To Log or Not to Log: Bootstrap as an Alternative to Parametric Estimation of Moderation Effects in the Presence of Skewed Dependent Variables

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Abstract

When gross deviations from parametric assumptions are observed, conventional data transformations are often applied with little regard for substantive theoretical implications. One such transformation involves using the logarithm of positively skewed dependent variables. Log transformations were shown to severely decrease estimates of true moderator effects using moderated regression procedures in a Monte Carlo simulation. Estimates of moderator effect sizes were substantially better estimates of the true latent moderator effect (i.e., larger by a multiple of 2.6 to 534) when estimated using a simple percentile bootstrap procedure in the original, positively skewed data. Conclusions regarding the presence or absence of a true moderator effect using a simple bootstrap procedure were unaffected by violation of parametric assumptions in the original, positively skewed data. In contrast, conclusions when moderated regression analysis was performed on a log transformed dependent variable severely increased Type II error. Implications are drawn for applied psychological and management research.

To Log or Not to Log: Bootstrap as an Alternative to Parametric Estimation of Moderation Effects in the Presence of Skewed Dependent Variables

At one time or another almost all investigators in applied psychological and management research have been concerned by assumptions required of common parametric statistical tests. Investigators typically assume their samples were drawn from a single population and rely on the power of the Central Limit Theorem and other parametric assumptions to draw inferences about latent relationships within that population. When violations of parametric assumptions are severe, investigators often use some data transformation designed to minimize the violation. For example, all three empirical studies reported in a recent special Academy of Management Journal forum on managerial compensation performed log transformations on compensation data (Conyon & Peck, 1998; Finkelstein & Boyd, 1998; Sanders & Carpenter, 1998) with no mention of the purpose or rationale behind these transformations. Presumably the log transformations were done to address the presence of heteroscedasticity, i.e., the lack of independence between the mean of Y given X ($\overline{Y}|X_i$) and the variance of Y given X ($\sigma^2_y|X_i$) that coincides with extreme positive outliers or severe positive skew (Winer, 1974). Winer's 1974 text has had a pervasive influence on organizational research as reflected in the fact it is the most highly cited publication in the Social Science Citation Index between 1957 and 1997 (Bennett, 1999) -- it is difficult to underestimate the effect Winer's text (and its subsequent updates) has had on organizational researchers. It could be argued that performing log transformations on positively skewed dependent variables has become a "convention" within applied psychology and management research.

One of the following characteristics is required of studies using parametric ordinary least squares (OLS) procedures to examine linear relationships between variables X and Y: 1) X and Y are random bivariate normal or 2) X is "fixed" and e is normal, where $e_i = Y_i - b_i X_1 - b_0$. In the former case, X is "random" in the sense that investigators do not specify or control levels of X treatment effects in advance. Instead, X values observed occur at a frequency dictated by the population probability distribution for X. Common survey methods employed in research examining voluntary employee turnover (e.g., Mobley, Griffeth, Hand, & Meglino, 1979), job satisfaction (Smith, Kendal, & Hulin, 1969), performance prediction (Bray, Campbell, & Grant, 1974; Owens & Schoenfeldt, 1979) and executive compensation (Finkelstein & Boyd, 1998) provide examples of random effects designs. Importantly, when X and Y are distributed bivariate normal, probabilistic inferences (e.g., conducting hypothesis test of H_o: $\rho = 0$, or estimating confidence intervals $\left\{ r_{xy} \pm t_{\alpha/2}\sqrt{\frac{(1-r_{xy}^2)^2}{N-2}} \right\}$) can be drawn due to ρ's presence in the bivariate normal density function described in Equation 1 below.

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\frac{x-\mu_x}{\sigma_x}\frac{y-\mu_y}{\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]}$$

Equation 1

If X is not normally distributed[1], as in the latter case, one may still use the Central Limit Theorem to assume $\sigma_y^2|X_i$ is normally distributed in order to test hypotheses about ρ. In these circumstances, X is often a "fixed effect" that takes on values occurring in some known frequency other than what one would have expected if values of X were drawn at random from the population (e.g., values of X the investigator selected for purposes of manipulation).

Importantly, regardless of study design, traditional parametric procedures cannot be used in conducting hypothesis tests or estimating confidence intervals if the true probability density function for prediction error (e) is unknown.

As noted above, one common violation of parametric assumptions occurs when the variance of Y given X ($\sigma_y^2|X_i$) is a function of the conditional mean ($\overline{Y}|X_i$). Efforts examining severely positively skewed Y distributions routinely occur in applied psychological research, particularly compensation research. Skewed compensation distributions are caused by a number of factors including the increasing span of pay ranges as the pay range mid-point increases (England & Pierson, 1990)[2] and the extreme levels of executive compensation typically reported in U.S. corporations. Both factors result in a lack of independence between $\overline{Y} \mid X_i$ and $\sigma_y^2 \mid X_i$, violating the homoscedasticity assumption (Winer, 1974).

Winer (1974, pp. 398-401) described a number of transformations that "correct" or at least lessen violations of some parametric assumptions. Log transformations of variables demonstrating highly positive or negative skew yield a more bell-shaped frequency distribution, where $\overline{Y}|X_i$ and σ_e are relatively uncorrelated. Winer (1974, p. 400) noted log transforms are particularly effective in stabilizing conditional variance of Y given X when independence of error terms is violated due to $\sigma_{y_i}^2 = k^2\overline{X}_i^2$, or when Y has a great deal of positive skew (Olds, Mattson, & Oldeh, 1956).[3]

The usual effect of such transforms is to lessen prediction error for values of Y occurring at the extreme "tail" of the positively skewed dependent variable, consequently increasing r_{xy}^2 in additive models used to predict log Y. The <u>SYSTAT6.0 for Windows: Statistics</u> manual described one such example (SPSS, Inc., 1996, pp. 252-257), where gross domestic product

(GDP) per capita (X) was used to predict military spending (Y) in a sample of 57 countries. In this example, r^2_{xy} goes from .417 to .734 in the presence of log transformation.

Importantly, the resultant model using the transformed data is $Y_{mil\$} = 10^{\beta_0 + \beta_1 \log_{10} X_{GDP} + \log_{10} e}$, which does not technically adhere to OLS characteristics (e.g., unbiased minimum variance parameter estimates). This model is perfectly serviceable if prediction is the investigator's main concern -- inferences about accuracy of prediction can be drawn from r_{xy}. Note, <u>probabilistic</u> inferences cannot be drawn for r_{xy}, b_o, b_1, or $\hat{Y}$ unless one assumes the $\log_{10}e$ term in $Y_{mil\$} = 10^{\beta_0 + \beta_1 \log_{10} X_{GDP} + \log_{10} e}$ is normally distributed. We are aware of no research stream (theory-based or otherwise) that holds the $\log_{10}$ of e is normal.[4] Regardless, the model must have some theoretical meaning if explanation is the investigator's main concern. For example, it is unclear what theory or policy implications should be drawn from finding the log of salary is differentially related to organizational tenure for men and women.[5] The authors are unaware of any studies examining interactive models providing a theoretical rationale justifying nonlinear (monotonic or nonmonotonic) transformations in applied psychological or management research (though concepts like the diminishing marginal utility of money may provide such a rationale in the future). Enhanced statistical elegance achieved via nonlinear transformations has not been accompanied by theory-based rationale justifying its use.

