Main Street versus Wall Street: Efficiency and Wealth Redistribution Effects of Insider Trading

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Abstract

By addressing the interaction between security market microstructure and shareholder-manager agency conflicts, we study the effects of informed insider trading on productive efficiency, price discovery, and wealth redistribution. Insider trading can lead simultaneously to production distortions and higher ex ante shareholder value because of a redistribution of wealth from uninformed liquidity traders to uninformed shareholders through the optimal design of managerial compensation contracts. Security market characteristics affect productive efficiency by influencing management incentive compensation while market liquidity is influenced by firm-specific characteristics. We identify the firm and market characteristics that determine the efficiency and wealth redistribution effects of insider trading.

Keywords: Insider trading; Generalized agency; Executive Compensation; Market Liquidity

JEL classification codes: G32, G34, D82
1 Introduction

Insider trading is a long-standing contentious issue that continues to generate intense interest. Opponents argue that trading by corporate executives is unambiguously damaging to public interest because it exploits uninformed investors, reduces market liquidity, and vitiates executive incentives for generating long-term shareholder value (Fishman and Hagerty, 1992; Easterbrook, 1981; Cox, 1986). These arguments form the basis of regulation outlawing insider trading “while in possession of material, non-public information [about the security].”1 Indeed, the Securities and Exchange Commission (SEC) considers the detection and prosecution of insider trading as an enforcement priority because it “…undermines investor confidence in the fairness and integrity of the securities markets.”

In contrast, proponents of insider trading, such as Manne (1966) and Carlton and Fischel (1983), argue that informed trading improves the informational efficiency of securities markets by facilitating price discovery and that it provides monetary incentives for managers to boost shareholder value.2 Moreover, Demsetz (1986) and Hu and Noe (1997) argue that insider trading is an efficient vehicle for compensating managers. These contending perspectives (on insider trading) have been highlighted recently in the filing of charges against 14 defendants for wide-spread and repeated insider trading in a number of hedge funds. The Securities and Exchange Commission’s (SEC) press release announcing the complaint argues that insider trading “…undermines the level playing field that is fundamental to our capital markets.”3 However, some have questioned the premise underlying the SEC’s complaint (e.g., Crovitz, 2009).

Regulating insider trading is therefore a complex issue because there are both costs and benefits. There are welfare costs because insider trading results in wealth transfers from uninformed to informed traders, and productive efficiency costs because it can distort managerial incentives. However, there are also benefits because insider trading improves price discovery

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1The definition of insider trading is quite uniform internationally (Padilla, 2002). U.S. security laws distinguish between official insiders — which include directors, officers, and blockholder with greater than 10 percent ownership — and tippees that come to the information as party of engagement with the firm (e.g., investment bankers, lawyers etc.). However, and as seen in the recent case of the hedge fund Galleon Group, outsiders are not protected from the charges of insider trading if they trade on nonpublic information.

2On the other hand, Fishman and Hagerty (1992) present conditions under which insider trading can actually lead to less efficient stock prices.

in security markets and the efficiency of executive compensation arrangements. Evaluating these costs and benefits is challenging as it requires a framework that integrates financial market microstructure with the managerial agency problem emanating from the separation of ownership and control. The literature provides little guidance in this respect. The basic question here is: What are the firm-specific and financial market related characteristics that determine the welfare impact of informed insider trading?

We address this question by developing an analytically tractable framework to examine the interaction between market microstructure and the firm’s information and contracting environment. We show that insider trading can have conflicting effects on productive efficiency and the informational efficiency of stock prices. We find that despite any productive distortions that insider trading may cause, it can still result in higher ex ante shareholder value. This result obtains because the informed manager makes trading profits at the expense of the noise or liquidity traders; therefore, the shareholders are, in effect, extracting profits from these traders by designing appropriate managerial compensation contracts. This redistribution of wealth between one set of uninformed agents, namely, liquidity traders and another set of uninformed agents, namely, outside shareholders via the optimal design of managerial compensation contracts has not been hitherto highlighted in the insider trading literature which, for the most part, has focused on the wealth transfer from uninformed traders to informed insiders.

Moreover, while our framework facilitates development of the various contending effects of insider trading, it also helps us determine the relationship between firm and security market related characteristics and the overall welfare effects of insider trading. We find that a few salient firm-specific and market factors, namely, the firm’s growth options, the volatility of the firm’s cash flows, and the trading noise (or volatility) in its stock determine the overall welfare impact of insider trading.

We study a generalized agency model (Myerson, 1982) where both moral hazard and adverse selection operate at the level of the firm’s management; the manager has private information on both the firm’s productivity and his own choice of productive action. In addition, the manager privately observes the actual cash flow realization before it becomes public, providing him with an opportunity to trade anonymously based on this information. Shareholders design equity-based incentive compensation for the manager. We augment this model with the Kyle (1985) market microstructure framework to study the effects of the
manager’s informed trading on price discovery and market liquidity.

An important insight from our analysis is that informed insider trading both exacerbates and ameliorates the agency problem, but along different dimensions. On one hand, profitable trading on privileged information \textit{ex post} increases the manager’s incentives to misrepresent his information and distort his decisions \textit{ex ante}. On the other hand, shareholders can rationally anticipate the profit that the manager expects to make from informed trading, and accordingly cut back on the compensation offered to managers, while still maintaining the incentives under the contract (Khanna et al., 1994; Baiman and Verrecchia, 1995, 1996).\(^4\) The net effect on \textit{ex ante} shareholder value is therefore ambiguous, which is in contrast to existing views in the literature that insider trading either unambiguously exacerbates the managerial agency problem, or unambiguously improves managerial incentives for value-creation.

Unlike in the usual incentive-mechanism design problem, in our model the manager’s informed trading \textit{ex post} influences the design of the incentive contract \textit{ex ante}. In turn, the stock market and the manager’s (insider’s) expected profits from informed trading are influenced by the manager’s incentive contract. Thus, this model allows us to assess the impact of firm-specific and security market characteristics on (i) the manager’s incentives and compensation, (ii) the informational efficiency of stock prices, and (iii) wealth redistribution.

We find that the stock liquidity depends on the endogenous equity-based compensation of the manager and \textit{vice versa}. That is, markets with high intrinsic liquidity (i.e., higher noise-to-signal ratios) need not necessarily be more liquid because the manager’s optimal equity share itself depends on these parameters. However, the standard intuition from Kyle (1985) that market liquidity is increasing with the noise-to-signal ratio holds if the firm’s growth options are sufficiently high and if \textit{a priori} beliefs on the firm’s productivity are agnostic. This is noteworthy because a large literature on insider trading posits — based mostly on informal arguments — a negative relationship between insider trading and liquidity.\(^5\) More importantly, the influence of firm-specific characteristics such as growth options should be of substantial interest to the growing empirical literature on market liquidity.

Meanwhile, the informed manager’s incentive constraints are tightened when markets

\(^4\)The negative relationship between insider trading and managerial compensation (holding other thing fixed) is in fact consistent with recent empirical evidence (Roulstone, 2003).

\(^5\)More recent empirical literature, however, finds no evidence of inimical effects of insider trading on liquidity (Cao et al., 2004).
are intrinsically more liquid (i.e., high noise-to-signal ratio) because the expected profits from manipulating the beliefs of market makers increase in such markets. Building on this insight, we derive refutable predictions on the effects of the noise-to-signal ratio on the optimal incentive contract for the manager. These effects depend on the firm’s growth options and the trading noise in its stock. In particular, for firms with sufficiently high growth options, the optimal equity-based (or high-powered) component of managerial compensation is negatively related to cash flow uncertainty and positively related to the trading noise. And since these are also the conditions when stock liquidity is increasing with the noise-to-signal ratio, the prediction is that pay-for-performance sensitivity of executive compensation will be positively related to stock liquidity, which is consistent with recent empirical results (Jayaraman and Milbourn, 2009).

We also offer a novel perspective on the relationship between pay-performance sensitivity and (cash flow) risk. As is well known, with a risk-averse manager, there is a negative relationship between pay-performance sensitivity and risk (e.g., Aggarwal and Samwick, 1999). However, the empirical evidence is ambiguous, motivating theoretical extensions of the standard agency model to include endogenous decentralization of decision-making authority to the agent (Prendergast, 2002). We find that when the generalized agency situation is extended to allow anonymous trading by a risk-neutral agent, the cash flow risk is indeed related to the optimal pay-performance sensitivity. However, the relationship between risk and pay-performance sensitivity can be positive or negative, depending on the firm’s growth options and the a priori optimism regarding the firm’s productivity.

Our generalized agency framework extends the agency-based literature on insider trading. In a model with adverse selection and two informed agents (an insider and an outsider), Khanna et al. (1994) examine the effects of insider trading on insider compensation and the equilibrium quality of outside information. Baiman and Verrecchia (1995, 1996) also use the Kyle (1985) framework to study the effects of insider trading on optimal compensation and financial disclosure with risk-averse managers when there is only moral hazard. These studies articulate the ‘compensation subsidy’ aspect of insider trading, but do not relate it to the possibility of wealth redistribution between sets of uninformed agents. Furthermore, because our framework allows for both adverse selection on firm productivity and manage-

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rial moral hazard on productive inputs, we clarify the tension between the productive and informational efficiency effects of insider trading.

We also highlight the interaction between managerial incentive compensation and stock market liquidity and present a broader perspective on the wealth transfers caused by insider trading. This interaction in our model is markedly different from that considered in the literature. For example, in Holmstrom and Tirole (1993), stock market liquidity affects the information acquisition incentives of speculators, and the optimal contract efficiently uses the acquired information reflected in the equilibrium stock price. In our setting, the informed trader is the manager himself and market liquidity affects the incentive constraints for effort and truth-telling directly. In particular, the optimal executive compensation contract and stock liquidity are jointly determined in our framework.

Overall, we extend the literature on insider trading by highlighting the wealth redistribution effects of such trading not only between informed insiders and uninformed investors (which has been the focus of the literature), but also between uninformed shareholders and uninformed liquidity traders. We also add to the literature on informed trading in competitive securities markets by allowing for endogenous incentives for informed trading by insiders through the design of optimal managerial compensation incentive contracts. We thus develop a structural model for market liquidity that depends not only on exogenous security market characteristics (such as the noise-to-signal ratio) but also on firm-specific characteristics such as the presence of growth options. These results speak particularly to the heightened interest in the determinants of financial market liquidity. Finally, we specify how optimal contracts in a generalized agency situation with both adverse selection and moral hazard (Myerson, 1982; Faynzilberg and Kumar, 1997, 2000) are influenced by the presence of informed insider trading, and derive refutable predictions on the effects of financial market microstructure on optimal executive compensation.

We organize the rest of the paper in the following fashion. Section 2 specifies the basic model and Section 3 records some benchmark equilibrium outcomes in the model with alternative informational and trading assumptions. Section 4 analyzes the equilibrium in the firm’s stock with insider trading and Section 5 characterizes the optimal managerial contract. Section 6 studies the welfare and efficiency effects of insider trading and Section 7 concludes.
2 The Model

We analyze an agency model of a widely-held firm that is controlled by a risk-neutral manager — the representative insider.\(^7\) There are five dates in the model, \(\tau = 0, 1, 2, 3, 4\), and four decision points at dates \(\tau = 0, 1, 2, 3\). The firm is liquidated at date \(\tau = 4\). For simplicity, all values are expressed in certainty equivalent units and the interest rate is normalized to be zero.

