Credit-Rating Shopping, Selection and the Equilibrium Structure of Ratings[*]

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Abstract

An important feature of the microstructure of credit ratings is the ability of a security issuer to choose which ratings to purchase. Such choices can reflect explicit or even implicit shopping for favorable credit reviews and induce "selection" effects in the structure of ratings. When there is considerable heterogeneity in views the issuer selects the ratings that are the most positive. Because the correlation among models is least when heterogeneity is greatest, we examine the influence of the correlation on the extent of ratings shopping and bias. Selectivity highlights the interaction between the decision about whether to rely on unsolicited ratings based upon coarser information and the potential for ratings shopping, illustrating the interaction between different types of potential conflicts of interest in the credit rating process. We use selection to provide an equilibrium interpretation for "notching" (formulaic haircutting of a credit rating agency's rating) by a rival. We point to the potential winner's curse that is implicit when the most favorable rating is the only rating published, raising the issue of how rating agencies, regulatory authorities and investors interpret solicited ratings. We conclude by presenting theoretical results and numerical solutions to illustrate qualitative aspects of rating shopping. For example, we show that higher cost of obtaining indicative ratings and regulatory mandates to charge fees for obtaining indicative ratings reduce the extent to which these are obtained, increasing the average published ratings. The higher cost of ratings also reduces the likelihood that an unpublished rating is below an existing published one.

1   Introductory Comments

Both the financial market crisis and the 2006 changes in the statutory framework underlying the role and structure of credit-rating agencies have focused attention upon the microstructure of credit ratings. Indeed, the failures of credit rating agencies during the market crisis had been the subject of Congressional hearings and Securities and Exchange Commission rulemakings, also motivating a number of potential regulatory proposals. Credit ratings are central to a broad array of valuation and regulatory issues in the marketplace. At the heart of the micro aspects of credit ratings of financial instruments are "selection effects" that arise at many levels. These occur because of the ability of security issuers to choose which credit ratings to purchase and have published. Such choices by an issuer can reflect *explicit* or even *implicit* shopping for more favorable credit ratings.[1] As a result of "selection," there is information not only in the ratings obtained for an issue, but also statistical information attributable to the absence of ratings from various credit-rating agencies. Interestingly, the shopping for credit ratings and their purchase from specific agencies by the issuer is closely associated with both the payment model in which the security issuer purchases one or more ratings and the absence in recent years of unsolicited ratings.[2] The incentive to obtain as high a rating as possible can reflect both market forces, to the extent that ratings are an important input influencing pricing and demand, as well as incentives from a variety of financial regulators. For example, net capital rules and other restrictions on allowed asset holdings of investors lead to regulatory incentives for issuers to obtain ratings above some threshold level, or to obtain as high a rating as possible.[3] In practice the issuers' incentives for high ratings gets reflected not only in the competition among rating agencies for market share, but also in a desire to rate financial products that have significant overall demand in the marketplace.[4]

One context in which the microstructure of ratings has played an especially prominent role is the debate involving "notching" by credit-rating agencies in which an agency would use a formula to "haircut" (reduce) ratings produced by its competitors when expressing the ratings on its own scale. When the U.S. Securities and Exchange Commission (SEC) considered its initial rules to implement the restructuring of the statutory framework underlying the rating agencies as established by the Credit Ratings Agency Reform Act of 2006, there was

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[1] Implicit shopping can arise because of the issuer's knowledge of the framework and likely ratings of different rating agencies. In some situations this may even extend to the issuer's choice of the rating agency's technique among a number of alternatives of varying costs. Transparency of the rating models promotes implicit shopping indirectly.

[2] As discussed later in Section 4, the difficulty of excluding investors from access to the ratings evaluation (the "free-rider" problem for selling information) lies at the heart of the payment model in which the issuer rather than the investor pays for the ratings evaluation.

[3] Of course, the cost of alternative ratings also would influence the objective.

[4] While we view the ratings designation from the rating agencies in this paper as passive, in practice the rating agencies can provide advice to influence the design of the instruments, which can be interpreted as heightening the potential conflict of interest.

considerable disagreement among market participants about whether to permit notching in the context of tranches in securitizations.[5] The issue, as addressed in the SEC releases, was essentially whether the SEC should require a credit-rating agency to treat its competitors' ratings as if these were its own or whether the agency could assign a rating by applying a "haircut" or "notch" downward the ratings of its competitors on CDO tranches that it had not rated itself. Of course, requiring a credit-rating agency to honor the higher ratings of one's rivals would limit the ability of an agency to compete in standard setting by developing its own quality standards.[6] When the SEC adopted its 2007 credit-rating agency rules it encouraged competition in standard setting by not objecting to "notching" per se, except when notching represented anti-competitive behavior in marketing and pricing ratings services.[7] Notice that the separate notions of competitive standard setting and competitive pricing have very different consequences.

The basic "selection" intuition offers an interesting equilibrium interpretation of notching that is not anti-competitive and even suggests that "mutual notching" can be consistent with equilibrium and pure competition. Not surprisingly, a credit-rating agency that did not rate a structure typically would have assigned a "virtual" (or shadow) rating for it that would be below the rating from agencies that did rate the structure; this is just a consequence of the *issuer's decision* to purchase the highest rating available! A rating agency whose rating was not selected would likely assign a lower rating than the one published by its rival if it were forced to assign a rating and hadn't originally provided a published rating. In essence, the selection effect that arises from rating shopping (even implicit shopping) implies that notching can be an equilibrium phenomenon. We also should emphasize that though our analysis offers an equilibrium interpretation and rationalization for notching, this does not imply that notching itself is necessarily inconsistent with anti-competitive behavior in trying to force the purchase of ratings (that are not desired). Of course, one example in which notching could be suggestive of such anti-competitive behavior is if the rating agencies used identical underlying models and assessed each issue identically, eliminating the selection effect being described here. It also is potentially consistent with the selection arguments for the rating agencies to mutually engage

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[5]This is reflected on the SEC web page for the rule-making (http://www.sec.gov/comments/S7-04-07/s70407.shtml) by the conflicting comment letters of the rating agencies, the diverse views and comment letters of academics, and the numerous memos there memorializing meetings among SEC officials and designees of the rating agencies. The extent of academic commentary was atypical; indeed, several academics told us that they had been encouraged to write comment letters, but declined to do so. In the spirit of our analysis, the absence of those letters (as well as the letters written) also conveys information.

[6]Analogously, in some pricing situations economists would view "price–matching" policies by firms in an industry as "anticompetitive," because the resulting pricing would provide the firm market power and allow it to act as a monopolist in setting prices. The application of this theme to financial markets in a market microstructure context is discussed by Biais, Glosten and Spatt [2005].

[7]The SEC's release and the adopted rule, Oversight of Credit Rating Agencies Registered as Nationally Recognized Statistical Rating Organizations is at http://www.sec.gov/rules/final/2007/34-55857fr.pdf.

in notching due to the diversity in their models—mutual notching can arise in equilibrium when each agency's own model overestimates the creditworthiness of different bond issues (assuming each model is *noisy* for the various bond issues) and the issuer retains the agency that offers the highest realized rating on each issue. The extent of *heterogeneity* among the valuations of different rating agencies emphasizes why notching arose as a significant issue for securitizations and tranching, but not for conventional corporate and municipal bonds. In our formal analysis we show explicitly that selection effects are especially strong when *heterogeneity is greatest* and *correlation* among the models *is least*. While different views among the rating agencies would limit the extent to which credit rating changes are a source of systemic risk, the reliance by regulators on credit ratings and rating shopping (biasing the publication of the rating that is published) can cause systemic risk to emerge.

In an interesting paper Skreta and Veldkamp [2008] examine cherry-picking in ratings by issuers who shop for the highest ratings in order to obtain the highest price when selling to myopic investors.[8] They highlight the influence of risk aversion in motivating the purchase of multiple ratings and the role of asset complexity and rating shopping in producing bias. Their paper emphasizes the importance of indicative ratings and ratings shopping for securitization, as does our analysis.[9] Rather than relying upon naive investors, we motivate the interest in ratings shopping by the regulatory advantages associated with high published ratings.[10] We also point to the importance of the "winner's curse" with respect to the selected rating when the regulator or rating agencies are sophisticated (and able to adjust for selection biases) for policy towards ratings and rating agencies. In effect, should rating agencies adjust for the "winner's curse" as the ratings tend to be purchased when they are the most favorable and if not, should regulatory requirements reflect the strength of the "winner's curse" in the actual ratings? To some degree this hinges on the meaning of credit ratings. More generally, we use a "selection" framework in which the issuer chooses whether to purchase a rating and if so, which ones to purchase, to provide an explanation for the differences between solicited and unsolicited ratings and to provide an equilibrium explanation for rating "notching" as well as to explore more generally the implications of ratings shopping. In our analysis we highlight surprising results about the impact of costs on behavior. For example, we show that though higher cost of obtaining indicative ratings reduces the extent to which these are obtained, the higher cost increases the average published rating and the higher cost of ratings reduces the

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[8]Another recent analysis of ratings shopping is Bolton, Freixas and Shapiro [2008], who highlight the role of market structure and rating agency reputation and examine related regulatory solutions. Farhi, Lerner and Tirole [2008] model the security issuers as initially attempting to receive certification from the most ambitious certification intermediary (a bit like the journal submission strategy of some academics).

[9]After writing the initial draft of our paper we became aware of the work by Skreta and Veldkamp [2008].

[10]Another form of regulatory shopping is the decision process of financial institutions in selecting the most favorable alternative regulatory approach and regulator for various activities. Secretary of the Treasury Timothy Geithner expressed concern about such regulatory shopping during Senate Budget Committee hearings on March 12, 2009.

probability of "notching," i.e., the likelihood that an unpublished rating is below an existing published one. This suggests that the settlement between the New York Attorney General and the three major rating agencies, which mandated fees for obtaining indicative ratings, potentially would increase published ratings.

In Section 2 we point to regulatory considerations that may motivate why an issuer will purchase ratings from agencies that offer the highest ratings.[11] This will include a brief discussion of net capital rules, rules for money market fund investments and restrictions on investment grade investments. For some regulatory purposes a single rating of an instrument at a specified level is necessary, while for other purposes at least two such ratings are required—highlighting the importance of the second highest rather than the highest rating in some contexts. We emphasize this aspect in more detail in Section 2. Consequently, credit ratings from multiple agencies and split ratings can be purchased and influence the market outcomes differently.[12] For example, fixing the number of rating agencies, additional ratings at the issue's maximal level reflect more positive information, lower coupon yields and lower anticipated losses to default. Then in Section 3 we discuss the nature of the informal interaction between the rating agencies and an issuer as well as the extent to which the models used by the rating agencies are known. Potential changes to the regulatory environment in this regard will be briefly described. We discuss the payment model for credit ratings and analyze the relationship between "solicited" and "unsolicited" ratings in Section 4, including how the structure of information influences the viability of "unsolicited" ratings and the role of selection in explaining empirical differences between solicited and unsolicited ratings. In Section 5 we examine the consequences of rating shopping and selection effects, including the implications for notching and various comparative statics predictions. The decision of an issuer about whether to purchase a rating (Section 4) and which rating to purchase (Section 5) are interrelated, analogous decisions. We formalize the role of the costs of ratings indications and the costs of obtaining ratings for purchase in Sections 6 and 7, obtaining some surprising comparative statics and presenting some numerical solutions. We conclude in Section 8 with a brief discussion of the implications of selection for tranching and securitization, including the importance of model heterogeneity for the magnitude of selection effects.

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[11]Sasseen [2008] argued that even though the SEC had proposed securities rules that reference the credit ratings substantially less often than in the past, the pervasiveness of credit ratings in various government regulations from a variety of regulators would still lead to a strong incentive to maximize the ratings, sometimes viewed as "regulatory arbitrage" (also see the references in the first paragraph of Section 2).

[12]See, for example, Mattarocci [2005], Livingston, Naranjo and Zhou [2005] and Bongaerts, Cremers and Goetzmann [2008].

2 Regulatory Uses of Credit Ratings

Coval, Jurek and Stafford [2009] is an excellent reference summarizing regulatory uses of credit ratings. Of course, these arise in banking (e.g., as in Basel II) and insurance as well as securities regulation. Among the most prominent uses are net capital rules (highly rated instruments require less capital charges) and suitability standards (both constraints on what money market funds can hold and various restrictions to owning only "investment grade" assets). The awkwardness of interpreting ratings similarly for corporate bonds and CDO tranches is highlighted by the analyses of Coval, Jurek and Stafford [2008] and Brennan, Hein and Poon [2008].[13] In fact, recently credit-rating agencies have begun to view corporate and municipal bond ratings in a more unified fashion, after criticism from municipalities in early 2008 that their bonds were judged much more harshly than were corporate bonds (see Creswell and Bajaj [2008]). The prior emphasis of the ratings agencies on relative evaluations within rather than across sectors is not consistent with regulatory standards that are based upon absolute norms and the tradition of regulatory reliance upon ratings.

