Three-way ANOVA

Divide and conquer

General Guidelines for Dealing with a 3-way ANOVA

- ABC is significant:
  - Do not interpret the main effects or the 2-way interactions.
  - Divide the 3-way analysis into 2-way analyses. For example, you may conduct a 2-way analysis (AB) at each level of C.
  - Follow up the two-way analyses and interpret them.
  - Of course, you could repeat the procedure for, say, the AC interaction at different levels of B.

Three-way ANOVA

- ABC is NOT significant, but all of the 2-way interactions (AB, AC, & BC) are significant:
  - You may follow up and interpret the two way interactions, but not the main effects.
  - Plot the AB interaction ignoring C to interpret it. You could also compare the means on the AB-table using post-hoc (or planned) comparisons.
  - You may repeat the procedure for the AC and BC interactions.

Three-way ANOVA

- ABC is not significant
- AB is not significant
- AC is not significant
- BC is significant
- A is significant

You can follow up interpret the BC interaction and the A main effect.

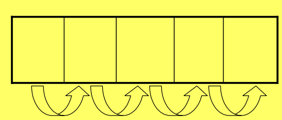
Three-way ANOVA

- ABC is not significant
- AB is not significant
- AC is not significant
- BC is not significant
- A is significant
- B is significant
- C is not significant

You can follow up and interpret the A and B main effects.

Repeated Measures Designs

- Simple repeated Measures Design: Uses the same subjects in all conditions.



Simple Repeated Measures Design

- The observations are not independent over conditions.
- It is an extension of the correlated (or paired) t-test.
- This analysis is also called a Within Design

SAS Setup for a Simple Repeated Measures Design

```
data repeated;
input ss y1-y3;
cards;
1 22 24 19
10 18 17 23
;
proc print; run;
proc means;run;
proc glm;
model y1-y3=  / nouni;
repeated repfact 3; run;
```

Means and Standard Dev.

Variable	N	Mean	Std Dev	Minimum	Maximum
ss	10	5.500	3.0276	1.00	10.00
y1	10	17.500	2.798	11.00	22.00
y2	10	18.900	2.685	15.00	24.00
y3	10	22.400	2.221	19.00	26.00

Multivariate Tests

Manova Test Criteria and Exact F Statistics for the Hypothesis of no repfact Effect
H = Type III SSCP Matrix for repfact
E = Error SSCP Matrix

S=1 M=0 N=3

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.421	5.49	2	8	0.0315
Pillai's Trace	0.578	5.49	2	8	0.0315
Hotelling-Lawley Trace	1.373	5.49	2	8	0.0315
Roy's Greatest Root	1.373	5.49	2	8	0.0315

The circularity assumption is not needed for the multivariate tests to be valid.

Source	DF	Type III SS	Mean Square	F Value	Pr > F	Adj Pr > F G - G	Adj Pr > F H - F
repfact	2	127.40	63.70	8.00	0.0033	0.0052	0.0033
Error(repfact)	18	143.26	7.95				

Circularity Assumption is Met when epsilon is one

Greenhouse-Geisser Epsilon	0.8712
Huynh-Feldt Epsilon	1.0626

Epsilon

- Epsilon is a (sample) measure of how well the circularity assumption has been met. It ranges from

  $1/df_{rep} \leq \varepsilon \leq 1$.

  In our previous example, the range is

  $1/2 \leq \varepsilon \leq 1$.

  When epsilon is one, the circularity assumption has been met. If epsilon is $1/df_{rep}$, circularity has been violated in a bad way.

More on Epsilon

- If epsilon is not one, the usual univariate F-test must be adjusted.
- When considering the univariate F-test we have three possibilities for adjusting the degrees of freedom:
  - Usual
  - Conservative
  - Adjusted

Adjusting the df's in the Univariate F-tests

- Usual F-test: use the usual dfs
  - a-1=2; (a-1)(s-1)=2*9=18;
  - df's=2,18
- Conservative F-test (assume that ε=.5)
  - Then the df's are 1 and 9.
  - $F_{.05,2,18}$=3.55 $F_{.05,1,9}$=5.12
- Epsilon corrected F-tests
  - Compute the sample epsilon and multiplied the dfs by this estimate.

Which Test is Best?

- Multivariate test makes less assumptions but it is not always more powerful.
- The e-adjusted test is a good alternative and can be more powerful than the multivariate tests.
- Ordinarily we look at both tests. If both of them are significant, then report the one.
- Never rely on the usual univariate F-test.

