Quantifying uncertainties in AVO forward modeling

Ayon K. Dey, CREWES, University of Calgary, Chandra Rai, EPTG, Amoco Corporation and Carl Sondergeld, EPTG, Amoco Corporation

Summary

AVO is a commonly used tool in hydrocarbon exploration. Previous work has shown that the forward modeled AVO response is sensitive to uncertainties in the input rock properties. This study quantifies the sensitivity to uncertainties in the rock properties. That is, we quantify the uncertainties in AVO forward modeling. The approach taken is to cross–plot the b_0–b_1 parameters from the AVO model and to summarize the dispersion pattern using the standard deviation method. This will define an elliptical region whose properties can be exploited to determine how well AVO will work as a hydrocarbon discriminant. Results from this analysis show that AVO has good success in discriminating between brine saturation and oil/gas saturation. These results also show that it is not as successful in trying to discriminate between different hydrocarbon saturation. Overall, the method provides a means by which a quantitative index can be computed for AVO risk assessment.

Introduction

A common tool used in the exploration of hydrocarbons is Amplitude-Versus-Offset (AVO) analysis. This particular method has been very effective in identifying major natural gas deposits. Sengupta et al., 1997 shows that the AVO response from data is sensitive to uncertainties in various rock properties. Specifically, the work of Sengupta et al., 1997 shows how the uncertainties in compressional-wave velocities (V_p), shear-wave velocities (V_s), and densities (ρ) project themselves as uncertainties in an AVO response. That is to say, these authors quantify how sensitive the AVO response is to uncertainties in the rock properties.

In this report there is a more detailed analysis of the uncertainties in the rock properties and its effect on the AVO response. The rock properties V_p, V_s, and ρ are used in AVO forward modeling and the results are analyzed. The focus of this work is to define confidence regions for various exploration scenarios and to examine how these different regions interact. The approach taken is to create an elliptical area that contains a certain amount of scatter data (parameters b_0 and b_1, in this case) and to examine to what degree the various areas overlap.

Methodology

The method employed in this investigation is to fit a *standard deviation ellipse* to the scatter data. This is done in order to summarize the dispersion in the point pattern and follows the method outlined in Ebdon, 1977.

A standard deviation ellipse is an ellipse centered about the mean center of a data set, with its long axis in the direction of maximum dispersion and the small axis in the direction of minimum dispersion. In order to fit such an ellipse, the following information must be known.

1. The length of the short axis.
2. The length of the long axis.
3. The orientation of the ellipse.

The length of the long axis and the length of the short axis are the values of the data's standard deviation in the x-direction and the y-direction, respectively. Data contained in such an ellipse is interpreted as being within at least *K* standard deviations of the mean center. The step-wise method is as follows.

- Calculate the co-ordinates $\left(\bar{x}, \bar{y}\right)$ of the mean center. Note that summations run from 1 to *n*; where *n* is the total number of data points under consideration. By default, the value of *n* is identical in both the $\bar{x}$ and $\bar{y}$ summations.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{i=n} x_i ,$$
$$\bar{y} = \frac{1}{n}\sum_{i=1}^{i=n} y_i .$$

- Calculate the co-ordinates of the data in the translated system by subtracting the mean from each of the original data co-ordinates.

$$\forall x, x' = x - \bar{x} .$$
$$\forall y, y' = y - \bar{y} .$$

- Calculate the tangent of the angle of rotation, θ. Note that from this point forward, all summations can be assumed to go from i=1 until i=n.

$$\tan(\theta) = \frac{\left(\sum x'^2 - \sum y'^2\right) + \sqrt{\left(\sum x'^2 - \sum y'^2\right)^2 + 4\left(\sum x'y'\right)^2}}{2\sum x'y'}$$

The arctangent of this value will give the rotation, or orientation, angle θ that is between the translated y-axis and the y-axis of the ellipse. The angle is measured clockwise from the translated y-axis. Note that this formulation to calculate the tangent of the orientation angle can give a negative result. If the tangent value happens to become negative, then the correct measure for the orientation angle θ is $\theta = 90^{\circ} - \arctan(-\tan(\theta))$.

