_t^M = \alpha,
$$

$$
c_t^M = -\frac{1}{\gamma^M} \ln \beta - \frac{1}{\gamma^M} \ln(-W_t^M) + \Psi(a_t^M),
$$

(A68)
where $\alpha$ and $\beta$ are determined by

\[
\beta = \gamma^M r \left[ \gamma^M \alpha (\psi \alpha - \mu_1) + 1 \right],
\]

\[
\left(1 + \frac{\gamma^L}{\gamma^M} \Theta \right) \left( \frac{\psi}{\mu_1} \alpha \beta \sigma \right)^2 - \left( \gamma^L r \Theta \sigma \right) \left( \frac{\psi}{\mu_1} \alpha \beta \sigma \right) - \left( r - \frac{\beta}{\gamma^M} \right) = 0.
\]  

(A69)

Note that the constants ($\alpha$ and $\beta$) depend upon $L$’s ownership level $\Theta$. By (A63) and optimal policies, we show that $M$’s promised payoff process follows

\[
d\ln(-W^M_t) = \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) dt - \sigma_W dZ_t,
\]  

(A70)

where

\[
\mu_W = \delta^M - \frac{\beta}{\gamma^M}; \quad \sigma_W = \frac{\psi}{\mu_1} \alpha \beta \sigma.
\]  

(A71)

Substituting the optimal policies and the derivatives of $F$ into the HJB equation (A66) yields

\[
G^*_t(t, \Theta) = rG^*_t(t, \Theta) - V^*(\Theta),
\]  

(A72)

where $L$’s net benefit flow from holding $\Theta$ is given

\[
V^*(\Theta) = \Theta \left( \mu(\alpha) - \left( -\frac{1}{\gamma^M} \ln \beta - \frac{1}{\gamma^M r} \left( \mu_W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right) \right) - \frac{1}{2} \gamma^L r \Theta^2 \left( \sigma - \frac{\sigma_W}{\gamma^M r} \right)^2.
\]  

(A73)

By integrating (A72) over the period, we obtain

\[
G^*(t, \Theta) = \phi(t)V^*(\Theta) + (1 - r\phi(t))G^*(t_{i+1}, \Theta).
\]  

(A74)

where $\phi(t) = \frac{1}{r} (1 - e^{-r(t_{i+1} - t)})$. For the last trading period $i = N$, we take the limit $t_{N+1} \to \infty$ and $\lim_{t \to \infty} G^*(t, \Theta) = 0$.

\textit{Proof for Proposition 6:} As $L$’s optimal policies are linear in $\ln(-W^M_t)$, we conjecture and verify that the equilibrium stock price in the agency contracting case has the following form of

\[
P^*(t, \Theta, W^M_t) = \Lambda^*(t, \Theta) + \frac{1}{\gamma^M} \ln(-W^M_t).
\]  

(A75)

We follow a similar approach to solve $S$’s optimal portfolio problem except that $S$’s budget constraint now subtracts $M$’s compensation payment in the firm’s incremental dividend, and we need to additionally consider the stochastic component of the stock price process. In equilibrium, $S$ rationally anticipates $L$’s optimal policies and the stock market clears, which derives the time deterministic component of the stock price, $\Lambda(t, \Theta)$, by

\[
d\Lambda^*_t - r\Lambda^*_t = -k^*(\Theta) dt,
\]  

(A76)
where

\[ k^*(\Theta) = \mu(\alpha) - \left( -\frac{1}{\gamma M} \ln \beta - \frac{1}{\gamma M} \left( \mu W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right) - \gamma S_{\infty}(1 - \Theta) \left( \sigma - \frac{\sigma_W}{\gamma M} \right)^2. \]  

(A77)

Note that the time-deterministic function \( \Lambda^*(t, \Theta) \) can be expressed as the following recursive format:

\[ \Lambda^*(t, \Theta) = \phi_i(t)k^*(\Theta) + (1 - r\phi_i(t))\Lambda^*(t_{i+1}, \Theta), \]  

(A78)

where \( \phi_i(t) = \frac{1}{r} \left( 1 - e^{-r(t_i+1-t)} \right) \). For the last trading period, \( \lim_{t \to \infty} \Lambda^*(t, \Theta) = 0 \). The detailed procedure to derive the above price equilibrium is similar the proof of Propositions 2 and 3 for the owner-manager case.