2.1 Profits and Informational Structure

The firm has a point input-point output technology that stochastically converts the manager’s effort \((e)\) into liquidating profits or cash flows.\(^8\) At date \(t = 1\), the manager privately observes a productivity parameter \(\theta \in \{\theta_L, \theta_H\}\), \(\theta_H > \theta_L\), and subsequently chooses the effort \(e\) at date \(t = 2\). The manager incurs a disutility \(\beta e^2\), \(\beta > 0\), from undertaking the effort \(e\), which is unobservable to outsiders.

At the beginning of date \(t = 3\), there is a profit shock \(\xi\), which has an infinite support, and the firm’s (liquidating) profits are determined as:

\[
\tilde{\pi}(e; \theta) = \theta e + \xi \tag{1}
\]

These profits are observed by the manager at date \(\tau = 3\), but not by outside shareholders and investors (or “outsiders”). At date \(\tau = 4\), the firm is liquidated and \(\tilde{\pi}\) are revealed publicly. Note that because \(\xi\) has an infinite support, the liquidating profits do not perfectly reveal the manager’s private information, namely, \((\theta, e)\). The common prior probability of the high-productivity state, \(\theta_H\), is \(q\).

2.2 Trading

For simplicity, we assume that the firm has one perfectly divisible share outstanding and equity trading occurs in a frictionless market populated with uninformed noise traders. At

\(^7\)For parsimony and notational ease, we assume throughout that the firm is unlevered; however, our results are qualitatively unaffected if there is debt in the capital structure.

\(^8\)We interpret managerial effort \((e)\) broadly to include all productive inputs that are costly for the manager — either through personal disutility (such as personal effort) or through opportunity costs (such as pecuniary investment).
date $\tau = 3$, subsequent to (privately) observing the cash flows \( \tilde{\pi} \), but prior to their public disclosure, the manager can trade in the firm’s stock. Our informational assumptions imply that the manager is fully informed of the stock’s liquidation value at the time of his trade.

The stock market is modeled as in Kyle (1985). The equilibrium stock price \( (P) \) is set by risk-neutral and competitive market makers. We let \( x \) denote the quantity traded by the manager that is chosen independent of the quantity traded by the noise traders, denoted by the random variable \( \eta \). Market makers observe only the total (market) order, \( y = x + \eta \), but not \( x \) and \( \eta \) separately. Free entry ensures that the equilibrium \( P \) equals the market makers’ conditional expectation of \( \tilde{\pi} \), given \( y \) and other elements of their information set that we denote by \( \Phi \).

We assume that the profit shock \( (\xi) \) and the quantity traded by the noise traders \( (\eta) \) are independent and have a bivariate normal distribution with zero means and variance \( \sigma^2_\xi \) and \( \sigma^2_\eta \), respectively. Moreover, \( (\xi, \eta) \) are conditionally independent of \( (\theta, e) \).

2.3 Manager’s Preferences

The manager chooses his effort \( (e) \) and subsequently stock trade \( (x) \) to maximize his expected utility. For simplicity, we normalize the interest rate to be zero. Then, under our assumptions, the manager’s expected utility depends on his expected wealth at date \( t = 4 \) and the disutility of effort. The expected wealth is itself composed of his possibly performance-contingent expected compensation from the firm and his expected profits from trading. We will quantify the manager’s expected utility momentarily. Finally, and without loss of generality, the manager’s reservation utility for managing the firm is set at zero.

2.4 Contracting

There are conflicts of interest between the shareholders and the self-interested manager because of the latter’s disutility for effort and the ability to trade on private information. These conflicts of interest give rise to a generalized agency problem (see ) because of the joint presence of hidden information (on \( \theta \) and \( \tilde{\pi} \)) and hidden actions (on \( e \) and \( x \)); this agency problem requires shareholders (or their representatives) to design appropriate incentive contracts for the manager.

Specifically, at date \( t = 0 \), an incentive contract is offered to the manager on the basis of
his communication regarding the privately observed productivity ($\theta$). Note that the manager has to be provided incentives to send informative signals on $\theta$, because (as we noted before) the eventual observation of the liquidating profits ($\tilde{\pi}$) does not yield a perfect inference on $\theta$. Consequently, shareholders cannot costlessly induce truthful reporting by threatening extreme penalties \textit{ex post}. On the other hand, requiring the manager to communicate $\tilde{\pi}$ at date $\tau = 3$ trivially forces him to completely reveal his private information on the liquidating profits and knocks out any basis for informed trading. We therefore assume that shareholders cannot directly proscribe the manager’s trading by forcing communication on the cash flows (see Baiman and Verrecchia (1995,1996) for a similar assumption). In our stylized model, this assumption captures the more realistic situation where outsiders are not able to precisely determine \textit{ex ante} the exact timing of the manager’s knowledge of $\tilde{\pi}$.$^9$

To facilitate analysis, we exploit the generalized revelation principle (Myerson, 1982; and Faynzilberg and Kumar, 1997, 2000) which allows us to restrict attention to the class of \textit{direct communication and obedience} contracts. In these contracts, it is incentive compatible for the manager to truthfully report the productivity $\theta$ and to obey instructions with respect to $\epsilon$. (For expositional ease, we will refer to such contracts as \textit{direct contracts}.) Furthermore, for tractability, we consider direct contracts where the manager’s incentive compensation is in the form of a fixed payment ($t$) and equity ($\gamma$) in the liquidating profits of the firm.

An admissible contract for the manager is therefore a menu $\delta \equiv \{\delta_i = (\gamma_i, t_i, e_i), i \in \{L, H\}\}$ that specifies, for each productivity report $\theta_i$, the managerial effort $e_i$, the fixed payment $t_i \in R$, and the fraction of the firm’s equity awarded to the manager, namely, $0 \leq \gamma_i \leq 1$. In the special case where $\gamma_i = 1$, outsiders sell the firm to the manager so that $t_i$ is negative and represents the sale price of the firm (Harris and Raviv, 1979)). In this case, we assume that the firm is “delisted” and there is no further trading in the firm, as the ownership passes to the manager.

The sequence of events is depicted in Figure 1.

(Insert Figure 1 here)

$^9$If outsiders could indeed renegotiate existing contracts (or write new ones), instantaneously as the manager gets to know $\tilde{\pi}$, then it is easy to show that there will exist a “no-trade” contract, where the manager does not trade, because he is provided the certainty equivalent of his profits from trading through direct transfers. Yet this is not the outcome that we observe in practice.
2.5 Equilibrium definition

Our notion of equilibrium includes an optimal choice of a direct contract by outsiders and a linear rational expectations equilibrium in the stock trading subgame. The contracting problem and the stock trading subgame are linked because market makers’ expectations of the stock’s liquidation value will depend on the contract and the manager’s productivity report (on \( \theta \)). To incorporate these interaction effects concisely, we develop some notation.

A contracting outcome \( \omega = (\delta, r) \) is a direct contract \( \delta \) and a subsequent report by the manager \( r \in \{\theta_L, \theta_H\} \). For notational convenience, we will denote by \( \omega_i, i = L, H \), the contracting outcome when \( r = \theta_i \). Contracting outcomes are public information and play an important role in the determination of trading equilibrium—along with the privately observed cash flows, they influence the manager’s optimal trading strategy, and they also influence the market makers’ beliefs on the stock’s liquidation value.

Specifically, fix some \( \omega_i \) (such that \( 0 \leq \gamma_i < 1 \)), \( i = L, H \), and let \( \Pi_i \equiv \theta_i e_i + \xi - t_i \) denote the realized cash flows net of the manager’s wage (or fixed compensation). Thus, the liquidating value of the firm’s stock is \( \tilde{V}_i \equiv (1 - \gamma_i)\Pi_i \). The manager’s wealth as a function of his trade \( x \) and any measurable pricing function \( P(y; \Phi) \) is

\[
W_i^M(x; \omega_i, \tilde{V}_i, P) = t_i + \gamma_i \Pi_i + x(\tilde{V}_i - P(y; \Phi))
\]

and his expected utility conditional on \( (\omega_i, \tilde{V}_i, P) \) is:

\[
\hat{U}_i^M(x; \omega_i, \tilde{V}_i, P) = E \left[ W_i^M(x; \omega_i, \tilde{V}_i, P) \right] - \frac{\beta e_i^2}{2}
\]

Since the manager has already undertaken his effort at the time of the trade, his optimal stock trading strategy \( X(\tilde{V}_i; \omega_i, P) \) maximizes his expected wealth \( E \left[ W_i^M(x; \omega_i, \tilde{V}_i, P) \right] \). A trading equilibrium in the stock market (or a “stock trading equilibrium”) is then given by \( X(\tilde{V}_i; \omega_i, P) \) and a pricing function \( P(y; \omega_i, X) \) such that \( P(y; \omega_i, X) = E[(1 - \gamma_i)\Pi_i \mid y, \omega_i, X] \).

We turn now to the determination of the optimal direct contract. Suppose that the true productivity is \( \theta_i \), but the manager reports \( r = \theta_j, i, j = L, H \). Let \( e_{ij}(\phi_j) \) be the manager’s optimal effort when the true productivity is \( i \), but he receives the allocation \( \phi_j = (\gamma_j, t_j) \). Accordingly, we write \( \Pi_{ij}(\phi_j) = [\theta_i e_{ij}(\phi_j) + \xi - t_j] \) and \( \tilde{V}_{ij}(\phi_j) = (1 - \gamma_j)\Pi_{ij}(\phi_j) \). Then the
manager’s expected utility (prior to the realization of $\bar{\Pi}_i$) is

$$U_{ij}^M(\delta; (X, P)) = \begin{cases} t_j + E_{\xi \eta} \left[ \gamma_j \bar{\Pi}_{ij}(\phi_j) + X(\bar{V}_{ij}(\phi_j) - P(y; \omega_i, X)) \right] - \frac{\beta |e_{ij}(\phi_j)|}{2} & \text{if } 0 \leq \gamma_j < 1 \\ E_{\xi} \left[ \bar{\Pi}_{ij}(\phi_j) \right] - \frac{\beta |e_{ij}(\phi_j)|}{2} & \text{if } \gamma_j = 1 \end{cases}$$

(4)

We note that unless the firm is sold to the manager (i.e., $\gamma_j = 1$), the manager’s expected utility at the contracting stage is defined only for a given stock trading equilibrium, $(X, P)$. For notational ease, we let $U_i^M = U_{ii}^M$ and $e_i = e_{ii}$.

The optimal direct contract, designed by outsiders ex ante, therefore solves

$$\max_{\delta} qV_H + (1 - q)V_L, \quad \text{s.t.,}$$

$$U_i^M \geq \bar{U}, \quad i = L, H, \quad \text{(6)}$$

$$U_i^M \geq U_{ij}^M, \quad i = L, H, \quad j = H, L. \quad \text{(7)}$$

Here, (6) and (7) are the individual rationality and the generalized incentive compatibility constraints, respectively. Here, we have combined the truth-telling and obedience constraints in (7) by ensuring that the manager’s effort choice (i.e., $e_{ij}(\phi_j)$) is incentive-compatible.

An equilibrium is the triple $(\delta, X, P)$, where $(X, P)$ is a stock market trading equilibrium and $\delta$ is an optimal direct contract i.e., it is a solution to (5)-(7).

## 3 Benchmark Outcomes

We begin our analysis two benchmark outcomes — one with complete information and the other with asymmetric information without insider trading — to facilitate intuition on the interaction between insider trading and the generalized agency problem. For notational ease, for each $i = L, H$, we will let the expected cash flows and equity value be, $\bar{\Pi}_i = \theta_i e_i - t_i$ and $\bar{V}_i = (1 - \gamma_i)\bar{\Pi}_i$, respectively. Clearly, the stock value falls as the manager’s equity share increases.

### 3.1 Equilibrium with Complete Information

Suppose that there is no information asymmetry, i.e., the firm’s productivity is common knowledge, the manager’s choice of effort is costlessly verifiable, and the manager has no
private information on the realization of the cash flows. In this case, there is no role for incentive contracts and there is no insider trading.