Nonlinear transformations can cause more uncertainty in interpreting tests of moderation than they resolve. Investigators generally need to estimate sample sizes required for replications and extensions of past research. Investigators examining previously reported data on the GDP-military spending relationships will solve $r_{xy} = t_{\alpha/2}\left(\sqrt{\frac{(1-r^2_{xy})}{N-2}}\right)$ for N, estimating they need a sample size when $r^2_{xy} = .417$ that is approximately four times as large as the sample required when $r^2_{xy} = .734$ at $\alpha = .05$. Again, absent theoretical rationale, arguments can be mounted for either estimate.

Importantly, nonlinear (monotonic and nonmonotonic) transformations of original data create a number of problems for OLS applications used to detect moderator effects. Busemeyer and Jones (1983) demonstrated monotonic transformations could be found that cause Y values generated from a truly additive model (e.g., $\hat{Y} = b_0 + b_1X_1 + b_2X_2$) to yield support for a multiplicative model (e.g., $\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_1X_2$) and vice versa. Hence, reports of significant and nonsignificant interaction effects after having performed log transformation on Y remain open to alternative interpretation (cf. Henderson & Fredrickson, 1996; Sanders & Carpenter, 1998).

In sum, investigators often face circumstances where data is clearly not bivariate normal, e is not normal, and/or heteroscedasticity is present. Nonlinear transformations generate unknown levels of distortion in the many estimates of moderator effects required to test theories in management and applied psychology (Busemeyer & Jones, 1983; Russell & Bobko, 1992). Investigators continued use of nonlinear transforms to test moderator effects (e.g., Henderson & Frederickson, 1996; Kuhn & Sweetman, 1998; Sanders & Carpenter, 1998) will result in literatures characterized by "mixed" findings containing frequent Type I and II errors. This will be especially true when other investigators do not use nonlinear transformations in studying the same phenomena (e.g., Gomez-Mejia, Tosi, & Hinkin, 1987). Severe consequences for theory development will result.

The bootstrap is a relatively new method of empirically estimating characteristics of population distributions from sample data (Efron, 1979) that holds remarkable implications for these applied research issues. Unfortunately, Mooney and Duval (1993) noted "the bootstrap is . . . foreign to most social scientists schooled in the traditional parametric approach to inference" (p. 27). The current study briefly reviews the bootstrap literature and reports the results of a Monte Carlo simulation demonstrating how log transformations can yield spuriously low estimates of moderator effect sizes (i.e., ΔR^2). Finally, a bootstrap approach to detect interaction effects when authors would otherwise employ log transformations and traditional OLS techniques is presented and implications for applied psychological and management research offered.

Bootstrap Estimation Procedures

Bootstrapping holds promise as a statistical estimation technique yielding precise estimates of population distributions from sample data. Bootstrapping estimates the population distribution of a statistic (e.g., r_{xy}) by iteratively resampling cases from a set of observed data. Basically, B "bootstrap" samples of size N are taken <u>with replacement</u> from the original sample of size N and saved to a file. An investigation using B = 1,000 bootstrap samples of size N is able to approximate the actual sampling distribution that would have been obtained if multiple independent samples of size N were drawn from the population (Efron & Tibshirani, 1993).

There are many advantages to using the bootstrap technique. First, it is not restricted by the normality assumptions of parametric tests. The percentile bootstrapping method (Efron & Tibshirani, 1993, Chapter 13) generates information about the latent population distribution permitting estimation of confidence intervals (CI) directly from the bootstrapped sampling distribution (e.g., if B = 1,000 bootstrap samples are taken, the bootstrap correlations $\{r_b\}$ representing the 25th and 975th largest values constitute the lower and upper points of a 95% CI). Graphical interpretation of r_b frequency distributions also yields insight into characteristics of the latent population distribution (Efron & Tibshirani, 1993). When the sample is drawn from a population with a single value of ρ, the Central Limit Theorem dictates the r_b frequency distribution will rapidly approximate the normal distribution as B and N increase. A multi-modal r_b frequency distribution would suggest the sample was drawn from multiple populations, each with its own value of ρ. Second, information concerning the form of the original sample is retained, with no loss of distributional information. Rasmussen (1987) noted loss of information does occur when nonparametric techniques convert data to ranks. Lunneborg (1985) described bootstrapping as falling between parametric and nonparametric procedures for making probabilistic inferences.

Rasmussen (1987) presented the following simple example of a bootstrap procedure. Suppose a researcher wants to test the null hypothesis that $\rho_{xy} = 0$ between first year grade point averages (GPA) and Graduate Record Exam (GRE) scores using data obtained from 10 graduate students (H_O: $\rho_{GPA,GRE} = 0$). First, an initial bootstrap sample (B_1) is randomly drawn with replacement from these ten observations, yielding the possibility of some observations being represented more than once in the bootstrap sample while other observations may not be included. A single bootstrap sample may include the following cases: 5, 2, 8, 6, 2, 7, 9, 6, 1, and 2. Note, due to <u>random</u> sampling with replacement, case "2" was included more than once while case "3" was not included in this first bootstrap sample (B_1). The 10 cases may result in a correlation of, say, $r_{b_i} = .59$. This procedure is repeated a large number of times (e.g., B = 1,000) and each r_b is saved to a separate file. Second, the bootstrap correlations (r_b) are rank ordered with the 25th and 975th r_{b_i} correlations representing 95% confidence interval end points. Finally, the null hypothesis H_o: $\rho_{GPA,GRE} = 0$ is tested by determining whether 0 falls within the confidence interval (Rasmussen, 1987).

Studies examining similarities in results obtained from bootstrap and normal theory approaches when parametric assumptions are met test the bootstrap's ability to estimate true latent population distributions (e.g., Diaconis & Efron, 1983; Efron, 1985, 1986; Lunneborg, 1985). These studies resulted in bootstrap statistics (e.g., estimates of confidence intervals) that were extremely close to those generated from parametric approaches. Bickel and Freedman (1981; Freedman, 1981) demonstrated the bootstrap was asymptotically valid for many statistics with known population probability distributions (e.g., t and OLS regression statistics). However, the procedure is perhaps of most value in drawing inferences about statistics with unknown population probability distributions (e.g., medians, or "mixed" samples drawn from multiple populations).

