

The Problem is not Mathematics, but Mathematicians: Plato and the Mathematicians Again

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In a recent essay¹ I have defended a reading of Plato's image of the Divided Line (henceforth 'the Line') according to which Plato's focus is methodological rather than ontological. Plato is less concerned to offer a fourfold ontology associated with the four sections of the Line,² than he is to describe the correct method of the greatest *mathêma* - the knowledge of the Form of the Good (*Republic* 6 504e-505b). In particular, Plato devotes the vast majority of his discussion of the Line to distinguishing between the method associated with *dianoia* [L3] and the method associated with *noêsis* [L4] - a distinction between what I call the dianoetic method and the dialectic method.³ According to Plato, however, I argue, the distinction is less a distinction between two different methods, than one between two different applications of the same method. Both the dianoetician and the dialectician apply or use the method of hypothesis, but the former does so inadequately and incorrectly. The dianoetician, unlike the dialectician, [1] mistakes her hypotheses for *archai* (first principles) and [2] inappropriately makes use of ordinary sensible objects. Let us call this the methodological reading of the Line.

Against such a methodological reading, it has been pointed out that far from finding fault with the dianoetic method as a misapplication of the dialectic method, Plato instead praises and indeed encourages the practice of the dianoetic method. The basis for this reading of the Line is Plato's alleged identification of the dianoetic method with the method of mathematics or the mathematical disciplines. Given this identification, when in the course of describing the education of philosophers in *Republic* 7 Plato recommends at least a ten year immersion in the mathematical disciplines, he can hardly be disparaging the mathematical method. Indeed, a closer look at the Line itself has been taken to indicate that Plato is not finding fault with the

¹(Benson forthcoming).

²Throughout I will be employing the labels La & Lb for the initial division into two and L1 and L2 for the subsections of La and L3 and L4 for the subsections of Lb, as has become relatively standard. See n. 8 below.

³Following (Murphy 1951:168).

dianoetic method, but correctly describing the necessary features of mathematics. According to this reading of the Line, then, Plato is not distinguishing between a correct and incorrect application of the same method (or even between a better and worse method, as a more common interpretation would have it⁴). Rather dianoetic is the correct application of the same method applied by dialectic. Dialectic is simply reserved for a further inquiry - an ontological inquiry - not pursued by dianoetic. Let us call such a reading of the Line the mathematical reading.⁵

I maintain that such a mathematical reading of the Line is incorrect, but not obviously so. Indeed, we will see that there is much to recommend it. Plato does indeed recommend the mathematical method, a method which he takes to be identical to the method of philosophy. Mathematics and philosophy do not differ in virtue of their method, according to Plato, but rather roughly in virtue of their subject matter, as the mathematical reading would suggest. But Plato does not identify the mathematical method with the dianoetic method of L3 in the Line. Rather, Plato takes the dianoetic method to be a misapplication of the mathematical method by a subset of practicing mathematicians. Thus, Plato's critique of dianoetic is a not a critique of mathematics, as such, but of mathematicians.⁶ If this is correct, then Plato's recommendation of mathematics in *Republic 7* does not serve as a reason for rejecting the methodological reading of the Line. It does not, that is, serve as a reason for rejecting a reading of the Line as a critique of the dianoetic method. Seeing why will provide a deeper understanding of the Line, and the dialectical method it is meant to be recommending.

⁴See n. 5 below.

⁵A particularly impressive defense of this reading of the Line is offered by Myles Burnyeat in a pair of influential essays: (Burnyeat 1987) and (Burnyeat 2000). Others who might be thought to promote a version of a mathematical reading of the Line, in the sense that they appear to identify dianoetic with the (correctly applied) mathematical method (the first thesis below), include (Adam 1902:esp. 159–160), (Joseph 1948:esp. 48–49), (Cherniss 1951:esp. 415–416) (Robinson 1953:esp. 155–157) (Rose 1963–64:esp. 434–435), (Cross and Woosley 1964:esp. 205–206), (Santas 1980:esp. 252–253, & 255), (Annas 1981:esp. 282), (Malcolm 1981:esp. 66), (Scolnicov 1988:esp. 92–96), (Mueller 1992:esp. 184–185), (Smith 1996:esp. 33), (Reeve 2003:esp. 41–42), (Bailey 2006:esp. 101), (Miller 2007:esp. 319), and (Denyer 2007:esp. 303). They do not, however, follow Burnyeat in maintaining that Plato is not being critical of the method or procedures of dianoetic (the second thesis below). Unlike, Burnyeat, they tend to take mathematics to be, at best, instrumentally valuable.

⁶This view is suggested by the first response of (Hare 1965:30). Similar views are suggested by (Vlastos 1965:144–145), (White 1976:98), and (Zhud 1998:241).

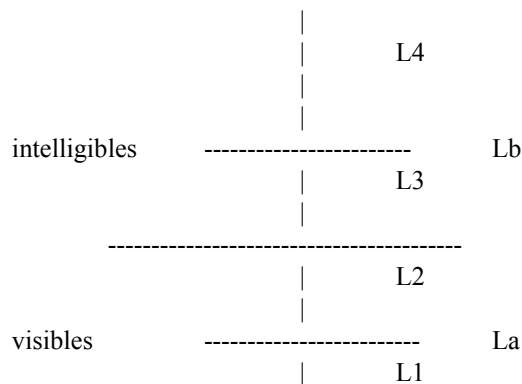
After briefly reviewing my methodological reading of the Line, I turn to the mathematical reading. I maintain that the latter reading relies on an insufficiently nuanced picture of mathematics as it was being practiced around the time of the composition of the *Republic*.⁷ I argue not only that Plato recognizes a distinction between a discipline and good and bad practitioners of that discipline, but also that he recognizes this distinction specifically in the case of the mathematical disciplines. Moreover, at least some of the passages which testify to this distinction reinforce the critique of dianoetic indicated by the methodological reading. The features which Plato associates with bad practitioners of the mathematical method correspond to the features associated with dianoetic in L3 as opposed to dialectic in L4. I will conclude by speculating concerning the reasons Plato may have had for focusing on the distinction between his recommended method and misapplications of it by practicing mathematicians in a central passage of the *Republic*.

A Methodological Reading of the Line

Socrates is made to summarize the distinction between the top two subsections of the Line⁸ as follows.

⁷I assume nothing more precise concerning the compositional date of the *Republic* than that it was composed by the mid fourth century or before. Indeed, nothing important hangs on this historical claim. Contemporary mathematicians are as susceptible (although not as commonly, one supposes) to the features associated with dianoeticians as were ancient ones.

⁸ The image of the Divided Line can be represented as follows:



Nothing in what follows hangs on representing the Line vertically, nor placing Lb above La. For some of the debate concerning these and other features of the Line see, e.g., (Cross and Woosley 1964:204), (Fogelin 1971:375), and esp. (Smith 1996:27–28).

In one subsection, the soul, [A1] using as images the things that were imitated before (τοῖς τότε μιμηθεῖσιν ὡς εἰκόσιν χρωμένῃ), [A2] is forced to investigate from hypotheses (ζητεῖν ἀναγκάζεται ἐξ ὑποθέσεων), [A3] proceeding not to a first principle but to a conclusion (οὐκ ἐπ’ ἀρχὴν πορευομένη ἀλλ’ ἐπὶ τελευτήν). In the other subsection, however, it [B1] makes its way to a first principle that is not a hypothesis (τὸ ἐπ’ ἀρχὴν ἀνυπόθετον), [B2] proceeding from a hypothesis (ἐξ ὑποθέσεως ἰοῦσα) [B3] but without the images used in the previous subsection, using forms themselves and making its investigation through them (καὶ ἄνευ τῶν περὶ ἐκεῖνο εἰκόνων, αὐτοῖς εἶδεσι δι’ αὐτῶν τὴν μέθοδον ποιουμένη). [*Republic* 510b4-9; Grube/Reeve trans.]⁹

For our present purposes it is important to notice that [A1] corresponds to [B3], [A2] to [B2], and [A3] to [B1]. That is, Plato indicates that both the dianoetic method and the dialectical¹⁰ method proceed from hypotheses ([A2] ἐξ ὑποθέσεων/ [B2] ἐξ ὑποθέσεως); their differences reside in the contrasts between [A1] and [B3] and [A3] and [B1]. Dianoetic uses as images the objects of L2 (ordinary sensible objects), while dialectic does not ([A1]/[B3]), and dianoetic does not proceed to a first principle, while dialectic does ([A3]/[B1]). To see what these differences amount to, however, we need to say a little bit about what the two methods have in common. We need to say, that is, a little bit about what it is to ‘proceed from hypotheses’.¹¹

Plato describes a method of proceeding from hypotheses in the *Meno* at 86e6-87b2 (see esp., λέγω δὲ τὸ ἐξ ὑποθέσεως ὧδε; 86e4) and again in the *Phaedo* at 100a3-8 and 101d1-e3. What emerges from these passages is a method consisting of two stages each consisting of two procedures. In the first, or proof, stage [1a] one seeks to identify a hypothesis from which an answer to the question whose answer one seeks to know can be derived, and then [1b] one shows how the hypothesis entails the answer to the question. In the second, or confirmation, stage one

⁹Of the 80 lines of text devoted to the Line, only 15 or so are devoted to the bottom two subsections (L1 and L2). Virtually all of the remaining 65 lines are devoted to this summary of L3 and L4 and its explication.