The optimal or first-best incentive mechanism is denoted by \( \delta_i^* = (\gamma_i^*, t_i^*, e_i^*) \), \( i \in \{L, H\} \), and is a solution to maximizing the objective function (5) subject to the individual rationality constraints (6). We can solve this program directly by incorporating the constraints into the objective function because the individual rationality constraints will be binding for each productivity type. Straightforward computations yield the optimal contract in productivity state \( i = L, H \), as:

\[
t_i^* = \frac{\beta (e_i^*)^2}{2}; \quad \gamma_i^* = 0; \quad e_i^* = \frac{\theta_i}{\beta}, \quad i \in \{L, H\}.
\]  

(8)

As one would expect, the managerial effort is positively related to the firm’s productivity. Hence, in the first-best arrangement, the (outside) equity value is \( V_i^* = \theta_i e_i^* - t_i^* \), \( i = L, H \).

However, the first-best is not achievable in the generalized agency case at hand. The best response to zero equity compensation, as seen in (8), for both types of managers (i.e., \( \theta_H \) and \( \theta_L \)) would be to put in zero effort, because it is personally costly and cash flows have an infinite support. In addition, the low-productivity manager would have an incentive to mimic the high-productivity manager and receive the compensation \( t_H^* > t_L^* \).

### 3.2 Asymmetric Information Without Insider Trading

Consider next the case where there is the generalized agency problem — that is, the manager has private information on the firm’s productivity and his choice of effort — but the manager does not have private information on the realized cash flows. Consequently, there is no insider trading, so that \( X \equiv 0 \), \( P(\omega_i, y = \eta) = V_i \), and

\[
U_{ij}^M(\delta; (X, P)) = t_j + E_\xi \left[ \gamma_j \tilde{\Pi}_{ij}(\phi_j) \right] - \frac{\beta (e_{ij}(\phi_j))^2}{2}, \quad 0 \leq \gamma_j \leq 1
\]  

(9)

The optimal direct contract is thus a solution to the optimization problem (5)-(7), when (9) replaces (4) in the constraint (7).

We briefly consider the polar cases of pure adverse selection — where there is private information on \( \theta \), but not on \( e \) — and pure moral hazard — where \( \theta \) is common knowledge, but \( e \) is only known by the manager. With adverse selection only, the truth-telling constraint is satisfied by flat (or non-performance-contingent) transfers to the manager. And with pure
moral hazard, the first-best outcome is achievable with pure moral hazard if the agent is risk-neutral—one simply “sells” the firm to the agent at a type-contingent price that makes the agent indifferent to accepting or rejecting the offer (Harris and Raviv, 1979). Thus, in this model, we can achieve the first-best allocations under either pure adverse selection or pure moral hazard settings.

But the first-best cannot be achieved when moral hazard and adverse selection are jointly present. Because of moral hazard, we cannot enforce the first-best by awarding a non-performance-contingent compensation to the manager, since such a reward structure induces the minimal effort from the manager. And, because of adverse selection, we cannot implement the first-best outcome by “selling” the firm, since the type of firm (and hence the correct sale-price) is unknown.

We therefore characterize the information-constrained or second-best optimal contract in the absence of informed insider trading, and denote this contract by \( \delta' \). As is typical in agency models, the individual rationality constraint of the low-type agent are binding; hence, the incentive-compatibility constraints are binding downwards, i.e., they are binding for the high-type agent, but not for the low-type agent. Therefore, it is optimal to sell the firm to the manager in the high-productivity state (with \( \gamma'_H = 1 \)), induce the first-best effort, and extract a higher sale price (subject to the binding incentive constraint). Finally, to relax the high-type agent’s incentive constraint, there is downward distortion in the low-type agent’s allocation, i.e., \( e'_L < e^*_L \).

**Theorem 1** Suppose that there is no asymmetric information on the cash flows. There exists some \( \epsilon' > 0 \) such that if \( \theta_H - \theta_L \geq \epsilon' \), then in the optimal contract \( \delta' \): (i) In the high-productivity state, the firm is sold to the manager (i.e., \( \gamma'_H = 1 \)) at the price \( -t'_H \), the first-best effort is achieved, but the manager makes information-based rents; (ii) In the low-productivity state, \( 0 < \gamma'_L < 1 \), \( e'_L < e^*_L \), \( t'_L < 0 \), and

\[
t'_L = -\frac{(\gamma'_L)^2 \theta^2_L}{2\beta(1-\gamma'_L)}, \quad \gamma'_L = \frac{(1-q)\theta^2_L}{q(\theta^2_H - \theta^2_L) + (1-q)\theta^2_L}, \quad e'_L = \frac{\gamma'_L \theta_L}{\beta}
\]

But if \( \theta_H - \theta_L < \epsilon' \), then it is preferable to offer a pooling contract where the firm is sold to both agent-types, i.e., \( \gamma'_j = 1 \), at the common price \( -t'_j = \frac{\vartheta^2_j}{\beta} \), \( j = L, H \).
The choice of the optimal equity award for the low-type ($\gamma'_L$) is subject to two offsetting considerations: increasing $\gamma'_L$ improves the low-type agent's incentives for effort, but also tightens the truth-telling constraint for the high-type agent. Consequently, as we see in (10), the optimal $\gamma'_L$ is increasing in $\theta_L$, but decreasing in $\theta_H$. It follows then that the effort distortion in the low state, i.e., $(e^*_L - e'_L) = [(1 - \gamma'_L)\theta_L]/\beta$, increases with firm's productivity range ($\theta_H - \theta_L$). Finally, the fixed transfers from the manager are chosen by the binding individual rationality and incentive compatibility constraints — for $t'_L$ and $t'_H$, respectively. Therefore, the sale price of the firm for the high-type manager is increasing in $\theta_H$.

The second part of the theorem establishes that when the spread between the two productivity types i.e., $\theta_H - \theta_L$ is sufficiently small, offering a pooling contract to the agent is preferable than offering the contract $\delta'$. The intuition here is that when the productivity spread is sufficiently small, the adverse selection problem is minimal, in which case the moral hazard problem is resolved by simply selling the firm (as we noted before). Henceforth, we will assume that the adverse selection problem is of a sufficient magnitude (i.e., $\theta_H - \theta_L$ is sufficiently large) so that offering the separating contract dominates offering the pooling contract.

We next turn to analysis of the generalized agency problem when the manager can also trade based on his private information (on the cash flows). Following the standard backward recursion approach, we first characterize the stock trading equilibrium, for a given contracting outcome. We then use this stock trading equilibrium to characterize the optimal contract.

4 Stock Market Equilibrium

Along the lines of Kyle (1985), we will construct a stock trading equilibrium $(\hat{X}, \hat{P})$, where $\hat{X}(\hat{V}_i)$ and $\hat{P}(y)$ are linear functions.\footnote{The stock trading equilibrium is relevant only if the contracting outcome $\omega_i$ is such that $0 \leq \gamma_i < 1$, which is what we assume for the remainder of this Section.} Such an equilibrium exists because, along the equilibrium path and conditional on any contracting outcome $\omega_i$, $i = L, H$, the equity value of the firm and the noise trading, i.e., $(\hat{V}_i, \eta)$, are multi-variate normal; specifically, $\hat{V}_i$ is normal with mean $\bar{V}_i$, variance $(1 - \gamma_i)^2 \sigma^2$, and is independent of the noise trading $\eta$. Moreover, the manager is a risk-neutral and strategic informed trader.

Then, suppose that the manager’s trading strategy is $X(\hat{V}_i; \omega_i, P) = A_i + B_i \hat{V}_i$ and the
pricing function is \( P(y; \omega_i) = C_i + D_i y, \) \( i = L, H. \) Taking \( P(y; \omega_i) \) as given, and conditional on \((\tilde{V}_i, \omega_i)\), the manager’s optimal trading strategy maximizes the quadratic objective function, \( x(\tilde{V}_i - (C_i + D_i x)) \). Hence, the optimal trade is \( x = (\tilde{V}_i - C_i) / 2D_i \), so that \( A_i = -C_i / 2D_i \) and \( B_i = \frac{1}{2D_i} \). Therefore, conditional on \( \omega_i \), the stock value \((\tilde{V}_i)\) and the aggregate market order \((y = x + \eta)\) are also bivariate normal, with the conditional covariance and correlation, respectively,

\[
\text{Cov} \left( \tilde{V}_i, y \mid \omega_i; X \right) = B_i (1 - \gamma_i) \sigma_{\xi}^2, \quad \text{Corr} \left( \tilde{V}_i, y \mid \omega_i; X \right) = \frac{B_i (1 - \gamma_i) \sigma_{\xi}}{\sqrt{B_i^2 (1 - \gamma_i)^2 \sigma_{\xi}^2 + \sigma_{\eta}^2}} \tag{12}
\]

The expressions in (12) clarify the effects of the manager’s optimal incentive contract \((\delta)\) on the stock market trading, and thereby also highlight the differences with with the standard linear trading equilibrium a à Kyle (1985). An increase in the manager’s equity share \(\gamma_i\), reduces the mean and variance of the stock value since \(\tilde{V}_i = (1 - \gamma_i) \Pi_i\) and \(\text{Var}(\tilde{V}_i) = (1 - \gamma_i)^2 \sigma_{\xi}^2\). Moreover, (12) implies that, holding fixed the manager’s trading strategy, the informativeness of the order flow observed by the market makers (i.e., \(\text{Corr}(\tilde{V}_i, y \mid \omega_i; X)\)) also falls with the manager’s equity share. This is because the manager’s optimal trade is an increasing function of the stock value \((\tilde{V})\); hence, as his equity share increases, the manager trades less aggressively, ceteris paribus, reducing the information content of the order flow for the market makers.

Thus, with a fixed trading strategy, the sensitivity of the equilibrium price to the order flow will be decreasing in \(\gamma_i\). That is, the market depth or liquidity — the order flow needed to induce a unit change in the stock price — will be increasing in the manager’s equity share \((\gamma_i)\). But, of course, in equilibrium, the manager’s trading strategy is endogenously determined along with the equilibrium stock price. Therefore, the manager will trade more aggressively in a more liquid market, and the net effect of \(\gamma_i\) on the manager’s trading is ambiguous ex ante.

We can specify a stock trading equilibrium which is unique among the class of equilibria with linear trading and pricing rules. This is because, using standard projection analysis, the conditional expectation of \(\tilde{V}_i\), given \((y, \omega_i)\), is linear in \(y\).

**Theorem 2** There exists a unique linear stock trading equilibrium \(\left\langle X(\tilde{V}_i; \omega_i), \ P(y; \omega_i) \right\rangle_{i=L}^{H} \),
where,

\[ X(V_i; \omega_i) = \frac{\sigma_y}{(1 - \gamma_i)\sigma_\xi} [V_i - \bar{V}_i] = \frac{\sigma_y (1 - \gamma_i)\xi}{(1 - \gamma_i)\sigma_\xi} = \frac{\sigma_y}{\sigma_\xi} \]

\[ P(y; \omega_i) = \bar{V}_i + \left[ \frac{(1 - \gamma_i)\sigma_\xi}{2\sigma_y} \right] y \quad (13) \]

Thus, in equilibrium, \( \hat{A}_i = -\bar{V}_i \left( \frac{\sigma_y}{(1 - \gamma_i)\sigma_\xi} \right), \hat{B}_i = \frac{\sigma_y}{(1 - \gamma_i)\sigma_\xi}, \hat{C}_i = \bar{V}_i, \) and \( \hat{D}_i = \frac{(1 - \gamma_i)\sigma_\xi}{2\sigma_y}. \)

In our model, the two offsetting effects of the manager's equity incentive award \( (\gamma_i) \) on his optimal trading — mentioned above — cancel out exactly, leaving the optimal trading strategy invariant to the manager’s incentive contract. Nevertheless, conditional on the contracting outcome \( \omega_i \), the equilibrium market liquidity, i.e., \( \left[ \frac{dP(y; \omega_i)}{dy} \right]^{-1} = 2\sigma_y [(1 - \gamma_i)\sigma_\xi]^{-1} \), is increasing in \( \gamma_i \). The reason is that as the stock price (i.e., the outside equity value) falls with the manager’s equity share \( (\gamma_i) \), ceteris paribus, it becomes less sensitive to variations in the order flow. Indeed, in the extreme case of \( \gamma = 1 \), where the manager retains all cash flows, the stock value is identically zero.