Some issues remain unresolved in using bootstrapping to conduct hypothesis testing, most revolving around the relative accuracy of parametric versus bootstrap procedures in estimating probability intervals at the extreme tails of known (i.e., normal) distributions. However, the simple percentile bootstrap method of estimating confidence intervals described above provides "good theoretical coverage properties as well as reasonable stability in practice" (Efron & Tibshirani, 1993, p. 169). Good "theoretical coverage" refers to confidence intervals that 1) accurately estimate probability of the population parameter falling within the confidence interval <u>and</u> 2) divide "coverage error" equally across the two tails.[6]

Empirical comparisons of bootstrap and traditional OLS regression procedures' abilities to detect moderator effects when the dependent variable is positively skewed are presented below.

Monte Carlo Simulation

Design

In typical random effects designs investigators do not know how independent variables and prediction error are distributed. In fixed effects designs, investigators typically control or specify independent variable levels, though the dependent Y distribution will be a function of the independent variable(s) and prediction error (e) distributions. Classical measurement theory presumes $Y_i = Y_{T_i} + e$, where Y_{T_i} is the true latent value of Y for person i. When Y_{T_i} is a function of some X (e.g., $Y = b_0 + b_1X_1 + e$ or $Y = b_0 + b_1X_1 + b_2X_2 + b_3X_1X_2 + e$), X_1, X_2, or e must be nonnormal in order for observed Y_i to be nonnormally distributed. Consequently, to simulate the kinds of data investigators might encounter in either random or fixed effect designs, data were generated in nine Monte Carlo simulations where independent variables X_1 and X_2 and prediction error (e) systematically varied across normal, uniform, and χ^2 distributions. Normal distributions were selected to simulate multivariate normal conditions in random effects designs. Uniform distributions were selected to simulate fixed effect experimental designs. χ^2 distributions for X and e simulated positively skewed Y distributions such as those found in compensation research.

Sample

Simulation data were generated for combinations of X_1, X_2, and e distributions using the SYSTAT9 computer package. Five thousand samples of N = 113 paired X_1, X_2 observations were drawn at random from all possible combinations of normal, uniform, and χ^2 population distributions X_1, X_2, and e (Guzzo, Jette, & Katzell, 1985, reported a mean N = 113 across studies in a meta-analysis of compensation-based intervention programs). Results are only reported for conditions where X_1 and X_2 were drawn from identical population distributions, though results when X_1 and X_2 were drawn from different population distributions were consistent with those reported below.[7] Note, 5000 samples of N = 113 were drawn for every combination of X, Y, and e distributions described below as per Mooney's (1997) suggestions for conducting Monte Carlo simulations, resulting in nine sets of 5000 samples of N = 113. All aspects of the Monte Carlo simulation were replicated using 5000 samples of N = 226 and N = 56 (i.e. using samples twice and one half as large as N = 113). Identical patterns of results emerged and are available from the first author on request.

When X_1 and X_2 observations were drawn at random from a normal population distribution, μ and σ were set at $\mu = 3$ and $\sigma = 1$. Variables X_1 and X_2 within each data set were then rounded to the nearest integer (yielding values ranging from 1 to 5, i.e., five point Likert scales) in order to simulate measurement circumstances commonly encountered in applied psychological and management research. Uniform X_1 and X_2 data sets were drawn from a population containing integer values between 1 and 5, inclusive. Additional X_1 and X_2 data sets were drawn from X^2 distributions with three degrees of freedom. These steps resulted in nine Monte Carlo data sets when the three possible X distributions (normal, uniform, χ^2) were combined with the three possible e distributions (normal, uniform, χ^2).

Three dependent variables were generated within each data set to reflect large, medium, and small effect sizes. Equations 2, 3, and 4 were used to generate values for Y_1, Y_2, and Y_3 within each of the nine data sets:

$$Y_1 = .75X_1X_2 + .25e$$
$$Y_2 = .50X_1X_2 + .50e$$
$$Y_3 = .25X_1X_2 + .75e$$

Equations 2, 3, & 4

Under the e = normal condition, prediction error e was drawn from a normal population with a mean and standard deviation set equal to the mean and standard deviation of the X_1X_2 product term with which it was paired. Under the e = uniform condition, e was randomly drawn from a uniform population distribution ranging from 1 to 20. Under the e = χ^2 condition, e was randomly drawn from a χ^2 population distribution with 9 degrees of freedom (where 9 is the mean population value for all X_1X_2 product terms regardless of sample X_1, X_2 distribution characteristics). Hence, three dependent variables Y_1, Y_2, and Y_3 reflecting large, medium, and small moderator effect sizes were available to be examined within each of the nine data sets.

Analyses

All tests of interaction effects used moderated regression analysis (Bobko, 1995; Darlington, 1968; Saunders, 1955, 1956). The F-test of H_o: $\Delta R^2 = 0$, where $\Delta R^2 = R^2_{multiplicative} - R^2_{additive}$ for the equations $\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_1X_2$ and $\hat{Y} = b_0 + b_1X_1 + b_2X_2$, respectively, constitutes the test of an interaction effect when X_1 and X_2 are interval scale measures. The strategy and organizational theory literatures commonly refer to this as the Chow test (Chow, 1960).

To provide a point of reference, samples of N = 50,000 for each combination of X_1, X_2 distribution were generated separately for purposes of estimating $E(\Delta R^2)$ when e = 0. When X_1 and X_2 were normal, uniform, and χ^2, $E(\Delta R^2)$ = .057, .077 , and .256, respectively. These values should be considered asymptotes or what would occur under circumstances of perfect, error free prediction. The addition of prediction error will slowly decrease $E(\Delta R^2)$, e.g., if when X_1 and X_2 are distributed as χ^2 the true prediction model is $Y = .1X_1X_2 + .9e$, then clearly $E(\Delta R^2) \neq .256$. Regardless, it should be noted that these are expected values of ΔR^2 and actual values observed might be larger or smaller when Y does or does not include prediction error (e.g., Russell & Bobko, 1992, observed ΔR^2 greater than $E\{\Delta R^2\}$ for some subjects).

