Temporal Clutter Filtering via Adaptive Techniques

1 Learning Objectives:

- *Students will learn about how to apply the least mean squares (LMS) and the recursive least squares (RLS) algorithm in order to build an adaptive digital filtering architecture that will remove clutter from radar returns. Complexity and convergence comparisons will be made for the two techniques.*

- *Students will learn how differentiation between the clutter and the useful signal is obtained by exploiting the different auto-correlation functions of the two signals. Students will learn how to devise a cost function so that the clutter may be recursively miminized on a sample by sample basis (i.e., for each point that comprises a range gate's spectrum).*

- *Most adaptive algorithms rely on using a statistical framework. Students will be exposed to new techniques in which theory necessary to consider adaptive signal processing using a recursive least squares algorithm – which does not depend on the ensemble statistics of a signal. This unique feature allows a broad class of signals to be filtered without regard to a particular signal model.*

- *After this module and classroom discussions, the student should also be in a position to describe various sources of ground based and sea based clutter. The statistical distributions of each are different and do influence the design of radar's receiver.*

2 Introduction

Radar (RAdio Detection And Ranging) was refined during World War II to counter enemy military forces, particularly airborne forces. Broader utility of radar was quickly recognized and the technology was soon applied to civilian aviation to meet its growing requirements. As the technology matured it became evident that radar could also be used in surveillance of weather phenomena [1], aiding immeasurably the meteorological and aviation safety communities. There have been many significant improvements to aircraft and weather radar systems since their initial fieldings that benefit mankind. As eloquently articulated in the passages of the book titled *Engineer of the 2020* [2], students will be expected to have a better understanding of the "natural world." Although natural disasters are beyond man's control, man's ability to predict them and adapt accordingly are essential to minimize impact, especially with observing systems such as radar.

2.1 A Discussion of Noise Cancellation and Adaptive Filters

This section summarizes a progression of the development of the adaptive filters. It is well known that the Wiener filter is the optimum filter for determining a desired signal in the mean squared sense, assuming that the signal is stationary. Here the objective is to explain how the Wiener filter structure may be augmented so that a filter architecture may be developed that is suitable for an adaptive digital filter.

The purpose of the Wiener filter is to calculate an estimated value of a desired signal. Given the noise cancelation scheme in Fig. 1, if $n_1(k)$ may be estimated by $\hat{n}_1(k)$ from $n_2(k)$, then this estimate may be subtracted from $s(k)$ to yield an estimate of the desired signal. If the signal is stationary, then the Wiener filter is the best choice since it is the optimum filter for estimating the signal.