In its 2003 report to Congress, the SEC reported on the role and function of credit ratings agencies in securities markets.[14] As described in the report, there are three significant areas of regulatory reliance on credit ratings agencies designated as "Nationally Recognized Statistical Ratings Organizations" (NRSROs) by the Commission. The first area of reliance is related to the Net Capital Rule, which sets the amount of regulatory capital a broker-dealer must retain against proprietary securities positions to ensure safety against losses that might be incurred due to the risk and illiquidity of those proprietary positions. The rule allows for less regulatory capital to be held where the securities owned by the broker-dealer are of investment grade as established by an NRSRO. The second area of reliance is in the holdings of money market mutual funds (Rule 2a-7). The rule requires that these types of funds limit their investment to short-term, high quality instruments, where part of the definition relies on ratings by *two* NRSROs in one of their two highest categories (or unrated securities of similar quality).[15] Finally, the Commission's rules allow for issuers of certain non-convertible debt, preferred securities and asset-backed securities to rely on Form S-3, the Commission's "short form", without satisfying a minimum public float test if the security is rated investment grade by at least one NRSRO.

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[13]Lowenstein [2008] highlights the incentive for generating AAA product for mortgage-backed security tranches. Credit-rating "arbitrage" also is discussed by Benmelech and Dlugosz (2008), which provides a detailed empirical description of the collateralized debt obligation (CDO) market and the ratings of those instruments.

[14]See, SEC [2003].

[15]Despite such rules, the Reserve Fund "broke the buck" in September 2008 and then suspended redemptions and liquidity. Many observers viewed the breaking of the buck and related suspension of the fund's liquidity as directly causing a half trillion dollar run on the money market fund industry and the related collapse of the commercial paper market as central to the crisis of confidence in the broader market.

Over time, the NRSRO concept has been included in a variety of financial legislation and regulation. For instance, Congress required, among other things, that a "mortgage related security" be rated in one of the highest two categories by at least one NRSRO. The U.S. Department of Education relies on NRSRO ratings to set standards of financial responsibility in certain circumstances. Insurance codes set by various state regulators rely on NRSRO ratings to determine appropriate investments for insurance companies. Private contracts also often include ratings triggers (a prominent example is the collateral requirements with various AIG credit-default swaps that were triggered by downgrades).

During 2008, the SEC proposed new amendments to rules governing credit ratings agencies designated as NRSROs. The first set of proposals were made in response to concerns about credit rating procedures and methodologies in "light of the role [NRSROs] played in determining credit ratings for securities collateralized by or linked to subprime residential mortgages."[16] The SEC's release described the purpose of the amendments as intended to address conflicts of interest that could arise from rating structured finance products and included proposals to increase disclosure, enhance comparability of ratings performance and strengthen internal controls.[17] Shortly afterwards, the Commission proposed an additional set of rule amendments designed to remove direct reference to NRSROs from certain of its rules, including rules related to net capital calculation, money market mutual fund holdings, and registration requirements for certain asset-backed securities.[18] The proposals addressed concerns that regulatory reliance on credit ratings contributed to insufficient due diligence on the part of investors and the rating agencies as well as concerns about creating private rents from defining public regulation, which itself could strengthen the conflict of interest.[19] A recent account in the *New York Times* (Labaton [2009]) suggested that the new chair of the SEC and other officials of the Obama Administration feel that an important source of conflict of interest among credit rating agencies is that they are "paid by companies to help them structure financial instruments, which the agencies then grade."[20] Of course, the payment model is closely related to ratings

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[16] See, SEC [2008a], Proposed Rules for Nationally Recognized Statistical Rating Organizations, Release No. 34-57967; (June 16, 2008), http://www.sec.gov/rules/proposed/2008/34-57967.pdf.

[17] Ibid., p.7. Implicit in such a proposal is that reputational forces are not adequate to achieve these ends. Perhaps, this is a reflection of a substantial loss in reputational capital by the rating agencies in recent years.

[18] See, SEC [2008b], References to Ratings of Nationally Recognized Statistical Rating Organizations, Release No. 34-58070; (July 1, 2008), http://www.sec.gov/rules/proposed/2008/34-58070.pdf; Release No. IC-28327, (July 1, 2008), http://www.sec.gov/rules/proposed/2008/ic-28327.pdf and SEC [2008c], Security Ratings, Release No. 33-8940 (July 1, 2008), http://www.sec.gov/rules/proposed/2008/33-8940.pdf.

[19] See, for example, President's Working Group on Financial Markets, Policy Statement on Financial Market Developments (March 2008), available at www.ustreas.gov ("PWG Statement"); The Report of the Financial Stability Forum on Enhancing Market and Institutional Resilience (March 2008), available at www.fsforum.org ("FSF Report"); Technical Committee of the International Organization of Securities Commissions, Consultation Report: The Role of Credit Rating Agencies in Structured Finance Markets (March 2008), page 9, available at www.iosco.org; Shadow Financial Regulatory Committee, February 11, 2008, "Statement on 'Reliance on Third-Party Credit Ratings';" and Financial Economists Roundtable, December 1, 2008, "Statement on Reforming the Role of the Statistical Ratings Organizations in the Securitization Process."

[20] See Labaton [2009, p. 1].

shopping. Rating shopping, which limits the diversity in signals from published ratings, as well as regulatory reliance on ratings, both could enhance the extent to which the ratings process contributes to systemic risk as a byproduct of changes in ratings assessments.

3   Interaction between Rating Agencies and Issuers

Credit-rating agencies have long published aspects of their methodology, including notes describing methodologies for rating different types of securities and the long-term performance of ratings to predict bankruptcy, thereby providing issuers with a perspective on the likely rating of a structure or product from particular agencies. Consequently, the shopping process by which the issuer may choose to hire a credit-rating agency that offers the most favorable rating may be fairly effective even absent explicit communication from the rating agency about how it would evaluate a particular situation. Of course, in some contexts the shopping process may be more direct because the credit-rating agency offers an indicative rating. Still more explicitly, but less frequently, the agency may offer a rationale for an indicative rating, which in the structuring or LBO context could guide adjustment of the financial design.

Some of the focus in SEC rule-making has been to attempt to enhance transparency of ratings in other dimensions, particularly for structured products—e.g., what was considered in the ratings process and what was the basis of the ratings? This would include making public all information provided to the ratings agency by all parties, including issuer, underwriter, sponsor, depositor or trustee, such that it includes all relevant information to determine an initial ratings and subsequent ratings monitoring.[21] These proposals identified as a goal enhancing the comparability of ratings performance via disclosure of rating histories for all rated securities. These features could potentially make it easier for entrants to compete by establishing their track records and reputations, but also make it easier for issuers to shop implicitly by shedding light on the models used by the ratings agencies and also facilitate potential collusion (e.g., in pricing) among the rating agencies.

Because the largest and most well-known credit rating agencies have "issuer-pay" business models, issuers typically only pay fees when ratings are published. That is, issuers can approach such a credit rating agency and request that a rating be generated. The credit rating agency then determines an indicative rating based on information provided by the issuer. The issuer can provide clarification and additional information to the agency and then determine if it wants to purchase the rating. If the rating is purchased, the credit rating agency then publishes the rating on its website, via press release and the rating may be included in materials provided by an underwriter.

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[21]SEC [2008a], p. 30.

As the fees have been paid upon issuer acceptance or publication, and not assessed by the rating agencies for any intermediate indications, it would be difficult to measure the shopping process directly (fee data would not appear to help) and distinguish these empirically. If the credit-rating agencies would be required to disclose their contacts with issuers, explicit communication would decline potentially and then signaling would increase. Interestingly, the usefulness of requiring such disclosures would be limited because of the transparency of the models themselves. Somewhat analogously, the issuers can hide indicative ratings they receive in equilibrium only if the decision to obtain a rating is unobservable rather than transparent (see, Faure-Grimaud, Peyrache and Quesada [2007]).

Of course, the above interpretation of rating shopping would exist only in agencies that do not assess the issuer when providing an informal indication, but only charge in conjunction with publication of a rating. Such a fee structure encourages explicit rating shopping as compared to an alternative fee structure that also charges at informal provision of an indicative rating. The recent settlement between the New York Attorney General and the three largest rating agencies could push towards this type of pricing structure (along with billing at the initial engagement) with the apparent aim of reducing explicit ratings shopping.[22] Yet, it is not obvious to what degree this would reduce selection or shopping effects because of the potential importance of implicit ratings shopping. In fact, selectivity could increase in such a regime, especially if the fee rules promote greater transparency about the models and less noise within them.[23] Analogously, mandating disclosure of the contacts between rating agencies and issuers could be counterproductive by leading to an increase in rating shopping as a result of increased transparency of the underlying model. Even if the degree of transparency were fixed, an increase in the cost of an indicative rating would decrease the number of these obtained, which leads to a higher threshold for accepting and publishing an individual indicative rating and potentially higher ratings.

4   Solicited and Unsolicited Ratings

A basic aspect of credit ratings concerns the underlying payment model in that these ratings have been purchased traditionally by the issuer of the security. An alternative model would be for the rating agency to charge the investors (users) and indeed variations of that arise within the market for credit ratings as well as in some other markets for information, such as the equity analyst market. Of course, charging the users can be awkward because of the inherent difficulty

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[22]For a description of the settlement, see Press Release, Office of the New York State Attorney General, "Attorney General Cuomo Announces Landmark Reform Agreements with the Nation's Three Principal Credit Rating Agencies" (June 2008). http://www.oag.state.ny.us/press/2008/june/june5a_08.html

[23]Of course, recent SEC proposals also potentially encourage greater transparency more directly. See SEC [2008a]

of excluding investors and the public goods aspect to information.[24]   In the credit-rating context this generic difficulty may have been reinforced because of regulatory impediments to exclusion, making it difficult to finance the cost of ratings from the users of the rating.[25] This may provide a basic explanation for why the rated firms usually finance the costs and why the users do not typically pay the costs directly.[26] Yet there are a number of advantages as well with an "investor pays" model—ratings shopping and selection complications are greatly mitigated when the security issuer does not decide which ratings should be published and the incentives for the rating agency to invest resources over time to ensure adequate secondary market pricing will be much more direct when the investors pay for the ratings as compared to the issuer.[27]

We next turn to economic aspects underlying "solicited" (i.e., purchased) vs. "unsolicited" ratings in which the issuing firm is not charged.[28] The use of unsolicited ratings, had been a standard practice in corporate ratings until a few years ago, but then in the face of widespread criticism the rating agencies backed away from this practice through a voluntary industry framework and changes to the IOSCO regulatory structure.[29] In light of this, the difficulty in obtaining unsolicited ratings is arguably in part regulatory, but also for the case of structured products the limited disclosure requirements in the past have suggested much less inherent ability for rating agencies to assign unsolicited ratings than for either public corporate or municipal bonds. Of course, a basic barrier to unsolicited ratings is the lack of financial payment that would accrue to the credit-rating agency, especially in light of the difficulties with an investor payment model. Particularly for structured products, the difficulty in gaining access to information describing the underlying assets in a pool represents an equally basic barrier. To the extent that regulatory policy has discouraged the use of unsolicited ratings, this may have indirectly encouraged ratings shopping because the security issuer decides whose

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[24]In a portfolio context the public goods problem associated with the investor purchasing information is illustrated by the Grossman and Stiglitz [1980] "Paradox," highlighting the limited value to investors of information that they purchase due to the spillover in market pricing.

[25]For instance, the statute and current SEC rules require that a credit rating agency registered as an NRSRO make its ratings available on the internet or through other readily accessible means, for a reasonable fee or free.

[26]As of August 2008 under the new regulatory regime, ten credit rating agencies have registered as NRSROs. They are: A.M. Best, DBRS, Fitch, Japan Credit Rating Agency, Moody's Investor Services, Rating and Investment Information, Standard & Poor's Rating Services, Egan-Jones Rating Company, LACE Financial and Realpoint. Of these, three are based on an "issuer-pay" compensation model (Egan-Jones, LACE and Realpoint) and of the three only one, Realpoint, provides ratings on structured products.

[27]Indeed, because many rating revisions after issuance are downgrades, the issuer has little incentive to compensate the rating agency for efforts to re-evaluate outstanding issues.

[28]A preliminary discussion of unsolicited ratings and the credit-rating agency payment model is in Spatt [2004].