Results

Table 1a reports results of moderated regression analyses performed on the three effect sizes (Y_1, Y_2, and Y_3) in the nine different combinations of X and e distributions (i.e., normal, uniform, and χ^2 X_1 and X_2 distributions paired with normal, uniform, and χ^2 e distributions). Moderator effect sizes are captured by the median ΔR^2 column, containing the 2500th largest value of ΔR^2 obtained from the 5000 samples of N = 113. While F statistics testing H_o: $\Delta R^2 = 0$ can be derived for median ΔR^2 values, only the ones derived for normally distributed prediction error meet parametric assumptions and are interpretable (i.e., statistics reported in the shaded area of Table 1a). Regardless, the 2.5 and 97.5 percentile values of ΔR^2 were identified within set of 5000 Monte Carlo N = 113 samples.[8] As the expected value of the F statistic testing H_o: $\Delta R^2 = 0$ is F = 1.0, one would reject H_o using logic underlying simple percentile bootstrap applications when the F statistic (i.e., $F_{1,109} = \dfrac{\Delta R^2 / (4-3)}{(1 - R^2_{\text{multiplicative}}) / (113-4)}$) for the moderator effect cutting off the lower 2.5% of the 5000 ΔR^2 is greater than 1 (i.e., when F = 1.0 falls outside of the 95% ΔR^2 CI). Median values of ΔR^2 reported in Table 1 for which the lower 2.5 percentile values generated F > 1.0 are indicated with asterisks.

Insert Table 1 about here

Interestingly, profiles of ΔR^2 for large, medium, and small effect sizes for interpretable equations in Table 1a (i.e., those meeting OLS assumptions) are .047/.024/.006, .067/.041/.009, and .221/.191/.087 for X_1, X_2 distributions drawn from normal, uniform, and χ^2 populations of X_1 and X_2, respectively. Not surprisingly, smaller values of ΔR^2 are observed as the effect size decreases across Y_1, Y_2, and Y_3. The pattern of effect sizes across normal, uniform, and X^2 distributions is consistent with McClelland and Judd's (1993) demonstration that multiplicative effect sizes are maximized in designs using extreme values of X_1 and X_2. Normally distributed X_1 and X_2 will have the fewest extreme X_1X_2 observations due to low probabilities in the extreme tails of the normal distribution. Uniform and χ^2 distributions for X_1 and X_2 will have increasingly more frequent extreme observations in the tails of an X_1X_2 distribution, respectively.

If X_1, X_2, or e are highly positively skewed, as they are when drawn from $\chi^2_{df=3}$ populations, Y will demonstrate some skewness. Investigators following Winer's (1974) convention would perform a log transform on Y in hope of permitting probabilistic inferences possible when parametric assumptions are met. Table 1b reports moderated regression results when Y_1, Y_2, and Y_3 were subjected to a $\log_{10}$ transformation for the five X_1, X_2, and e combinations involving skewed χ^2 distributions (when X_1, X_2, or e are positively skewed, Y will be positively skewed). Moderated regression effect sizes for the non-transformed Y_1, Y_2, and Y_3 (Table 1a) are 2.7 to 15 times larger than effect sizes observed for log transformed Y_1, Y_2, and Y_3 (Table 1b). Perhaps most interestingly, effect sizes for the one data set that meets parametric assumptions (X_1 and X_2 distributed as χ^2, e distributed normally) go from $\Delta R^2 = .221$ to $\Delta R^2 = .032$ for Y_1 and $\log_{10}Y_1$, respectively, from $\Delta R^2 = .191$ to $\Delta R^2 = .041$ for Y_2 and $\log_{10}Y_2$, respectively, and from $\Delta R^2 = .087$ to $\Delta R^2 = .025$ for Y_3 and $\log_{10}Y_3$, respectively. Hence, moderated regression effect sizes are 3.5 to 6.9 times larger and more likely to correctly detect the true "latent" population moderator effect when estimated from the nontransformed data, though investigators following convention would have log transformed Y_1, Y_2, and Y_3 before conducting the analyses. The stronger the moderator effect, the larger the difference between effect sizes derived from nontransformed versus log transformed Y's.

In sum, moderated regression effect sizes derived from a Monte Carlo simulation of 5000 N = 113 samples drawn from normal, uniform, and χ^2 e and X distributions are 2.7 to 15 times more likely to detect true latent moderator effects (i.e., reject H_o: $\Delta R^2 = 0$) when the dependent variable has not been subjected to a log transformation. The final portion of this study demonstrates how primary researchers would apply a simple bootstrap procedure in analyzing data obtained from a single sample and confirming implications of the Monte Carlo results (i.e., that inferences drawn from bootstrap-generated confidence intervals about moderator effects are expected to exhibit less Type II error).

Bootstrap Demonstration

Samples

As a rule, researchers generally face circumstances in which they have data gathered from a single sample, not 5000 samples. Hence, to simulate what individual researchers typically encounter, nine samples of N = 113 paired X_1, X_2 observations were created at random from normal, uniform, and χ^2 population distributions using the SYSTAT9 computer package. When X_1 and X_2 observations were drawn at random from a normal population distribution, μ

and σ were set at $\mu = 3$ and $\sigma = 1$. As in the Monte Carlo simulation and consistent with measurement circumstances commonly encountered in applied psychological and management research, X_1 and X_2 data sets were rounded to the nearest integer yielding values from 1 to 5. Uniform X_1 and X_2 data sets were drawn from a population containing integer values between 1 and 5, inclusive, yielding $\overline{X}_1 = 3.012$, $\sigma_{X_1} = 1.438$, and $\overline{X}_2 = 2.889$, $\sigma_{X_2} = 1.394$, respectively. Finally, X_1 and X_2 data sets were drawn from X^2 distributions with three degrees of freedom, yielding $\overline{X}_1 = 2.986$, $\sigma_{X_1} = 2.344$ and $\overline{X}_2 = 3.008, \sigma_{X_2} = 2.660$. Three dependent variables were generated within each sample using Equations 2, 3, and 4 described in the Monte Carlo simulation above. Error terms (e) were drawn from the same populations as described in the Monte Carlo simulation above, with their means and standard deviations set equal to the X_1X_2 product term means and standard deviations.

Analyses

Tests of interaction effects using moderated regression analysis were performed using dependent variables Y_1, Y_2, Y_3, $\text{Log}Y_1$, $\text{Log}Y_2$, and $\text{Log}Y_3$ in each of the nine samples. Additionally, B = 1000 bootstrap estimates of ΔR^2 were derived for all dependent variables in each of the nine samples using the percentile bootstrap method described above.[9]

Results

Table 2a reports results of moderated regression analyses performed on the nine samples of N = 113 containing different combinations of X_1, X_2, and e distributions (i.e., normal, uniform, and χ^2 distributions of X_1 and X_2 paired with normal, uniform, and χ^2 e distributions). While F statistics are reported for moderation effects in all nine combinations, only the three derived for normally distributed prediction error meet parametric assumptions and are interpretable (i.e., statistics reported in the shaded area of Table 2a). Moderator effect sizes are captured by the ΔR^2 column (the F statistic tests H_o: $\Delta R^2 = 0$, Bobko, 1995; Darlington, 1968).

Insert Table 2 about here

ΔR^2 for Y_1 in the three interpretable equations in Table 2a are .049, .068, and .216 for X_1, X_2 distributions drawn from normal, uniform, and χ^2 populations, respectively. This profile of effect sizes is again consistent with the observation that normal X_1, X_2 will have the fewest extreme X_1X_2 observations due to low probabilities in the extreme tails of the normal distribution and results reported in the Monte Carlo study reported above.