Thus, unlike the Kyle (1985) model where the equilibrium market liquidity is determined completely by the exogenously given noise-to-signal ratio, in our framework market liquidity is also influenced by the endogenous equity-based compensation of the manager; and, of course, the signal variance here is the cash flow uncertainty of the firm, i.e., \( \sigma_\xi \). It is useful to distinguish between the influence of the exogenous signal-to-noise ratio \( \nu \equiv \frac{\sigma_\xi}{\sigma_y} \) and the endogenous equity compensation \( \gamma_i \), and write the equilibrium market liquidity as (conditional on \( \omega_i \)):

\[ \alpha_i \equiv \frac{2}{(1 - \gamma_i)\nu}. \quad (14) \]

However, the optimal incentive contract will itself depend on \( \nu \), so that the equilibrium relationship between \( \alpha_i \) and \( \nu \) is ambiguous ex ante. By contrast, the equilibrium liquidity in the Kyle (1985) model is \( 1/(2\nu) \) and is unambiguously decreasing with the signal-to-noise ratio.

The manager’s expected trading profits in the stock market equilibrium are (for each
Thus, in equilibrium, market makers correctly anticipate — and adjust for — the expected stock value based on the contracting outcome. However, the realized equilibrium stock price only captures one-half of the ex ante value uncertainty (Kyle, 1985), i.e., market makers can not completely adjust for the deviations from expected value through the trading mechanism. Consequently, the manager profits from trading at the expense of the noise traders, and, in equilibrium, these profits increase with the earnings and trading noise, but decrease with the manager’s equity share ($\gamma_i$).

Importantly, the market makers’ perception about the firm’s productivity ($\theta$) and the manager’s effort ($e$) depend on their conjectures regarding $\delta$ — in particular, whether they take the contract to satisfy the manager’s truth-telling and obedience constraints. But if it is common knowledge that the market makers’ expectations depend on their conjectures regarding $\delta$, then the manager may have an incentive to defect in order to exploit these expectations. Such incentives will not exist in a pure moral hazard model, or where there is no hidden information on a value-relevant parameter, because the market makers will correctly anticipate the equilibrium effort (Yung, 2005). However, it turns out that when both hidden information and hidden action are jointly present, such incentives will exist, as we explicate in the next section.

5 The Optimal Contract with Insider Trading

In this section, we characterize the optimal contract when the manager can trade in the stock market; we denote this contract by $\hat{\delta}_i = \hat{\delta}_i, \hat{\ell}_i, \hat{e}_i, i \in \{L, H\}$. In the presence of insider trading, the incentive compatibility constraints require that the manager not have incentives to defect from the equilibrium path and simultaneously misrepresent the firm’s productivity and alter his effort in order to exploit the market makers’ expectations. We start by specifying the manager’s optimal effort policy both on- and off-the-equilibrium paths.
5.1 The Manager’s Effort Policy

5.1.1 Equilibrium Effort Policy

A consequence of (15) is that the manager’s expected profits from stock trading along the equilibrium path is independent of his effort. Thus, given a contract $\delta$ and the productivity state $\theta_i$, $i = L, H$, in equilibrium the manager’s optimal effort choice is to:

$$\text{Max}_e \left\{ \gamma_i (\theta_i e - t_i) - \frac{\beta e^2}{2} \right\}.$$  

The optimal effort is $\hat{e}_i(\phi_i) = \frac{\gamma_i \theta_i}{\beta}$. Note that this equilibrium effort policy is identical to the optimal effort policy without insider trading (cf. Theorem 1).

5.1.2 Manager’s Payoffs from Defection

Fix some candidate equilibrium with the direct contract $\delta$. Consider the off-equilibrium-path event where the true productivity is $\theta_i$, but the manager defects by reporting $\theta_j$ and chooses the effort $e_{ij}$, $j = L, H$. The market makers will still use the equilibrium pricing rule that is consistent with the contracting outcome $\omega_j$, i.e., $\hat{P}(y; \omega_j) = \hat{C}_j + \hat{D}_j y$ (cf. Theorem 2). In response, the manager’s optimal stock trading strategy is, $\hat{X}(\tilde{V}; \omega_j) = \hat{A}_j + \hat{B}_j \tilde{V}_{ij}$, where $\tilde{V}_{ij} \equiv (1 - \gamma_j) \Pi_{ij}$. Since $V_{ij}(\phi_j) = (1 - \gamma_j) [\theta_j \hat{e}_j(\phi_j) - t_j]$, we can compute the manager’s expected trading profit from defection:

$$\Gamma_{ij} = E_{\xi, \eta} \left[ X(\tilde{V}_{ij} - P(X + \eta; \omega_j, X)) \right] = \frac{(1 - \gamma_j)}{2\nu} \left[ (\theta_i e_{ij} - \theta_j \hat{e}_j(\phi_j))^2 + \sigma_{\xi}^2 \right]$$  

(16)  

Conditional on the contract $\delta$, the manager’s expected trading profits following the defection are decreasing with the signal-to-noise ratio ($\nu$) or increasing with the market liquidity (cf. (14)).

The manager’s expected utility from the defection is $U_{ij}^M(\delta; (\hat{X}, \hat{P})) = E \left\{ t_j + \gamma_j \Pi_{ij} + \Gamma_{ij} \right\} - \frac{\beta \nu^2}{2}$. Substituting the equilibrium effort policy $\hat{e}_j(\phi_j) = \frac{\gamma_j \theta_j}{\beta}$ in (16) and then optimizing $U_{ij}^M(\delta; (\hat{X}, \hat{P}))$ with respect to the off-equilibrium effort policy $e_{ij}$ yields

$$\hat{e}_{ij}(\phi_j) = \frac{\gamma_j \theta_i \beta(\sigma_{\xi} - \sigma_{\eta}(1 - \gamma_j)\theta_j^2)}{\beta(\sigma_{\xi} - \sigma_{\eta}(1 - \gamma_j)\theta_j^2)}$$  

(17)
The second order sufficient condition for the optimality of \( \hat{\epsilon}_{ij}(\phi_j) \) is that \( v > \frac{\sigma_H}{\beta} \), i.e., the signal-to-noise ratio be bounded below; or, referring to (14), the market liquidity be bounded above. This condition follows from (16): if liquidity is unboundedly large, then the manager can trade aggressively without materially affecting the stock price and no interior solution for \( e_{ij} \) may exist. We may interpret the ratio \( \frac{\sigma_H}{\beta} \) as the high-type manager’s benefit-cost ratio for increasing expected trading profits by under-reporting productivity and increasing effort, since \( \beta \) is the manager’s effort disutility parameter. Note that \( \hat{\epsilon}_{ij}(\phi_j) > 0 \) if \( v > \frac{\sigma_H}{\beta} \) and we will henceforth assume this condition holds.

It is instructive to compare the off-equilibrium-path effort when the manager can trade with the manager’s optimal effort following a defection when trading is proscribed i.e., \( e'_{ij}(\phi_j) = \frac{\gamma_i}{\beta} \) (refer to the proof of Theorem 2 in the appendix). We can re-write (17) as

\[
\hat{\epsilon}_{ij}(\phi_j) = Z_{ij} e'_{ij}(\phi_j), \quad Z_{ij} \equiv \frac{\beta v - (1 - \gamma_j) \theta^2_H}{\beta v - (1 - \gamma_j) \theta^2_i},
\]

(18)

Note that \( Z_{ij} = 1 \) in the polar case \( \sigma_\eta = 0 \), i.e., when there is no noise trading, there is no informed trading as well. Moreover, if the high-productivity manager mimics the low-type, so that \( \theta_i = \theta_H > \theta_j = \theta_L \), then \( Z_{HL} > 1 \) and \( \hat{\epsilon}_{HL} > e'_{HL} \). The reason is that the high-type manager can increase expected trading profits by lowering market makers’ expectations on the stock value by announcing low productivity (\( \theta_L \)) and simultaneously increasing effort to maximize the deviation between the realized and expected value. Thus, the opportunity for informed trading amplifies the high-type agent’s incentive to defect from truth-telling and obedience to increase expected trading profits by manipulating the market makers’ beliefs. In particular, one can infer directly the effect of \( v \) on \( \hat{\epsilon}_{ij}(\phi_j) \) from (18).

**Proposition 1** \( \hat{\epsilon}_{ij}(\phi_j) \) is increasing in \( v \) if \( \theta_j > \theta_i \), and conversely if \( \theta_j < \theta_i \).

Proposition 1 implies that \( \hat{\epsilon}_{HL} \) decreases with \( v \). The reason is that the high-productivity manager’s expected trading profits from defecting are lower, ceteris paribus, if \( v \) is high (cf. (16)). That is, the (generalized) incentive compatibility constraints are tightened in markets with greater trading noise, because the manager can aggressively trade without materially affecting the stock price in such markets. Thus, the influence of \( v \) on \( \hat{\epsilon}_{ij}(\phi_j) \) facilitates intuition on the effects of stock market liquidity — which is inversely related to \( v \) — on the manager’s incentive contract when informed trading is possible. In particular, the
incentive to defect from truth-telling and obedience decreases with the signal-to-noise ratio, and increases with liquidity.

5.2 Contract Characterization

We next characterize \( \hat{\delta} \). We know from the preceding analysis that the signal and noise parameters (i.e., \( \sigma_\zeta \) and \( \sigma_\eta \)) influence the optimal contract with insider trading, \( \hat{\delta} \). In some aspects, the structure of \( \hat{\delta} \) remains similar to \( \delta' \) (cf. Theorem 1): Only the incentive constraint for the high-type agent is binding and the firm is sold to this agent-type; the low-type agent receives an incentive compensation (with \( 0 < \hat{\gamma}_L < 1 \)), but makes zero rents. Thus, in the optimal contract, only the low-type manager trades along the equilibrium path.

However, informed trading by the low-type manager has a substantial impact on the design of the optimal contract for both agent-types. Specifically, the optimal contract extracts the expected trading profits from the low-type agent (because his participation constraint is binding). But the opportunity to trade tightens the incentive constraints of the high-type agent, and this affects the (incentive compatible) sale price of the firm for this agent-type. In a related vein, \( \hat{\gamma}_L \) differs substantially from \( \gamma'_L \) because the manager’s equity share influences the expected trading profits, as we see in (16).