- Calculate the standard deviation along the x-axis of the ellipse. This is a measure of the dispersion in the x-direction for the scatter data.

$$\sigma_x = \sqrt{\frac{\sum \left(x'\cos(\theta) - y'\sin(\theta)\right)^2}{n}}.$$

- Calculate the standard deviation along the y-axis of the ellipse. This is a measure of the dispersion in the y-direction for the scatter data.

$$\sigma_y = \sqrt{\frac{\sum \left(x'\sin(\theta) + y'\cos(\theta)\right)^2}{n}}.$$

These formulae will generate a 1-σ ellipse. That is, the data contained in the ellipse will be within one standard deviation of the mean. By introducing a user defined constant *K*, and multiplying the standard deviations in the *x* and *y* directions by it, the algorithm can fit a *K*-σ ellipse. Therefore, ellipses of any size that the user wants can be fit to the data at hand. Following this method will yield the information necessary to plot an ellipse. Data that lies within this ellipse will be within *K* standard deviations of the mean center.

The approach taken to analyze the overlap of ellipses from different reflection interfaces is far less elegant. Rather than develop a method that solves for the points of intersection and then implement some type of numerical integration routine, a *brute force* method is chosen. This method exploits the fact that the regions of interest are ellipses and it is computationally cheap. Quite simply, the method examines each point in the two data sets of interest and determines whether or not the point is within both ellipses. If it is within both ellipses, then it means that the point is within *K* standard deviations of both mean centers and cannot be definitely assigned to one region or the other. The algorithm is given below.

- $\overline{x}_1, a_1, \overline{y}_1, b_1, \theta_1, \overline{x}_2, a_2, \overline{y}_2, b_2, \theta_2$ are computed values determined from ellipse fitting for the two interfaces. The a_i and b_i values are the σ_x and σ_y values, respectively.
- For each point *(x,y)* in both data sets, compute:

$$e_1 = \frac{\left[\left(x-\overline{x}_1\right)\cos(\theta_1) + \left(y-\overline{y}_1\right)\sin(\theta_1)\right]^2}{a_1^2} + \frac{\left[-\left(x-\overline{x}_1\right)\sin(\theta_1) + \left(y-\overline{y}_1\right)\cos(\theta_1)\right]^2}{b_1^2}$$

and

$$e_2 = \frac{\left[\left(x-\overline{x}_2\right)\cos(\theta_2) + \left(y-\overline{y}_2\right)\sin(\theta_2)\right]^2}{a_2^2} + \frac{\left[-\left(x-\overline{x}_2\right)\sin(\theta_2) + \left(y-\overline{y}_2\right)\cos(\theta_2)\right]^2}{b_2^2}$$

- If $e_1<1$ and $e_2<1$, then the data point *(x,y)* is within both ellipses.

By bookkeeping the number of points that fall within both ellipses, the percentage of total data that lies in an area common to both confidence regions can be computed.

Results

This section shows the results of a number of different tests on the ellipse generation algorithm and the overlap algorithm. Figures 2 through 7 show results for investigations into what occurs when there is a simple stratigraphic scenario of a shale overlying a sandstone that is saturated with various fluids. An isotropic, elastic wavefield's AVO response is forward modeled as it encounters a shale-sandstone interface. The sandstone itself is saturated with various fluids. This is geologically simple in that there are only two layers and there is no dip. This scenario is schematically represented in Figure 1 below. Figures 2–7 show examples that serve to illustrate the AVO sensitivity to pore fluids. They display how this method can be used to determine whether or not AVO will be successful tool in discriminating between different fluid saturation. The plots all have 1.5-σ ellipses that contain approximately 68% of the total data. Table 1 outlines the AVO parameters used.

shale layer

sandstone layer

Figure 1: Simple stratigraphy through which the AVO response of an elastic and isotropic wavefield is modeled.