Figure 1 demonstrates when X_1, X_2, or e were highly positively skewed, as they are when drawn from $\chi^2_{df=3}$ populations, Y exhibited some positive skewness. Investigators following convention would perform a log transform on Y hoping to permit the probabilistic inferences possible when parametric assumptions are met. Table 2b reports moderated regression results when Y was subjected to a $\log_{10}$ transformation for the five X_1, X_2, and e combinations with skewed χ^2 distributions (i.e., skewed Y distributions appear only when X_1, X_2, or e distributions were positively skewed). Consistent with the Monte Carlo findings reported above, moderated regression effect sizes for the original non-transformed data were two to seven times larger than effect sizes observed for log transformed data. Effect sizes for the one data set that met parametric assumptions (X_1 and X_2 distributed as χ^2, e distributed normally) went from ΔR^2 = .216 to ΔR^2 = .030 when Y was subjected to log transformation. Hence, moderated regression effect size was 7.2 times larger when (correctly) estimated from nontransformed data.

Insert Figure 1 about here

However, confidence intervals around ΔR^2 can be derived via bootstrapping procedures regardless of how X_1, X_2, e, or Y are distributed. Table 3 reports bootstrap estimates of the 2.5^{th} percentile values of the moderated regression effect size ΔR^2 taken from B = 1000 bootstrap samples of size N = 113 for the five situations where Y is positively skewed, i.e., those subject to log transformation using current methodological convention. Interestingly, median effect sizes across 1000 bootstrap samples were between 2.6 and 534 times larger than ΔR^2 effect sizes resulting from analyses conducted after Y was log transformed Table 2b. This suggests that to be equally likely to be detected, moderator effect sizes when Y is skewed and log transformed must be 2.6 to 534 times as large as those observed under conditions when Y is not log transformed and ΔR^2 is estimated from the median bootstrap ΔR^2 value. Put another way, other things being equal, the sample size needed to correctly reject H_o: $\Delta R^2 = 0$ would need to be 6.76 to 285,156 times as large when Y is skewed and log transformed in these samples. Investigators using OLS moderated regression and log transformed dependent variables would be much more likely to fail to detect true interaction effects (Type II error).

Insert Table 3 about here

Discussion

This study demonstrated a fundamental problem in detection of latent moderation effects when log transforms are used to “correct” positively skewed dependent variables. Specifically, increased probability of Type II error was demonstrated in both a Monte Carlo simulation generating 5000 samples from known population distributions and subsequent bootstrap analysis of individual simulated samples. Results suggested severe decrements in statistical power required to test moderation regression effects, i.e., H_o: $\Delta R^2 = 0$, resulted from log transformations. These decrements occurred when parametric assumptions were in fact met (i.e., the shaded rows of Tables 1a and 2a) as well as when parametric assumptions were not met. Graphically, log transformations change the Y distribution shape, effectively decreasing Y variance by reducing the degree to which extremely positive Y values deviate from the mean. If these extreme Y values were created by an interaction between one or more positively skewed independent variables (e.g., when X_1 and X_2 are distributed as χ^2), log transformations of Y effectively "disguise" the extreme values of Y that should result from the product of extreme X_1, X_2 values as less extreme values, effectively yielding a log Y variable exhibiting less variance than the original raw Y observations. While Type I error is always possible (cf. Aguinis & Pierce, 1998), it is clear that log transformations of positively skewed dependent variables greatly enhances Type II error probability.

Fortunately, results also indicated bootstrapping procedures provide a viable alternative to traditional parametric statistical procedures for detecting moderator effects regardless of how X_1, X_2, and e are distributed. In fact, in situations where convention dictates Y should be subjected to log transformation, log transformations caused extremely severe decrements in statistical power for parametric OLS procedures relative to bootstrap procedures. Data simulated here are commonly found in compensation research, where parametric procedures are commonly used after Y is routinely subjected to log transformation (e.g., Henderson & Frederickson, 1996; Sanders & Carpenter, 1998).

Of course log transformations could be justified on some theoretical basis. The authors are unaware of any theoretical rationale put forth by compensation theory or any other area of applied psychological or management research to justify such a transformation in the presence of a multiplicative model. Further, the authors have never seen any discussion of the theoretical underpinnings of latent models that result from such a transformation, such as $\hat{Y} = 10^{\beta_0 + \beta_1 \log_{10} X}$ (SPSS, 1996). As a result, any gains in statistical elegance and predictive power (i.e., for additive models) stemming from log transformations are not currently matched by gains in theoretical insight. Null results for tests of moderation in studies employing log transformations are expected to frequently reflect Type II error when a true latent moderation process is present.

In sum, when hypothesized models involve interaction effects, applied psychological and management research would benefit from routine application of bootstrap procedures. While not replacing common parametric procedures, bootstrap applications are appropriate when parametric assumptions are not viable (e.g., when heteroscedasticity is present due to a positively skewed dependent variable). Nonlinear monotonic transformations may achieve necessary statistical conditions for parametric inferences in OLS applications to additive models (Busemeyer & Jones, 1983; Winer, 1974). Current results indicated nonlinear monotonic transformations also erode investigator's capacity to assess theory-based predictions of moderation effects (e.g., estimates of moderation effect ΔR^2). Importantly, bootstrapping provides an alternative method of assessing <u>theory-based</u> inferences of moderation effects from data that cannot be assessed with comparable statistical power by conventional procedures.

References

Aguinis, H. & Pierce, C.A. (1998). Heterogeneity of error variance and the assessment of moderating effects of categorical variables: A conceptual review. Organizational Research Methods, 1, 296-305.

Bennett, J.L. (1999). Personal communication, July 12, 1999.

Bergman, T.J., Scarpello, V.G., & Hills, F.S. (1998). Compensation Decision Making (3rd Ed.). San Diego: Dryden Press.

Bickel, P., & Freedman, D.A. (1981). Some asymptotic theory for the bootstrap. Annals of Statistics, 9, 1196-1217.

Bobko, P. (1995). Correlation and regression: Principles and applications of industrial/organizational psychology and management. New York: McGraw-Hill Inc.

Bray, D.W., Campbell, R.J., & Grant, D.L. (1974). Formative years in business. Malabar, FL: Robert Krieger Publishing.

Busemeyer, J.R. & Jones, J.E. (1983). Analysis of multiplicative combination rules when the causal variables are measured with error. Psychological Bulletin, 93, 549-562.

Chow, G. (1960). Test of equality between sets of coefficients in two linear regressions. Econometrica, 28, 591-604.

Conyon, M.J. & Peck, S.I. (1998). Board control, remuneration committees, and top management compensation. Academy of Management Journal, 41, 146-157.