For notational ease, let

\[
\hat{\Pi}_{ij} = [\theta_i \hat{e}_{ij}(\phi_j) - \hat{t}_j], \quad \hat{\Pi}_{ii} = \hat{\Pi}_i, \quad \text{and}
\]

\[
\hat{\Gamma}_{ij} = \frac{(1 - \hat{\gamma}_j)}{2\hat{v}} \left[ (\theta_i \hat{e}_{ij} - \theta_j \hat{e}_j)^2 + \sigma_\varepsilon^2 \right], \quad i, j = L, H.
\]

Theorem 3 Suppose that the manager privately observes the cash flows of the firm and subsequently trades in the stock market. There exists some \( \hat{\varepsilon} > 0 \) such that if \( \theta_H - \theta_L \geq \hat{\varepsilon} \), then the optimal contract \( \hat{\delta} \) is such that: (i) In the high-productivity state, the firm is sold to the manager (i.e., \( \hat{\gamma}_H = 1 \)) at the price \( -\hat{t}_h \), the first-best effort is achieved, but the manager makes information-based rents. (ii) In the low-productivity state, \( \hat{\varepsilon}_L = \hat{\gamma}_L \theta_L / \beta \), \( 0 < \hat{\gamma}_L < 1 \) (if \( \hat{\varepsilon} > (\theta_H^2 - \theta_L^2)^2 + \beta^2 \sigma_\varepsilon^2 \hat{v} \)) and \( \hat{\gamma}_L \) is implicitly given by

\[
F[\hat{\gamma}_L] = \hat{\Pi}_L + (1 - q) \left[ \frac{(1 - \hat{\gamma}_L) \theta_L^2}{\beta} - \hat{\Pi}_L \right] - q \left( \hat{\Pi}_{HL} + \frac{\partial \hat{\Gamma}_{HL}}{\partial \gamma_L} \right) = 0
\]
Moreover, the optimal contract extracts the expected trading profits of the low-type manager through the design of $\hat{t}_L$ (cf. (20)). But since the manager — the strategic informed trader in our model — makes trading profits at the expense of the noise traders, the shareholders effectively use the design of the manager’s optimal compensation contract to extract profits from the noise or liquidity traders. Since the shareholders themselves are uninformed, it follows that in equilibrium there is a redistribution of expected profits between two types of uninformed agents: The shareholders of the firm and the agents who trade in the stock of the firm for liquidity and other reasons. Notably, this redistribution occurs through the optimal incentive contract for the informed agent whose trading links the two types of uninformed agents.

Furthermore, the sale price of the firm for the high-type agent has to be reduced in order to relax the incentives of this agent-type to defect and make trading profits by falsely reporting his type. Hence, there is a loss in the shareholder value of the firm in the high state, and the net effect of insider trading on the expected value of the firm (prior to contracting) is ambiguous. Moreover, the high-type manager makes greater information-based rents as a consequence of informed trading, and these additional rents are extracted from the noise traders as well.

We now consider the effects of insider trading on the manager’s equity incentive award. The optimal $\hat{\gamma}_L$ attempts to relax the incentive constraint of the high-type agent by reducing $\hat{\Gamma}_{HL}$ — his expected trading profits following a defection. But since the share value is decreasing in $\hat{\gamma}_L$, ceteris paribus, there is an incentive to increase the low-type manager’s equity share to reduce the high-type manager’s expected trading profits from defection. Indeed, while $\hat{\gamma}_L$ (the optimal equity award for the low-type manager without insider trading) lies strictly between zero and one (cf. Theorem 1 and (10)), $\hat{\gamma}_L$ is always positive, but requires a lower bound on the signal-to-noise ratio for it to be strictly less than one. The reason is that the marginal effect of $\gamma_L$ on $\hat{\Gamma}_{HL}$, i.e., $\frac{\partial \hat{\Gamma}_{HL}}{\partial \gamma_L} \propto -1/v$, is decreasing (in absolute terms)

\[\hat{t}_L = -\frac{1}{2} \left[ \frac{\hat{\gamma}_L^2 \theta^2_L}{\beta (1 - \hat{\gamma}_L)} + \sigma \eta \sigma \xi \right] ; \hat{t}_H = - \left[ \frac{(1 - \hat{\gamma}_L^2) \theta^2_H}{2 \beta} - (1 - \hat{\gamma}_L) \hat{t}_L - \hat{\Gamma}_{HL} \right].\]  

12 Our maintained assumption (from Section 5.1.2) is that $v > \frac{\theta^2_L}{\beta}$. However, the lower bound given in Theorem 3 may be smaller or larger than $\frac{\theta^2_L}{\beta}$, depending on the relative sizes of $\sigma \eta \sigma \xi$ and $(\theta^2_H - \theta^2_L)$. 

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with $v$. Thus, the optimal $\gamma_L$ approaches one if $v$ is not strictly bounded below.

In general, however, the impact of the signal-to-noise ratio $v$ on $\hat{\gamma}_L$ for $0 < \hat{\gamma}_L < 1$ is not clear cut because there are two conflicting effects of increasing $v$ on $\hat{\gamma}_L$. Since $\hat{e}_{HL}$ is negatively related to $v$ (cf. Proposition 1), the high-type manager's expected payoffs from the firm’s profits (following a defection) are negatively related to $v$, as well. Hence, a higher $v$ relaxes this agent-type’s incentive compatibility constraints and increases $\hat{\gamma}_L$, ceteris paribus. However, as we emphasized above, the marginal effect of increasing $\gamma_L$ on $\hat{\Gamma}_{HL}$ is reduced as $v$ increases—inducing a negative relation between $\hat{\gamma}_L$ and $v$. This negative effect is amplified if $\theta_H$ is large relative to $\theta_L$. Therefore, one may conjecture that $v$ will be negatively related to $\hat{\gamma}_L$ if $\theta_H$ is sufficiently large relative to $\theta_L$. The following proposition established this intuition analytically for the case of symmetric prior beliefs on $\theta$ (i.e., $q = \frac{1}{2}$) given conditions under which $\hat{\gamma}_L$ is sufficiently large (i.e., $\hat{\gamma}_L > \frac{3}{4}$).

**Proposition 2** Suppose that prior beliefs on $\theta$ are symmetric i.e., $q = \frac{1}{2}$. Then, ceteris paribus, the relationship between $v$ and $\hat{\gamma}_L$ is negative if (i) $\theta_H$ is sufficiently large relative to $\theta_L$, and (ii) $\sigma_H^2 > \sigma^*$. 

In Figure 2, we plot the effects of varying $v$ and the a priori beliefs $q$ on $\hat{\gamma}_L$ for a suitable parameterization of the model. Here, $\hat{\gamma}_L$ is clearly decreasing in $v$. Interestingly, the relationship between $\hat{\gamma}_L$ and $v$ is convex, so that increases in the signal-to-noise ratio have a successively weaker effect on the equity compensation. Meanwhile, as the prior beliefs on the firm’s productivity ($q$) the equilibrium compensation also increases. But the negative relationship between $\hat{\gamma}_L$ and $v$ appears to hold over a reasonable range of the prior beliefs, suggesting that our restriction on the prior beliefs in Proposition 2 is not substantive.

Figure 3 plots the effects of varying $v$ and the high productivity parameter $\theta_H$ on $\hat{\gamma}_L$. Note that, for a fixed $\theta_L$, the firm’s growth options increase with $\theta_H$. For this parameterization, $\hat{\gamma}_L$ is increasing in $\theta_H$. The reason is that high values of $\theta_H$ relax the high-type manager’s incentive constraints since his surplus from owning the firm (after paying are relaxed for $\hat{t}_H$) are increasing in $\theta_H$.

(Insert Figures 2 and 3 here)

By characterizing the relation between signal-to-noise ratio $v$ ($= \frac{\sigma_{\epsilon}}{\sigma_{\eta}}$) and the optimal equity-based compensation $\hat{\gamma}_L$, Proposition 2 presents refutable predictions on the effects of trading noise ($\sigma_{\eta}$) and cash flow uncertainty ($\sigma_{\epsilon}$) and equity-based executive compensation.
Specifically, for firms with significant growth options — proxied by the difference in the productivities \((\theta_H - \theta_L)\) — the optimal equity-based compensation is negatively related to cash flow uncertainty and positively related to trading noise. And, since market liquidity is increasing in the signal-to-noise ratio if growth options are sufficiently high (Kyle, 1985), this result predicts that pay-for-performance sensitivity of executive compensation will be positively related to stock liquidity, which is borne out by recent empirical results (Jayaraman and Milbourn, 2009).

Proposition 2 also offers a fresh perspective on the relationship between pay-performance sensitivity and (cash flow) risk. While standard agency theory does not offer any link between cash flow risk and incentives for risk-neutral agents, it does posit a negative relationship for risk-averse managers (Aggarwal and Samwick, 1993). By extending the standard model to include informed trading by the agent, we find that the cash flow risk is indeed related to the optimal pay-performance sensitivity (with the pay measured by the manager’s equity share) even for a risk-neutral manager.13

It is also of interest to compare the optimal equity-based compensation with and without insider trading. In our model, it is not generally true that the low-type manager’s equity-based compensation increases with informed trading, i.e., that \(\hat{\gamma}_L > \gamma'_L\). The reason is that \(\gamma_L\) has two conflicting effects on the high-type agent’s incentives to defect and mimic the low-type agent. A higher \(\gamma_L\) limits the high-type manager’s expected profits from informed trading ceteris paribus (as we noted above). But a higher \(\gamma_L\) also increases the high-type manager’s expected compensation from managing the firm while pretending to be the low-type agent. The relationship between \(\hat{\gamma}_L\) and \(\gamma'_L\) is therefore ambiguous. However, under conditions of Proposition 2, the first effect dominates the second one, because the marginal effect of the equity share on the manager’s expected compensation is low.

**Proposition 3** Suppose that prior beliefs on \(\theta\) are symmetric i.e., \(q = \frac{1}{2}\). Then \(\hat{\gamma}_L > \gamma'_L\) provided (i) \(\theta_H\) is sufficiently large relative to \(\theta_L\) such that \(\frac{\theta^2_L}{\theta^2_H} \leq \frac{2}{3}\), and (ii) \(\sigma^2_{\eta} > \sigma^*\).

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13 Prendergast (2002) derives a connection between risk and incentives with a risk-neutral agent by relating uncertainty to the delegation decision; in his framework, uncertainty influences whether decision-making is delegated to the agent.
5.2.1 Market Liquidity with Optimal Contracting

As we mentioned above, in our model equilibrium market liquidity and equity award ($\hat{\gamma}$) are inherently linked. In particular, having endogenized the managerial compensation contract (cf. Theorem 3), the equilibrium market liquidity in the low-state is $\hat{\alpha}_L \equiv 2[(1 - \hat{\gamma}_L)v]^{-1}$ (cf. 14). But since the relation between $\hat{\gamma}_L$ and $v$ need not generally be monotonic, it follows that with informed insider trading market liquidity need not be monotonically related to $v$, as is known to be the case in the standard Kyle (1985) framework. In fact,

$$\frac{\partial \hat{\alpha}_L}{\partial v} \propto -(1 - \hat{\gamma}_L) + v \frac{\partial \hat{\gamma}_L}{\partial v}$$

(21)

Therefore, $\hat{\alpha}_L$ is guaranteed to be decreasing in $v$ only if $\hat{\gamma}_L$ is negatively related to $v$. We can therefore use the sufficient conditions in Proposition 2 to conclude that

**Proposition 4** Suppose that prior beliefs on $\theta$ are symmetric, i.e., $q = \frac{1}{2}$, and (i) $\theta_H$ is sufficiently large relative to $\theta_L$, and (ii) $\sigma^2_{\eta} > \sigma^*$. Then, equilibrium market liquidity in the low productivity state ($\hat{\alpha}_L$) is strictly increasing with the ratio of trading noise to cash flow risk ($\frac{\sigma^2_{\eta}}{v^2} = \frac{1}{v}$).

In other words, the usual intuition of liquidity being higher in markets with high trading noise or low uncertainty on share values need not generally apply in the presence of strategic informed trading by insiders who control share values through their actions. This said, however, Proposition 4 confirms that the standard intuition does apply for firms with high growth options and investors have agnostic *a priori* beliefs on their productivity. Notice also that the sufficient conditions of Proposition 4 are identical to that of Proposition 2. Hence, Proposition 2 implies that the equity-based component of optimal compensation is increasing in stock liquidity (for the low-type firm).