¹⁰It should be noted that Plato does not use the word ‘διαλεκτική’ or any of its forms in the Line, but he does use the phrase ‘τῇ τοῦ διαλέγεσθαι δυνάμει’ in his elaboration of the method associated with [L4] at 511b4, which (Robinson 1953:69) maintains is equivalent to ‘διαλεκτική’. See also, (Kahn 1996:293) and (Annas 1981:276).

¹¹I concede that Plato may simply mean by ‘proceeding from hypotheses’ (ἐξ ὑποθέσεων) roughly ‘assuming a hypothesis and deriving consequences from it’, but as a method of acquiring knowledge on one’s own when one is ignorant, as it is explicitly introduced in the *Meno* (or at least so I argue; see (Benson 2003)), such a method is particularly lame. Even so, I concede that Plato’s use of ἐξ ὑποθέσεων here in the *Republic* may not be referring to the method introduced in the *Meno* or the only slightly more fully described method in the *Phaedo*. My purpose in this and other essays, however, is to investigate whether we can gain a deeper understanding and appreciation of various passages (like Plato’s Line) if we assume that Plato is referring to the same method he introduced and described in the *Meno* and *Phaedo*. I believe we can. But the final judgment on this issue as always must be left to the reader.

seeks to confirm the truth of the hypothesis, [2a] first by identifying a further hypothesis from which the original hypothesis can be derived and showing how this derivation goes until one reaches ‘something adequate’¹², and then¹³ [2b] by testing the consequences of the hypothesis to see whether they agree with one another.¹⁴

If this is what Plato has in mind by ‘proceeding from hypotheses’, it is fairly easy to see what the second difference between dianoetic and dialectic amounts to. According to [A3] dianoetic does not carry out the first procedure of the confirmation stage, [2a], far enough, i.e., all the way to a first principle, while according to [B1], dialectic does. Indeed, according to Plato, dialectic carries out this first procedure all the way to the unhypothetical first principle of everything (τοῦ ἀνυποθέτου ἐπὶ τὴν τοῦ παντὸς ἀρχὴν; 511b6-7). This suggests that according to Plato at some stage in the process of confirming one’s hypothesis by appealing to higher hypotheses, [2a], dianoeticians take their higher hypothesis to be a first principle when it is not, while dialecticians do not. Dianoeticians, that is, take their hypotheses to be known and clear to all when they are not (... ὡς εἰδότες ... ὡς παντὶ φανερόν; 510c6-d1), while dialecticians do not. To this extent dianoeticians fail to apply the method of hypothesis correctly, and their method (dianoetic) is flawed relative to dialectic.

The first difference between dianoetic and dialectic, however, remains obscure. How does dianoetic’s use of ordinary sensible objects as opposed to dialectic’s use of Forms alone

¹²This portion of the second stage is simply a repetition of the proof stage directed at the hypothesis identified in [1a].

¹³(Bailey 2006:102) takes [2b] to temporally precede [2a]. This is evidently the suggestion of *Phaedo* 101d1-102a2, but this is not how the method gets practiced in the *Meno* and the *Republic*. I suspect that the temporal order does not matter. See (Benson 2003) and (Benson 2008) for a more detailed justification for this general structure of the method of hypothesis.

¹⁴Characterized in this way, hypotheses appear to be premises of roughly deductive arguments and so to be propositional in character. Nevertheless, the examples of hypotheses that Plato offers in this passage look more like objects - the odd and the even and the various figures. This may suggest that Plato has in mind by these examples, not the objects themselves but their definitions. See (Annas 1981:287–289) for a succinct statement of the two lines of interpretation suggested by these remarks, as well as her salutary warning that “Plato has no precise terms for ‘proposition’, ‘imply’, ‘derive’, and no explicit distinctions between premises, conclusions, and rules of inference.” I suspect that neither Plato nor Aristotle has very clearly distinguished all of the issues surrounding the differences among objects, definitions, propositions, explanations, and proofs. Indeed, they are in the process of working these things out. But I cannot defend this suspicion here.

apply to the method of hypothesis, as I have described it above? Socrates' discussion of the Form of the Good at 534b8-d2, where Plato is plausibly referring to a testing of the consequences of the Form of the Good to see whether they accord or fail to accord with one another, [2b], provides some guidance. There Plato distinguishes between testing consequences according to opinion (*κατὰ δόξαν*) and testing consequences according to being (*κατ' οὐσίαν*). I maintain that this distinction parallels the Line's distinction between using ordinary sensible objects and using Forms alone. The idea is that dianoetic, in virtue of studying the Forms by means of their images (e.g., Justice as instantiated, Philosophy as instantiated, Virtue as instantiated), mistakes what are not in fact consequences of its hypotheses as genuine consequences. It takes features of the Form that follow from the contingent nature of the world as genuine features of the Form. It then tests all of these consequences - the contingent or artificial ones as well as the genuine ones - for consistency. Dialectic, by contrast, does not mistake features of instances of Forms for those features that follow from the Form's being or nature. It does not mistake contingent or artificial consequences for genuine consequences. Rather, it tests only the genuine consequences for consistency and responds to the contrary evidence of the artificial or contingent consequences.

Thus, according to the methodological interpretation of the Line, Plato distinguishes between a correct and incorrect application of the method of hypothesis. A dianoetic application of the method [A3] mistakes what are in fact hypotheses in need of further confirmation from yet higher hypotheses as first principles, known and clear to all, and [A1] mistakes what are in fact artificial or contingent consequences of hypotheses as genuine consequences as a result of studying the images of Forms rather than the Forms themselves and tests these consequences - both the contingent or artificial and the genuine ones - for consistency. A dialectical application [B1] confirms its hypotheses all the way to the first principle of everything and [B3] does not mistake contingent or artificial consequences for genuine consequences as a result of studying only Forms, tests only those genuine consequences for consistency, and explains away the contrary evidence (the contingency or artificiality of the alleged contrary consequences).

Nevertheless, a number of considerations tell against this reading of the Line, not the least of which derive from the mathematical reading.

The Mathematical Reading of the Line

The essence of the mathematical reading of the Line relies on two fundamental theses. According to the first thesis, Plato identifies the dianoetic method with the method of mathematics. According to the second thesis, far from finding fault or critiquing the method of mathematics, Plato promotes and recommends it. It follows from these two theses that Plato cannot be finding fault with the dianoetic method as the methodological reading would suggest. If Plato is reserving dialectic to the philosopher-ruler it must be for reasons other than those provided by the methodological reading.

Let us briefly consider the argument for these two theses. The identification of the dianoetic method with the method of mathematics is indicated when Socrates elaborates his initial description of the methodology of L3. Socrates immediately appeals to the practitioners of geometry, calculation, and the rest (οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς καὶ τὰ τοιαῦτα πραγματευόμενοι) as examples of dianoeticians. The identification is reinforced when Glaucon summarizes Socrates' elaboration. Glaucon reiterates the example by describing the cognitive state of geometers as *dianoia* since it falls between *doxa* and *noûs*. And the identification appears assured when in the course of describing the propaedeutic mathematical disciplines comprising the initial stages of the philosopher-rulers education in Book 7 Socrates associates these disciplines with *dianoia*. Socrates concludes his account of geometry by writing that “geometry draws the soul towards truth (Ὀλκὸν ἄρα, ὃ γενναῖε, ψυχῆς πρὸς ἀλήθειαν) and produces philosophic *dianoia* (ἀπεργαστικὸν φιλοσόφου διανοίας) by directing upwards what we now wrongly direct downwards (πρὸς τὸ ἄνω σχεῖν ἃ νῦν κάτω οὐ δέον ἔχομεν)” [*Republic* 7 527b9-12], and later he writes that “From force of habit, we've often called these crafts [i.e., the propaedeutic mathematical disciplines] sciences or kinds of knowledge (ἐπιστήμας), but they need another name, clearer than opinion (ἐναργεστέρου μὲν ἢ δόξης), darker than knowledge (ἀμυδροτέρου δὲ ἢ ἐπιστήμης). We called them *dianoia* (διάνοιαν) somewhere before”

[*Republic* 7 533d1-7; adapted from G/R trans.]. This is the first thesis of the mathematical reading.

The idea that Plato approves of the mathematical disciplines and presumably their method - the second thesis of the mathematical reading - is reflected in Plato's description of the education of the philosopher-ruler in *Republic* 7. As is well-known following Plato's image of the Cave, Plato provides a detailed description of the education of the philosopher-rulers whose formal training begins at the age of 20 with a 10 year immersion into the mathematical disciplines - beginning with arithmetic¹⁵, next geometry, stereometry, astronomy, and harmonics, followed by five years of dialectic (or 'pure dialectic' as I would prefer to call it for reasons which will emerge below). After 15 years of practical training, the future philosopher-ruler at age 50 returns to dialectic and ascends to the Form of the Good. What is apparent from this description of the education of the philosopher-ruler is the respect Plato shows to the mathematical disciplines. Not only does he devote 10 years to their study, but he repeatedly stresses the necessity of these disciplines for leading the soul towards knowledge, truth, and being.¹⁶ These hardly look like critical remarks.