Table 1: Uncertainties in the parameter values for the shale-sandstone models.

Layer and Pore Fluid	V_p ± 5% *(ft/s)*	V_s ± 10% *(ft/s)*	ρ ± 5% *(ft/s)*
shale	8700	4000	2.15
brine	8510	4320	2.08
100% black oil	7385	4430	1.98
50% black oil	7568	4374	2.03
20% lean gas condensate	7028	4372	2.03
100% lean gas condensate	7213	4600	1.83

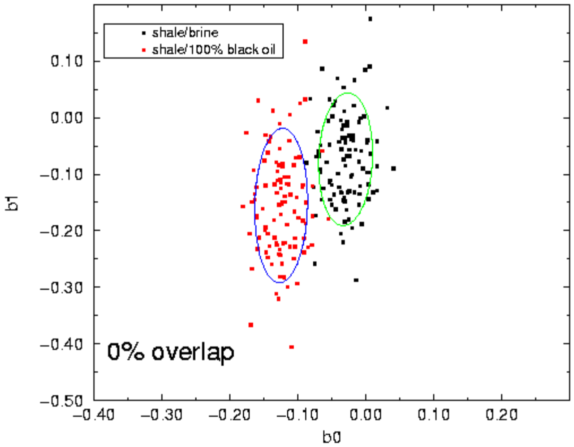

Figure 2: Cross-plot of AVO parameters for brine and 100% black oil saturation in the sandstone layer.

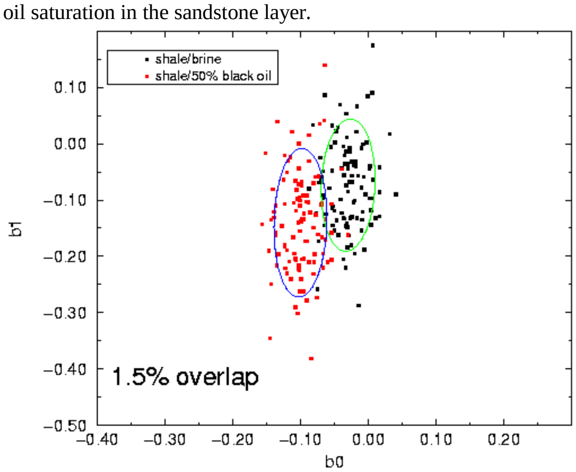

Figure 3: Cross-plot of AVO parameters for brine and 50% black oil saturation in the sandstone layer.

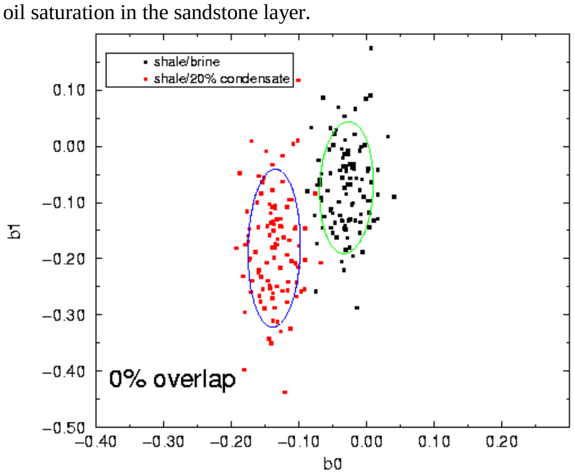

Figure 4: Cross-plot of AVO parameters for brine and 20% lean gas condensate saturation in the sandstone layer

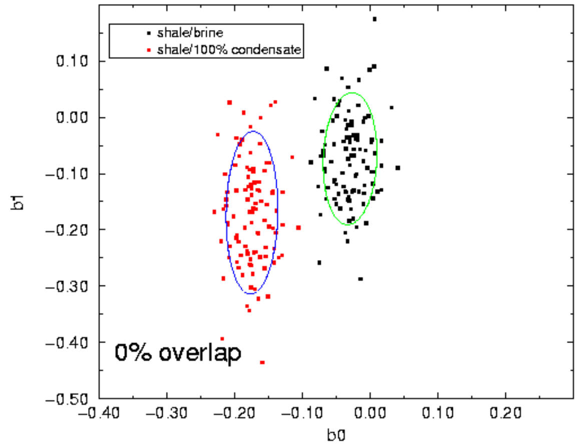

Figure 5: Cross-plot of AVO parameters for brine and 100% lean gas condensate saturation in the sandstone layer.