6 Informed Insider Trading and Welfare

Informed trading by the manager has productive, informational, and wealth distribution implications. First, insider trading has productive effects because it influences the choice of the managerial effort ($\epsilon$) through the design of the optimal incentive contract. Second, informed insider trading improves the informational efficiency of the equilibrium stock price.
Finally, with informed trading, the relative welfare (or expected utility) of the shareholders and the manager is altered. In addition, information trading results in profit expropriation from the noise traders toward the shareholders and the manager. In this section, we analyze these effects both analytically and numerically. Our analysis is facilitated by comparing the optimal contract with trading \((\hat{\delta})\) to the benchmark optimal contract without trading \((\delta')\).

Let us examine first the productive consequences of insider trading through the change in (gross) expected profits \(E[\hat{\pi}(e; \theta)] = \theta e\). Thus, the productive effects of insider trading in state \(i = L, H\) are \(\Delta_i^\pi \equiv \theta_i (\hat{e}_i - e'_i)\). Therefore, an immediate implication of Theorems 1 and 3 is that insider trading has no productive effects in the high-productivity state since the first-best effort is achieved with or without insider trading. Hence, \(\Delta^\pi = (1 - q)\theta_L (\hat{e}_L - e'_L)\).

But since \(\hat{e}_L - e'_L = (\hat{\gamma}_L - \gamma'_L) (\theta_L / \beta)\), it follows from the foregoing results that

**Proposition 5** Suppose that prior beliefs on \(\theta\) are symmetric, i.e., \(q = \frac{1}{2}\), and (i) \(\theta_H\) is sufficiently large relative to \(\theta_L\), and (ii) \(\sigma^2_H > \sigma^*\). Then, \(\Delta^\pi\) is decreasing in \(v\); equivalently, the productive distortion due to insider training is decreasing in \(v\).

In general, \(\hat{\gamma}_L\) can be greater or less than \(\gamma'_L\), the effect of \(v\) on the productive distortion introduced by informed insider trading is ambiguous. For example, for firms with high growth options, the productive distortions may **increase** with \(v\) if \(\hat{\gamma}_L < \gamma'_L\). However, under the conditions specified in Proposition 5, the productive distortion is actually higher in more liquid markets, i.e., with greater noise trading and lower cash flow uncertainty.

Turning to the effects of insider trading on shareholder value, as we mentioned above (cf. Theorem 3), the net effect of insider trading in the low-productivity state is ambiguous because the distortion in managerial effort is set against the reduction in the transfer payment to the manager (i.e., \(\hat{t}_L < t'_L\)). We denote the change in the firm’s equity value due to insider trading in state \(i = L, H\), by \(\Delta_i^V \equiv \hat{V}_i - V'_i\), where

\[
\Delta_L^V = (1 - \hat{\gamma}_L) \hat{\Pi}_L - (1 - \gamma'_L) \Pi'_L
\]

Figure 4 plots \(\Delta^V\). Notice that, for the chosen parameterization, shareholder value in the low state increases with informed insider trading. In particular, we find that \(\Delta^V\) is increasing in \(v\) because \(\hat{\Pi}_L\) is increasing in the signal-to-noise ratio. Thus, interestingly, insider trading makes shareholders especially better off for firms with high cash flow uncertainty or for firms...
with low liquidity trading in their stock. On the other hand, $\Delta^V_H$ is essentially invariant with respect to $\theta_H$.

(Insert Figure 4 here)

Since shareholders sell the firm to the high-type agent, $\Delta^V_H = t'_H - \hat{t}_H$. But note that, in equilibrium, the effects of insider trading on the manager’s expected utility are simply the opposite of $\Delta^V_H$. If $\Delta^M_i$ denotes the change in managerial expected utility due to insider trading in state $i = L, H$, then,

$$
\Delta^M_i \equiv (\hat{\gamma}_i \hat{\Pi}_i + \hat{t}_i) - (\gamma'_i \Pi'_i + t'_i) - \left(\frac{\beta}{2}\right)[\hat{\epsilon}^2_i - (\epsilon'_i)^2]
$$

(23)

Since the low-type agent receives his reservation utility (zero) with or without trading, insider trading has no impact on his welfare (i.e., $\Delta^M_L = 0$). The high-type agent in both cases — that is, with or without trading — chooses the first-best effort and receives complete ownership. Hence, $\Delta^M_H = \hat{t}_H - t'_H$ (i.e. $- \Delta^V_H$).

As we noted in the discussion following Theorem 3, $\Delta^M_H > 0$. Figure 5 plots $\Delta^V_H$ (i.e., $-\Delta^M_H$) with respect to $v$ and $\theta_H$ for the same model parameterization that we used in Figures 2-4. We find that $\Delta^V_H$ (or $\Delta^M_H$) increases (decreases) sharply in $v$, but decreases (increases) with respect to $\theta_H$. The reason is that, under the assumed parametric assumptions, the high-type agent’s incentive constraints loosen as $v$ increases, leading to a higher sale price for shareholders. On the other hand, the high-type manager’s expected profits from owning the firm increase with $\theta_H$.

(Insert Figure 5 here)

For the maintained parameterization, Figure 6 plots the change in expected shareholder value due to informed insider trading, i.e., $\Delta^V = q\Delta^V_H + (1-q)\Delta^V_L$ with respect to $v$ and $\theta_H$, for $q = 0.5$. Notice that for an open set of parameters $\Delta^V > 0$, so that ex ante shareholder value increases with insider trading. Moreover, under the assumed parameterization, $\Delta^V$ is increasing in $v$ and in $\theta_H$. Intuitively, as $v$ increases (or intrinsic liquidity decreases), the high-type agent’s incentive constraints loosen, and shareholders can extract a higher sale price. On the other hand, as $v$ increases (or intrinsic liquidity decreases), the low-type agent’s expected trading profits fall and so does shareholders’ ability to extract these profits.
through the compensation contract. In Figure 6, the former effect dominates the latter effect. Figure 6 also illustrates that firms having greater growth options are more likely to benefit from insider trading, other things held fixed.

(Insert Figure 6 here)

Finally, we examine the ‘pure’ efficiency effects of insider trading. Keeping aside the re-distribution issues from liquidity traders to shareholders and managers, insider trading can potentially have two conflicting effects on efficiency relative to the constrained efficient arrangement without insider trading—an improvement in informational efficiency of stock prices but an increase in production inefficiency or distortions. But depending on the relative values of \( \hat{\gamma}_L \) and \( \gamma'_L \), the information efficiency effect and the productive efficiency effect could well complement each other (for example, under conditions of Proposition 3, \( \hat{\gamma}_L > \gamma'_L \)). There is an improvement in price discovery since the ex ante uncertainty regarding the liquidating equity value is reduced, conditional on the equilibrium stock price \( P \). Straightforward calculations from Theorem 2 show that

\[
\text{Var}(\tilde{\Pi}_L) - \text{Var}(\tilde{\Pi}_L \mid P_L) = \frac{(1 - \hat{\gamma}_L)\sigma^2}{2}
\]  

(24)

Ex-ante, the gain from informational efficiency is therefore \( \Delta^I \equiv (1 - q) \frac{(1 - \hat{\gamma}_L)\sigma^2}{2} \). The productive effects of insider trading are \( \Delta^s \). Therefore, the total efficiency impact of informed insider trading is given by \( \Delta^S \equiv \Delta^I + \Delta^s \). Straightforward substitution shows that

\[
\Delta^S = (1 - q) \left[ \frac{(1 - \hat{\gamma}_L)\sigma^2}{2} + \frac{\theta_L^2}{\beta} (\hat{\gamma}_L - \gamma'_L) \right]
\]  

(25)

**Proposition 6** Suppose that prior beliefs on \( \theta \) are symmetric (\( q = \frac{1}{2} \)). Then, \( \Delta^S \) is increasing in \( \sigma^2 \) under conditions of Proposition 3.

### 7 Summary and Conclusions

Insider trading and its regulation continue to generate intense interest. While proponents of regulation argue that insider trading exploits uninformed investors, reduces market liquidity, and vitiates insiders’ incentives for generating long-term shareholder value, opponents of
regulation contend that informed trading improves the informational efficiency of securities markets, provides monetary incentives for managers to enhance shareholder value, and is an efficient method for compensating managers and large shareholders. To analyze these conflicting effects of insider trading, we develop a framework that not only places insider trading in the context of the wider agency problem between managers and shareholders — where insiders’ private information on stock value is part of a broader pattern of asymmetric information — but also incorporates the interaction between security market microstructure and firm’s information and contracting environment.

We find that insider trading can lead simultaneously to production distortions (or vitiation of managerial incentives) and higher ex ante shareholder value, in contrast to the literature that views insider trading as either unambiguously worsening the managerial agency problem or unambiguously improving managerial incentives for value-creation. Moreover, we point out that shareholders effectively use the design of the manager’s compensation contract to extract profits from liquidity traders, something that is rarely highlighted in the literature that focuses on the redistribution of wealth from uninformed traders to informed insiders.

We also assess the role of firm-specific characteristics — such as the production technology and cash flow risk — and security market characteristics — such as the variance of the liquidity trading in the firm’s stock — on the manager’s incentives and compensation and on the informational efficiency of stock prices. Specifically, we derive refutable predictions regarding the effect of cash flow risk and the variance trading on equity-based compensation that are novel to the executive compensation literature. Moreover, we find that equilibrium market liquidity will depend on the firm’s growth options. Finally, we link the net welfare impact of informed insider trading to the firm’s growth options, cash flow risk, and the variance of liquidity trading.
Appendix

Proof of Theorem 1

We start by conjecturing that the individual rationality constraint with respect to the high-productivity type is not binding and characterize the solution. We then confirm that this is in fact the case. We also initially conjecture that \( \gamma_j < 1 \), for \( j = L, H \) (i.e., the firm is not sold to the manager in either state), and then examine whether there is a corner solution. Now, let \( \lambda_L \) and \( \omega_L \) and \( \omega_H \) be the respective non-negative Kuhn-Tucker multipliers on the (6) and (7) constraints, respectively. Put, \( \Pi'_i(\phi_i) \equiv [\theta_i e_{ij}(\phi_i) - t_j] \), \( i = L, H \). Let \( \Pi'_i(\phi_i) = \Pi'_H(\phi_i) \). Then, with judicious substitutions, we can write the Lagrangian as:

\[
\mathcal{L} = q (1 - \gamma_H) \Pi'_H(\phi_H) + (1 - q) (1 - \gamma_L) \Pi'_L(\phi_L) + \lambda_L \left\{ t_L + \gamma_L \Pi'_L(\phi_L) - \frac{\beta(e_L(\phi_L))^2}{2} \right\} - 0 \]

\[
+ \omega_L \left\{ t_L + \gamma_L \Pi'_L(\phi_L) - \frac{\beta(e_L(\phi_L))^2}{2} \right\} - \left\{ t_H + \gamma_H \Pi'_H(\phi_H) - \frac{\beta(e_L(\phi_H))^2}{2} \right\} \]

\[
+ \omega_H \left\{ t_H + \gamma_H \Pi'_H(\phi_H) - \frac{\beta(e_H(\phi_H))^2}{2} \right\} - \left\{ t_L + \gamma_L \Pi'_L(\phi_L) - \frac{\beta(e_H(\phi_L))^2}{2} \right\} \]  

where, \( e_{LH}(\phi_H) = \frac{\gamma_H \theta_H}{\beta} \), \( e_{HL}(\phi_L) = \frac{\gamma_L \theta_L}{\beta} \), \( e_H(\phi_H) = \frac{\gamma_H \theta_H}{\beta} \), \( e_L(\phi_L) = \frac{\gamma_L \theta_L}{\beta} \). Then, the adjoint conditions for \( t_L \) and \( t_H \) are, respectively,

\[
(1 - \gamma_L)[-(1 - q) + \lambda_L + \omega_L - \omega_H] = 0, \quad (29)
\]

\[
(1 - \gamma_H)[-q - \omega_L + \omega_H] = 0. \quad (30)
\]