Building upon these remarks, Myles Burnyeat has persuasively rejected what he calls 'a long-established tradition of interpretation' according to which in the *Republic* (and especially the Line passage) Plato is concerned to criticize the procedure of the mathematical disciplines and 'in particular their reliance on hypotheses'.¹⁷ According to Burnyeat, Plato's problem with

¹⁵According to (Fowler 1999:16), this discipline should be understood as number theory, *pace* (Mueller 1991:93 n. 18). I will be translating λογιστική τε καὶ ἀριθμητική as 'arithmetic' without commitment to a resolution of this debate.

¹⁶*Republic* 523a1-4, 525b1, 525b11-c6, 527b9-12, and 532c3-6.

¹⁷See (Burnyeat 1987:218). N.B. that the phrase 'their reliance on hypotheses' is ambiguous between 'finding fault with the way in which the mathematical disciplines use hypotheses' and 'finding fault with the fact that they use hypotheses at all'. Like Burnyeat, I do not take Plato to be objecting to the fact that mathematicians use hypotheses, but, unlike Burnyeat, I do take Plato to be objecting to the way in which mathematicians use their hypotheses. It is interesting to note that the only two scholars Burnyeat cites on behalf of the long-established tradition according to which Plato criticizes mathematics for its procedure and reliance on hypotheses are (Robinson 1953:146–156) and (Annas 1981:277–279). But neither of these scholars, at least if I understand them correctly, take Plato to be criticizing mathematics for employing the method of hypothesis. On the contrary they take him to be criticizing mathematics for not employing the method of hypothesis. This is explicit in (Robinson 1953:152).

the mathematical disciplines is not their method (i.e., their use of hypotheses), but rather their ontology.¹⁸ The mathematical disciplines leave unaddressed and undetermined the nature of the objects of their disciplines. This ‘ontological’ inquiry is the task of a distinct methodology - dialectic. Even so, Burnyeat does not suppose that Plato is critical of mathematics as such. To disparage mathematics for failing to pursue an ontological inquiry would be to ask it to be something it is not. Mathematics, according to Plato, accomplishes everything it can and should accomplish. If further inquiries are required that is a task for some other discipline.

Burnyeat’s fundamental argument is as brief as it is forceful. The idea that Plato is criticising mathematics for using hypotheses should have been ‘killed off’ by the occurrence of one word - ἀναγκάζεσθαι at 510b5 in the description of the dianoetic method. As we have seen, Plato writes concerning this subsection of the Line:

In one subsection [L3], the soul, [A1] using as images the things that were imitated before, [A2] is **forced** to investigate from hypotheses (ζητεῖν ἀναγκάζεται ἐξ ὑποθέσεων), [A3] proceeding not to first principle but to a conclusion. [510b4-6; Grube/Reeve trans.]

Later Plato makes it clear that the individuals he has in mind here are ‘the practitioners of geometry, calculation and the like’ (οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς καὶ τὰ τοιαῦτα πραγματευόμενοι; 510c2-3) and Burnyeat correctly points out that far from criticizing these practitioners of mathematics for using hypotheses, Plato maintains that they are forced to use hypotheses. It can hardly be a criticism of mathematics to do what it must do.

Lest it be thought that this is to put too fine a point on Plato’s words, Plato repeats the point in Socrates’ concluding summary of his elaboration of the above passage. Socrates summarizes as follows:

This, then, is the kind of thing that, on the one hand, I said is intelligible, and, on the other, is such that the soul [A2] is **forced** to use hypotheses in the investigation of it (ὑποθέσει δ’ ἀναγκαζομένην ψυχὴν χρῆσθαι περὶ τὴν ζήτησιν αὐτοῦ), [A3] not travelling up to a first principle, since it cannot reach beyond its hypotheses, [A1] but using as images those very things of which images were made in the section below, and which, by comparison to their images, were thought to be clear and to be valued as such. [511a3-b2; Grube/Reeve trans.]

¹⁸(Burnyeat 1987:218); cf. also (Bowen 1983) for a view which has considerable sympathies with Burnyeat’s view.

That mathematicians are forced to use hypotheses is evidently not a throw away line.

Burnyeat rightly goes on to explain that in maintaining that mathematicians are forced to use hypotheses, Plato is quite right. As Burnyeat puts it “... hypotheses are intrinsic to the nature of mathematical thought. There is no other way of doing deductive mathematics than by deriving theorems and constructions from what is laid down at the beginning” (Burnyeat 2000:37); cf. also (Burnyeat 1987:219). So Burnyeat concludes that Plato cannot be taken to be criticizing the mathematicians for doing what he correctly believes they must do, i.e., use hypotheses.¹⁹ This, then, is the second thesis of the mathematical reading.

When these two theses are conjoined, we appear to have strong reason for rejecting the methodological reading of the Line. Plato cannot be critiquing the dianoetic method for misapplying the method of hypothesis. He identifies the dianoetic method with the procedures of mathematics and he rightly promotes and recommends those procedures, in particular its application of the method of hypothesis. This, in brief, is the mathematical reading of the Line.

Mathematics and Mathematicians

Before responding to this formidable reading of the Line, let’s pause to review what the mathematical and the methodological readings have in common. It is no objection to the methodological reading to maintain that Plato approves of the method of hypothesis (or even the use of ordinary sensible objects, although this is more complicated), as employed by mathematics or any other discipline. Recall that according to the methodological reading Plato does not find fault with dianoetic for employing the method of hypothesis. That is a feature of dianoetic that it shares with dialectic ([A2]/[B2]), and no one to my knowledge takes Plato to be finding fault with dialectic in the Line. Plato’s critique, if there is one, concerns not the use of the method of hypothesis, but its misuse. It concerns employing the method of hypothesis incorrectly.

¹⁹(Burnyeat 1987:219) also takes the use of ἀναγκαίως at 527a6 to indicate that Plato takes the use of ordinary sensible objects, in the form of diagrams, as necessary for mathematics and that Plato is correct to do so. (See also (Netz 2003:306 n. 22), *pace* (Mueller 1992:185). We will return to the mathematicians’ use of ordinary sensible objects below.

Nor is it an objection to the methodological reading to maintain that Plato approves of the procedures of mathematics or the mathematical method. The methodological reading simply does not mention the mathematical method. According to the methodological reading, Plato disapproves of dianoetic, not mathematics. Thus, the methodological reading accepts the second thesis of the mathematical reading. Plato is not finding fault with the mathematical method, and especially not for its use of the method of hypothesis. In Book 7 of the *Republic* Plato is impressed with the virtues²⁰ of the mathematical disciplines, requiring the philosopher-rulers to devote 10 years of their educational training to their study and indicating that they lead the soul toward knowledge, truth, and being. In Book 6, he points out that these same disciplines employ the same method (the method of hypothesis) as his preferred method (dialectic), the method of philosophy.

Nevertheless, according to the methodological reading, in Book 6 Plato appears to fault the mathematicians for failing to confirm their hypotheses all the way to the unhypothetical *arché* and for in some way inappropriately using ordinary sensible objects. How are we to understand these critical remarks about the mathematicians, on the methodological reading, in light of Plato's evident respect for the mathematical disciplines? Putting the question this way, however, already reveals (perhaps begs) an answer.

The methodological reading rejects the first thesis of the mathematical reading, the identification of dianoetic with the mathematical method. On the methodological reading, in finding fault with dianoetic in L3 of the Line, Plato is not finding fault with the mathematical method. Dianoetic and the mathematical method are not identical. Rather, Plato is finding fault with the mathematicians - those who practice (πραγματευόμενοι; 510c3) these disciplines, for example, the geometers (τῶν γεωμετρικῶν; 511d3). The problem is with the mathematicians - how they in fact practice their discipline (or apply their method), not mathematics. Plato characterizes the mathematicians as failing to proceed to the unhypothetical *arché*, (or at least for -----

²⁰Whatever they are. (Burnyeat 2000) forcefully argues that they are not merely instrumental, but include content relevant to the Form of the Good. See also (Netz 2002:257–259).

failing to recognize that their hypotheses remain unclear and unknown until they have arrived at the unhypothetical *archê*) and for failing to use ordinary sensible objects correctly. These failures are not features of the mathematical disciplines as such. They are failures of (at least some of) its practitioners.²¹ The mathematical disciplines employed correctly compose ten years of the philosopher-rulers' education. So practiced they do not misuse sensible objects and they hand over their results to a higher discipline²², all the way up to the Form of the Good, the unhypothetical *archê*. As such, these disciplines are not inferior to dialectic, but constitute the beginning stages of dialectic, at least in one of its guises.²³

Burnyeat recognizes the distinction between the practitioners of mathematics and the discipline itself when he writes “Such criticism [of mathematics] as there is comes later and concerns, not the procedures of geometry and number theory, but the mathematicians’ failure to exploit those procedures to their full potential in stereometry (528bc), astronomy (529aff.), and harmonics (531ac)” (Burnyeat 1987:218 n 14). Burnyeat here allows that Plato is critical of the practitioners of stereometry, astronomy, and harmonics, but not the disciplines themselves. He does not, however, believe that Plato is criticizing geometers or arithmeticians, the two mathematical disciplines well on their way to ‘completion’ at the time of Plato’s writing of the *Republic*. Indeed, Burnyeat anticipates the idea that Plato may be criticizing the mathematicians in general as opposed to mathematics and offers the following rebuttal

Even when Socrates says that mathematicians set down hypotheses ὡς εἰδότες (510c6) and give no account of them ὡς παντὶ φανερωῶν (510d1), this should be read as straight description, not criticism. ... it neither says nor implies, what would be false of Euclid,

²¹It might be thought that the failure of the mathematical disciplines to arrive at an unhypothetical *arche* is an essential feature of the disciplines as such on the grounds that these disciplines have been proving theorems from hypotheses for centuries without in fact arriving at the *archê*. I find such an inference rather suspect, but even so, it does not follow that the failure to recognize that the hypotheses from which the theorems are derived are unclear and unknown until they have been derived from such an *archê* is an essential feature of the mathematical disciplines.