[29]The Code of Conduct Fundamentals for Credit Ratings Agencies, IOSCO as published in December 2004 also includes the provision that each unsolicited rating made by an agency should be identified as such and that each credit rating agency should disclose policies and procedures regarding unsolicited ratings. A consideration of the role of unsolicited ratings for structured products can be found as part of the May 2008 IOSCO report, The Role of Credit Rating Agencies in Structured Finance Markets.

rating to validate (this aspect is more central to the conflict of interest than who formally pays) and arguably, there is no objective arbiter if the ratings are solicited.[30] As previously noted, recently proposed rules by the SEC have highlighted the potential usefulness of unsolicited ratings in offering alternative views of a security or securitization, promoting disclosure of the basis for other ratings, thereby potentially making it more feasible for competing agencies to offer their views despite limited information being supplied through the instrument's creator.[31]

The traditional rationale by rating agencies for providing unsolicited ratings is that they are "providers of opinion" and that the users (investors) are interested in relative opinions, which the rating agencies cannot provide for the issuers that solicit them without a broad enough base of ratings. Along these lines, the well-known First Amendment attorney, Floyd Abrams, provided a related rationale for unsolicited ratings: "The fact of unsolicited ratings is what makes a rating firm journalistic in nature."[32] Interestingly, the credit-rating agencies are viewed by courts as "financial publishers" with some First Amendment (freedom of the press) protection,[33] though why the credit-rating agencies are entitled to such preferred treatment compared to equity analysts and others information providers is not obvious from an economist's perspective. Similarly, commentators such as former SEC Chairman Arthur Levitt have noted that the First Amendment protection that the rating agencies enjoy did not disappear when credit-rating agencies stopped issuing unsolicited ratings.[34]

Of course, unsolicited ratings reflect the rating agency's incentive to rate an instrument, even if the rating agency is not being paid. In face of such a potential incentive why would the security issuer possess an incentive to purchase a rating? One reason is the possibility (or in some cases the likelihood) that a particular instrument would not otherwise be rated. If the actual ratings in the solicited and unsolicited case were not distinguishable from the issuer's perspective, then there would be no incentive to pay for a rating unless the likelihood of receiving such an unsolicited rating was not sufficiently high. The greater the likelihood and use of unsolicited ratings the more problematic is the incentive constraint determining the issuer's purchase decision and the less that can be extracted from those who purchase ratings.

In addition to increasing the probability (to one) that an instrument will be rated (assum-

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[30] In effect, there is a built-in conflict of interest associated with the "issuer pays" model; also see Coffee [2008].

[31] Given the intrinsic difficulties with the payment model for unsolicited ratings, the extent to which these would be provided is certainly an open question.

[32] Abrams, who has served as counsel to a major credit-rating agency, expressed this perspective at a conference on credit ratings at the American Enterprise Institute in September 2005.

[33] For example, Jefferson County School District No. R-1 vs. Moody's Investor's Services, United States Court of Appeals for the Tenth Circuit, May 4, 1999, 175 F. 3d 848 and Compuware vs. Moody's Investors Services, United States Court of Appeals for the Sixth Circuit, No. 05-1851, August 23, 2007.

[34] Former SEC Chairman Arthur Levitt [2007] characterized the claims by credit-rating agencies of First Amendment protection for solicited ratings as being on "very shaky ground" as compared to when they just offered unsolicited ratings.

ing that an unsolicited rating would not be universally provided), in some instances soliciting a rating may increase the issue's anticipated rating. Indeed, several empirical studies document that solicited ratings exceed unsolicited ratings.[35] The agencies have emphasized that when a rating is solicited the agencies are better positioned to drill deeper and generate finer information. While some skepticism is warranted as to whether finer information should be inherently value enhancing on average (e.g., in light of the "law of iterated expectation"), the selection effect points in this direction. Specifically, the firms that want to provide finer information to the ratings agency would be those for whom the finer information would tend to increase the ratings. For a given issuer, those types who chose not to have a solicited rating are (negatively) signaling to the rating agency. If the solicited rating were based upon finer information that was costless to obtain, then standard theoretical arguments imply that all types of issuers would willingly obtain such a rating to avoid adverse selection and actually being viewed as the worst type (e.g., Akerlof [1970] and the analyses of disclosure in Grossman [1981] and Milgrom [1981]). If the private information of the firm to be rated were adverse relative to the agency's likely unsolicited rating, then it would be optimal to not pay. Indeed, in analyses of verification costs (such as Townsend [1980]) and disclosure costs (such as Verrecchia [1983]), there is a threshold below which verification or disclosure is not optimal. For example, if the firm's anticipated rating was actually drawn from the interval $[a, b]$ and the issuer received a signal that the rating would be x, then it would be optimal for the issuer to solicit a rating only if $x > x^*$, where the critical margin x^* reflects the ratings cost.[36] Consequently, a solicited rating would typically not drop below the unsolicited one based upon coarser information, as the issuer has the option of whether to solicit a rating and facilitate providing finer information. In the spirit of the analyses incorporating verification and disclosure costs, the increase in the firm's valuation shouldn't just be non-negative, but actually compensate for the ratings cost for the marginal anticipated type. This argument is reinforced further here by the presence of multiple rating agencies as the firm also has a choice as to which rating(s) to solicit, thereby going beyond the analysis in Verrecchia [1983]. As we argue elsewhere in this paper, the firm will purchase a rating only from agencies that are anticipated to offer a relatively high rating (compared to the rival agencies).

Of course, yet another interpretation related to the unsolicited ratings being lower than the solicited ratings is that otherwise there would be no incentive for issuers to purchase ratings. One could view this as punishment to those who do not pay the rating agency[37] or allowing

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[35]See, for example, the empirical evidence in Butler and Rodgers [2003], Byoun and Shin [2002] and Bannier, Behr and Guttler [2008]. An anecdotal example also is discussed in Klein [2004]. The study by Bannier, Behr and Guttler [2008] uses empirical default rates and so can explicitly analyze adverse selection and strategic motives that would help explain the overall empirical results in different subsamples.

[36]The higher the cost the smaller the proportion of purchased ratings, but the higher the unsolicited rating as well as the average solicited rating.

[37]For example, this perspective appears to underlie Klein [2004].

the ratings agency to be compensated for its information production costs, especially given the awkwardness and free-rider aspects associated with attempting to sell information.[38] Earlier regulatory concerns about unsolicited ratings implicitly reflect the perspective that these were artificially low, perhaps reflecting punitive decisions by rating agencies, while the most recent attention to ratings shopping suggests that shopping leads to solicited ratings that could be artificially high. The overall context points to a delicate aspect in managing potential conflicts of interest. To the extent that there are a variety of conflicts one may play an important role in mitigating another and so one should not decide to eliminate a particular type of conflict in isolation.[39] Of course, since in our formal analysis the rating agency always discloses the signal that it obtains there is not a true conflict of interest.

5 Selectivity and Ratings Determination

To address the implications of selectivity we first consider a setting in which there are n credit-rating agencies, each possessing its own model that produces a signal about the distribution of the payoff on a one-period instrument. Suppose initially that the underlying models are common knowledge to all the rating agencies and the signals themselves are known by the issuer and that the only role of the signal is to classify the asset's risk for regulatory purposes. Additionally, we suppose that the regulator treats signals purchased from each of the credit-rating agencies equivalently—but that the higher the signal the lower the regulatory capital required to be reserved by the issuer and therefore, the greater the net value to the issuer. Because the issuer's objective is to maximize the net value that they receive for the security and since the various models and signals are known to the issuer, the issuer will purchase a rating only from an agency that would offer the highest signal or rating at equivalent pricing.[40] Consequently, indicative ratings that are not purchased from other agencies on an instrument will not exceed (and often will be strictly below) those that are purchased.[41] This setting suggests a number of immediate implications such as the equilibrium interpretation of notching summarized in our Introduction and the discussion in the prior section of solicited vs. unsolicited (not purchased) credit ratings.

The setting also raises an interesting issue about how to think about potential regulation as

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[38]Upon hearing a few years ago about the issue of solicited vs. unsolicited ratings, one prominent game theorist remarked that presumably the unsolicited ratings were the "B" (i.e., not the top) ratings, consistent with these interpretations (including a selection framework) and evoking the punishment theme.

[39]In the theory of the second best some frictions can help mitigate others.

[40]Indeed, Lucchetti [2007] reports that Moody's market share "dropped to 25% from 75% in rating commercial mortgage deals after it increased standards."

[41]This conclusion follows directly assuming equivalent pricing. Of course, the rating agency offering the higher rating could potentially extract the incremental value relative to the next highest rating through its pricing for the rating.

the structure of rating agencies changes. As one increases the number of potential ratings by introducing additional rating agencies and potential models, the interpretation of the maximum signal changes. As there are more potential ratings (including the use of a broader set of ratings techniques that the issuer can select), the distribution of the first-order statistic exhibits first-order dominance relative to the distribution of that statistic with a smaller number of potential ratings. This raises an interesting issue about the rating process in the presence of ratings shopping—i.e., whether the credit-rating agencies should adjust for the "winner's curse" because their ratings are only purchased by the issuer when they are the outlier.[42] Of course, the specific winner's curse adjustments, if an agency were to adjust its own assessment of the rating provided by another agency without undertaking its own direct due diligence, would depend upon the nature of the knowledge of the rating agencies about the models of the other agencies. Absent correction for "the winner's curse," this suggests that the regulators should adjust any relevant regulatory standard as a function of the set of potential credit-rating agencies. Of course, the magnitude of such adjustments, like the winner's curse, should depend upon the extent of cross-sectional dispersion in the signals and the number of rating agencies (or bidders), as in auction theory. Absent significant heterogeneity in the models and signals, this effect would not be very important. The empirical analysis in Becker and Milbourn [2008] suggests that just the *reverse* of the winner's curse happens in practice—with the entry of Fitch the ratings of the incumbent agencies *rose* rather than *declined*![43] This is natural given ratings shopping and inadequate correction for the winner's curse—with more ratings agencies there could be more competition to provide the highest rating.

Suppose instead that the underlying models of the different credit-rating agencies are not known by the issuer, but that the issuer approaches various agencies to seek a preliminary rating for an instrument. Then based upon these underlying preliminary rating signals the issuer can, as earlier, purchase the highest rating. In this scenario the issuer is explicitly shopping rather than implicitly shopping based upon his underlying knowledge of the ratings process. However, absent costs of approaching the various agencies, the selection effects associated with explicit shopping are identical to those in the implicit shopping scenario as in both instances the issuer acquires a rating from the rating agency that offers the highest rating. Central differences between explicit and implicit shopping could arise either because (a) implicit shopping uses less reliable (and more noisy) judgments about the underlying models of the various rating agencies (and therefore, sometimes misjudges which agency would offer the most favorable

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[42]The winner's curse plays a prominent role in auction theory because in a common-value auction the bidders recognize that their bid is valuation relevant only if their bid is successful (of course, in many specifications the other bidders have less favorable signals than the winning bidder). Consequently, in a common-value auction the ex ante bids reflect this. An important early reference on common-value auctions is Milgrom and Weber [1982].

[43]Another interesting recent study of ratings and competition, focusing upon the introduction of a rival to compete with a monopolist is Doherty, Kartasheva and Phillips [2008].

rating) or (b) the relative costliness of explicit shopping limits the extent of sampling (as in economic models of search). Either of these features would limit the degree of selection bias because the issuer isn't as easily able to identify especially high potential ratings.

There are a number of natural directions to pursue in this treatment of selectivity and equilibrium notching. For example, the basic concerns that lead to selectivity and notching extend beyond environments in which the regulatory rules focus upon the issuer purchasing a single rating. If the relevant regulation requires at least two ratings at a specified level, then the issuer would purchase the two highest ratings (following our discussion above) and the analysis of selectivity and notching would focus upon the second-order statistic.[44] The degree of selection bias will be more limited for the second-order statistic. Indeed, in the extreme with only two rating agencies and each rating by just a single technique the relevant regulation would be equivalent to the minimum rating being at least the specified level.[45]

As emphasized herein, the extent of the phenomenon depends upon the degree of heterogeneity in the underlying signals. Focusing again on the case in which a single rating at a critical level is required, we wish to explore the comparative static associated with selectivity and rating notching in the presence of asymmetric rating agencies. If one credit-rating agency tends to produce higher assessments than another (i.e., biased assessments among the agencies), then (not surprisingly) our theory implies that there should be greater downward notching of its ratings than the downward adjustments that would be justified for the ratings of an agency that produced relatively lower ratings. Suppose that for a particular type of instrument an agency's evaluations attract significantly greater market share compared to the rival agencies because its ratings are artificially higher. That suggests that the ratings of the issue whose rating agency has the substantially higher market share should be notched to a greater degree to control the larger upward bias. This leads to predictions about selection bias based upon such considerations as ratings level (high ratings for a fixed ease of valuation are more likely to be upward biased), market share and type of instrument.[46]

Next, we suppose that the different agencies produce ratings of varying precision. The greater imprecision of one agency's signal suggests that under the circumstances in which it provides the maximum rating there will be a need to notch (reduce) its rating to a greater degree because of the greater adverse selection associated with its being the maximal rating compared to a situation in which an agency whose rating was more precise produced the maximal rating. Of course, this is exactly what would emerge in the analysis of common-value

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[44]This again assumes equivalent cost for the various ratings. Of course, in principle an issuer would select a lower rating than the highest two if that were sufficiently less expensive.

[45]Depending upon one's definition of the selectivity bias this would either eliminate it (since there is no potential for ratings shopping) or reverse it (since the minimum rating would reflect a "loser's curse") and be below an unconditional forecast.

[46]However, as discussed above lower-rated bonds often are harder to value.

auctions. An extreme example in the opposite direction would arise in the case in which the rating agency with the maximal rating would produce an extremely precise rating. Then there would be little reason to haircut (reduce) its rating in equilibrium. Both the cases of bias across the rating agencies and differing precision among the agencies point to differences in selectivity and equilibrium notching. Notice that a credit-rating agency's ratings would appear to be more conservative, if its underlying ratings technology is relatively more precise compared to its rivals (or more obviously, if its ratings are biased lower).