**Proof for Proposition 7:** As shown earlier, \( L^* \)’s value function at \( t_i \) right after she chooses the ownership level of \( \Theta \) is then given by

\[ W_{L_i}^{L} = -\frac{1}{\gamma L^L} e^{-\gamma L_i} \left[ r(B_i^L + G^*(t_i, \Theta)) + \frac{\Theta_i}{\gamma M} \ln(-W_{M_i}^M) + \frac{\delta_i L_i^L}{\gamma L^L} \right], \]  

(A79)

where \( M^* \)’s promised payoff from the incentive contract determined given \( L^* \)’s ownership choice \( \Theta \) is \( W_{L_i}^M \). We now consider \( L^* \)’s optimal equity stake choice at \( t_i^- \), given her current shareholdings, \( \Theta_{t_{i-1}}^{L*} \). First, note that the renegotiation-proof constraint requires that the incentive contract provide \( M \) with at least \( M^* \)’s promised payoff resulting from the previous period,

\[ (IR) : W_{L_i}^M(c^M, a^M) \geq W_{L_i}^M, \]  

(A80)

which holds with equality. Second, from the previous proofs, recall that the time-deterministic functions \( G \) and \( \Lambda \) in \( L^* \)’s value function and in the stock price process, respectively, are

\[ G^*(t, \Theta) = \phi_i(t)V^*(\Theta) + (1 - r\phi_i(t))G^*(t_{i+1}, \Theta), \]  

(A81)

\[ \Lambda^*(t, \Theta) = \phi_i(t)k^*(\Theta) + (1 - r\phi_i(t))\Lambda^*(t_{i+1}, \Theta). \]  

(A82)

As in the proof of Proposition 4, \( L^* \)’s optimal trading policy at \( t_i^- \), maximizes her certainty-equivalent continuation payoff after making a new trade in (A79) plus the proceeds from the trade, \( B_i^L + (\Theta_{t_{i-1}}^{L*} - \Theta)P^*(t_i, \Theta, W_{L_i}^M) + G^*(t_i, \Theta) + \frac{\Theta_i}{\gamma M} \ln(-W_{M_i}^M) \). By (A81) and (A82), we can define the time-deterministic function \( G \) at \( t_i^- \) from \( L^* \)’s optimal decision as below:

\[ G^*(t_i^-, \Theta_{t_{i-1}}^{L*}) = \max_{\Theta} (\Theta_{t_{i-1}}^{L*} - \Theta)\Lambda^*(t_i, \Theta) + G^*(t_i, \Theta) \]

\[ = \max_{\Theta} (\Theta_{t_{i-1}}^{L*} - \Theta)\Lambda^*(t_i, \Theta) + \phi_i(t_i)V^*(\Theta) + (1 - r\phi_i(t_i))G^*(t_{i+1}, \Theta), \]  

(A83)

whose FOC is then derived as
α \text{ (A84)}

where the last term is, due to the envelope theorem and continuity in the stock price,

$$\frac{\partial G^*(t_{i+1}, \Theta)}{\partial \Theta} = \Lambda^*(t_{i+1}, \Theta^L_{t_{i+1}}). \quad \text{(A85)}$$

By (A78), we rewrite the FOC above as

$$\text{FOC : (} \Theta^L_{t_{i-1}} - \Theta \text{)} \frac{\partial \Lambda^*(t_i, \Theta)}{\partial \Theta} - \Lambda^*(t_i, \Theta) + \phi_i(t_i) V''(\Theta) + (1 - r \phi_i(t_i)) \frac{\partial G^*(t_{i+1}, \Theta)}{\partial \Theta} = 0.$$ 