²²For the image of mathematicians (γεωμέτραι καὶ οἱ ἀστρονόμοι καὶ οἱ λογιστικοί) handing over their results to dialecticians see *Euthydemus* 290b7-c7.

²³‘Dialectic’ (ἡ διαλεκτικὴ ἢ τῆ τοῦ διαλέγεσθαι δυνάμει) as used by Plato and his commentators is equivocal. It can mean ‘the discipline of dialectic’ which is equivalent to philosophy as distinct from mathematics, for example. It can also mean ‘the method of dialectic’ which is equivalent to the method of hypothesis. And, it can mean ‘the entire road to wisdom’ which is equivalent to the greatest *mathēma* or the education of the philosopher-ruler. Hence, we might designate the first meaning of ‘dialectic’, ‘pure dialectic’ which as such begins for the future philosopher-rulers following the ten years of mathematical study.

that the mathematicians openly claim to have knowledge or claim that their hypotheses are clear to anyone. If, unbeknownst to us, mathematicians earlier than Euclid did advance such claims, they would of course be open to criticism from the standpoint of the *Republic* - but only for epistemological rashness, not for mathematical malpractice. (Burnyeat 1987:219 n 18).

Burnyeat here anticipates the interpretation that Plato is criticizing mathematicians, but not mathematics, and rejects it for two reasons. First, we have no evidence that there are such practitioners of geometry and arithmetic (if not stereometers, astronomers, and harmonists) at whom such criticisms could be reasonably or fairly directed. We have, that is, no likely target for Plato's criticism. Second, even if we were to find such a target, the criticism is not methodological, but epistemological. It is an objection to drawing the wrong conclusions from the method, not to employing the wrong method. We will return to both of these points below, but first let us be clear that Plato distinguishes between a discipline and its method, on the one hand, and good and bad practitioners of that discipline, on the other.²⁴

The Conclusion of the *Euthydemus*

In the *Euthydemus*, after describing the concluding scene of the third eristic debate between the two eristic brothers (Euthydemus and Dionysodoros) and Socrates, Socrates invites Crito to join him in attending the brothers' classes. Crito, however, warns Socrates of a complaint he received concerning the eristic performances Socrates had just participated in. According to Crito the complainant criticized both the discipline (philosophy, cf. *Euthydemus* 307b6-c4 below) and those who practiced it (τὸ πρᾶγμα αὐτὸ καὶ οἱ ἄνθρωποι οἱ ἐπὶ τῷ πράγματι διατρίβοντες; *Euthydemus* 305a6-7) as being worthless and laughable. Crito, however, doubts the correctness of the complaint concerning the discipline (τὸ πρᾶγμα), but can't help agreeing about the practitioners; *Euthydemus* 305a8-b3. After Socrates impugns the motives of the complainant, he concludes the dialogue with the following advice

SOCRATES: My dear Crito, don't you realize that in every pursuit most of the practitioners are paltry and of no account whereas the serious men are few and beyond price (ἐν παντὶ ἐπιτηδεύματι οἱ μὲν φαῦλοι πολλοὶ καὶ οὐδενὸς ἄξιοι, οἱ δὲ σπουδαῖοι ὀλίγοι καὶ παντὸς

²⁴We might characterize a discipline as defined by its method and its subject matter, although these are difficult issues. In particular, it is not clear that Plato has anything helpful to say about the distinction between practicing a discipline poorly or incorrectly and not practicing that discipline (but another discipline) at all.

ἄξιου)? For instance, doesn't gymnastics strike you as a fine thing? And money making and rhetoric and the art of the general?

...

SOCRATES: Well then, in each of these cases don't you notice that the majority give a laughable performance of their respective tasks?

...

SOCRATES: And just because this is so, do you intend to run away from all these pursuits and entrust your son to none of them?

...

SOCRATES: Then don't do what you ought not to, Crito, but pay no attention to the practitioners of philosophy, whether good or bad. Rather give serious consideration to the thing itself (τοὺς ἐπιτηδεύοντας φιλοσοφίαν, εἴτε χρηστοί εἰσιν εἴτε πονηροί, αὐτὸ τὸ πρᾶγμα βασανίσας καλῶς τε καὶ εὖ): if it seems to you negligible, then turn everyone from it, not just your sons. But if it seems to you to be what I think it is, then take heart, pursue it, practice it, both you and yours, as the proverb says. [*Euthydemus* 307a3-c4; Sprague trans.]

Plato here explicitly distinguishes between the practitioners of a discipline and the discipline itself, and allows that the practitioners may be fairly criticized without the discipline being at fault. Indeed, he even provides a target for Plato's critical remarks about mathematicians. He says that 'in every pursuit' most of its practitioners are worthy of criticism. Presumably, that includes arithmetic and geometry, and the other mathematical disciplines. Indeed, at 290b7-c7 Socrates explicitly includes the geometers, arithmeticians, and astronomers, saying that the ones who are not completely senseless (perhaps the ones applying their method correctly) hand over their discoveries to the dialecticians, while the completely senseless ones do not. But what is perhaps most striking about Socrates' concluding remarks is that he appears to allow that the eristic brothers may be fairly characterized as practicing philosophy, but doing so poorly. Plato here seems to explicitly endorse what the methodological reading of the Line would suggest: the method of philosophy (whatever that is thought to be) can be practiced well, (L4), or poorly, (L3).

Nevertheless, one might worry that the end of the *Euthydemus* raises more difficulties for the methodological reading of the Line than it resolves. According to this reading, it is the method of hypothesis that is being applied poorly by the mathematicians in L3 and applied well by the philosophers in L4. So, according to the methodological reading, the method of hypothesis is the method of philosophy. But if Plato is suggesting at the end of the *Euthydemus*

that the eristic brothers are applying the method of philosophy - albeit poorly, it hardly looks like the method of philosophy is the method of hypothesis. While scholars have found the method of hypothesis employed in a variety of dialogues (including, perhaps surprisingly, the *Charmides* and *Euthyphro*, among others),²⁵ no one to my knowledge has found it lurking in the *Euthydemus*. So how can the *Euthydemus*'s suggestion that the eristic brothers are practicing the method of philosophy - albeit poorly - be compatible with the Line's suggestion that the mathematicians are practicing the method of philosophy - albeit poorly? The method of the eristics and the method of the mathematicians are radically different. If the two groups are applying the method of philosophy - albeit poorly - the method of philosophy in the *Euthydemus* and the *Republic* is radically different.

Here I want to call the reader's attention to what might be described as a difference of focus on Plato's part in a group of dialogues that have frequently been considered among Plato's early works and a group of dialogues associated with Plato's middle period. In the former group Plato's methodological remarks appear to be focused on a methodology of short questions and answers that has come to be called the *elenchus*. Moreover, Plato tends to depict Socrates in those dialogues as practicing this method of *elenchus*. On the other hand, in the so-called middle dialogues Plato tends to focus his methodological remarks on a different methodology. Determining the precise features of this distinct methodology is the project of which the current essay forms a part. To say anything very specific about them, then, could only be done at this stage by begging a variety questions. Suffice it to say that the method that appears to capture Plato's attention in the so-called middle dialogues relies less on testing the knowledge claims of others and more on discovering and/or defending a variety of views from a common state of professed ignorance. Let us call this method dialectic. Moreover, just as in the case of the so-

²⁵See, for example, (Kahn 1996:184) and (Novak 2005:5).

called early dialogues, Plato tends to depict Socrates in the so-called middle dialogues as practicing dialectic.²⁶

I mention all of this because I take it that at least part of Plato's purpose in the *Euthydemus* is to distinguish the eristic method from the method he depicts Socrates practising in the elenctic dialogues - the *elenchus*.²⁷ Whatever else is going on in the *Euthydemus*, Plato appears concerned to contrast the method of the eristic brothers depicted in the three eristic debates (274d-278c, 282d-288a, and 292e-304b) and the method of *elenchus* practised by Socrates in the two protreptic displays (278d-282c and 288b-292d) sandwiched between those eristic debates. This places the *Euthydemus* among those dialogues whose focus is the philosophical method of the *elenchus* rather than those dialogues whose focus is dialectic - whether that focus is to be explained by a change or development in Plato's views²⁸ or a proleptic decision on Plato's part to display the complete philosophical method one piece at a time.²⁹ What is salient for our present purposes is not the shift in focus - however it is to be explained - but Plato's explicit recognition at the end of the *Euthydemus* of the distinction between the practitioner of a method and the method itself, as well as the recognition that such practitioners can be distinguish by those who practice the method well and those who practice it poorly, even in the case of the method of philosophy itself. This is precisely the sort of thing we should expect Plato to recognize if the methodological reading of the Line is correct.