Two-way ANOVA
One Within and One Between

- Lets say B is the within factor
- And that A is the between factor

	B_1	B_2	B_3	B_4
A_1	s_1 ... s_{n1}			
A_2	s_{n1+1} ... s_{n2}			

F-test for the Groups by trials

Source	df	F-test
A	(a-1)	
S/A (error for A)	(n-1)a	
B	(b-1)	
B*A	(b-1)(a-1)	
B*S/A (error for B and B*A)	(b-1)(s-1)a	

Weight Training Data

```
data wtsmiss;
   input subj program$ s1 s2 s3 s4 s5 s6 s7;
   datalines;
1    CONT   85   85   86   85   87   86   87
2    CONT   80   79   79   78   78   79   78
3    CONT   78   77   77   77   76   76   77
4    CONT   84   84   85   84   83   84   85
5    CONT   80   81   80   80   79   79   80
6    CONT   76   78   77   78   78   77   74
7    CONT   79   79   80   79   80   79   81
8    CONT   76   76   76   75   75   74   74
9    CONT   77   78   78   80   80   81   80
10   CONT   79   79   79   79   77   78   79
```

SAS Setup for Groups by Trials

```
/* Test of homogeneity of var-cov matrices for the Multivariate tests */
proc discrim pool=test;
class program;
Var s1-s7;
run;
/* Obtainin the corrected univariate and multivariate tests */
Proc glm data=wtsmiss;
class program;
model s1-s7= program / nouni;
repeated time 7 /printe summary;
means program;
run;
```

Looking at the "Programs"

```
/* Running the simple main effect tests on the programs*/
proc glm data=wtsmiss;
class program;
model s1-s7= program;
means program /tukey;
run;
```

Assessing the Homogeneity Assumption

Within Covariance Matrix Information

program	Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
CONT	7	0.40441
RI	7	2.06269
WI	7	-0.33541
Pooled	7	1.86089

The Chi-square Test

Chi-Square	DF	Pr > ChiSq
55.863503	56	0.4800

Interaction Effect

Manova Test for the Hypothesis of no Time*program Effect
H = Type III SSCP Matrix for time*program
E = Error SSCP Matrix

S=2 M=1.5 N=23.5

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.7316	1.38	12	98	0.1880
Pillai's Trace	0.2818	1.37	12	100	0.1943
Hotelling-Lawley Trace	0.3481	1.40	12	73.199	0.1847
Roy's Greatest Root	0.2825	2.35	6	50	0.0442

Time Effect

Manova Test Criteria and Exact F Statistics for the Hypothesis of no time Effect
H = Type III SSCP Matrix for time
E = Error SSCP Matrix

S=1 M=2 N=23.5

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.5584	6.46	6	49	<.0001
Pillai's Trace	0.4415	6.46	6	49	<.0001
Hotelling-Lawley Trace	0.7905	6.46	6	49	<.0001
Roy's Greatest Root	0.7905	6.46	6	49	<.0001

Epsilon

Greenhouse-Geisser Epsilon	0.4233
Huynh-Feldt Epsilon	0.4624

Test of Circularity for the Repeated Factor

Sphericity Tests

Variables	DF	Mauchly's Criterion	Chi-Square	Pr > ChiSq
Transformed Variates	20	0.0009992	357.70745	<.0001
Orthogonal Components	20	0.0403737	166.18471	<.0001

Contrast Variable: time_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	0.3792618	0.3792618	0.14	0.7093
program	2	10.7185464	5.3592732	1.98	0.1474
Error	54	145.8428571	2.7007937		

Univariate Analysis: The Between Effect

Source	DF	Type III SS	Mean Square	F Value	Pr > F
program	2	419.435	209.717631	3.07	0.0548
Error	54	3694.690	68.420186		

Univariate Tests: Time & Interaction

Source	DF	Type III SS	Mean Square	F Value	Pr > F	Adj Pr > F G - G	Adj Pr > F H - F
time	6	53.3542	8.8923	7.43	.0001	.0003	.0002
time*program	12	43.0002	3.5833	2.99	.0005	.0130	.0104
Error(time)	324	387.7867	1.1968				

Program Effect

Source	DF	Type III SS	Mean Square	F Value	Pr > F
program	2	419.435262	209.717631	3.07	0.0548
Error	54	3694.690051	68.420186		

Contrast Variable: time_1 vs. Time_7

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	69.4671822	69.4671	14.92	0.0003
program	2	51.9285401	25.9642	5.58	0.0063
Error	54	251.4398810	4.6562		

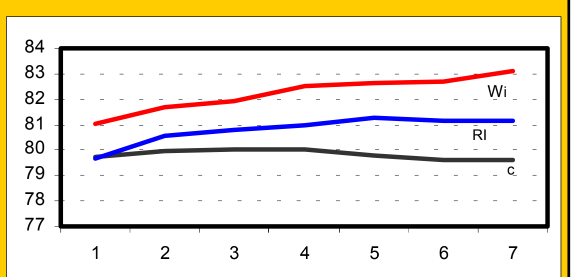
Contrast Variable: time_2 vs. Time_7

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	16.8194570	16.8194570	3.50	0.0669
program	2	32.4047306	16.2023653	3.37	0.0418
Error	54	259.6303571	4.8079696		

Contrast Variable: time_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	7.5978248	7.5978248	2.54	0.1168
program	2	26.0331767	13.0165883	4.35	0.0177
Error	54	161.4755952	2.9902888		

Plot of Means



Repeated Factor

		T1	T2	T3	T4	T5	T6	T7
Between	Cont							
	RI							
	WI							

A

s/A

B

BA

B*s/A

Simple Main Effects in Repeated Measures Designs

Simple Main Effect with Repeated Factors

- When going across the repeated factor at a level of the between factor: Use the error term for the repeated factor.
- When going across the between factor at a level of the within factor: Pool the between and within error terms.

$$MS_p = \frac{SS_{s/A} + SS_{Bs/A}}{df_{s/A} + df_{Bs/A}}$$