It must then be the case that either \( \gamma_H = 1 \), or \(-q - \omega_L + \omega_H\) = 0. We will conjecture that \( \gamma_H = 1 \) i.e., the firm is sold to the manager in the high productivity state, and the high-productivity manager will implement the first-best effort level \( e^*_H = \frac{\theta_H}{\beta} \). This implies that \( t_H < 0 \) and represents the price the high-type manager pays to acquire the firm. The net payoff to the high type manager is \( t_H + \frac{\theta_H}{2\beta} \). Consistent with standard adverse selection models, we also conjecture that the individual rationality constraint for the low-type manager is binding. i.e.,

\[
\{ t_L + \gamma_L \Pi'_L(\phi_L) - \frac{\beta(e_L(\phi_L))^2}{2} \} = 0 \]

\[
\Leftrightarrow \tilde{t}_L = -\frac{\gamma_L^2 \theta_L^2}{(1 - \gamma_L)^2 \beta}.
\]

With \( \gamma_H = 1 \), from (30) it must be the case that \(-q - \omega_L + \omega_H = 0\). We know that \( \omega_H \geq 0 \). Let us suppose that \( \omega_H > 0 \); that is, the associated incentive compatibility constraint, (28), is binding, or the high-type manager is indifferent between reporting his type as low-type and reporting his type truthfully. If the high-type manager misreports his type, his expected payoff will
be

\[ t_L + \gamma_L \bar{\Pi}'_{HL}(\phi_L) - \frac{\beta(e_{HL}(\phi_L))^2}{2} \]

\[ = t_L(1 - \gamma_L) + \frac{\gamma_1^2 \theta_H^2}{2\beta} \]

\[ = \frac{\gamma_L^2 (\theta_H^2 - \theta_L^2)}{2\beta}. \]

On the other hand, by reporting the truth, his expected utility will be \( t_H + \frac{\theta_H^2}{2\beta} \). Therefore, a binding incentive compatibility constraint implies

\[ t_H + \frac{\theta_H^2}{2\beta} = \frac{\gamma_L^2 (\theta_H^2 - \theta_L^2)}{2\beta}, \text{ or} \]

\[ \bar{t}_H = \frac{-(1 - \gamma_L^2) \theta_H^2 + \gamma_L^2 \theta_L^2}{2\beta}. \]

However, substituting the values for \( t_H \) and \( t_L \) into the constraint (27) for \( \gamma_L < 1 \) to see if this constraint is satisfied, we should have

\[ 0 \geq t_H + \frac{\theta_L^2}{2\beta} \]

\[ > \frac{-(1 - \gamma_L^2) \theta_H^2 - \gamma_L^2 \theta_L^2}{2\beta} + \frac{\theta_L^2}{2\beta} \]

\[ = \frac{-(1 - \gamma_L^2) (\theta_H^2 - \theta_L^2)}{2\beta}. \]

which establishes that the constraint (27) will not be binding, or \( \omega_L = 0 \). The equations (29) and (30) then yield

\[ (1 - \gamma_L)[-(1 - q) + \lambda_L - \omega_H] = 0, \]

\[ (1 - \gamma_H)[q - \omega_H] = 0. \]

We can see that with \( \gamma_L < 1 \) and \( \gamma_H = 1 \), setting \( \omega_H = q \) and \( \lambda_L = 1 \) satisfies these two conditions. Moreover, using the envelope condition, \( (\gamma_L \frac{\partial \bar{\Pi}'_{HL}(\phi_L)}{\partial e_{HL}} - \beta e_{HL}) = 0 \), the adjoint condition for \( \gamma_L \) is,

\[ \frac{\partial \mathcal{L}}{\partial \gamma_L} = -(1 - q)\bar{\Pi}'_L(\phi_L) - \omega_H\bar{\Pi}'_{HL}(\phi_L) + \lambda_L\bar{\Pi}'_L(\phi_L) \]

\[ + (1 - q)(1 - \gamma_L) \frac{\partial \bar{\Pi}'_L(\phi_L)}{\partial e_L} \frac{\partial e_L}{\partial \gamma_L} = 0. \]
Making appropriate substitutions from above, it can be shown with some algebra that

\[ \gamma'_L = \frac{(1-q)\theta_L^2}{q(\theta_H^2 - \theta_L^2)} + (1-q)\theta_L^2 < 1. \]

We next show that there exists some \( \epsilon' > 0 \) such that if \( \theta_H - \theta_L \geq \epsilon' \), then implementing the mechanism \( \delta'_i = \langle \eta_i, \bar{t}_i, \bar{e}_i; i = L, H \rangle \) maximizes shareholder value. On the other hand, if \( \theta_H - \theta_L < \epsilon' \), offering the pooling contract, where the firm is sold to both agent-types, i.e., \( \gamma'_j = 1 \) at the common price \( -t'_j = \frac{\theta_H^2}{\beta} \), \( j = L, H \), is preferable. Define \( \zeta = \theta_H - \theta_L \); or \( \theta_H = \theta_L + \zeta \). Notice that for a given \( (q, \theta_L) \), shareholder value under \( \delta' \) is

\[
SV(\zeta; \delta') = q(-t'_H(1) + (1-q)(1-\gamma'_L)[\theta_L e_L - \bar{t}_H])
\]

\[
= q \left[ \frac{(1-\gamma'_L)(\theta_L + \zeta)^2 + \gamma'_L \theta_L^2}{2\beta} \right]
+ (1-q) \left( 1 - \frac{(1-q)\theta_L^2}{q(\theta_L + \zeta)^2 - \theta_L^2} + (1-q)\theta_L^2 \right)
\]

and

\[ SV(\text{Pooling}) = \frac{\theta_L^2}{\beta}. \]

It is easy to show that \( SV(\zeta; \delta') \) is increasing in \( \zeta \), and

\[ SV(\zeta; \delta') < SV(\text{Pooling}), \text{ for } \zeta \rightarrow 0, \text{ and } \]

\[ SV(\zeta; \delta') > SV(\text{Pooling}), \text{ for some } \zeta > 0. \]

Thus, the intermediate value theorem guarantees the existence of \( \epsilon' > 0 \) such that when \( \zeta = \theta_H - \theta_L < \epsilon' \), a pooling contract is preferred; otherwise the contract \( \delta' \) is preferred. \( \blacksquare \)

**Proof of Theorem 2:**

Since \( X(\tilde{V}_j; \omega_j, P) = A_j + B_j \tilde{V}_j \) and \( P(y; \omega_j) = C_j + D_j y, j = L, H \), the conditional expectation of \( \tilde{V}_j \) is linear in \( y \). Using the fact that \( \tilde{V}_j = (1-\gamma_j) \left( \theta_j e_j + \tilde{\xi} - t_j \right) \), \( \sqrt{\text{Var}(\tilde{V}_j \mid \omega_j)} = (1-\gamma_j)\sigma_{\xi} \), and \( \sqrt{\text{Var}(y \mid \omega_j)} = \sqrt{B_j^2(1-\gamma_i)\sigma_{\xi}^2 + \sigma_y^2} \), we have,

\[ E \left[ \tilde{V}_j \mid y; \omega_j \right] = E \left[ \tilde{V}_j \mid \omega_j \right] + \frac{\text{Corr} \left( \tilde{V}_j, y \mid \omega_j; X \right) (1-\gamma_j)\sigma_{\xi}}{\sqrt{B_j^2(1-\gamma_i)\sigma_{\xi}^2 + \sigma_y^2}} [y - E \left[ y \mid \omega_j \right]] \]

\[ = V_j + \frac{B_j(1-\gamma_j)^2\sigma_{\xi}^2}{B_j^2(1-\gamma_i)^2\sigma_{\xi}^2 + \sigma_y^2} [y - (A_j + B_j \tilde{V}_j)] \quad (31) \]

where, in (31), we have substituted the expression for the conditional correlation from the text [cf.(12)], and used the fact that \( E \left[ y \mid \omega_j \right] = (A_j + B_j \tilde{V}_j) \), under the assumed trading strategy \( X \) and the assumption of zero mean noise trading. Since \( P(y; \omega_j) = E \left[ \tilde{V}_j \mid y; \omega_j \right] \), rearranging
terms in (31) and matching yields,
\[ C_j = \frac{\sigma_\eta^2 V_j - A_j B_j (1 - \gamma_j)^2 \sigma_\xi^2}{B_j^2 (1 - \gamma_j)^2 \sigma_\xi^2 + \sigma_\eta^2}; \quad D_j = \frac{B_j (1 - \gamma_j)^2 \sigma_\xi^2}{B_j^2 (1 - \gamma_j)^2 \sigma_\xi^2 + \sigma_\eta^2} \]  (32)

The manager’s trading strategy \( X(\tilde{V}_j; \omega, J) \) is determined by
\[ \text{Max}_x \left[ x \left( \tilde{V}_j - (C_j + D_j x) \right) \right]. \]

The corresponding first order condition yields
\[ \hat{A}_j = -\frac{\dot{C}_j}{2\dot{D}_j}, \quad \hat{B}_j = \frac{1}{2\dot{D}_j}. \]  (33)

Using (33) and (32), we therefore get, \( \hat{B}_j = \frac{\sigma_\eta}{(1-\gamma_j)\sigma_\xi} \) and \( \hat{D}_j = \frac{(1-\gamma_j)\sigma_\xi}{2\sigma_\eta} \). Plugging these values in the expression for \( C_j \) in (32) and using (33) again then yields, \( \hat{A}_j = -V_j \frac{\sigma_\eta}{(1-\gamma_j)\sigma_\xi} \) and \( \hat{C}_j = V_j \).