Targets for Plato's Criticism in L3

²⁶For some defense of this last claim see (Benson 2006). A similar shift in focus takes place in the later dialogues where Plato tends to discuss and depict Socrates (or his other apparent spokesmen) as practicing what has come to be called the method of collection and division. I have used imprecise and hedge words like 'tends', 'so-called', and 'focus' deliberately in order to avoid begging the question between the so-called developmentalists and so-called unitarians. See nn. 28 & 29 below. While I am sympathetic with the developmental approach, I doubt that anything I maintain in this essay hangs on that sympathy. Indeed, while most scholars of a developmentalist persuasion are inclined to see these three methodologies as representing a shift in Plato's judgment concerning the ideal methodology of philosophy, I am inclined to see these three methodologies as components of a single methodology whose components occupy Plato's attention in different groups of dialogues. For the beginning of an argument in that direction see (Benson 2005).

²⁷See, for example, (Hawtrey 1981:6-7).

²⁸For example, (Vlastos 1991) (although Vlastos understood the *Euthydemus* to be transitional) and many others sympathetic to his general approach.

²⁹For example, (Kahn 1996) and many others sympathetic to his general approach.

So the *Euthydemus* passage should make us more comfortable about maintaining that Plato distinguishes the method of mathematics from the practitioners of mathematics. Indeed, it should even make us more comfortable about maintaining that Plato distinguishes the method of philosophy from the practitioners of philosophy. But comfort does not amount to evidence and one would like some evidence that Plato would draw such a distinction in the case of mathematics in particular (besides the general claim in the *Euthydemus* that the distinction applies to every discipline).³⁰

When Plato mentions mathematicians, we think of Euclid and Eudoxus, but who is Plato thinking of? Certainly not Euclid. He hasn't yet been born.³¹ Perhaps Eudoxus.³² Almost certainly, Theaetetus. But is that all? I doubt it. I wish I knew much more about the history of Greek mathematics than I do. But one does not need to know much about Greek mathematics to suspect that when Plato thought about the mathematicians of his day he had in mind not only those associated with the Academy (or who were otherwise accomplished), but also less accomplished, if not incompetent, individuals. Indeed, I think we have evidence from the dialogues themselves that he did sometimes have in mind such less accomplished individuals as practitioners of mathematics.

In three different dialogues³³ Plato associates the sophist Hippias with mathematical expertise. At *Hippias Minor* 366c-d he depicts Hippias as professing to be experienced (ἐμπειρος), most powerful (δυνατώτατος), wisest (σοφώτατος), and best (ἄριστος), concerning

³⁰Although, recall *Euthydemus* 290b-c.

³¹Euclid's dates are 325 to ca. 270. Plato's are ca.428 to 347 B.C.E. (Burnyeat 2000:24) provides a persuasive case for taking 'Euclid as a our best guide for contextualizing' Plato's discussion of mathematics in the *Republic*. The case depends on the plausible view that the best work in mathematics during Plato's life essentially amounted to 'early draft[s] of Euclid's *Elements*.' (See also (Reeve 2003:40–41), perhaps *pace* (Netz 2002:259 n 2).) Consequently, Euclid provides an excellent guide to Plato's views concerning the best work in mathematics during his day. Clearly, it is the best work in mathematics during his day that Plato has in mind in Book 7 as he describes the propaedeutic disciplines. The question before us, however, is whether Plato also has the best work in mathematics during his day in mind as he describes L3 of the Line.

³²For a brief account of Eudoxus' association with the Academy see (Vlastos 1980:23 n 24). For a more complete and some what less Platonically friendly account see (Zhmud 1998:227–232).

³³Assuming as I do that the *Hippias Major* is authentically Platonic following (Woodruff 1982) et al., *pace* (Kahn 1988).

arithmetic (λογισμῶν καὶ λογιστικῆς) and arithmetical matters (τὰ λογιστικά). In the *Hippias Major* Socrates describes Hippias as professing to know (ἐπίστασαι) astronomy (τὰ περὶ τὰ ἄστρα τε καὶ τὰ οὐράνια πάθη), geometry (γεωμετρίας), arithmetic (λογισμῶν), and harmonics (ἁρμονιῶν;) (*Hippias Major* 285b5-d2). And finally, in the *Protagoras*, Protagoras insinuates that unlike Hippias, he will not require his students to learn from him the *technai* of arithmetic, astronomy, geometry, and music (λογισμούς τε καὶ ἀστρονομίαν καὶ γεωμετρίαν καὶ μουσικὴν) (*Protagoras* 318d9-e4).³⁴ We tend to think of Hippias as a sophist, as he almost certainly was. But these passages suggest that Plato may have viewed him as a mathematician, or at least as a self-professed one.

It is interesting to note that if Hippias is identified as a mathematician, he may be characterized by the flaws of such mathematicians indicated by the description of the dianoetic method in L3. Certainly, Hippias is prone to thinking he knows things he does not and so presumably prone to taking as *archai*, i.e., as known and clear to all, mathematical hypotheses that in fact require further confirmation, [A3]. And while we know virtually nothing about his work in mathematics, his work in harmonics or euphony may suggest an inappropriate appeal to ordinary sensible objects [A1].³⁵ While the evidence is disputed, Hippias may even be credited with a minor mathematical discovery which may have required the employment of hypotheses, if Burnyeat is correct about the necessity of ‘proceeding from hypotheses’ for the mathematical disciplines.³⁶ So with Hippias, perhaps, we do have a target for Plato’s criticisms of mathematicians in L3 - if criticisms are what they are.

But Hippias, whose claim to being a mathematician is tenuous at best, is a rather feeble target for Plato’s disparagement in L3. Nevertheless, Theodorus may provide a better target. There is little doubt that Plato considered Theodorus to be a credible mathematician. Consider

³⁴See (Burnyeat 2000:4) who comments on this passage as evidence for a popular disapproval of mathematical studies, but who apparently does not appreciate that Hippias appears to be considered a mathematician in this passage.

³⁵See (Woodruff 1982:42 n 38 & 126).

³⁶See (Woodruff 1982:134 n 3). See also (Mueller 1997:309–310) who concludes his summary of the history of Greek mathematics in the fifth and sixth centuries B.C.E. with Hippias.

how Socrates describes him in the *Theaetetus* as an expert in geometry, astronomy, arithmetic and music (... γεωμετρικός; ... ἀστρονομικός καὶ λογιστικός τε καὶ μουσικός; *Theaetetus* 145a5-8), and yet later he has Theaetetus contrast Theodorus' mathematical practice with his own (*Theaetetus* 147d3-148b4). Theaetetus characterizes Theodorus as drawing (ἔγραφε)³⁷ a number of figures one at a time which exemplified the general principle that the square roots of non-square numbers are incommensurable until he reached the square root of 17, at which point he became entangled and stopped (ἐνέσχετο).³⁸ Theaetetus, however, offers a fully general definition of powers, i.e., the square roots of non square numbers, without any hint of drawing. Unfortunately, the details of this example are rather obscure,³⁹ and we would do well to avoid drawing any conclusions concerning the relative flaws and merits of Theodorus' and Theaetetus' procedures. But what is not obscure in this example is that Plato recognizes more and less accomplished practitioners of a mathematical (arithmetical) discipline.

Here, then, we have two potential targets of Plato's critique in L3 - Hippias and Theodorus - with whom Plato is clearly familiar. But while Hippias' claim to being a mathematician is tenuous, Theodorus' flaws (if he has any) are obscure. Nevertheless, more robust targets of Plato's critique are available in the *Republic* itself. As Burnyeat perceptively shows, Plato does not include all of contemporary mathematics in the propaedeutic studies making up the educational curriculum of the future philosopher-rulers. For example, he excludes the harmonic theory of two Pythagorean mathematicians whose written work he could have studied - Philolaus and Archytas.⁴⁰ The issues here are complex⁴¹ but Plato's reasons for

³⁷Concerning ἔγραφε, see, for example, (Burnyeat 1978:505 n. 57).

³⁸Concerning ἐνέσχετο, see, for example, (Hackforth 1957:128) and (Burnyeat 1978:503–504).

³⁹It is not clear, for example, what precisely Theodorus was trying to do, and so not clear how Theaetetus more successfully achieves it. What is clear is that Theodorus aims to be showing something about powers and Theaetetus offers a fully general definition of powers. See (Cherniss 1951:411–412), (Brown 1969), (Burnyeat 1978), and (Mazur 2007).

⁴⁰Burnyeat's identification of Philolaus and Archytas as the mathematicians whose work Plato means to be excluding from the philosopher-ruler's education is not essential to my thesis, but I found Burnyeat argument for this identification to be persuasive. What is essential to my thesis is that Plato is excluding the work of some contemporaneous mathematicians presumably because their practice of mathematics is in some way flawed.

⁴¹In addition to Burnyeat's discussion of these passages, other helpful discussions can be found in (Vlastos 1980), (Mueller 1980), and especially (Mourelatos 1980). The view I attempt to defend in this essay comes closest to (Mueller 1980:esp. 116) who, as I understand him, suggests that Plato would object to harmonics and astronomy as

excluding the work of these mathematicians appear to be summed up in the following passage:

They seek out the numbers that are to be found in these audible consonances (τοὺς γὰρ ἐν ταύταις ταῖς συμφωνίαις ταῖς ἀκουομέναις ἀριθμοὺς ζητοῦσιν), but they do not make the ascent to problems (ἀλλ' οὐκ εἰς προβλήματα ἀνίστασιν). They don't investigate, for example, which numbers are consonant and which aren't or what the explanation is of each (ἐπισκοπεῖν τίνες σύμφωνοι ἀριθμοὶ καὶ τίνες οὐ, καὶ διὰ τί ἑκάτεροι). But that would be a superhuman task (Δαιμόνιον ... πρᾶγμα). [531c1-5; G/R trans.]