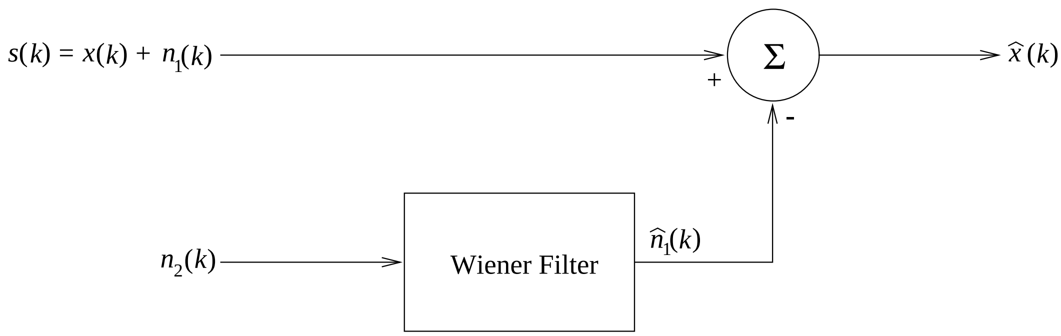

Figure 1: *Noise Cancelation, Case 1.*

This noise cancelation scheme is suitable provided: (1) $x(k)$, $n_1(k)$, and $n_2(k)$ are stationary, (2) the statistics about $n_1(k)$ and $n_2(k)$ are known, and (3) $n_2(k)$ is a noise signal that is correlated with $n_1(k)$ [3]. The block diagram depicted in Fig. 2 illustrates an architecture that is different from the previous for the following two reasons: (1) it will accommodate signals which are non-stationary, (2) the statistics of $x(k)$, $n_1(k)$, and $n_2(k)$ need not be known other than the fact that $n_2(k)$ must be uncorrelated with $x(k)$. $n_2(k)$ is a reference signal needed to estimate $n_1(k)$. The cabins of fighter jet aircrafts have employed such a two sensor scheme in some instances. One microphone is used to detect the pilot's voice, while another microphone is employed to capture the noise in the cabin of the aircraft.

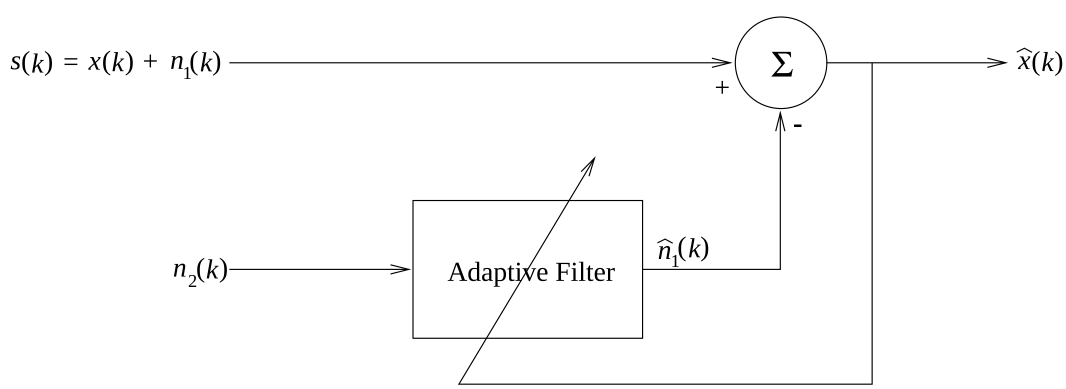

Figure 2: *Noise Cancelation, Case 2.*

When source $n_2(k)$ is not available, the block diagram in Fig. 3 depicts the final variation of the filter architectures that will account for the absence of a second sensor. The single sensor will capture both the desired signal and the noise. The purpose of the signal path that contains the delay element is to generate a reference signal that is used to estimate $\hat{x}(k)$. This architecture has the following characteristics: (1) it does not require any statistics about the desired signal, (2) the desired signal need not be stationary, and (3) it only requires one input source, which makes it suitable to be used as a compact, low-complexity filter. The Wiener filter does require the statistics about the signal in order to compute its coefficients. Moreover, even though the Wiener filter is the optimum filter, it does require computing the inverse of the sum of the signal autocorrelation and noise autocorrelation matrices to determine its coefficients – this is a very costly mathematical operation for real time applications.