By the definitions of $V^*(\Theta)$ and $k^*(\Theta)$ in (A73) and (A77),

$$V^*(\Theta) = \Theta \Omega_1(\Theta) + \frac{\Theta}{\gamma M r} \Omega_2(\Theta) - \frac{1}{2} \gamma^L r \Theta^2 \Omega_3(\Theta)$$

$$k^*(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma M r} \Omega_2(\Theta) - \gamma^S r (1 - \Theta) \Omega_3(\Theta),$$

where $\Omega_1(\Theta) \equiv (\mu(\alpha) + \frac{1}{\gamma M} \ln(\beta) - \Psi(\alpha))$, $\Omega_2(\Theta) \equiv (\mu_W - \frac{1}{2} \sigma_W^2)$, and $\Omega_3(\Theta) \equiv \left(\sigma - \frac{\sigma_W}{\gamma M r}\right)^2$. Note that the manager’s contractual variables, $\alpha$ and $\beta$, are also functions of $\Theta$ and that $\beta$ is directly obtained by its relation with $\alpha$: $\beta = \beta(\alpha) = \gamma M r [\gamma M \alpha(\psi \alpha - \mu_1) + 1]$. Also note that the variables $\mu_W$ and $\sigma_W$ are functions of $\alpha$.

The derivative of $V^*(\Theta)$ with respect to $\Theta$ is

$$V''(\Theta) = \Theta \Omega_1(\Theta) + \frac{\Theta}{\gamma M r} \Omega_2(\Theta) + \frac{1}{\gamma^M r} \left( \Omega_2(\Theta) + \Theta \frac{\partial \Omega_2(\Theta)}{\partial \Theta} \right) - \gamma^L r \Theta \Omega_3(\Theta) - \frac{1}{2} \gamma^L r \Theta^2 \frac{\partial \Omega_3(\Theta)}{\partial \Theta}. \quad \text{(A87)}$$

Consider the three derivative terms above:

$$\Theta \frac{\partial \Omega_1(\Theta)}{\partial \Theta} + \frac{1}{\gamma M r} \Theta \frac{\partial \Omega_2(\Theta)}{\partial \Theta} - \frac{1}{2} \gamma^L r \Theta^2 \frac{\partial \Omega_3(\Theta)}{\partial \Theta}$$

$$= \Theta \left( \mu_1 \alpha'(\Theta) + \frac{1}{\gamma M} \frac{\beta'(\alpha) \alpha'\Theta(\Theta)}{\beta(\alpha)} - \psi \alpha(\Theta) \alpha'\Theta(\Theta) \right)$$

$$+ \frac{1}{\gamma M} \Theta \left( \mu_W'(\alpha) \alpha'(\Theta) - \sigma_W(\alpha) \sigma_W'(\alpha) \alpha'(\Theta) \right) - \gamma^L r \Theta^2 \left( \sigma - \frac{\sigma_W(\alpha)}{\gamma M r} \right) \left( - \frac{1}{\gamma M} \sigma_W'(\alpha) \alpha'(\Theta) \right) = 0. \quad \text{(A88)}$$

By the envelope theorem, the above three terms in (A88) that arise from the dependence of $\Theta$ through $\alpha$ should be zero. To see this more clearly, recall that we have chosen the manager’s optimal effort $\alpha$ for a given $\Theta$ to maximize $L$’s value function in (A74), that is, $V^*(\Theta)$. We thus have

$$\frac{\partial V''(\alpha)}{\partial \alpha} = \Theta \left( \mu_1 + \frac{1}{\gamma M} \frac{\beta'(\alpha)}{\beta(\alpha)} - \psi \alpha(\Theta) \right).$$
\[
+ \frac{1}{\gamma M^r} \Theta \left( \mu_W(\alpha) - \sigma_W(\alpha) \sigma_W(\alpha) \right) - \gamma L r \Theta^2 \left( \sigma - \frac{\sigma_W(\alpha)}{\gamma M^r} \right) \left( -\frac{1}{\gamma M^r} \sigma_W(\alpha) \right) = 0,
\]
which results in
\[
V''(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma M^r} \Omega_2(\Theta) - \gamma L r \Theta \Omega_3(\Theta).
\]
(A89)