Notice that Plato appears to be singling out two criticisms corresponding to the two criticisms we found in L3. First, the Pythagorean harmonists seek out numbers in *audible* consonances. That this is a criticism is indicated by the preceding lines in which Plato ridicules these mathematicians for putting their ears before their understanding (ᾄτα τοῦ νοῦ προστησάμενοι) and for torturing their strings. However else we understand Plato's criticism here he is evidently criticising these mathematicians for an inappropriate use of ordinary sensible objects, [A1]. Second, the Pythagorean harmonists do not 'ascend to problems'. Again, considerable controversy surrounds what Plato has in mind by this criticism, and in particular what Plato means by 'problems' (προβλήματα). Burnyeat (Burnyeat 2000:15 n. 18) correctly (in my view) follows (Mueller 1980:120 n 13) in rejecting the view that Plato has in mind the technical sense of 'problem' according to which problems are associated with constructions in geometry.⁴² Plato's use of 'problems' here in the *Republic* should instead be understood in light of Aristotle's subsequent discussion in *Posterior Analytics* II 14-17.⁴³ There the use of 'problems' evidently refers to a method of justifying, confirming, or identifying the premises used in Aristotelian demonstrations until one arrives at Aristotelian demonstrative *archai*.⁴⁴ But even if one does not

in fact practiced by Euclid in the *Phenomena* and *Sectio*. Mueller understands Plato's objection to be the attempt of the mathematicians to justify their hypotheses; this is the job of dialectic, not mathematics, as Mueller understands Plato. I suggest that the problem Plato finds with these mathematicians is that they take their justifications to be sufficient; not that they attempt to justify, but that their attempt to justify is inadequate. They take what are not *archai* and not known, as *archai* and known, and they let the evidence of the senses influence their judgments in inappropriate ways.

⁴² Cf. Proclus *Commentary on the First Book of Euclid's Elements* 77.7-81.22. *Pace* (Mourelatos 1980:62) and (Cherniss 1936).

⁴³This is suggested in part by Plato's apparent association of not 'ascending to problems' with the failure to seek the διὰ τί.

⁴⁴ See especially, (Lennox 1994) and (Ferejohn 1991). Lennox correctly sees these chapters in the *Posterior Analytics* as corresponding roughly to what Ferejohn refers to as the 'framing stage' of Aristotelian demonstrative science. While I cannot defend the point here, Ferejohn's 'framing stage' is an Aristotelian development of the upward paths, [1a] and [2a], of Plato's method of hypothesis as described above.

understand ‘problems’ in this way, Plato’s use of ‘ascent’ (ἀνίασιν) suggests that contemporary harmonists do not go high enough, just as the mathematicians in L3 do not ascend to higher hypotheses by which their lower hypotheses are to be confirmed until they reach the unhypothetical *archê*. In either case, they appear to treat as *archai*, i.e., as known and clear to all, what are in fact problems in need of further resolution or confirmation, [A3]. So, once again, we seem to have a target for Plato’s critique in L3.

But these are not the only mathematicians Plato finds fault with in this portion of the *Republic*. Plato tells us that contemporary astronomers are also to be criticized for similar mistakes.⁴⁵ Again, the details here are complex, but Plato’s critique of astronomy is evidently summed up in the following recommendation

Then if, by really taking part in astronomy, we're to make the naturally intelligent part of the soul useful instead of useless, let's study astronomy by means of problems, as we do geometry, and leave the things in the sky alone. (Προβλήμασιν ἄρα, ἣν δ' ἐγώ, χρώμενοι ὡσπερ γεωμετρίαν οὕτω καὶ ἀστρονομίαν μέτιμεν, τὰ δ' ἐν τῷ οὐρανῷ ἐάσομεν, εἰ μέλλομεν ὄντως ἀστρονομίας μεταλαμβάνοντες χρήσιμον τὸ φύσει φρόνιμον ἐν τῇ ψυχῇ ἐξ ἀχρήστου ποιήσειν.) [530b6-c1; G/R trans.]

Plato here suggests that if astronomy is to be practiced correctly, i.e., in a way that turns the soul in the direction of truth and being,⁴⁶ it must make use of problems and cease making use of the (somewhat less than) ordinary sensible objects in the sky. Precisely what Plato means by ‘leaving alone the things in the sky’ has been the subject of immense controversy,⁴⁷ but all would agree that he is criticizing contemporary astronomy for in some way making inappropriate use of their observations of the motions of the stars and other heavenly objects, [A1]. And again, whatever ‘making use of problems’ is ultimately understood to mean, it is clear it corresponds to the second fault of contemporary harmonists, which we have seen to be plausibly associated with failing to proceed to the unhypothetical *archê*, [A3]. So, again we see Plato finding fault with

⁴⁵(Burnyeat 2000:14) proposes two late fifth century astronomers Meton and Euctemon with whom Plato may have been familiar. See (Vlastos 1980:20 n. 8) who thinks Plato may have in mind Hippias. Again, the identification of these astronomers is not essential to my thesis.

⁴⁶See *Republic* 518b-519d and 521c-d.

⁴⁷See, for example, the debate between (Mourelatos 1980) and (Vlastos 1980), et al. concerning the appropriate use of such observations.

the practitioners of a discipline, although not the discipline itself, for failures in keeping with the features that make the dianoetic method inferior to dialectic in the Line.

Finally, before Plato turns to the discipline of harmonics he indicates that there may be other disciplines within mathematics which he is excluding from the philosopher-rulers' curriculum.⁴⁸ In doing so he appears to refer back to his critique of contemporary astronomy, writing

That those whom we are rearing should never try to learn anything incomplete (*ἀτελὲς ἐπιχειρῶσιν ἡμῖν μανθάνειν*), anything that doesn't reach the end that everything should reach (*οὐκ ἐξῆκον ἐκεῖσε ἀεὶ, οἷ πάντα δεῖ ἀφίκειν*)—the end we mentioned just now in the case of astronomy. [530e5-7; G/R trans.]

Here Plato indicates that anything that does not achieve the goal he mentioned as the goal of astronomy should be considered incomplete (*ἀτελὲς*), and should not be learned by the future philosopher-rulers. But what was the goal of astronomy he mentioned earlier? Evidently, it is making the naturally *phronimos* part of the soul useful, rather than useless, i.e., turning the soul toward truth and being. But we saw in the case of astronomy that to accomplish this goal Plato did not recommend abandoning astronomy. Rather he recommended practicing astronomy correctly by making use of problems, i.e., confirming its hypotheses up to the unhypothetical *archê*, and by leaving alone the things in sky, i.e., not making inappropriate use of sensible objects. The current passage suggests, however, that Plato may recognize some mathematical disciplines whose very nature prohibits them from accomplishing this goal by their correct use. And it is for this reason that he excludes them altogether from the curriculum of the future philosopher-rulers. By contrast, however, Plato evidently thinks that astronomy and harmonics, though they are not currently practiced correctly by their practitioners, can be practiced correctly so as to achieve the goal of turning the soul toward truth by using/ascending to problems and not inappropriately using sensible objects.

⁴⁸I owe this point, like many others in this paper, to (Burnyeat 2000:15–18) careful and insightful exegesis of *Republic* 7 530c-e. Burnyeat suggests that Plato may be excluding mathematical mechanics and mathematical optics.

Of course, Burnyeat, for one, might not disagree with much of what I have maintained concerning the practitioners of harmonics and astronomy. Indeed, much of it was encouraged by my reading of Burnyeat. Nevertheless, he will point out that none of this suggests that Plato has a target for the Line's critique of arithmetic and geometry. But, we must be careful here. Plato's critique of astronomy and harmonics here in Book 7 is not that some astronomers and harmonists practice their discipline incorrectly. Rather it is that they all do. Consequently, when he encourages the practice of astronomy and harmonics as part of the philosopher-rulers' education he is quick to point out that he does not mean the way it is practiced in his day. For he maintains that no one in his day is practicing it correctly. Rather, he means the way it should be practiced. So, when Plato fails to critique arithmetic and geometry in a similar way in the course of describing the philosopher-rulers' education, we should not take that as evidence that all arithmeticians and geometers are practicing their disciplines correctly - which is both *prima facie* implausible and in conflict with Plato's claim in the *Euthydemus* that in every pursuit most of its practitioners practice it incorrectly, as well as with Plato's assessment of Hippias and perhaps Theodorus.⁴⁹ Rather he is simply crediting those arithmeticians and geometers who are practicing their discipline correctly, and recommending their example - perhaps Theaetetus and Eudoxus.