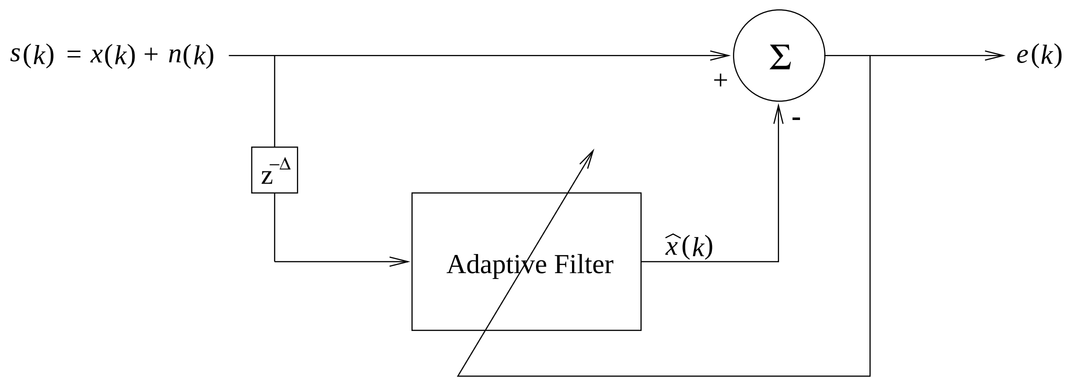

Figure 3: *Noise Cancelation, Case 3.*

Another difference between this filter architecture and the previous two is the fact that a second sensor does not exist. However, if the input signal is delayed then a reference signal can be produced. Thus if two signals, $x(k)$ and $x(k-\Delta)$, are correlated, then $x(k)$ may be estimated by $\hat{x}(k)$ from $x(k-\Delta)$ [3]. The signal path with the delay element is important to provide a reference signal, and Δ ensures that $n(k)$ and $n(k-\Delta)$ are not correlated. It is also assumed that $n(k)$ is not correlated with $x(k)$. By minimizing the mean squared error of $e(k)$, $\hat{x}(k)$ will be the best estimate of $x(k)$ as evident from the mathematics below.

$$E[e^2(k)] = E[(x(k) - \hat{x}(k) + n(k))^2] \tag{1}$$

$$E[e^2(k)] = E[(x(k) - \hat{x}(k))^2 + 2(x(k) - \hat{x}(k))n(k) + n^2(k)] \tag{2}$$

$$E[e^2(k)] = E[(x(k) - \hat{x}(k))^2] + 2E[(x(k) - \hat{x}(k))n(k)] + E[n^2(k)] \tag{3}$$

Assuming $E[x(k)n(k)] = 0$ and $E[n(k)n(k-\Delta)] = 0$, then

$$2E[(x(k) - \hat{x}(k))n(k)] = 0. \tag{4}$$

Consequently, the mean squared error is

$$J = E[e^2(k)] = E[(x(k) - \hat{x}(k))^2] + E[n^2(k)] \quad . \tag{5}$$

Minimizing J is equivalent to minimizing $E[(x(k) - \hat{x}(k))^2]$. Therefore, when $x(k)$ and $\hat{x}(k)$ are random processes, minimizing J will cause $\hat{x}(k)$ to be the minimum mean-square estimate of $x(k)$ [3].

Estimating $\hat{x}(k)$ depends on several factors including the type of filter, either IIR or FIR, and the strategy of how the cost function is to be minimized, be it either least mean squares or recursive least squares [3]. For this laboratory, the type of filter will be either FIR or IIR, and it will minimize J predominately based on the least mean squares or recursive least squares algorithm. It should be noted that the signals to be processed are random, since a specific function that defines the signal does not exist. Historically, the least squares filter has been described as a deterministic filter since each individual incoming sample influences the filter coefficients, whose values are a function of time. Therefore, the filter coefficients are not a function of ensemble averages, as is the case for least mean squared error filters. Minimizing the mean squared error between two stationary stochastic processes will yield one particular filter. This filter may be used to minimize the difference between many realizations of the stationary stochastic process. However, a recursive least squares filter will provide a unique filter for each realization of the stationary stochastic process. Therefore, the algorithm will produce different filters for random signals having the same ensemble statistics [3]. Despite their computational complexity, the main reason that recursive least squares filters are selected is because they converge about an order of magnitude faster than their least mean squares counterparts – this makes them more suitable for non-stationary signals.

Expressed from another point of view, the solution to the least-squares optimization problem is considered to be deterministic, since no statistical information about the input signal is needed [4]. In summary, the IIR recursive least squares filter presented in this work will not rely on the statistics of the signal, but it will update its coefficients as each new piece of sampled data is received.