Darlington, R. (1968). Multiple regression in psychological research and practice. Psychological Bulletin, 69, 161-187.

Diaconis, P. & Efron, B. (1983, May). Computer-intensive methods in statistics. Scientific American, 116-130.

Efron, B. (1979). Computers and the theory of statistics: Thinking the unthinkable. Society for Industrial and Applied Mathematics Review, 21, 460-480.

Efron, B. (1985). Bootstrap confidence intervals for a class of parametric problems. Biometrica, 72, 45-58.

Efron, B. (1986). How biased is the apparent error rate of a prediction rule? Journal of the American Statistical Association, 81, 461-470.

Efron, B. & Tibshirani R.J. (1993). An Introduction to the Bootstrap. New York: Chapman & Hall.

England, J.D. & Pierson, D.A. (1990). Salary ranges and merit matrices: The time targeting approach. Compensation and Benefits Review, 107(October/December), 25.

Finkelstein, S. & Boyd, B.K. (1998). How much does the CEO matter? The role of managerial discretion in the setting of CEO compensation. Academy of Management Journal, 41, 179-199.

Freedman, D.A. (1981). Bootstrapping regression models. Annals of Statistics, 9, 1218-1228.

Gomez-Mejia, L.R., Tosi, H., & Hinkin, T. (1987). Managerial control, performance, and executive compensation. Academy of Management Journal, 30, 51-70.

Guzzo, R., Jette, Q., & Katzell, R. (1985). The effects of psychologically based intervention programs on workers' productivity: A meta-analysis. Personnel Psychology, 38, 275-291.

Henderson, A.D. & Frederickson, J.W. (1996). Information-processing demands as a determinant of CEO compensation. Academy of Management Journal, 39, 575-606.

Kuhn, P. & Sweetman, A. (1998). Wage loss following displacement: The role of union coverage. Industrial and Labor Relations Review, 51, 384-400.

Lunneborg, C.E. (1985). Estimating the correlation coefficient: The bootstrap approach. Psychological Bulletin, 98, 209-215.

McClelland, G.H. & Judd, C.M. (1993). Statistical difficulties of detecting interactions and moderator effects. Psychological Bulletin, 114, 376- 390.

Milkovich, G.T. & Newman, J.M. (1996). Compensation (5th ed.). Boston, MA: Irwin McGraw-Hill.

Mobley, W.H., Griffeth, R.W., Hand, H.H., & Meglino, B.M. (1979). Review and conceptual analysis of the employee turnover process. Psychological Bulletin, 86, 493-522.

Mooney, C.Z. (1997). Monte Carlo simulation. Newbury Park, CA: Sage Publications Inc.

Mooney, C.Z. & Duval, R.D. (1993). Bootstrapping: A Nonparametric Approach to Statistical Inference. Newbury Park, CA: Sage Publications Inc.

Olds, E.G., Mattson, T.B., & Oldeh, R.E. (1956). Notes on the use of transformations in the analysis of variance. WADC Tech Report 59-82, Wright-Patterson Air Force Base, Ohio.

Owens, W.A. & Schoenfeldt, L.F. (1979). Toward a classification of persons. Journal of Applied Psychology, 65, 569-607.

Rasmussen, J. L. (1987). Estimating correlation coefficients: Bootstrap and parametric approaches. Psychological Bulletin, 101, 136-139.

Rasmussen, J.L. (1989). Data transformation, type I error rate and power. British Journal of Mathematical and Statistical Psychology, 42, 203-213.

Russell, C.J. & Bobko, P. (1992). Moderated regression analysis and Likert scales: Too coarse for comfort. Journal of Applied Psychology, 77, 336-342.

Sanders, W.G. & Carpenter, M.A. (1998). Internationalization and firm governance: The roles of CEO compensation, top team composition, and board structure. Academy of Management Journal, 41, 158-178.

Saunders, D.R. (1955). The "moderator variable" as a useful tool in prediction. In Proceedings of the 1954 Invitational Conference on Testing Problems (pp. 54-58). Princeton, NJ: Educational Testing Service.

Saunders, D.R. (1956). Moderator variables in prediction. Educational and Psychological Measurement, 16, 209-222.

Smith, P.C., Kendall, L.M., & Hulin, C.L. (1969). The measurement of satisfaction in work and retirement. Chicago, IL: Rand McNally.

SPSS, Inc. (1996). SYSTAT 6.0 for Windows: Statistics. Chicago, IL: Author.

Winer, B.J. (1974). Statistical principles in experimental design. New York: McGraw Hill.

Table 1a
Monte Carlo Moderated Regression Analyses for Different X and e Distributions: 5,000 random samples of N = 113[1]

Table 1a: $Y_1 = .75X_1X_2 + .25e$, $Y_2 = .5X_1X_2 + .5e$, $Y_3 = .25X_1X_2 + .75e$

X_1, X_2 Distribution Characteristics	e Distribution Characteristics		Median $R^2_{multiplicative}$	$-R^2_{additive}$	$= \Delta R^2$
Normal, $\mu = 3, \sigma = 1$, rounded to nearest integer.	**Normal:** $\bar{e} = 9, \sigma = 5^2$	Y_1	.943	.918	.047*
		Y_2	.786	.771	.024
		Y_3	.298	.288	.006
	URN: Uniform random number between 1 & 18.	Y_1	.919	.893	.047*
		Y_2	.726	.709	.025*
		Y_3	.293	.281	.007
	$\mathbf{X^2}$, df = 9^3	Y_1	.943	.918	.047*
		Y_2	.732	.712	.029*
		Y_3	.383	.373	.008
URN: Uniform random distribution from 1 to 5, rounded to nearest integer.	**Normal:** $\bar{e} = 9, \sigma = 5^2$	Y_1	.956	.920	.067*
		Y_2	.803	.778	.041*
		Y_3	.360	.347	.009
	URN between 1 & 20	Y_1	.960	.924	.068*
		Y_2	.648	.616	.041*
		Y_3	.431	.420	.009
	$\mathbf{X^2}$, df = 9^3	Y_1	.969	.933	.070*
		Y_2	.713	.680	.046*
		Y_3	.496	.484	.012
$\mathbf{X^2}$, df = 3, rounded to nearest integer	**Normal:** $\bar{e} = 9, \sigma = 5^2$	Y_1	.991	.873	.221*
		Y_2	.910	.798	.191*
		Y_3	.561	.477	.087*
	URN between 1 & 18	Y_1	.992	.874	.220*
		Y_2	.924	.815	.190*
		Y_3	.658	.589	.086*
	$\mathbf{X^2}$, df = 9^3	Y_1	.992	.874	.221*
		Y_2	.939	.827	.197*
		Y_3	.647	.561	.104*

* the F statistic for the 2.5%tile value of ΔR^2 was greater than 1.
[1] N = 113 is the average N across k = 330 effect sizes reported in a meta-analysis of Guzzo, Jette, and Katzel (1985).
[2] Mean and SD for all normally distributed error terms were set to be equal to the mean and SD for the product X_1X_2.
[3] Expected value of a X^2 distribution is equal to its df. Hence, with df = 9, the expected midpoint of the error distribution is equal to the mean of the X_1X_2 product term.