So, there are plausible targets of Plato's critique of mathematics in L3. Not Euclid, for obvious reasons, nor those mathematicians whose work amounted to early drafts of Euclid either - perhaps Eudoxus, Theaetetus, and others. Nevertheless, plausible targets remain - Hippias, perhaps Theodorus, Pythagoreans like Archytas and Philolaus, and a host of others whose failure

⁴⁹See also Socrates' claim at 523a1-4 that 'no one uses [arithmetic] correctly, as being fitted for drawing one toward being' (χρησθαι δ' οὐδεις αὐτῷ ὀρθῶς, ἐλκτικῷ ὄντι παντάπασιν πρὸς οὐσίαν). Of course, if we take Plato's use of 'no one' literally, then arithmetic will be no different than astronomy and harmonics. Indeed, if one takes Plato to be maintaining that all of the practitioners of geometry are laughable (γελοῖως) at 527a1-b2, then geometry too will be no different than astronomy and harmonics. But this will provide no comfort to Burnyeat and the mathematical reading of the Line. (For Burnyeat's reading of γελοῖως at 527a6, see (Burnyeat 1984:219).) It makes no difference to the methodological reading of the Line whether all or some of the practitioners of the mathematical disciplines practice their disciplines incorrectly. What is important for the methodological reading is simply that they can be practiced correctly or incorrectly, and that Plato disparages the incorrect practice and recommends the correct practice.

to practice their disciplines correctly might very well have consigned them to obscurity. Moreover, Plato does distinguish between finding fault with the discipline itself and finding fault with its practitioners. Finally, in the case of those disciplines whose goal is to lead the soul to truth and being, Plato indicates that many of their practitioners are at fault for something like not pursuing their hypotheses all the way to the unhypothetical *archê*, [A3], and for inappropriately using sensible objects, [A1], although precisely what these faults amount to is not developed in the course of Plato's description of the propaedeutic disciplines.

Epistemology and Methodology

We can come to understand these faults more fully by turning to Burnyeat's second point that even if targets of Plato's critique could be found, Plato's critique does not concern the method of the mathematicians, but their epistemological rashness. This seems to me to be right, but to see why we will need to return to the two failures of dianoetic according to the methodological reading of the Line with which we began.

Recall that, of the two, the failure of pursuing one's hypotheses all the way to an unhypothetical *archê* was easier to diagnose. As Socrates is made to elaborate this first failure,⁵⁰ those individuals who apply the dianoetic method of L3 (e.g., geometers, arithmeticians, and the like) make hypotheses (e.g., the odd and the even, the various figures, and the like) as though they knew them (ὡς εἰδότες), not thinking it necessary to give a *logos* of them (οὐδένα λόγον οὔτε αὐτοῖς οὔτε ἄλλοις ἔτι ἀξιοῦσι περὶ αὐτῶν διδόναι), as though they were clear to all (ὡς παντὶ φανερῶν). Under these conditions, the dianoeticicians begin from such hypotheses (ἀρχόμενοι) validly⁵¹ going through the steps until they reach the *teleutê*, i.e., an answer to the question with which they began (τελευτῶσιν ὁμολογουμένως ἐπὶ τοῦτο οὗ ἂν ἐπὶ σκέψιν ὁρμήσωσι; 510d2-3). Plato contrasts these dianoeticicians with dialecticians at 511b3-8. According to Socrates, dialecticians make their hypotheses not as first principles, but as really

⁵⁰For an account of the structure of Socrates' elaboration of the difference between L3 and L4 see (Benson forthcoming).

⁵¹See (Benson forthcoming) and n. 14 above.

hypotheses (τὰς ὑποθέσεις ποιούμενος οὐκ ἀρχὰς ἀλλὰ τῶ ὄντι ὑποθέσεις), like stepping stones (οἷον ἐπιβάσεις τε καὶ ὀρμάς), in order to reach the unhypothetical *archê* of everything (ἵνα μέχρι τοῦ ἀνυποθέτου ἐπὶ τὴν τοῦ παντὸς ἀρχὴν ἴω). Having grasped this *archê* dialectic comes down to a *teleutê* (πάλιν αὖ ἐχόμενος τῶν ἐκείνης ἐχομένων, οὕτως ἐπὶ τελευτὴν καταβαίνει; 511b7-8).

Notice that the contrast between dianoeticians and dialecticians does not reside in arguing (validly) to a conclusion, i.e., a *teleutê*. Rather it resides in how they treat the starting points of those arguments, i.e., how they treat their hypotheses. Dianoeticians treat them as *archai*,⁵² as already known, as not needing a *logos*, as already clear to all,⁵³ when they are not. Dialecticians treat them as the genuine hypotheses that they are - as assumptions from which one can derive the *teleutê* but which in order to be known require confirmation up to the genuine *archê* of everything. That is, dianoetic does not treat its hypotheses as requiring confirmation as it should, while dialectic does. So understood, Burnyeat is quite right in understanding Plato's criticism of the mathematically inclined dianoeticians as epistemological rashness. They too hastily take their hypotheses to be known, when they are not. They have not yet been derived from (or seen in light of) the unhypothetical *archê*.⁵⁴ Indeed, we have seen that some individuals whom Plato recognizes as plausible mathematicians are susceptible to such rashness. Certainly, Hippias is susceptible to hasty knowledge claims, while Theodorus (who may not take himself to know the mathematical theorems he had 'proved' up to 17) compares poorly to his student Theaetetus who proceeds to a higher, more general, hypothesis. So, when Plato finds fault with contemporary astronomers and harmonists for not ascending to problems, here too we should understand Plato's critique to amount to taking a proposition in need of proof or justification as having already been proven or established.⁵⁵ So, epistemological rashness looks like a good way of

⁵²See 511c6-7 in Glaucon's summary.

⁵³See *Republic* 7. 533c1-3.

⁵⁴Perhaps the Form of the Good which gives the power to know to the knower (τῶ γινώσκοντι τὴν δύναμιν ἀποδιδόν τὴν τοῦ ἀγαθοῦ ἰδέαν φάθι εἶναι; *Republic* 6 508e2-3)

⁵⁵See (Lennox 1994:58): "In the dialectical context of the *Topics*, a statement might be referred to either as a premise or a problem, depending on whether it was a commonly accepted assumption of debate or a statement being debated. In the proof context of the *Analytics*, a statement might be referred to as either a conclusion or a problem depending on whether it was derived from premises or is in need of such derivation."

characterizing the inferiority of the dianoetician to the dialectician, at least with respect to this first failure.

Before turning to the other failure of dianoetic, understanding the previous failure in the way I have suggested opens a middle ground between dianoetic and dialectic as described in the Line. One need not be seeking and certainly one need not already have arrived at the unhypothetical *archê* of everything in order to avoid falling into dianoetic. Rather, one simply needs to recognize that until one arrives at this *archê*, the theorems one has established by means of one's hypotheses are preliminary, unknown, and in need of further confirmation. Indeed, a mathematician - Euclid or Eudoxos or Theaetetus perhaps - who realized his elements were merely hypotheses until they had been derived from the unhypothetical *archê* would be practicing dialectic, or at least not dianoetic in its unfavorable guise. He might, however, simply choose to leave this higher pursuit of deriving his elements from the unhypothetical *archê* to others. The point is that one can engage in the method of mathematics, i.e., the method of hypothesis, without engaging in blameworthy dianoetic, on the one hand, or pure dialectic, on the other. One can apply the method of hypothesis without taking one's hypotheses as known and not in need of further confirmation and without having derived those hypotheses from the unhypothetical *archê*. We might be inclined to call this middle ground 'dialectic' in one of its guises, i.e. *qua* method of acquiring robust philosophical knowledge, as opposed to dialectic as the finished product of the philosopher-rulers' education.⁵⁶ We will be inclined to do so insofar as we take the formal education of the philosopher-ruler beginning at age 20 to represent dialectic in one of its guises. On the other hand, we might be inclined to call this middle ground 'philosophical dianoetic', as Plato indicates at *Republic* 7 527b9-12. Recognizing this middle ground between dianoetic and dialectic as described in the Line provides an alternative reading of the two passages from *Republic* 7 that appeared to indicate an identity of dianoetic and mathematics.⁵⁷ In these passages, Plato is not identifying dianoetic of the Line with mathematics. Rather, he is

⁵⁶See n. 23 above.

⁵⁷*Republic* 527b9-12 and 533d1-7. See p. 000 above.

distinguishing the mathematical disciplines when practiced correctly from pure dialectic which aims to confirm the hypotheses of mathematics, among other things. But this is not the distinction Plato means to be drawing in the Line.

What about dianoetic's failure to use ordinary sensible objects in an appropriate way? As we have said above, this failure of dianoetic is considerably harder to diagnose. All Plato appears to tell us about this failure in the discussion of the Line is that dianoetic 'uses ordinary sensible objects' in the process of giving a *logos* of something else. When he elaborates on this feature at 510d5-511a2, he adds only that the dianoetician uses these objects to think and talk about Forms, e.g., the square itself and the diagonal itself.⁵⁸ In contrasting dianoetic with dialectic in this respect at 511b7-c2, we are simply told that dialectic uses only Forms in proceeding to the *teleutê*. Consequently, this feature of dianoetic has commonly been taken to be referring to mathematics' use of sensible diagrams and constructions. But it is difficult to see why the use of sensible diagrams and constructions should be thought to be problematic.