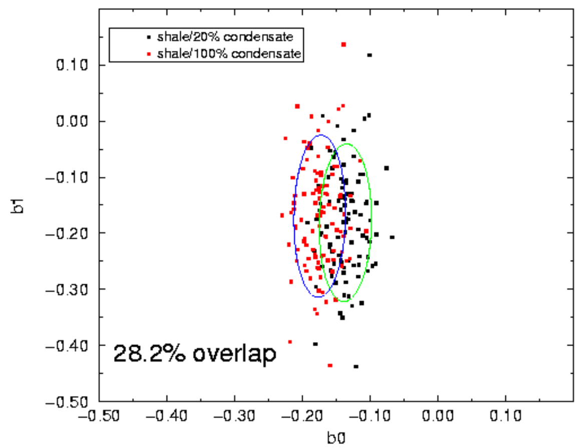

Figure 6: Cross-plot of AVO parameters for 20% lean gas condensate and 100% lean gas condensate saturation in the sandstone layer.

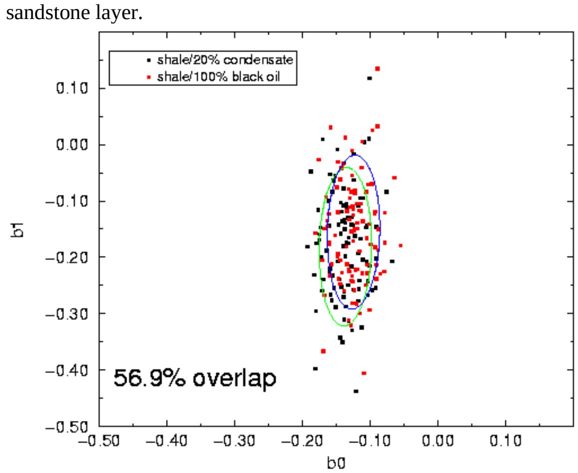

Figure 7: Cross-plot of AVO parameters for 20% lean gas condensate and 100% black oil saturation in the sandstone layer.

Figures 2 through 7 indicate that the type of fluid saturation in the sandstone can greatly impact the amount of overlap and interaction that occurs between the confidence regions for various AVO responses. Specifically, AVO can be a useful tool in discriminating between a brine saturation and black oil or lean gas condensate saturation. This, however, is not the case when trying to discriminate between different condensate responses or condensate and black oil responses. In these cases, there is a significant amount of overlap between the confidence regions and the interaction between them must be accounted for.

Conclusions

This project yields results that are consistent with some of the intuitive notions that arise from AVO forward modeling. That is, the best-fit estimate to a confidence region for b_0-b_1 scatter data is an ellipse. Furthermore, the specific fitting of standard deviational ellipses to various scatter data quite adequately approximates the general best-fit ellipse to the data and at far less cost. In addition, one can exploit the geometric properties of these ellipses to quickly and cheaply test to see if a data point has an equal probability of belonging to either response of interest. That is, the data can be efficiently divided into two clear groups; those that unambiguously belong to a certain response and those that have an equal probability of belonging to either response being considered. In addition, the entire process can be applied to any set of data where the underlying objective is to quantify the observed dispersion. Overall, this method can provide a quantitative index for AVO risk assessment. The ultimate goal is to create a general method to best-fit ellipses and calculate overlap that is quick and stable. While there is an elegant and simple analytical theory to achieve this, current technology does not allow this theory to be effectively translated into the discrete language of digital computing.

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