Table 1b: $Log_{10}(Y_1) = .75X_1X_2 + .25e$, $Log_{10}(Y_2) = .5X_1X_2 + .5e$, $Log_{10}(Y_3) = .25X_1X_2 + .75e$

X_1, X_2 Distribution Characteristics	e Distribution Characteristics		Median		
			$R^2_{multiplicative}$	$-R^2_{additive}$	$=\Delta R^2$
Normal	**X^2**, df = 9^3	**$Log_{10}Y_1$**	.876	.873	.006*
		$Log_{10}Y_2$	.749	.743	.007
		$Log_{10}Y_3$	.343	.334	.005
URN	**X^2**, df = 9^3	**$Log_{10}Y_1$**	.948	.944	.008
		$Log_{10}Y_2$	.724	.714	.010
		$Log_{10}Y_3$	.440	.431	.008
X^2, df = 3	**Normal**: $\bar{e} = 9, SD = 5^2$	**$Log_{10}Y_1$**	.874	.856	.032
		$Log_{10}Y_2$	.748	.720	.041
		$Log_{10}Y_3$	.501	.475	.025
	URN between 1 & 20	**$Log_{10}Y_1$**	.862	.843	.032
		$Log_{10}Y_2$	.716	.688	.040
		$Log_{10}Y_3$	.498	.473	.025
	X^2, df = 9^3	**$Log_{10}Y_1$**	.829	.808	.035
		$Log_{10}Y_2$	.805	.772	.051
		$Log_{10}Y_3$	.618	.585	.039

* the F statistic for the 2.5%tile value of ΔR^2 was greater than 1.

Table 2a
Moderated Regression Analyses for Different X and e Distributions: N = 113[1]
Table 2a
$Y_1 = .75X_1X_2 + .25e$, $Y_2 = .5X_1X_2 + .5e$, $Y_3 = .25X_1X_2 + .75e$

X_1, X_2 Distribution Characteristics	e Distribution Characteristics		$R^2_{multiplicative}$	$- R^2_{additive}$	$= \Delta R^2$	$F_{1,109}$
Normal, $\mu = 3$, $\sigma = 1$, rounded to nearest integer.	**Normal:** $\bar{e} = 8.789, SD = 4.996$ [2]	Y_1	.799	.750	.049	5.616, p < .01
		Y_2	.587	.561	.026	2.910, p > .05
		Y_3	.331	.322	.009	0.990, p > .05
	URN: Uniform random number between 1 & 20.	Y_1	.880	.821	.059	6.834, p < .01
		Y_2	.601	.580	.021	2.357, p > .05
		Y_3	.291	.282	.009	0.990, p > .05
	$\mathbf{X^2}$, df = 9[3]	Y_1	.932	.887	.045	5.136, p < .05
		Y_2	.599	.567	.032	3.603, p > .05
		Y_3	.333	.329	.004	0.438, p > .05
URN: Uniform random distribution from 1 to 5, rounded to nearest integer.	**Normal:** $\bar{e} = 8.702, SD = 6.029$ [2]	Y_1	.825	.757	.068	7.953, p < .01
		Y_2	.633	.590	.043	4.898, p < .05
		Y_3	.436	.425	.011	1.110, p > .05
	URN between 1 & 20	Y_1	.831	.787	.044	4.561, p < .05
		Y_2	.599	.580	.019	2.111, p > .05
		Y_3	.395	.383	.012	1.324, p > .05
	$\mathbf{X^2}$, df = 9[3]	Y_1	.813	.792	.071	9.269, p < .01
		Y_2	.701	.651	.050	5.737, p < .05
		Y_3	.521	.479	.042	4.779, p < .05
$\mathbf{X^2}$, df = 3, rounded to nearest integer	**Normal:** $\bar{e} = 8.982, SD = 6.163$ [2]	Y_1	.846	.630	.216	30.030, p < .001
		Y_2	.620	.460	.160	12.976, p < .001
		Y_3	.461	.355	.106	12.924, p < .001
	URN between 1 & 20	Y_1	.906	.688	.214	29.677, p < .01
		Y_2	.782	.642	.140	17.744, p < .01
		Y_3	.506	.446	.060	6.957, p < .01
	$\mathbf{X^2}$, df = 9[3]	Y_1	.949	.799	.150	19.235, p < .01
		Y_2	.682	.581	.101	12.246, p < .01
		Y_3	.466	.426	.040	4.542, p < .05

Note, only statistics appearing in shaded areas are interpretable under parametric assumptions and F statistics test H_0: $\Delta R^2 = 0$.
[1] N = 113 is the average N across k = 330 effect sizes reported in a meta-analysis of Guzzo, Jette, and Katzel (1985).
[2] Mean and SD for all normally distributed error terms were set to be equal to the mean and SD for the product X_1X_2.
[3] Expected value of a X^2 distribution is equal to its df. Hence, with df = 9, the expected midpoint of the error distribution is approximately the same as that of the X_1X_2 product term.

Table 2b
$Log_{10}(Y_1) = .75X_1X_2 + .25e$, $Log_{10}(Y_2) = .5X_1X_2 + .5e$, $Log_{10}(Y_3) = .25X_1X_2 + .75e$

X_1, X_2 Distribution Characteristics	e Distribution Characteristics		$R^2_{multiplicative}$	$- R^2_{additive}$	$= \Delta R^2$	$F_{1,109}$
Normal	$\mathbf{X^2}$, df = 9^3	**$Log_{10}Y_1$**	.757	.736	.021	2.338, $p > .05$
		$Log_{10}Y_2$	.511	.500	.011	1.100, $p > .05$
		$Log_{10}Y_3$	.409	.402	.007	0.768, $p > .05$
URN		**$Log_{10}Y_1$**	.880	.877	.003	0.328, $p > .05$
		$Log_{10}Y_2$	.567	.566	.001	0.109, $p > .05$
		$Log_{10}Y_3$	.399	.399	.000	0.087, $p > .05$
$\mathbf{X^2}$, df = 3	**Normal**: $\bar{e} = 8.808, SD = 6.029^2$	**$Log_{10}Y_1$**	.553	.523	.030	3.371, $p > .05$
		$Log_{10}Y_2$	.399	.385	.014	1.548, $p > .05$
		$Log_{10}Y_3$	.278	.269	.009	0.990. $p > .05$
	URN between 1 & 20	**$Log_{10}Y_1$**	.611	.550	.061	7.081, $p < .05$
		$Log_{10}Y_2$	.456	.425	.031	3.487, $p > .05$
		$Log_{10}Y_3$	.342	.331	.011	1.100, $p > .05$
	$\mathbf{X^2}$, df = 9^3	**$Log_{10}Y_1$**	.678	.588	.090	10.780, $p < .01$
		$Log_{10}Y_2$	.444	.399	.045	5.136, $p < .05$
		$Log_{10}Y_3$	.367	.348	.019	2.111, $p > .05$