3 Hands-On Activities

- Describe different types of clutter that may influence a radar's receiver. Discuss traditional ground clutter that influences ground based WSR-88D radars and sea clutter that influences the IPIX radar in Canada. Your discussion should be cast in light of the statistical distributions of the clutter and elevation angle of the radar's beam.

- Download this data:
  - www.ou.edu/radar/data0305100343.m
  - www.ou.edu/radar/data0305100355.m

- Download additional data from the Canadian IPIX radar:
  - http://soma.crl.mcmaster.ca/ipix/dartmouth/index.html

- By assuming that $\mathbf{v}$ describes a vector of inputs, i.e. $\mathbf{v}(i) = [x(i), x(i-1), ..., x(i-M)]^T$, its autocorrelation matrix may be expressed as:

$$\begin{aligned} \mathbf{r}_{vv}(k) &= \sum_{i=0}^{k} \mathbf{v}(i)\mathbf{v}^T(i) \\ &= \sum_{i=0}^{k-1} \mathbf{v}(i)\mathbf{v}^T(i) + \mathbf{v}(k)\mathbf{v}^T(k) \\ &= \mathbf{r}_{vv}(k-1) + \mathbf{v}(k)\mathbf{v}^T(k) \quad . \end{aligned} \tag{6}$$

  By using the *matrix inversion lemma* or Woodbury's identity, prove that the inverse of the autocorrelation matrix may be recursively calculated and written as:

$$\mathbf{r}_{vv}^{-1}(k) = \mathbf{r}_{vv}^{-1}(k-1) - \frac{\mathbf{r}_{vv}^{-1}(k-1)\mathbf{v}(k)\mathbf{v}^T(k)\mathbf{r}_{vv}^{-1}(k-1)}{1 + \mathbf{v}^T(k)\mathbf{r}_{vv}^{-1}(k-1)\mathbf{v}(k)} \quad . \tag{7}$$

- Design an adaptive FIR filter for Figure 3 that will minimize $E[(x(k) - \hat{x}(k))^2]$. Do this for both an LMS and RLS system. For the RLS system, assume that past inputs are exponentially weighted by a forgetting factor known as λ. Let the weight vector that defines the filter be described by $\mathbf{w} = [b_0, b_1, ..., b_M]^T$. Hint: for the RLS case, it is noted that:

$$\begin{aligned} \mathbf{w}(k) &= \mathbf{w}(k-1) - \left[\frac{\mathbf{r}_{vv}^{-1}(k-1)\mathbf{v}(k)}{\lambda + \mathbf{v}^T(k)\mathbf{r}_{vv}^{-1}(k-1)\mathbf{v}(k)}\right]\mathbf{v}^T(k)\mathbf{w}(k-1) \\ &\quad + s(k)\left[\frac{\mathbf{r}_{vv}^{-1}(k-1)\mathbf{v}(k)}{\lambda + \mathbf{v}^T(k)\mathbf{r}_{vv}^{-1}(k-1)\mathbf{v}(k)}\right] \quad . \end{aligned} \tag{8}$$

- Using your adaptive FIR filter that is govened either by an LMS or RLS algorithm, devise a technique in which clutter may be suppressed from a radar's received signal. Assume that the data are in terms of I and Q samples, which are organized as range gates. Use the data from above. State all assumptions, results, and convergence properties of your filters.

- BONUS: Following the same design requirements above, design an adaptive IIR filter for Figure 3 that will minimize $E[(x(k) - \hat{x}(k))^2]$ based on the LMS and RLS algorithms.

References

[1] R. Doviak and D. Zrnic, *Doppler Radar and Weather Observations*. Dover Publishers, Mineola, N.Y. 1150pp., 2006.

[2] National Academy of Engineering, *The Engineer of 2020*. National Academy Press, 2004.

[3] Monson H. Hayes, *Statistical Digital Signal Processing and Modeling*. New York: John Wiley and Sons, 1996.

[4] James V. Candy, *Signal Processing, the Model Based Approach*. New York, New York: McGraw-Hill Book Company, 1986.