One interesting insight that emerges in the mutual fund performance literature is that funds with especially strong realized performance often have relatively high risk (e.g., Brown. Goetzmann, Ibbotson and Ross (1992))—the high risk is why they are especially likely to emerge having an outlier return. Analogously, rating agencies whose models are relatively imprecise and as emphasized by Coval, Jurek and Stafford [2008] the use of instruments that are difficult to value can be viewed as a response to the regulatory-induced demand for high ratings.

In practice there will be much less precision in the ratings of longer-term bonds and especially lower-rated bonds and tranches. Therefore, we would expect much stronger selectivity effects and greater equilibrium notching at such levels, especially in the context of hard-to-model securitizations. Indeed, actual notching schedules used incorporate the feature that there is greater notching on lower-rated bonds of securitizations.[47]

6   A Formal Model

We turn to a more formal model that highlights the potential costliness of obtaining indicative ratings and the stochastic nature of ratings. We begin by summarizing the main assumptions of the model we are solving.

Players

- The issuer of a one-period instrument with random final payoff $X \backsim N(\mu_X, \sigma_X^2)$.

- N credit rating agencies (each denoted n).

Information

- Each of the N credit rating agencies privately observes a signal $S_n = X + \varepsilon_n$, with $\varepsilon_n \backsim N(0, \sigma_\varepsilon^2)$. The error terms in the signals that agencies observe are potentially

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[47]For an example of the rules that govern the inclusion of unrated instruments in a CDO, see the bond indenture for Bernoulli High Grade CDO II, LLC, http://www.ifsra.ie/data/in_mark_prosp/9195Bernoulli%20CDO.pdf

correlated: let $\rho = corr(\varepsilon_n, \varepsilon_m)$, for all $n, m = 1...N$ and $n \neq m$. On the other hand, all other random variables in the model are assumed to be uncorrelated.

- The issuer privately observes some information $S_0 = X + \varepsilon_0$, with $\varepsilon_0 \backsim N(0, \sigma_0^2)$. Also, the issuer observes a vector of signals $(\hat{S}_1, ..., \hat{S}_N)$ in which the generic element satisfies $\hat{S}_n = S_n + \delta_n$, with $\delta_n \backsim N(0, \sigma_\delta^2)$.

Actions and payoffs

- Whenever the issuer contacts a rating agency (say, agency n), she pays a cost c in order to get an *indicative rating:* after paying the cost, the rating agency discloses to the issuer its signal S_n. At this point the issuer can pay an additional cost χ to publish the rating, otherwise the indicative rating remains private information.

- Whenever the issuer solicits rating agency n to publish a rating, the resulting rating equals the conditional expectation of the payoff given the rating agency's information: $\mathbb{E}[X|S_n] = \mu_X + \frac{\sigma_X^2}{\sigma_S^2}(S_n - \mu_X)$. In this specification of the model we do not adjust for the possibility of the winner's curse, which is elaborated upon below.

- If the issuer decides not to contact any agency or not to make any indicative rating public, then the issuer obtains an unsolicited rating at zero cost. The unsolicited rating equals the prior mean μ_X.

- The issuer's objective is to maximize the rating net of the costs. Denote by $V_{i,n}^j$ the payoff of the issuer, where the superscript $j = s, u$ denotes whether the issuer solicited a rating (s) or not (u); the subscript i denotes the number of indicative ratings she solicited, and the subscript n the rating agency chosen by the issuer to publish the rating (in case a rating is finally solicited). It follows from the assumptions that the issuer will not solicit more than one rating.

Timing and payoffs

- Stage 0: the issuer and rating agencies privately observe their signals. The issuer decides whether to contact at least one agency or not. If she decides not to contact any agency the process ends and the issuer gets an unsolicited rating at zero cost. In this case the payoff of the issuer is $V_0^u = \mu_X$. Alternatively, the issuer can decide to contact a rating agency; in this case she pays the cost c and moves to stage 1.

- Stage 1: the issuer observes the indicative rating S_1[48]. At this point the issuer has 3 options: (i) decide not to solicit any rating, in which case the payoff is $V_1^u = \mu_X - c$; (ii) solicit the

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[48]We index the rating agencies by the order in which are contacted by the issuer.

rating from the rating agency, in which case the payoff is $V_1^s = -c - \chi$, or (iii) contact another rating agency, in which case she pays c and moves to stage 2.

- Stage 2: the issuer observes the indicative rating S_2. At this point the issuer has 4 options: (i) decide not to solicit any rating, in which case the payoff is $V_2^u = \mu_X - 2c$; (ii) solicit the rating from the second rating agency, in which case the payoff is $V_{2,2}^s = \mathbb{E}[X|S_2] - 2c - \chi$; (iii) go back and solicit the rating to the first rating agency, in which case the payoff is $V_{2,1}^s = \mathbb{E}[X|S_1] - 2c - \chi$ or (iv) contact another rating agency, in which case she pays c and moves to stage 3.

- Stage 3:...

The process goes on until the issuer decides not to solicit any rating or to solicit one; in any case there are at most N stages. The timing of the model and the corresponding payoffs for the issuer are sketched in Figure 1 for the case $N = 2$.

Some comments on the model's assumptions.

- The assumption that the noise terms in rating agencies signals are potentially less than perfectly correlated captures the degree of "heterogeneity" among the rating technologies. Clearly, as ρ approaches unity, there is effectively only one rating available.

- The assumption that the issuer observes a vector of signals $(\hat{S}_1, ..., \hat{S}_N)$ captures the possibility of *implicit shopping*, whereby the variance σ_δ^2 of the error terms δ_n measures the degree of transparency in the rating process. As $\sigma_\delta^2 \downarrow 0$ the process is very transparent and the issuer will immediately anticipate which rating agency will issue the highest rating; as $\sigma_\delta^2 \uparrow \infty$ the issuer is ex ante indifferent among rating agencies because of the symmetry assumption.

- We are implicitly assuming that the rating agencies are not correcting for the winner's curse. Given the sequential structure of the model, if the issuer decides to solicit a rating she will solicit the highest among the observed indicative ratings. Then the ability of agencies to correct for the winner's curse will depend on additional assumptions regarding the extent to which contracting between agencies and the issuer is public, that is, if agencies know how many indicative ratings the issuer asked for (or even whether any were sought for evaluating an unsolicited rating) and if indicative ratings are observed by other rating agencies, etc. (Note that in the presence of a winner's curse the issuer may even seek incremental rating(s) to assure the investing public that a signal purchased is not a fluke.) If contracting is completely private, then the ability to correct for the

winner's curse may be very limited.[49]

- Given the assumption underlying our formal analysis that the rating agencies are not correcting for the winner's curse the ratings they provide are identical in our formal sequential game as they would be in a model in which the ratings are provided simultaneously or in a model in which the agencies do not know the sequencing in which they are approached by the issuer.

The baseline model

The specific version of the model we analyze corresponds to the case when $N = 2$ and $\sigma_\delta^2 \uparrow \infty$. In this case the issuer is indifferent ex ante between rating agencies, and if the issuer decides to contact one will select it at random. Before examining the details of the model's solution, we first define the notation used throughout. The issuer's conditional expectation and conditional variance at stage i of a variable x are denoted respectively as $\mathbb{E}_i[x]$ and $\sigma_{x|i}^2$; so for example $\mathbb{E}_0[x] = \mathbb{E}[x|S_0]$, $\sigma_{x|0}^2 = \mathbb{V}ar[x|S_0]$ and $\mathbb{E}_1[x] = \mathbb{E}[X|S_0 \wedge S_1]$, $\sigma_{x|1}^2 = \mathbb{V}ar[x|S_0 \wedge S_1]$. The functions $f(t)$ and $\Phi(t)$ denote respectively the probability density function and the cumulative distribution function of a standard normal valued at t. Finally, we denote $\sigma_S^2 = \sigma_X^2 + \sigma_\varepsilon^2$ and normalize $\mu_X = 0$.[50]

7 Model's solution

In what follows we provide a description of the results; all the details of the model's solution are contained in the Appendix. We solve the model backwards, starting from **stage 2**. Notice that at this stage the issuer's available information amounts to both the indicative ratings (S_1, S_2) plus its own signal S_0.

Lemma 1. *There exists a publication threshold $\bar{S}$ such that the issuer's choice among the available options is:*

(2i) *not to solicit (purchase) any rating if $S_1 \leq \bar{S}$ and $S_2 \leq \bar{S}$;*

(2ii) *solicit a rating from the rating agency she contacted in stage 1 if $S_1 > \bar{S}$ and $S_1 \geq S_2$;*

(2iii) *solicit a rating from the rating agency she contacted in stage 2 if $S_2 > \bar{S}$ and $S_1 < S_2$.*

Conditional on being in stage 2, the issuer will decide to solicit the highest between the two indicative ratings (options $ii - iii$) only if this is high enough to cover the costs of publishing

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the rating (i.e., higher than the publication threshold $\bar{S}$); otherwise the issuer will opt for the unsolicited rating (option i).

Then we return to **stage 1**, where the issuer's available information amounts to the first indicative rating S_1 plus its own signal S_0. In order to compare the different options available at this stage, the issuer uses available information to form an expectation of next period's payoff $\mathbb{E}_1[V_2]$.

Lemma 2. *There exist two threshold values for* S_1*, namely* $\overline{S_1^a} \leq \bar{S}$ *and* $\overline{S_1^b} \geq \bar{S}$*, which are implicit functions of* S_0*, such that the issuer's choice among the available options is:*

(1i) *not solicit any rating if* $S_1 \leq \overline{S_1^a}$*;*

(1ii) *solicit the observed rating if* $S_1 \geq \overline{S_1^b}$*;*

(1iii) *enter into next stage otherwise, namely if* $S_1 \in \left(\overline{S_1^a}, \overline{S_1^b} \right)$*.*

The expected benefit of asking for an additional rating at this stage refers to the possibility that S_2 will be higher than S_1 and in any case high enough (i.e. higher than $\bar{S}$) that it it will be worth soliciting its publication. Crucially, the issuer's conditional expectation of S_2 depends on both S_1 and S_0. If $S_1 \leq S_1^a$, then not only the first indicative rating is below the publication threshold $\bar{S}$; it is also low enough that the perceived possibility of the second indicative rating S_2 being higher than $\bar{S}$ is not enough to cover the additional cost c. As a consequence the issuer decides not to solicit any rating and to stop. On the other hand, if $S_1 \geq \overline{S_1^b}$, then not only the first indicative rating is worth publishing; it is also sufficiently high that the perceived possibility of the second indicative rating S_2 being higher than S_1 is not enough to cover c. In this case the issuer decides to publish the first rating. Only if $S_1 \in \left(\overline{S_1^a}, \overline{S_1^b} \right)$, then the expected benefit of entering into next stage are high enough that the issuer is willing to pay for an additional indicative rating and moves into next stage.

Finally, **stage 0**, in which the issuer's only information is its own signal S_0.

Lemma 3 *There exists a threshold value* $\overline{S_0}$ *such that the issuer asks for an indicative rating if* $S_0 > \overline{S_0}$ *and opts for an unsolicited rating otherwise.*

7.1   The value of shopping and the decision to ask for indicative ratings

We now explore the conditions under which the possibility of shopping between two agencies (as opposed to the existence of only one agency) is relevant for the issuer's decision to ask for the first indicative rating in stage 0. The next Lemma provides a decomposition of the payoff in stage 1 that highlights the value of shopping. We denote by $\hat{V}_1$ the payoff in stage 1 in the

case in which there is only one rating agency, and by $V_2 - V_1$ the *extra* payoff from going into stage 2 from stage 1.

Lemma 4. *The expected payoff of stage 1 can be written as*

$$\mathbb{E}_0[V_1] = \mathbb{E}_0[\hat{V}_1] + (\mathbb{E}_0[V_2 - V_1])^+ , \tag{1}$$

and there exist thresholds $\hat{S}_0$ and $\bar{S}_0'$ such that $E_0[\hat{V}_1] > 0$ iff $S_0 > \hat{S}_0$ and $(\mathbb{E}_0[V_2 - V_1])^+ > 0$ iff $S_0 > \bar{S}_0'$. Both $\hat{S}_0$ and $\bar{S}_0'$ are increasing in the costs c and χ.

The expected rating is increasing in the issuer's information, therefore for the expected payoff from soliciting one rating to exceed the cost c, the signal S_0 has to be greater than some threshold $\hat{S}_0$. Lemma 4 also shows that the expected extra-benefit of having a *second* rating agency instead of only one, i.e., the value of shopping, is positive whenever the issuer's signal is above some threshold $\bar{S}_0'$. The intuition follows from the anticipated behavior in stage 1. The net value of stage 2 reflects the possibility that the second indicative rating might be higher than the first one and in any case high enough to be worth publishing, and the issuer's perceived likelihood of this is increasing in its own signal.[51] Therefore, for the anticipated benefit of shopping to be high enough and exceed the cost c, the issuer's signal has to be higher than some threshold $\bar{S}_0'$. Not surprisingly, both thresholds are increasing in c and χ as the issuer anticipates that, being more expensive, ratings must be higher in order to be profitable. The next Lemma summarizes the effects of the cost structure onto the issuer's decision at stage 0.