As a result, the FOC (A86) further reduces to
\[
FOC : (\Theta^L* - \Theta) \frac{\partial \Lambda^*(t_i, \Theta)}{\partial \Theta} + \phi_i(t_i) \left[ \gamma^S - (\gamma^L + \gamma^S) \Theta \right] r \Omega_3(\Theta) = 0,
\]
(A90)

where note that \( \Omega_3(\Theta) = \sigma^2_r(\Theta) = \left( \sigma - \frac{\sigma_W}{\gamma M^r} \right)^2 \geq 0. \)

**Proof of Corollary 6**: It is evident that, in the steady-state equilibrium, \( L \)’s optimal ownership level \( \Theta^L* \) solves \( V''(\Theta) - k^*(\Theta) = 0 \). From the above derivation, we have
\[
V''(\Theta) = \gamma^S - (\gamma^L + \gamma^S) \Theta | r \Omega_3(\Theta) = 0,
\]
which leads to the steady-state \( L \)’s ownership, \( \Theta^L* = \gamma^S / (\gamma^L + \gamma^S) \).

Q.E.D..

**A2 Infinite Trading Rounds for Large Shareholder**

In our main analysis, we follow DU by assuming that the number \( N \) of \( L \)’s trading rounds is finite. In this appendix, we briefly describe the analysis of the scenario in which the number of trading dates for \( L \) is infinite, but each trading interval has finite length, \( \Delta \). The scenario with infinite trading rounds provides a rigorous underpinning for the steady state equilibrium that we describe in the main body where we consider the limit as the number of trading dates, \( N \to \infty \).

As in the model with a finite number of trading dates for \( L \), an equilibrium is described by the vector of processes
\[
\{(\Theta^L*, B^L*, c^L*); (\theta^S*, Y^S*, c^S*); \Pi^*; P^*\}
\]
(A91)

where \( (\Theta^L*, B^L*, c^L*) \) is the vector of processes representing \( L \)’s share ownership, money market account balance, and consumption, respectively; \( (\theta^S*, Y^S*, c^S*) \) is the vector of processes representing a small shareholder \( S \)’s share ownership, total wealth, and consumption, \( \Pi^* \) is the manager’s contract, and \( P^* \) is the stock price process.

As we discussed in Section 2.4, \( L \)’s ownership, consumption and contracting choices must be sequentially optimal and, therefore, solve (10) subject to the budget constraint given by (9). The recursive characterization of renegotiation-proofness in Section 2.3 continues to hold in the setting with an infinite number of trading rounds (see Giat and Subramanian (2013)).

We can derive the equilibrium using dynamic programming by guessing the value function for \( L \) and then applying a verification theorem. We outline the procedure, but omit the details of each
1. Optimal contracting between $L$ and $M$, given $L$’s trading policy $\Theta$ and stock price process $P$: $M$’s problem to choose an optimal effort given a contract is the same as before. In the $L$’s optimal consumption and contracting problem, to ensure that our HJB equation approach and, therefore, the guessed value function and optimal policies are valid (that is, the verification theorem can be applied), we need to make sure that the consumption process of $L$ is well-behaved. As in the main model, to avoid “Ponzi schemes” (otherwise, $L$ could keep increasing her consumption by borrowing money without repaying it), we impose the restriction on $L$’s time deterministic process $G^*(t)$: $\lim_{t \to \infty} G^*(t, \Theta) = 0$. We also need to make sure that $L$’s trading gains do not explode. While $L$’s holdings are within $[0, 1]$, the price itself could explode so that we impose a condition that there is no “bubble” in the price process: $\lim_{t \to \infty} e^{-\delta t} P(t) = 0$. Given these two conditions, our solutions to the HJB equation of the large shareholder in the main body continue to hold.