As Burnyeat points out and Plato appears to recognize at 527a6-b1, the use of diagrams and constructions is plausibly essential to the practice of mathematics.⁵⁹ So, like its use of the method of hypothesis, mathematics can hardly be criticized for doing what it must do. Moreover, being critical of the use of ordinary sensible objects in one's pursuit of knowledge of Forms is at odds with those passages in which Socrates is made to encourage their use in this pursuit. Recall that in the recollection argument in the *Phaedo* (73b3-77a5), Socrates indicates the value of perceiving equal sticks and stones in pursuit of the knowledge of the Form of Equality through recollection. They appear to serve as some sort of catalyst in the recollection process. Moreover, in the course of describing the educational process of the philosopher-rulers in *Republic 7* (523a1-524d6) Socrates famously distinguishes those features of ordinary sensible objects which do not turn the soul to towards truth and knowledge, and those that do. Miriam Byrd has

⁵⁸I here assume that Plato does not have in mind mathematical intermediates, but no part of my general thesis depends on this assumption.

⁵⁹See (Burnyeat 1987:219) and (Burnyeat 2000:40–41). See nn. 19 and 49 above.

helpfully dubbed such sensible objects or properties as ‘summoners’.⁶⁰ Plato appears not to find fault with the mere use of ordinary sensible objects, and but rather with their inappropriate use (as I have been describing this failure throughout).⁶¹ Consequently, in my earlier paper and above, I described this failure as roughly mistaking contingent or artificial consequences of one’s hypothesis for genuine or natural consequences. The idea is that, in addition to attempting to derive one’s hypothesis from a higher more secure hypothesis, Plato’s method of hypothesis also requires testing one’s hypothesis against the empirical evidence. One searches out apparent counter-evidence.⁶² But one should not too hastily take the occurrence of such counter-evidence to suffice for rejecting one’s hypothesis.⁶³ Rather one needs to determine whether the counter-evidence genuinely derives from the hypothesis.⁶⁴ For example, one might take the hypothesis that the circumference of a circle is equivalent to two times the product of π and the radius (or the hypotheses from which this hypothesis is derived) to be refuted by measuring the circumference of a given drawn (or otherwise instantiated) circle. But to do so would be an inappropriate use of sensible objects. The length of the circumference of a circle based on physical measurements is not to be explained by the nature of a circle, but rather by various contingent features of the measurement apparatus and the drawn circle. This apparent counter-evidence, i.e., the measured length of the circumference, does not refute the hypothesis because it is not a natural or essential

⁶⁰(Byrd 2007)a and (Byrd 2007)b.

⁶¹None of this is of course to deny the apparently purely *a priori* side of Plato’s epistemology found especially in the beginning pages of the *Phaedo*. This only serves to confirm my current point. Plato appears ambivalent about the role of the senses in knowledge acquisition. Sometimes he is harshly critical to the point of forbidding any role to perception; sometimes he recognizes its role in stimulating and reinforcing knowledge acquisition. I propose that this ambivalence can be explained at least partially by understanding Plato to be recommending the appropriate use of perception and discouraging its inappropriate use. Of course, the devil is in the details - explaining precisely the distinction between appropriate and inappropriate use. See also the interpretation of Platonic ‘real astronomy’ of (Vlastos 1980), as opposed to (Mourelatos 1980).

⁶²See, for example, the apparent counter-evidence of the lack of teachers of virtue to the hypothesis that virtue is knowledge in the *Meno* and the viciousness and uselessness of philosophers to the hypothesis that philosophers are rulers and rulers philosophers in *Republic* 6. For more on these passages see (Benson 2003) and (Benson 2008).

⁶³As Socrates does in the *Meno*.

⁶⁴Or, from the nature of the object or objects the hypothesis is about, as Socrates does in *Republic* 6 at 487d-497a. See (Benson 2008).

consequence of the hypothesis.⁶⁵ It has been objected to such an understanding of the second failure of the dianoetic method that no geometer worth her salt would make such a mistake, certainly not Euclid.⁶⁶ With this objection I agree. But I have been at pains to argue in this essay that it is not Euclid and other accomplished geometers or mathematicians that Plato has in mind as his targets when finding fault with dianoetic. It is rather unaccomplished geometers and mathematicians who may be just the ones inclined to make such a mistake. Consequently, here too the failure of the dianoetic method is more like epistemological rashness, than methodological failure. It is a too hasty reliance on the evidence of one's senses.

Conclusion

In conclusion, one might wonder why Plato should devote such a prominent position to the dianoetic method, if it is simply the method of unaccomplished mathematicians, as methodological interpretation of the Line suggests. The image of the Line is one of a handful of the most enduring images in Plato (whether Plato anticipated this or not) and yet if the methodological reading of it is correct its primary purpose appears to be to distinguish Plato's recommended method of knowledge acquisition (dialectic) from a method of knowledge acquisition practiced by unaccomplished mathematicians. Why should Plato be concerned to do that?

The image of the Line provides part of Plato's answer to the question "what is the greatest *mathêma* (μέγιστον μάθημα)?" which every philosopher-ruler must acquire. Socrates' initial answer to this question is straightforward: The greatest *mathêma* is the Form of the Good (ἡ τοῦ ἀγαθοῦ ἰδέα; *Republic* 6 505a2). But, when pressed to say what the Good is, Socrates professes ignorance and resorts to images. In the first image, Socrates focuses on the subject

⁶⁵Making precise this distinction between the contingent counter-evidence and the natural or essential counter-evidence is a difficult matter and beyond our present concerns, but for some start in the direction see (Benson forthcoming).

⁶⁶I owe this objection to a valuable correspondence with Jerry Santas. Perhaps a more sophisticated example would make the mistake easier to fathom. Consider the '64=65 Geometry Paradox' (<http://brainden.com/forum/index.php?/topic/139-64-65-geometry-paradox/>) which might be taken as providing conflicting consequences of the commutative principle. I owe this example to Jon Kvanvig.

matter of this *mathêma*, while in the second and third he focuses on its method and acquisition. This method, however, was not a Platonic invention, but a method borrowed and perfected by Plato, just as the method of short question and answer refutation was not invented by Socrates but borrowed and perfected by him. In describing this latter method in the elenctic dialogues Plato was especially concerned to distinguish its correct application from its application by sophists, rhetors, and eristics.⁶⁷ The differences were subtle, as Socrates testifies in the *Apology*, but profound. So here in the *Republic*, as Plato describes the method associated with philosophy and the greatest *mathêma*, Plato is concerned to distinguish its correct application from its application by those who misused it. Given the care with which Plato draws this distinction in the case of the *elenchus* in the elenctic dialogues, it should not surprise us that the distinction in the case of the method of hypothesis should get Plato's attention in these middle books of the *Republic*. The issues of the nature of philosophical method are complex and often obscure. And Plato is right to devote considerable attention to detailing its correct and incorrect applications.

Thus, in Plato's image of the Divided Line, Plato is not objecting to mathematics or the mathematicians for employing the method of hypothesis or for making use of diagrams and constructions. These are essential features of mathematics as such, and mathematics is not flawed for doing what it must do. Nevertheless, the method of hypothesis, as well as the appropriate use of ordinary sensible objects are also an essential features of the method of philosophy, and the greatest *mathêma*. Thus, when Plato distinguishes between dianoetic in L3 and dialectic in L4 Plato is not distinguishing between two different methods, the method of mathematics and the method of philosophy. Nor is he distinguishing between a method that can pursue an ontological inquiry and one that cannot. Rather, he is distinguishing between two different applications of the same method, the incorrect application of many contemporary

⁶⁷See, for example, *Gorgias*, *Protagoras*, and *Euthydemus*, as well as the *Apology*. Plato may have thought that incorrect applications of the method were a consequence of character flaws possessed by the practitioners - pomposity, conceit, competitiveness, etc., and Plato may be more focused on the character flaws in these dialogues than on the resulting misapplications. But what is salient for my present purposes is that Plato allowed for the correct and incorrect practice of philosophy, as well as correct and incorrect practice of mathematics, and was concerned to explain the difference.

mathematicians and the correct application of accomplished mathematicians and budding philosopher-rulers. Of course, none of this shows that Plato would not reserve the ontological inquiry for pure dialectic. Here again, I think, we are indebted to Burnyeat's careful analysis. But in doing so, Plato is not indicating that the *method* of mathematics is incapable of pursuing such an ontological inquiry. Rather, the claim is descriptive rather than prescriptive. Mathematics (when it is contrasted with pure dialectic) or, perhaps better, philosophical dianoetic pursues its inquiry only so far, recognizes that its procedure is incomplete, and so hands over its results to the pure dialectician. But the problem here - if that is the right word⁶⁸ - is not with mathematics or its method, but with the mathematician.⁶⁹

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⁶⁸The issue here is not really a problem, but a predilection. A Euclid, who turns over his hypotheses to the pure dialectician to confirm by means of a complete ontological inquiry and concentrates on filling out the theorems of the discipline of geometry, is not to be disparaged, any more than a cobbler who sticks to his or her cobblery. He is simply displaying his proclivity for geometry, rather than the more ethereal realms of ontology. We might describe such a Euclid as practicing philosophical dianoetic to distinguish what he is doing from the pure dialectician who has arrived at the unhypothetical *archê* as well as from a Euclid who takes his hypotheses to be clear and known to all. This latter Euclid is to be disparaged as a practitioner of pure dianoetic, perhaps like the craftsmen of the *Apology* who take themselves to know other great things that they do not because of the knowledge that they do have; see *Apology* 22d6-7.

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