Lemma 5 *There exist values $\bar{\chi}$ and c^* such that the issuer is more likely to ask for the first indicative rating in the presence of a second agency iff $c < c^*$. If $\chi > \bar{\chi}$ then the issuer's decision at stage 0 is unaffected by the presence of a second rating agency.*

Next, we analyze the relationship between the value of shopping and the issuer's decision to ask for the second indicative rating. By the same decomposition in (1), a necessary condition for the issuer to go into stage 2 from stage 1 for a given realization of S_0 is that $\mathbb{E}_0[V_2 - V_1] > 0$, as this is equivalent to the length of the interval $\left[\overline{S_1^a}, \overline{S_1^b}\right]$ around $\bar{S}$ being strictly positive.[52] This is illustrated in the top panel of Figure 2, in which the thresholds $\overline{S_1^a}$ and $\overline{S_1^b}$ are plotted against the realizations of S_0. If S_0 is low enough (i.e. lower than $\bar{S}_0'$, by the definition in Lemma 4), then both $\overline{S_1^a}$ and $\overline{S_1^b}$ equal $\bar{S}$, and for no realization of the first indicative rating the issuer finds it profitable to ask for a second rating.

As intuition would suggest, the length of the interval $\left[\overline{S_1^a}, \overline{S_1^b}\right]$ can be shown[53] to be

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[51] Look at the derivation of (18)-(20) for details.

[52] This statement can be formally proved by direct comparisons of definitions (8), (10)-(11), (12)-(13) with (20) in the Appendix.

[53] Apply the Implicit Function Theorem to the definition of S_1^a, S_1^b in (10)-(11) to show that $dS_1^a/dc > 0$ and

monotonically decreasing in the costs c and χ. Clearly if c is large enough then $\overline{S_1^b} - \overline{S_1^a}$ converges to zero, while if c is very small then $\overline{S_1^b} - \overline{S_1^a}$ becomes very large as the issuer always asks for all indicative ratings if they are very cheap. For intermediate values of c, there are cases[54] in which the issuer would publish the second rating if she reached the second stage, but because of the cost for indicative ratings she stops at stage 1 instead.

Another interesting comparative static result refers to the value of shopping and the precision in the rating agencies' signals σ_ε^2.

Lemma 6 *The value of shopping is non monotonic in σ_ε^2. As $\sigma_\varepsilon^2 \downarrow 0$ or $\sigma_\varepsilon^2 \uparrow \infty$ we have* $(\mathbb{E}_0[V_2 - V_1])^+ = 0$.

The intuition for this result is very simple: in both the extreme cases the two ratings coincide. As the signals become extremely noisy both ratings equal the prior mean of the payoff (and therefore coincide with the unsolicited rating); as the signals become very precise both ratings converge to the true payoff's value. As the two ratings coincide there is no value in shopping for ratings. In this case actually the solution to the model becomes trivial: no rating will be published if signals are very imprecise, while if signals are very precise only firms with good enough information will decide to contact the rating agency[55].

7.2 The cost structure, heterogenous rating technologies and ratings' bias

Motivated by the empirical relevance of solicited versus unsolicited ratings, and even more importantly by the apparent ratings inflation reflected in the recent financial market crisis, we now turn to our model's predictions of the effects of changes in the environment in which ratings are determined for the equilibrium structure of published ratings. Rather than the characterization of the model's properties conditional on the issuer being at various stages of the rating game, we now are interested in analyzing the *unconditional* value of endogenous quantities. Specifically, we are interested in the ex-ante probability that ratings are eventually published at the end of the rating game, and in the level of these ratings.

An initial question we ask is how the outcome of the rating process might be influenced by the cost structure. Figure 3 illustrates the effects of an increase in the cost of obtaining indicative ratings. Not surprisingly, an increase in this cost makes ratings less likely to be published. As shown in Lemma 4, the minimum threshold value that determines the decision

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$dS_1^b/dc < 0$ and $dS_1^a/d\chi > 0$ and $dS_1^b/d\chi = 0$.

[54]Formally, these two regions are: *i)* when $S_1 \geq \overline{S_1^b}$ and $S_2 > S_1$; *ii)* when $S_1 \leq \overline{S_1^a}$ and $S_2 > \bar{S}$.

[55]It can be easily confirmed that in this case the issuer asks for the indicative rating if

$$ S_0 > \frac{\sigma_X^2 + \sigma_0^2}{\sigma_X^2}(c + \chi), $$

and then publishes the rating if $X > \chi$.

to solicit the first rating increases in the costs, implying that issuers are more likely ex ante to opt for unsolicited ratings. Given the sequential structure of the model, it is also intuitive that the contribution of shopping to the probability that ratings are issued (measured by the vertical distance between the thick and the dashed curves) decreases in the costs. The expected level of published ratings instead, is increasing in the cost. Being more expensive, anticipated ratings need to be higher in order to be profitable to obtain and therefore to be published.

A second parameter with respect to which we want to do comparative statics is the correlation among the error terms in rating agencies' signals. This parameter captures the degree of heterogeneity in rating technologies. As we argue in more detail in the next section, the presence of such heterogenous frameworks might have severely affected the magnitude of selection effects, especially in the context of securitization structures. Figure 4 illustrates these effects. When the correlation between agencies' signals tends to one, there is effectively no difference between the two ratings. As this correlation decreases, rating technologies become more diverse and the issuer is potentially faced with different ratings to pick among, so that the value of shopping increases. As a result, the probability of ratings being issued increases. Crucially, such an increase in structures being rated is accompanied by more ratings inflation: as ratings become more heterogenous, the mean of the first-order statistic increases, resulting in higher ratings being published, on average. As evident from comparing the case of one versus two agencies, heterogeneity in rating technologies greatly enhances selection effects on the equilibrium structure of ratings.

Finally, Figure 5 plots the expected ratings' bias, conditional on publication, as a function of the cost of obtaining indicative ratings (top panel) and the correlation among the error terms in rating agencies' signals (bottom panel). An immediate observation is that the level can be negative. The bias is defined by subtracting from the rating the realized value of the fundamental.[56] As a consequence, conditional on publication, the expected bias reflects the fact that it is issuers with higher than average fundamentals that self-select into the rating process. Since ratings in our model are conditional expectations based on unbiased signals, ratings are less sensitive to a higher payoff than the payoff itself, which potentially results in a negative bias. In the case of one rating agency for example, the average bias conditional on publication is indeed always negative.[57] With two rating agencies, the level of the bias is shifted upward as an effect of shopping, resulting in a positive average bias if the cost is not too high. The top panel shows that higher costs imply lower bias. Behind this result there are two forces: firstly, higher costs reduce shopping, as illustrated by the vertical distance between the one vs. two agencies cases; secondly, higher costs imply higher average fundamentals conditional on publication,

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[56]More formally, given a published rating $\mathbb{E}[X|S]$ and underlying fundamental X, the bias b is defined as $b = \mathbb{E}[X|S] - X$.

[57]Unless the cost for indicative rating is zero or the issuer has no prior information on the quality of the asset, in which case the average bias is zero.

which in turn imply lower bias. Consequently, higher costs (e.g., higher indicative fess) imply that weaker issuers are less likely to get their products rated. From a policy perspective this is not obviously a desirable outcome as it discourages information production about lower quality products. Finally, the bottom panel shows that the higher the heterogeneity (i.e., the lower the correlation), the higher the extent of shopping and the higher the bias (the less negative the bias, if the bias is negative) in the case of two agencies.

8   Concluding Comments: Implications for Tranching and Securitizations

The basic insights in this draft can be cast in terms of a simple framework in which one-period instruments are being evaluated and tranching of instruments and securitization are not introduced explicitly. Nevertheless ratings' shopping is a central phenomenon and introduces strong selection bias in equilibrium. In fact, these effects can be interpreted in terms of "notching," where the own estimate (virtual rating) of a rating agency that does not rate a particular instrument is below that of the agencies that rate the issue. This is a natural consequence of shopping by the issue or security creator, whether implicit or explicit, for the highest potential rating to maximize the net value of selling the instruments. In fact, the various agencies would potentially each notch or haircut the ratings of its rivals to control the selection effects associated with different agencies being asked to supply ratings for evaluating different instruments. As the analysis in this paper highlights, tranching per se is not crucial conceptually to the existence of selection effects or notching. However, the presence of relatively heterogeneous frameworks is important to the magnitude of such effects, pointing to why notching emerged as a concern in the context of tranching and securitization (but not in the context of traditional corporate or municipal bonds). Indeed, in Figure 6 we see that the probability of notching (i.e., the probability that a non-published rating is lower than the published rating) increases with greater heterogeneity in ratings methodology and lower correlation. Figure 6 also shows that raising costs reduces the probability of notching, which is highest (i.e., one) when the cost is zero. Of course, with a zero cost we always have notching because the non-published comparison (virtual) rating is always below the published one. It is well known that these securitization structures are much more difficult to value than traditional corporate and municipal bonds and that valuation often hinges on relatively fine details.

Much of this paper has focused upon the choice of the security issuer of which rating to purchase and resulting selectivity implications. Yet, we have not highlighted the role of alternative ratings technologies (and potential pricing issues for acquiring these ratings) and to what degree the market is able to distinguish among these. To the extent that the issuer is in a position to cherry-pick among these techniques (ideally net of any ratings cost) in addition

to selecting among rating agencies for the purpose of optimizing a rating, this reinforces the extent of selectivity and notching. It is precisely in the context of securitizations that the rating agencies have made available a range of techniques for ratings (reflecting the inherent complexity and trying to provide low-cost alternatives), but this will lead to much greater selectivity and a larger role for equilibrium notching. Of course, securitizations are the context in which concerns about notching have been emphasized.

Obviously, the issuer will select the technique that yields the largest payoff for issuing the security net of the ratings cost. Given the presence of several alternatives in the tranching context that reinforces the extent of selectivity bias and the degree to which the rating of the rival agency should be haircut. Of course, the notched rating is of relevance only in scenarios when that rating would be purchased by the issuer and provided to the capital market.

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Appendix

Proof of Lemma 1.

At stage 2 the issuer chooses the option that has maximum value, namely:

(2i) not solicit any rating if $V_2^u \geq V_{2,1}^s \wedge V_2^u \geq V_{2,2}^s$ which, given the definitions of the payoffs in the description of the timing of the model, is equivalent to

$$S_1 \leq \bar{S} \quad \wedge \quad S_2 \leq \bar{S},$$

where

$$\bar{S} := \frac{\sigma_S^2}{\sigma_X^2}\chi;$$

(2ii) solicit a rating from the rating agency she contacted in stage 1 if $V_2^u < V_{2,1}^s \wedge V_{2,1}^s \geq V_{2,2}^s$, which is equivalent to

$$S_1 > \bar{S} \quad \wedge \quad S_1 \geq S_2;$$

(2iii) solicit a rating from the rating agency she contacted in stage 2 if $V_2^u < V_{2,2}^s \wedge V_{2,1}^s < V_{2,2}^s$, which is equivalent to

$$S_2 > \bar{S} \quad \wedge \quad S_1 < S_2.$$

Proof of Lemma 2.