**Proof of Theorem 3:**

We proceed exactly as we did in the proof of Proposition 1, conjecturing that only the individual rationality constraint with respect to the high-productivity type is not binding and that \( \gamma_j < 1 \), for \( j = L, H \) (i.e., the firm is not sold to the manager in either state). As before, let \( \lambda_L \) and \( \omega_L \) and \( \omega_H \) be the respective non-negative Kuhn-Tucker multipliers on the (6) and (7) constraints, respectively. Put, \( \hat{\Pi}_{ij}(\phi_i) \equiv [\theta_i e_{ij}(\phi_i) - \bar{t}_j] \), \( i = L, H \). Let \( \hat{\Pi}_i(\phi_i) = \hat{\Pi}_{ii}(\phi_i) \). Then, with \( v = \frac{\sigma_\xi}{\sigma_\eta} \) and judicious substitutions, the Lagrangian is:

\[
\mathcal{L} = q (1 - \gamma_H) \hat{\Pi}_H(\phi_H) + (1 - q) (1 - \gamma_L) \hat{\Pi}_L(\phi_L) + \left[ t_L + \gamma_L \hat{\Pi}_L(\phi_L) - \frac{\beta(\hat{e}_L(\phi_L))^2}{2} \right] + \lambda_L \left[ \left\{ \bar{t}_L + \gamma_L \hat{\Pi}_L(\phi_L) - \frac{(1 - \gamma_L)\sigma_\eta\sigma_\xi}{2} - \frac{\beta(\hat{e}_L(\phi_L))^2}{2} \right\} \right] \tag{34}
\]

\[
+ \omega_L \sum_{i=L,H} \left\{ \bar{t}_L + \gamma_i \hat{\Pi}_{iL}(\phi_L) - \frac{(1 - \gamma_i)\sigma_\eta\sigma_\xi}{2} - \frac{\beta(\hat{e}_{iL}(\phi_L))^2}{2} \right\} - \left[ \bar{t}_L + \gamma_L \hat{\Pi}_{HL}(\phi_L) - \frac{(1 - \gamma_L)\sigma_\eta\sigma_\xi}{2} - \frac{\beta(\hat{e}_{HL}(\phi_L))^2}{2} \right] \tag{35}
\]

where (from the text)

\[
\hat{e}_j(\phi_j) = \frac{\gamma_j \theta_j}{\beta}, \quad \hat{e}_{ij}(\phi_j) = \frac{\gamma_j \theta_i [\beta \sigma_\xi - \sigma_\eta (1 - \gamma_j) \theta_i^2]}{\beta [\beta \sigma_\xi - \sigma_\eta (1 - \gamma_j) \theta_i^2]} \] (37)

31
Then, the adjoint conditions for \( \tilde{t}_L \) and \( \tilde{t}_H \) are, respectively,

\[
(1 - \gamma_L)[-(1 - q) + \lambda_L + \omega_L - \omega_H)] = 0, \tag{38}
\]

\[
(1 - \gamma_H)[q - \omega_H] = 0. \tag{39}
\]

It must then be the case that either \( \gamma_H = 1 \), or \([−q − \omega_L + \omega_H]\) = 0. We will conjecture that \( \gamma_H = 1 \) i.e., the firm is sold to the manager in the high productivity state, and the high-productivity manager will implement the first-best effort level \( e^*_H = \frac{\theta_H}{\beta} \). This implies that \( \tilde{t}_H < 0 \) and represents the price the high-type manager pays to acquire the firm. The net payoff to the high type manager is \( \tilde{t}_H + \frac{\delta_H}{2} \). The binding individual rationality constraint for the low-type manager is

\[
\left\{ \tilde{t}_L + \gamma_L \tilde{\Pi}_L(\phi_L) - \frac{\beta(e_L(\phi_L))^2}{2} \right\} + \frac{(1 - \gamma_L)\sigma_\eta\sigma_\xi}{2} = 0
\]

\[
\Leftrightarrow \tilde{t}_L = -\frac{\gamma_L^2\theta_L^2 + (1 - \gamma_L)\beta\sigma_\eta\sigma_\xi}{(1 - \gamma_L)2\beta}.
\]

With \( \gamma_H = 1 \), from (30) it must be the case that \(-q - \omega_L + \omega_H = 0 \). We know that \( \omega_H \geq 0 \). Let us suppose that \( \omega_H > 0 \); that is, the associated incentive compatibility constraint, (28), is binding, or the high-type manager is indifferent between reporting his type as low-type and reporting his type truthfully. If the high-type manager misreports his type, his expected payoff will be

\[
U^M_{HL} = \tilde{t}_L + \gamma_L \tilde{\Pi}_{HL}(\phi_L) - \frac{\beta(e_{HL}(\phi_L))^2}{2}
\]

\[
+ \frac{(1 - \gamma_L)}{2v} \left[ (\theta_H\tilde{e}_{HL}(\phi_L) - \theta_L\tilde{e}_L(\phi_L))^2 + \sigma_\xi^2 \right],
\]

where the effort choices \( \tilde{e}_{HL}(\phi_L) \) and \( \tilde{e}_L(\phi_L) \) are given by the expressions in (37). On the other hand, by reporting the truth, his expected utility will be \( \tilde{t}_H + \frac{\delta_H}{2\beta} \). Therefore, a binding incentive compatibility constraint (36) implies \( \tilde{t}_H + \frac{\delta_H}{2\beta} = U^M_{HL} \). We can solving this equation for \( \tilde{t}_H \) after making appropriate substitutions (derivation omitted). Note that \( \tilde{t}_H < 0 \). Moreover, substituting the values for \( \tilde{t}_H \) and \( \tilde{t}_L \) into the constraint (35), it can be shown that the constraint will be non-binding, or \( \omega_L = 0 \).

We can see that with \( \gamma_L < 1 \) and \( \gamma_H = 1 \), setting \( \omega_H = q \) and \( \lambda_L = 1 \) satisfies the equations (38) and (39). Using the envelope condition as in the proof of Proposition 1, \( \gamma_L \frac{\partial \tilde{\Pi}_{HL}(\phi_H)}{\partial e_{HL}} - \beta e_{HL} = 0 \), the adjoint condition for \( \gamma_L \) is:

\[
F[\gamma_L] = -(1 - q) \tilde{\Pi}_L(\phi_L) + (1 - q)(1 - \gamma_L) \frac{\partial \tilde{\Pi}_L(\phi_L)}{\partial e_L} \frac{\partial e_L}{\partial \gamma_L} + \lambda_L \tilde{\Pi}_L(\phi_L) \tag{40}
\]

\[
- \omega_H \left( \frac{\partial \tilde{\Pi}_{HL}(\phi_L)}{\partial \gamma_L} \left[ \frac{(1 - \gamma_L)}{2v} \left[ (\theta_H\tilde{e}_{HL}(\phi_L) - \theta_L\tilde{e}_L(\phi_L))^2 + \sigma_\xi^2 \right] \right] \right) = 0.
\]
It can be shown that
\[ F(0) \propto \left[ 2(1 - q)\theta_L^2 + q\beta v\sigma_n^2 \right] > 0, \]
and
\[ F(1) \propto -q \left[ (2\beta v + \theta_H^2 - \theta_L^2)(\theta_H^2 - \theta_L^2) - \beta^2 v^2\sigma_n^2 \right]. \]

With some algebra it follows that \( F(1) < 0 \) provided
\[ \sigma_n < \frac{(2\beta v + \theta_H^2 - \theta_L^2)(\theta_H^2 - \theta_L^2)}{\beta^2 v^2}. \]

Under this condition, \( \hat{\gamma}_L \) is interior. \( \blacksquare \)

**Proof of Proposition 2**

From (40), and using the implicit function theorem, it is straightforward to show that \( \frac{\partial \hat{v}_L}{\partial \nu} < 0 \) if
\[ \frac{\partial FOC}{\partial \nu} = \frac{1}{2}q \left[ -\gamma_L (\theta_H^2 - \theta_L^2)^2 \left[ (2 + (-3 + \gamma_L)\gamma_L)\theta_H^2 + (-2 + 3\gamma_L)\beta v \right] \right] \left[ (-1 + \gamma_L)\theta_H^2 + \beta v \right]^3 + \sigma_n^2 < 0 \quad (41) \]

Because \( \beta v > \theta_H^2 \) (maintained assumption), the term \( \left[ (-1 + \gamma_L)\theta_H^2 + \beta v \right]^3 < 0. \) Thus, a necessary condition for (41) will be true is that
\[ (2 + (-3 + \gamma_L)\gamma_L)\theta_H^2 + (-2 + 3\gamma_L)\beta v > 0, \]
which holds if \( \gamma_L > \frac{2}{3}, \) or if \( F\left(\frac{2}{3}\right) > 0. \) Using the expression for \( F(\cdot) \) from (40), it can be shown that
\[ F\left(\frac{2}{3}\right) = \frac{-2\theta_H^2 + 3\theta_L^2 + \frac{2\sigma_n^2}{\beta^2 v^2}}{6\beta} + \frac{\nu\sigma_n^2}{4}. \]

Simplifying, \( F\left(\frac{2}{3}\right) > 0 \) if
\[ (\theta_H^2 - 3\beta v)^2 \left[ 3\beta v\sigma_n^2 - 4\theta_H^2 + 6\theta_L^2 \right] + 4\theta_H^2(\theta_H^2 - \theta_L^2) > 0. \]

Given our maintained assumption that \( \beta v > \theta_H^2, \) this inequality will hold if \( \sigma_n^2 \geq \sigma^* = 4/3. \) Referring back to the expression (41) above this condition establishes that the term
\[ -\gamma_L (\theta_H^2 - \theta_L^2)^2 \left[ (2 + (-3 + \gamma_L)\gamma_L)\theta_H^2 + (-2 + 3\gamma_L)\beta v \right] \left[ (-1 + \gamma_L)\theta_H^2 + \beta v \right]^3 < 0. \]

Note that this term is sufficiently negative to more than offset \( \sigma_n^2 \) for the inequality (41) to be met. For any given \( \sigma_n^2 \geq \sigma^* = 4/3, \) we can always choose \( \theta_H - \theta_L \) to be large enough to meet this condition. \( \blacksquare \)

**Proof of Proposition 3**

Referring to Theorem 1, the optimal equity compensation to the low-type manager \( \gamma_L' \) is given
by
\[ \gamma'_L = \frac{(1 - q)\theta^2_L}{q(\theta^2_H - \theta^2_L) + (1 - q)\theta^2_L}. \]

When \( q = \frac{1}{2} \), \( \gamma'_L = \frac{\theta^2_H}{\theta^2_H} \). We know that under conditions of Proposition 2, \( \hat{\gamma}_L > \frac{2}{3} \) (refer to the proof of Proposition 2 above). So, as long as \( \theta_H - \theta_L \) is large enough such that \( \frac{\theta^2_H}{\theta^2_H} \leq \frac{2}{3} \), \( \gamma'_L < \hat{\gamma}_L \).
References


<table>
<thead>
<tr>
<th>$\tau = 0$</th>
<th>$\tau = 1$</th>
<th>$\tau = 2$</th>
<th>$\tau = 3$</th>
<th>$\tau = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outsiders offer contract to the manager: $\delta$</td>
<td>The manager privately observes $\theta \in {\theta_L, \theta_H}$</td>
<td>The manager supplies effort $e$</td>
<td>The manager observes $\tilde{\pi}$ and trades</td>
<td>Outsiders observe $\tilde{\pi}$ Manager paid</td>
</tr>
</tbody>
</table>

Figure 1: Sequence of events
Figure 2
This figure presents a graph of the optimal equity compensation to the low-type manager, $\gamma_L$, as a function of the signal-to-noise ratio, $v$, and \textit{a priori} beliefs of outsiders that the manager is high-type, $q$. For this graph, the parameter values are set at $\theta_L = 1$, $\theta_H = 3$, $\beta = 2$, $\sigma_\eta = 0.1$.

Figure 3
This figure presents a graph of the optimal equity compensation to the low-type manager, $\gamma_L$, as a function of the signal-to-noise ratio, $v$, and high-type manager’s productivity, $\theta_H$. For this graph, the parameter values are set at $\theta_L = 1$, $q = 0.5$, $\beta = 2$, $\sigma_\eta = 0.1$.  
Figure 4
This figure presents a graph of the change in the firm’s equity value due to insider trading by the low-type manager as a function of the signal-to-noise ratio, $v$, and high-type manager’s productivity, $\theta_H$. For this graph, the parameter values are set at $\theta_L=1$, $q=0.5$, $\beta=2$, $\sigma_n=0.1$.

Figure 5
This figure presents a graph of change in the firm’s equity value with respect to the high-type manager as a function of the signal-to-noise ratio, $v$, and high-type manager’s productivity, $\theta_H$. For this graph, the parameter values are set at $\theta_L=1$, $q=0.5$, $\beta=2$, $\sigma_n=0.1$. 
Figure 6
This figure presents a graph of the change in expected shareholder value as a function of the signal-to-noise ratio, $\nu$, and high-type manager’s productivity, $\theta_H$. For this graph, the parameter values are set at $\theta_l = 1$, $q = 0.5$, $\beta = 2$, $\sigma_\eta = 0.1$. 