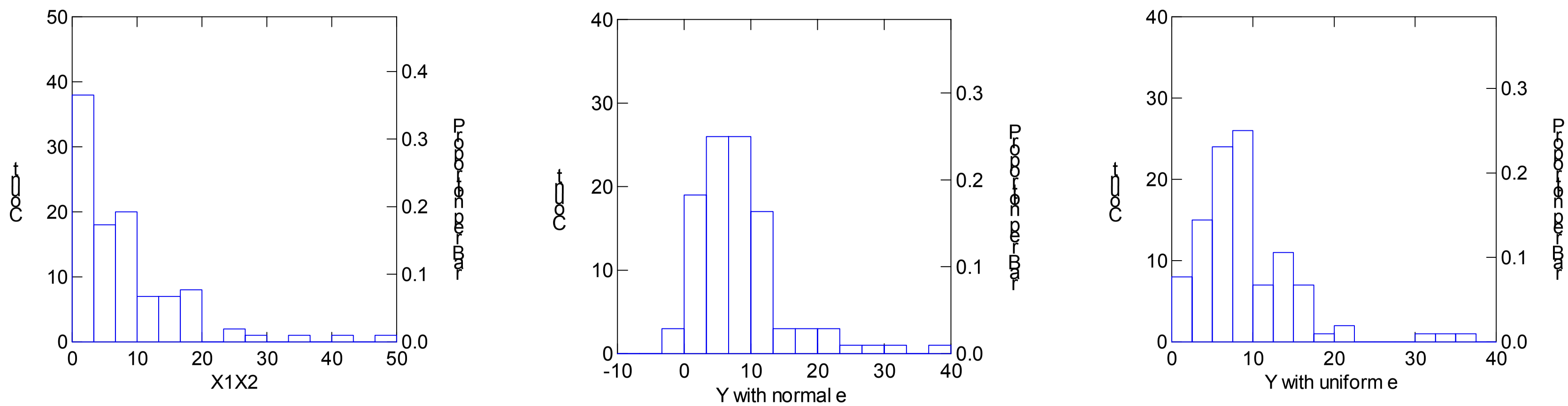
Table 3

Bootstrap 95% CIs of ΔR^2 Moderated Regression Analyses:
B = 1000, N = 113[1], and Simulation Data Derived from $Y_1 = .75X_1X_2 + .25e$, $Y_2 = .5X_1X_2 + .5e$, $Y_3 = .25X_1X_2 + .75e$

X_1, X_2 Distribution Characteristics	e Distribution Characteristics		$R^2_{multiplicative}$	$- R^2_{additive}$	$= \Delta R^2$
Normal	$\mathbf{X^2}$, df = 9[3]	$\mathbf{Y_1}$	.833	.663	.170*
		$\mathbf{Y_2}$	.602	.482	.120*
		$\mathbf{Y_3}$	.433	.343	.090
URN		$\mathbf{Y_1}$	.917	.700	.217*
		$\mathbf{Y_2}$	.689	.529	.160*
		$\mathbf{Y_3}$	.450	.340	.110*
$\mathbf{X^2}$, df = 3	**Normal**: $\bar{e} = 8.808, SD = 6.029$[2]	$\mathbf{Y_1}$	.817	.607	.210*
		$\mathbf{Y_2}$	.555	.422	.133*
		$\mathbf{Y_3}$	.311	.178	.133*
	URN between 1 & 20	$\mathbf{Y_1}$	.862	.666	.196*
		$\mathbf{Y_2}$	.499	.349	.150*
		$\mathbf{Y_3}$	.311	.201	.111*
	$\mathbf{X^2}$, df = 9[3]	$\mathbf{Y_1}$	.890	.656	.234*
		$\mathbf{Y_2}$	.609	.448	.161*
		$\mathbf{Y_3}$	.522	.402	.120*

* the F statistic for the 2.5%tile value of ΔR^2 was greater than 1.
[1] N = 113 is the average N across k = 330 effect sizes reported in a meta-analysis of Guzzo, Jette, and Katzel (1985).
[2] Mean and SD for all normally distributed error terms were set to be equal to the mean and SD for the product X_1X_2.
[3] Expected value of a X^2 distribution is equal to its df. Hence, with df = 9, the expected midpoint of the error distribution is approximately the same as that of the X_1X_2 product term.

Figure 1
Frequency Distribution for X_1X_2 and Y, where X_1 and X_2 are χ^2 Distributed and $Y = .75X_1X_2 + .25e$ *



* e drawn from random normal or random uniform distributions with $\bar{e}$ and σ_e set equal to $\overline{X_1X_2}$ and $\sigma_{X_1X_2}$, respectively.

Biographical Paragraphs

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Craig J. Russell is the J.C. Penney Chair of Business Leadership at the Price College of Business, University of Oklahoma and holds a joint appoinment as Professor of Psychology. He received his Ph.D. in business administration from the University of Iowa and pursues research in management and leadership development, personnel selection, and research methods.

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Notes

[1] Note, if X is not normally distributed and Y is a linear function of X, Y will also likely not be normally distributed.

[2] Bergman, Scarpello, and Hills (1997) and Milkovich and Newman (1996) noted how pay ranges are generally a constant or increasing percentage of the range midpoint. Hence, as pay range mid-point (and mean) increases, the variation in observed salaries around the mid-point increases.

[3] Of course, weighted least square (WLS) procedures would also resolve the heteroscedasticity problem. However, it would do so by migrating what was a nonlinear transformation paired with OLS into the internal optimal weighting procedures characteristic of WLS.

[4] We thank an anonymous reviewer for bringing this to our attention.

[5] We thank an anonymous reviewer for this example.

[6] See Efron and Tibshirani (1993), chapter 14 (pp. 178-201) for a discussion of alternatives to the "simple" bootstrap. Specifically, the bias-corrected and accelerated (BC_a) and the approximate bootstrap confidence (ABC) interval methods are marginally more complex techniques that overcome most shortcomings associated with the simple bootstrap.

[7] The first author will provide these results on request.

[8] ΔR^2 for the 2.5 and 97.5 percentile values are available from the first author on request.

[9] Again, see Efron and Tibshirani (1993), chapter 14 (pp. 178-201) for more elaborate bootstrap procedures exhibiting certain statistical elegancies that might yield more robust CI intervals.