In order to compare the value of the alternatives at stage 1, we compute the conditional expectation of next period's payoff $E_1[V_2]$. In order to do so we denote with $f_{S_2|1}$ the PDF of S_2 conditional on the information available at stage 1, i.e. (S_0, S_1), which is given by

$$f_{S_2|1}(S_2) = \frac{1}{\sqrt{2\pi\sigma_{S_2|1}^2}} \exp\left(-\frac{(S_2 - \mathbb{E}_1[S_2])^2}{2\sigma_{S_2|1}^2}\right),$$

where

$$E_1[S_2] = E[S_2|S_0 \wedge S_1] = \frac{\sigma_X^2\sigma_\varepsilon^2(1-\rho)}{\sigma_X^2\sigma_\varepsilon^2 + \sigma_0^2\sigma_S^2}S_0 + \frac{\sigma_0^2\sigma_X^2 + (\sigma_0^2 + \sigma_X^2)\sigma_\varepsilon^2\rho}{\sigma_X^2\sigma_\varepsilon^2 + \sigma_0^2\sigma_S^2}S_1,$$

$$\sigma_{S_2|1}^2 = Var[S_2|S_0 \wedge S_1] = \frac{\sigma_\varepsilon^2(1-\rho)(\sigma_X^2\sigma_\varepsilon^2(1+\rho) + \sigma_0^2(2\sigma_X^2 + \sigma_\varepsilon^2(1+\rho)))}{\sigma_X^2\sigma_\varepsilon^2 + \sigma_0^2\sigma_S^2}.$$

Also we define the indicator function

$$\mathbf{1}_{(A)} = \begin{cases} 1 & \text{if } A \text{ is true,} \\ 0 & \text{otherwise.} \end{cases}$$

30

Now we derive the conditional expectation of next period's payoff $E_1[V_2]$:

$$E_1[V_2] = \int_{-\infty}^{\infty} \left(V_2^u \mathbf{1}_{(S_1 \leq \bar{S} \wedge S_2 \leq \bar{S})} + V_{2,1}^s \mathbf{1}_{(S_1 > \bar{S} \wedge S_2 \leq S_1)} + V_{2,2}^s \mathbf{1}_{(S_2 > \bar{S} \wedge S_2 > S_1)} \right) f_{S_2|1}(S_2) dS_2;$$

that can be rewritten as

$$\mathbb{E}_1[V_2] = \begin{cases} \int_{-\infty}^{\infty} \left(V_2^u \mathbf{1}_{(S_2 \leq \bar{S})} + V_{2,2}^s \mathbf{1}_{(S_2 > \bar{S})} \right) f_{S_2|1}(S_2) dS_2 & \text{if } S_1 \leq \bar{S}, \\ \int_{-\infty}^{\infty} \left(V_{2,1}^s \mathbf{1}_{(S_2 \leq S_1)} + V_{2,2}^s \mathbf{1}_{(S_2 > S_1)} \right) f_{S_2|1}(S_2) dS_2 & \text{if } S_1 > \bar{S}. \end{cases} \tag{2}$$

We derive the two parts of (2) separately, starting with the case $S_1 \leq \bar{S}$. We use the change of variable

$$z = \frac{S_2 - \mathbb{E}_1[S_2]}{\sigma_{S_2|1}},$$

and the definition

$$t_1 := \frac{\mathbb{E}_1[S_2] - \bar{S}}{\sigma_{S_2|1}}, \tag{3}$$

jointly with the definitions of the different payoffs V to write the first line in (2) as

$$
\begin{aligned}
E_1[V_2] &= \int_{-\infty}^{\infty} \left((-2c)\mathbf{1}_{(z \leq -t_1)} + (\mathbb{E}[X|S_2] - 2c - \chi)\,\mathbf{1}_{(z > -t_1)} \right) f(z) dz \\
&= -2c + \int_{-t_1}^{\infty} \left(\frac{\sigma_X^2}{\sigma_S^2} S_2 - \chi \right) f(z) dz \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \int_{-t_1}^{\infty} \frac{(S_2 - \bar{S})}{\sigma_{S_2|1}} f(z) dz \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left[\frac{(\mathbb{E}_1[S_2] - \bar{S})}{\sigma_{S_2|1}} \int_{-t_1}^{\infty} f(z) dz + \int_{-t_1}^{\infty} z f(z) dz \right] \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left[t_1 \left(1 - \Phi(-t_1) \right) + \int_{-t_1}^{\infty} z f(z) dz \right] \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left[t_1 \Phi(t_1) + f(t_1) \right].
\end{aligned}
$$

For the case $S_1 > \bar{S}$ we use the same change of variable and the definition

$$t_2 := \frac{\mathbb{E}_1[S_2] - S_1}{\sigma_{S_2|1}}, \tag{4}$$

to write the second line in (2) as

$$
\begin{aligned}
\mathbb{E}_1[V_2] &= \int_{-\infty}^{\infty} \left((\mathbb{E}[X|S_1] - 2c - \chi)\mathbf{1}_{(z \le -t_2)} + (\mathbb{E}[X|S_2] - 2c - \chi)\mathbf{1}_{(z > -t_2)} \right) f(z)dz \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1} \int_{-\infty}^{\infty} \left(\frac{S_1 - \bar{S}}{\sigma_{S_2|1}}\mathbf{1}_{(z \le -t_2)} + \frac{S_2 - \bar{S}}{\sigma_{S_2|1}}\mathbf{1}_{(z > -t_2)} \right) f(z)dz \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1} \left(\frac{S_1 - \bar{S}}{\sigma_{S_2|1}}\Phi(-t_2) + \int_{-\infty}^{\infty} \left(t_2\mathbf{1}_{(z \le -t_2)} + z\mathbf{1}_{(z > -t_2)} \right) f(z)dz \right) \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1} \left(\frac{S_1 - \bar{S}}{\sigma_{S_2|1}}(1 - \Phi(t_2)) + t_2\Phi(t_2) + f(t_2) \right) \\
&= -2c + \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1} \left(\frac{S_1 - \bar{S}}{\sigma_{S_2|1}} + t_2\Phi(t_2) + f(t_2) \right).
\end{aligned}
$$

Hence we obtained

$$
\mathbb{E}_1[V_2] =
\begin{cases}
-2c + \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1}\left[t_1\Phi(t_1) + f(t_1)\right] & \text{if } S_1 \le \bar{S}, \\
-2c + \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1}\left(\frac{S_1 - \bar{S}}{\sigma_{S_2|1}} + t_2\Phi(t_2) + f(t_2) \right) & \text{if } S_1 > \bar{S}.
\end{cases}
\tag{5}
$$

Using (5) and the definitions of the payoffs in the description of the timing of the model, we can compare the value of the alternative options at stage 1. Accordingly, the issuer will:

(1i) not solicit any rating if $V_1^u \ge V_1^s \wedge V_1^u \ge E_1[V_2]$, which is equivalent to

$$
S_1 \le \bar{S} \quad \wedge \quad \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1}\left(t_1\Phi(t_1) + f(t_1)\right) \le c,
\tag{6}
$$

(1ii) solicit the observed rating if $V_1^u < V_1^s \wedge V_1^s \ge E_1[V_2]$, which is equivalent to

$$
S_1 > \bar{S} \quad \wedge \quad \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1}\left(t_2\Phi(t_2) + f(t_2)\right) \le c,
\tag{7}
$$

(1iii) enter into next stage if $V_1^u < E_1[V_2] \wedge V_1^s < E_1[V_2]$, which is equivalent to either of the following two:

$$
\begin{aligned}
S_1 \le \bar{S} \quad &\wedge \quad \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1}\left(t_1\Phi(t_1) + f(t_1)\right) > c, \\
S_1 > \bar{S} \quad &\wedge \quad \frac{\sigma_X^2}{\sigma_S^2}\sigma_{S_2|1}\left(t_2\Phi(t_2) + f(t_2)\right) > c.
\end{aligned}
\tag{8}
$$

It is left to to obtain the equivalent characterization of conditions (6)-(8) reported in the text of

the Lemma in terms of threshold values for S_1. We introduce the following shorter notation:

$$\beta_1 := \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left[t_1 \Phi(t_1) + f(t_1)\right]; \quad \beta_2 := \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left(t_2 \Phi(t_2) + f(t_2)\right); \tag{9}$$

Next, define the following implicit functions

$$S_1^a(S_0) \quad : \quad S_1 \text{ such that } \beta_1|_{S_1^a(S_0)} = c \text{ for all } S_0; \tag{10}$$

$$S_1^b(S_0) \quad : \quad S_1 \text{ such that } \beta_2|_{S_1^b(S_0)} = c \text{ for all } S_0. \tag{11}$$

The functions defined in (10)-(11) are shown to exist and to be unique as follows. The function $g(t) := t\Phi(t) + f(t)$ is positive, monotonically increasing for all t and

$$\lim_{t \downarrow -\infty} g(t) = 0; \quad \lim_{t \uparrow \infty} g(t) = \infty.$$

Moreover, t_1 and t_2 defined in (3) and (4) are linear increasing and decreasing functions respectively in S_1 for all values of S_0,[58] and the functions β_1 and β_2 are proportional to g. As a consequence we have

$$\lim_{S_1 \downarrow -\infty} \beta_1 = 0; \quad \lim_{S_1 \uparrow \infty} \beta_1 = \infty;$$
$$\lim_{S_1 \downarrow -\infty} \beta_2 = \infty; \quad \lim_{S_1 \uparrow \infty} \beta_2 = 0.$$

The above limits and monotonicity establish existence and uniqueness. Next, let

$$\overline{S_1^a(S_0)} \quad : \quad = \min\left\{S_1^a(S_0), \bar{S}\right\}; \tag{12}$$

$$\overline{S_1^b(S_0)} \quad : \quad = \max\left\{S_1^b(S_0), \bar{S}\right\}. \tag{13}$$

In the following, as well as in the text, we suppress the explicit dependence of the functions (10)-(11) and (12)-(13) from S_0. Then by construction we have

$$\beta_1 - c > 0 \Leftrightarrow S_1 > \overline{S_1^a};$$
$$\beta_2 - c > 0 \Leftrightarrow S_1 < \overline{S_1^b},$$

implying the equivalence between (6)-(8) and the statement in the Lemma.

Proof of Lemma 3 and Lemma 4.

We derive the conditional expectation of next period's payoff $E_0[V_1]$. Similarly to the previous case

---

[58] We are implicitly assuming the following parameter restriction: $\rho > -\frac{\sigma_0^2 \sigma_X^2}{\sigma_\varepsilon^2 (\sigma_X^2 + \sigma_0^2)}$.

$f_{S_1|0}$ denotes the PDF of S_1 conditional on S_0, which is given by

$$f_{S_1|0}(S_1) = \frac{1}{\sqrt{2\pi\sigma_{S_1|0}^2}} \exp\left(-\frac{(S_1 - \mathbb{E}_0[S_1])^2}{2\sigma_{S_1|0}^2}\right),$$

where

$$E_0[S_1] = E[S_1|S_0] = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_0^2} S_0,$$

$$\sigma_{S_1|0}^2 = Var[S_1|S_0] = \frac{\sigma_0^2\sigma_X^2 + \sigma_0^2\sigma_\varepsilon^2 + \sigma_X^2\sigma_\varepsilon^2}{\sigma_X^2 + \sigma_0^2}.$$

Using the notation in (9) we have

$$
\begin{aligned}
E_0[V_1] &= \int_{-\infty}^{\infty} \left(V_1^u \mathbf{1}_{(S_1 \leq \bar{S} \wedge c \geq \beta_1)} + V_1^s \mathbf{1}_{(S_1 > \bar{S} \wedge c \geq \beta_2)} \right. \\
&\quad \left. + \mathbb{E}_1[V_2] \left(1 - \mathbf{1}_{(S_1 \leq \bar{S} \wedge c \geq \beta_1)} - \mathbf{1}_{(S_1 > \bar{S} \wedge c \geq \beta_2)} \right) \right) f_{S_1|0}(S_1) dS_1 \\
&= \int_{-\infty}^{\infty} \left(V_1^u \mathbf{1}_{(S_1 \leq \bar{S} \wedge c \geq \beta_1)} + V_1^s \mathbf{1}_{(S_1 > \bar{S} \wedge c \geq \beta_2)} \right. \\
&\quad \left. + (V_1^u + \beta_1 - c) \mathbf{1}_{(S_1 \leq \bar{S} \wedge c < \beta_1)} + (V_1^s + \beta_2 - c) \mathbf{1}_{(S_1 > \bar{S} \wedge c < \beta_2)} \right) f_{S_1|0}(S_1) dS_1 \\
&= \int_{-\infty}^{\bar{S}} V_1^u f_{S_1|0}(S_1) dS_1 + \int_{\bar{S}}^{\infty} V_1^s f_{S_1|0}(S_1) dS_1 + \\
&\quad \int_{-\infty}^{\infty} \left((\beta_1 - c)\mathbf{1}_{(S_1 \leq \bar{S} \wedge c < \beta_1)} + (\beta_2 - c)\mathbf{1}_{(S_1 > \bar{S} \wedge c < \beta_2)} \right) f_{S_1|0}(S_1) dS_1 \\
&= E_0[\hat{V}_1] + (\mathbb{E}_0[V_2 - V_1])^+, \quad\quad\quad\quad (14)
\end{aligned}
$$

where in the last equality (14) we define $\hat{V}_1$ as the payoff in stage 1 in the case in which there is only one rating agency, instead of two, and $(x)^+ := \max\{x, 0\}$, so that $(\mathbb{E}_0[V_2 - V_1])^+$ represents the positive part of the expected extra payoff from going into stage 2 from stage 1. In order to proceed with the computation we use the following change of variable

$$z = \frac{S_1 - \mathbb{E}_0[S_1]}{\sigma_{S_1|0}},$$

and the definition

$$t_0 := \frac{\mathbb{E}_0[S_1] - \bar{S}}{\sigma_{S_1|0}}, \quad\quad\quad\quad (15)$$

to write

$$
\begin{aligned}
E_0[\hat{V}_1] &= \int_{-\infty}^{-t_0} (-c)\, f(z)dz + \int_{-t_0}^{\infty} (\mathbb{E}[X|S_1] - c - \chi)\, f(z)dz \\
&= -c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_1|0} \int_{-t_0}^{\infty} \left(\frac{S_1 - \bar{S}}{\sigma_{S_1|0}} \right) f(z)dz \\
&= -c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_1|0} \left(\left(\frac{\mathbb{E}_0[S_1] - \bar{S}}{\sigma_{S_1|0}} \right) \int_{-t_0}^{\infty} f(z)dz + \int_{-t_0}^{\infty} z f(z)dz \right) \\
&= -c + \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_1|0} \left(t_0 \Phi(t_0) + f(t_0) \right).
\end{aligned}
\tag{16}
$$