2. Optimal portfolio problem of $S$ and equilibrium stock price $P$, given $L$’s trading policy $\Theta$ and optimal contracting: For small shareholder $S$’s problem, we also impose the same conditions: $\lim_{t \to \infty} J^*(t, \Theta) = 0$ and $\lim_{t \to \infty} e^{-\delta t} P(t) = 0$.

3. Optimal trading Policy of $L$, given stock price process: Given well-behaved contracting and price process, the remaining problem is to characterize $L$’s optimal trading policy. The procedure for showing the existence of the solution to the discrete time, infinite horizon problem of $L$’s trading rounds and obtaining it is largely identical to those in Abreu et al. (1986) and GHH (2014).

Through these steps, we can prove the equivalence between the equilibrium solution to the infinite trading round problem and the solution that we have obtained for the finite trading round problem in the main body.
Table 1: Baseline Parameters

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Baseline Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time interval between L’s successive trading dates ((\Delta))</td>
<td>1/4</td>
</tr>
<tr>
<td>Risk-free rate ((r))</td>
<td>0.02</td>
</tr>
<tr>
<td>Subjective discount rate ((\delta_L = \delta_M = \delta_S))</td>
<td>0.0404</td>
</tr>
<tr>
<td>Unit cost of effort ((\psi))</td>
<td>1</td>
</tr>
<tr>
<td>Constant term ((\mu_0)) in the firm’s mean cash flow (\mu(a_t))</td>
<td>0.17</td>
</tr>
<tr>
<td>Productivity of effort ((\mu_1)) in the firm’s mean cash flow (\mu(a_t))</td>
<td>0.03</td>
</tr>
<tr>
<td>Firm output (cash flow) volatility ((\sigma))</td>
<td>0.35</td>
</tr>
<tr>
<td>L’s absolute risk aversion ((\gamma_L))</td>
<td>224.25</td>
</tr>
<tr>
<td>M’s risk aversion ((\gamma_M))</td>
<td>297.79</td>
</tr>
<tr>
<td>S’s absolute risk aversion ((\gamma_S))</td>
<td>74.75</td>
</tr>
<tr>
<td>M’s mean promised payoff in steady state ((\ln(-\bar{W}^M)))</td>
<td>-9.26</td>
</tr>
</tbody>
</table>

Table 2: Actual and Model-Predicted (Steady-State) Moments

<table>
<thead>
<tr>
<th>L’s ownership</th>
<th>Stock return volatility</th>
<th>Sharpe ratio</th>
<th>Market-to-book ratio</th>
<th>Dividend-price ratio</th>
<th>(\text{PPS}_{RT})</th>
<th>CEO pay-to-market value ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>0.250</td>
<td>0.194</td>
<td>0.320</td>
<td>1.236</td>
<td>0.045</td>
<td>0.003</td>
</tr>
<tr>
<td>Predicted</td>
<td>0.250</td>
<td>0.126</td>
<td>0.320</td>
<td>1.235</td>
<td>0.062</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 3: Model-Predicted (Steady-State) Equilibrium in the Agency Contracting and Owner-Manager Cases

<table>
<thead>
<tr>
<th>L’s ownership</th>
<th>Effort</th>
<th>Expected stock return</th>
<th>Stock return volatility</th>
<th>Sharpe ratio</th>
<th>Market-to-book ratio</th>
<th>Dividend-price ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agency Contracting Case (dollar return)</td>
<td>0.250</td>
<td>0.005</td>
<td>0.040</td>
<td>0.320</td>
<td>1.235</td>
<td>0.062</td>
</tr>
<tr>
<td>Owner-Manager Case (dollar return)</td>
<td>0.250</td>
<td>0.008</td>
<td>0.034</td>
<td>0.381</td>
<td>2.081</td>
<td>0.043</td>
</tr>
</tbody>
</table>

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Figure 1: Effects of Large Shareholder Ownership

Figure 2: Productivity, Stock Returns and Incentives

Figure 3: Cash Flow Volatility, Stock Returns and Incentives
Figure 4: Productivity and Large Shareholder Ownership

Figure 5: Cash Flow Volatility and Large Shareholder Ownership