Again, for future reference we introduce the following shorter notation

$$
\beta_0 := \frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_1|0} \left(t_0 \Phi(t_0) + f(t_0) \right).
\tag{17}
$$

Inspection of (16) and (15) reveals that $E_0[\hat{V}_1]$ is monotonically increasing in S_0 and

$$
\lim_{S_0 \downarrow -\infty} \mathbb{E}_0[\hat{V}_1] = -c, \quad \lim_{S_0 \uparrow \infty} \mathbb{E}_0[\hat{V}_1] = \infty;
$$

therefore the exists a value, denote it $\hat{S}_0$, such that $E_0[\hat{V}_1] > 0$ iff $S_0 > \hat{S}_0$. Next we deal with the second term in the r.h.s. of (14) and rewrite it as

$$
(\mathbb{E}_0[V_2 - V_1])^+ = \int_{-\infty}^{\bar{S}} (\beta_1 - c)^+ f_{S_1|0}(S_1)dS_1 + \int_{\bar{S}}^{\infty} (\beta_2 - c)^+ f_{S_1|0}(S_1)dS_1.
\tag{18}
$$

Then, define the following threshold values of S_0 :

$$
\begin{aligned}
S_0^a &: \quad S_0 \text{ such that } S_1^a(S_0^a) = \bar{S}; \\
S_0^b &: \quad S_0 \text{ such that } S_1^b(S_0^b) = \bar{S},
\end{aligned}
$$

where S_1^a and S_1^b are given in (10)-(11). From the definitions (3)-(4), (9) and (10)-(11) follows that

$$
\frac{dS_1^a(S_0)}{dS_0} < 0; \quad \frac{dS_1^b(S_0)}{dS_0} > 0,
$$

implying that

$$
\begin{aligned}
\overline{S_1^a(S_0)} &< \bar{S} \Leftrightarrow S_0 > S_0^a; \\
\overline{S_1^b(S_0)} &> \bar{S} \Leftrightarrow S_0 > S_0^b.
\end{aligned}
$$

Notice that, by construction, if $S_1 = \bar{S}$ we have that $t_1 = t_2$, and therefore $\beta_1 = \beta_2$, implying that

$S_0^a = S_0^b$. Denote such a value as $\bar{S}_0'$. The above discussion implies that we can can rewrite (18) as

$$(\mathbb{E}_0[V_2 - V_1])^+ = \int_{S_1^a}^{\bar{S}} (\beta_1 - c)^+ f_{S_1|0}(S_1)dS_1 + \int_{\bar{S}}^{\overline{S_1^b}} (\beta_2 - c)^+ f_{S_1|0}(S_1)dS_1, \qquad (19)$$

where the intervals of integration, and therefore the positive part of the net value of stage 2, are positive if $S_0 > \bar{S}_0'$ and zero otherwise. Therefore we can equivalently write (19) as

$$(\mathbb{E}_0[V_2 - V_1])^+ = \begin{cases} 0 & \text{if } S_0 \leq \bar{S}_0', \\[2ex] \left(\int_{S_1^a}^{\bar{S}} \left(\frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left(t_1 \Phi(t_1) + f(t_1) \right) - c \right) f_{S_1|0} dS_1 \right. \\[2ex] \left. + \int_{\bar{S}}^{S_1^b} \left(\frac{\sigma_X^2}{\sigma_S^2} \sigma_{S_2|1} \left(t_2 \Phi(t_2) + f(t_2) \right) - c \right) f_{S_1|0} dS_1 \right) & \text{if } S_0 > \bar{S}_0'. \end{cases} \qquad (20)$$

By (16) and (18) and the relative discussion follows that $E_0[V_1]$ is monotonically increasing in S_0, and that there exists a value $\bar{S}_0$ such that $E_0[V_1] > 0$ iff $S > \bar{S}_0$. It remains to show that $\hat{S}_0$ and $\bar{S}_0'$ are increasing in the costs c and χ. Consider the following functions:

$$\begin{aligned} G_1 &= \beta_0 - c, \\ G_2 &= \beta_1|_{S_1 = \bar{S}} - c; \end{aligned}$$

where β_1 and β_0 are defined in (9) and (17), and notice that by construction $\hat{S}_0$ and $\bar{S}_0'$ are found by equating G_1 and G_2 to zero respectively. The claim is easily verified by applying the Implicit Function Theorem to the functions G_1 and G_2.

Proof of Lemma 5 (sketch). The Lemma can be restated more formally as: there exist values $\bar{\chi}$ and c^* such that $\bar{S}_0' > \hat{S}_0$ if $\chi > \bar{\chi}$ and $\bar{S}_0' > \hat{S}_0$ iff $c > c^*$. Applying the Implicit Function Theorem to the functions G_1 and G_2 we have

$$\lim_{\chi \uparrow \infty} \hat{S}_0 = \lim_{\chi \uparrow \infty} \bar{S}_0' = \infty.$$

In order to assess the behavior of $\hat{S}_0 / \bar{S}_0'$ as $\chi \uparrow \infty$, by applying l'Hôpital's rule and the Implicit Function Theorem to the functions G_1 and G_2 it can be shown that $\lim_{\chi \uparrow \infty} \left(\hat{S}_0 / \bar{S}_0' \right) < 1$. For the existence of c^* as stated in the Lemma, the following conditions can be shown: $i)$ β_0 and $\beta_1|_{S_1 = \bar{S}}$ are strictly increasing in S_0, with $\lim_{S_0 \downarrow -\infty} \beta_0 = \lim_{S_0 \downarrow -\infty} \beta_1|_{S_1 = \bar{S}} = 0$ and $\lim_{S_0 \uparrow \infty} \beta_0 = \lim_{S_0 \uparrow \infty} \beta_1|_{S_1 = \bar{S}} = \infty$, $ii)$ β_0 and $\beta_1|_{S_1 = \bar{S}}$ cross only once at some S_0^* with $\beta_0 > \beta_1|_{S_1 = \bar{S}} \Leftrightarrow S_0 > S_0^*$. The previous conditions imply $c^* = \beta_0|_{S_1 = S_0^*}$.
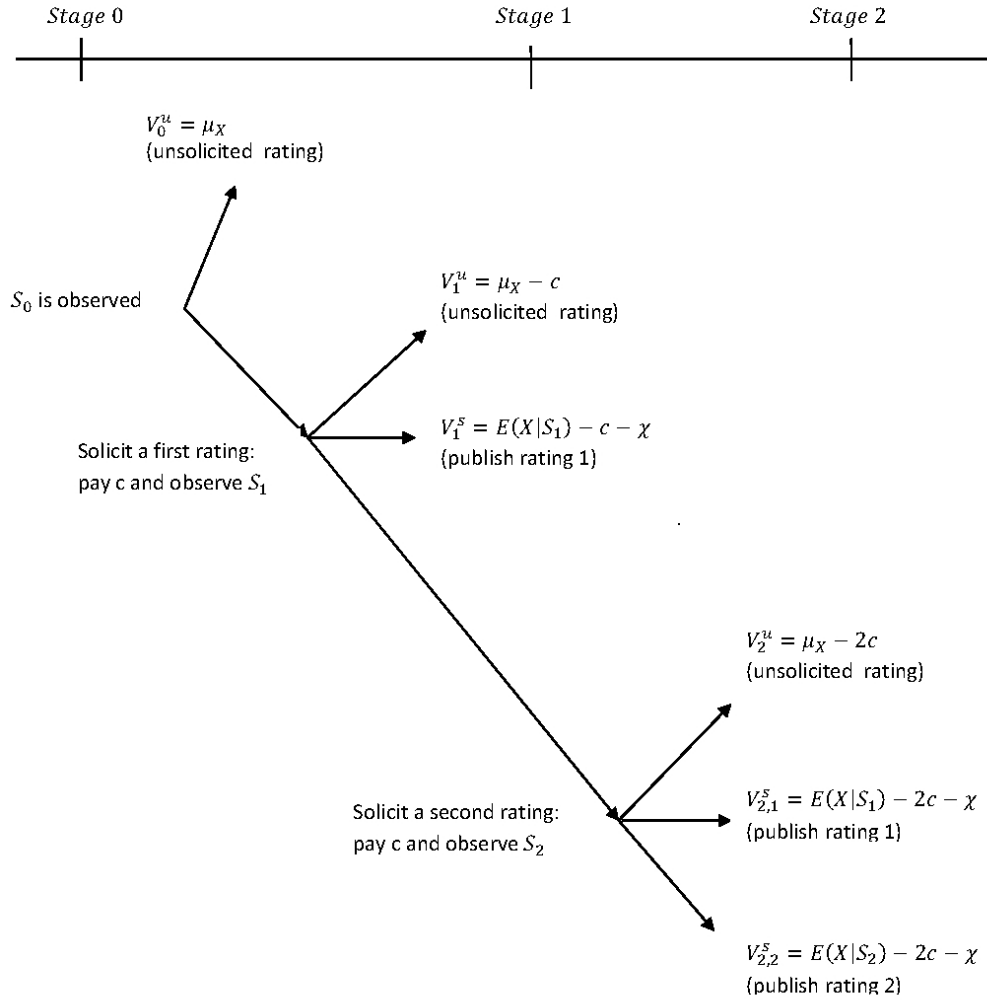
Figure 1: The rating game

Figure 2: The top graph plots the thresholds $\bar{S}$, $\bar{S}_1^a$ (dotted) and $\bar{S}_1^b$ (dashed) from Lemma 2 at stage 1 as a function of the realization of S_0. The bottom graph plots the threshold value $\bar{S}_0$ (from Lemma 3) at stage 0 as a function of the cost c. All variances are set equal to 1, the publishing cost equal to 0.14 and the correlation to zero; in the top graph the cost c is set equal to 0.1.
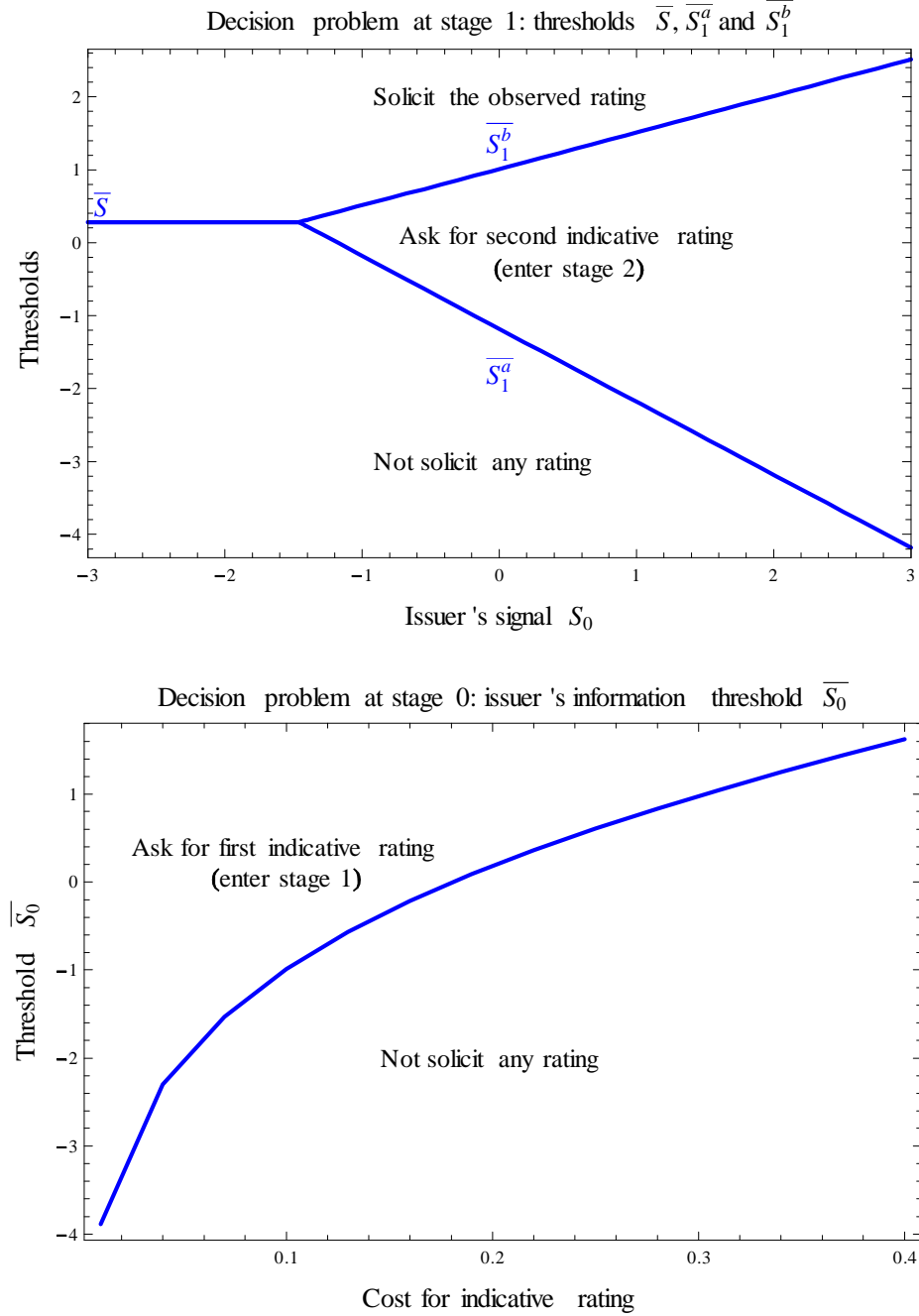
Figure 3: The dashed line represents the case of one rating agency, the thick line the case of two rating agencies. All variances are set equal to 1, the publishing cost equal to 0.14 and the correlation to zero.
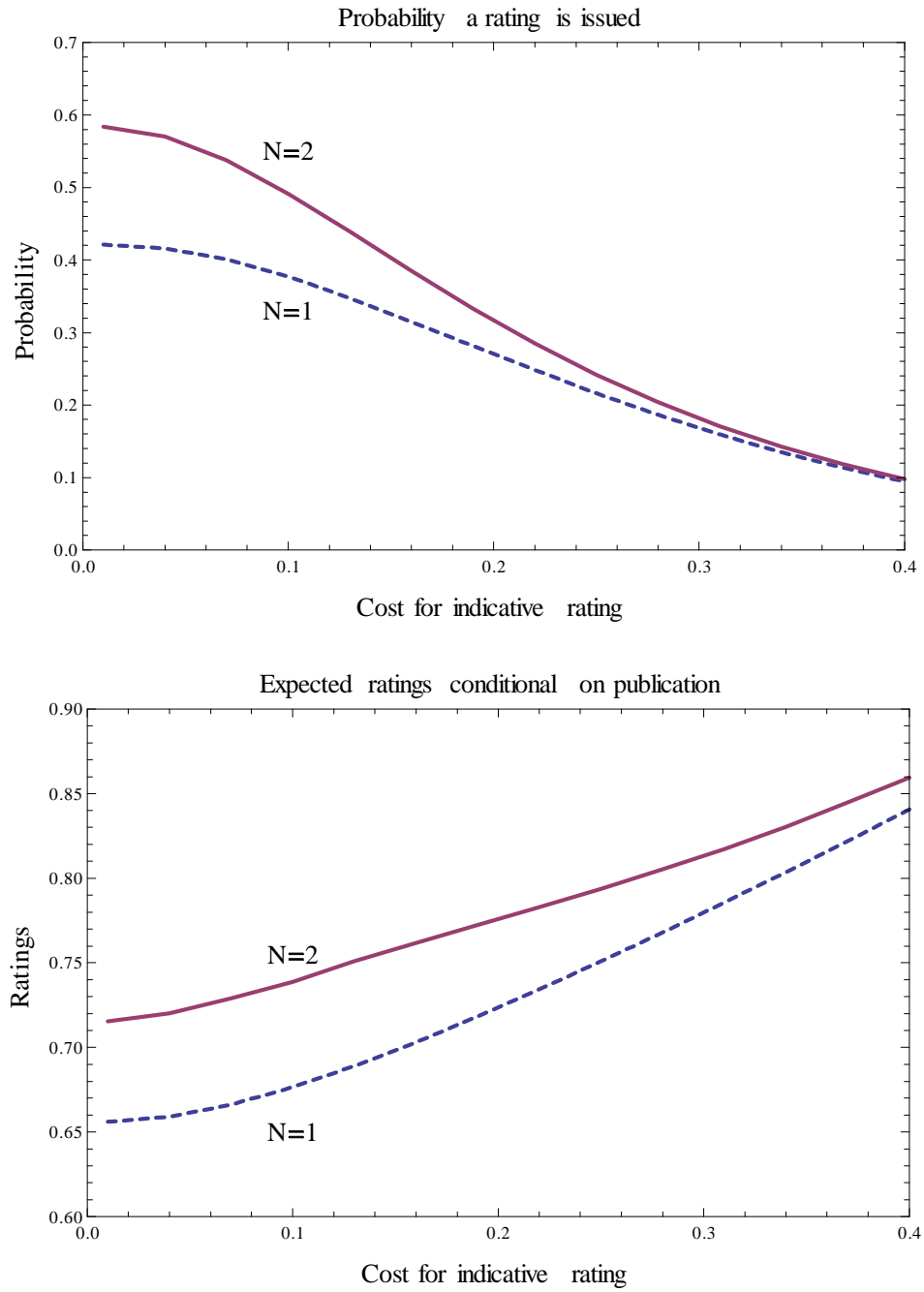
Figure 4: The dashed line represents the case of one rating agency, the thick line the case of two rating agencies. All variances are set equal to 1 and the costs equal to 0.14.
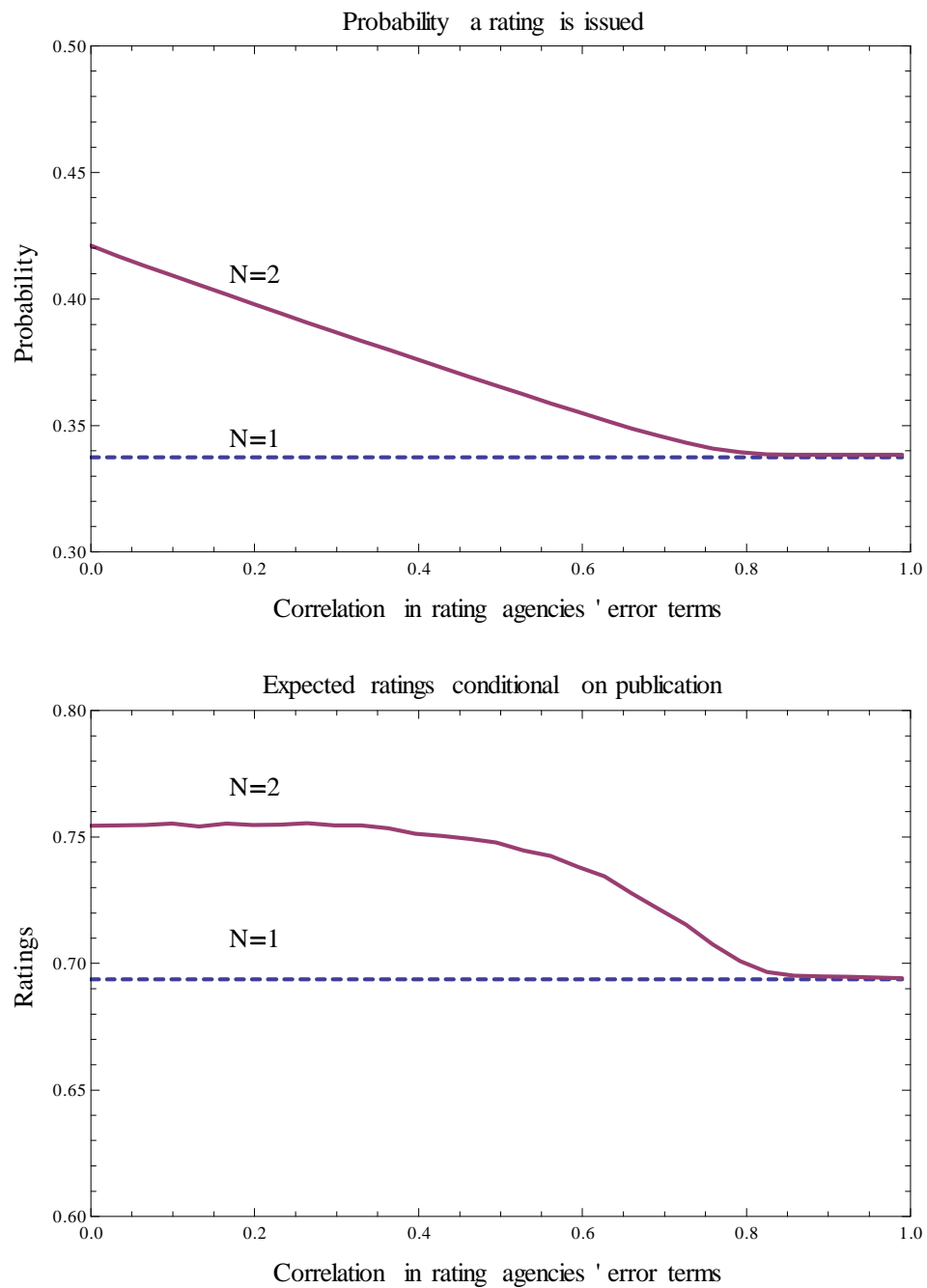

Probability a rating is issued

Expected ratings conditional on publication

Figure 5: The dashed line represents the case of one rating agency, the thick line the case of two rating agencies. All variances are set equal to 1 and the publishing cost equal to 0.14; in the top panel the correlation is set to zero and in the bottom panel the cost for indicative ratings is set to 0.14 .
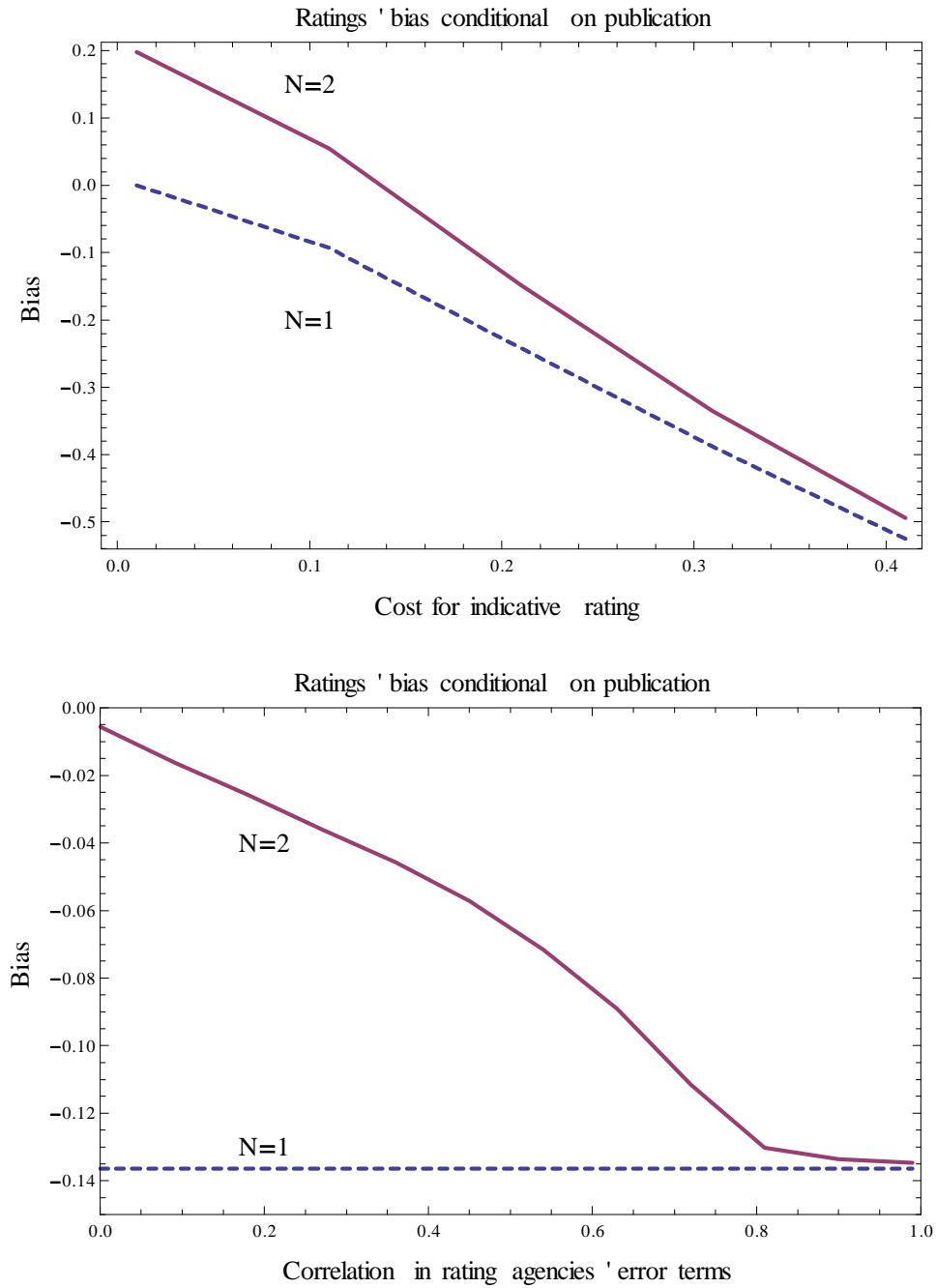
Figure 6: The graph plots the probability of notching (i.e., the probability of the non published rating being lower than the published rating) as a function of the cost for indicative rating c. The different curves are for different values of the correlation coefficient:0, 0.3 (dashed), 0.6 (dotted) and 0.9 (dashed-dotted). All variances are set equal to 1 and the publishing cost equal to 0.14

Probability of Notching (non−published rating < published rating)