# CHEMICAL ENGINEERING DESIGN & SAFETY

## CHE 4253

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Fluid Flow Pressure Drop and Pipe Design

# FLUID FLOW

## *Mechanical Energy Balance*

$$g\Delta Z + \int \frac{dp}{\rho} + \Delta\left(\frac{V^2}{2}\right) = W_o - \sum F$$

potential energy change | expansion work | kinetic energy change | work added/ subtracted by pumps or compressors | sum of friction losses

Note that the balance is per unit mass. In differential form:

$$dp = -\rho\left(g \cdot dZ - V \cdot dV - \delta F + \delta W_o\right)$$

# FLUID FLOW

## Mechanical Energy Balance

Divide by *dL*, (L is the length of the pipe)

$$\left.\frac{dp}{dL}\right|_{Tot} = -\rho g \frac{dZ}{dL} + \rho V \frac{dV}{dL} + \rho \frac{\delta F}{\delta L} - \rho \frac{\delta W_o}{\delta L}$$

or:

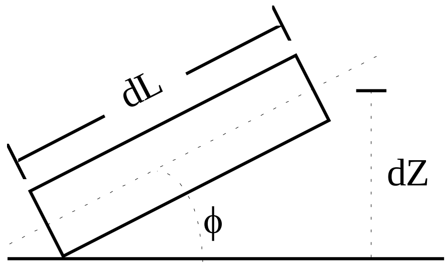
$$\left.\frac{dp}{dL}\right)_{Tot} = \left.\frac{dp}{dL}\right)_{elev} + \left.\frac{dp}{dL}\right)_{accel} + \left.\frac{dp}{dL}\right)_{frict}$$

$\dfrac{\delta W_o}{\delta L}$ is usually ignored, as the equation applies to a pipe section.

The above equation is an alternative way of writing the mechanical energy balance. It is not a different equation

# Mechanical Energy Balance

Potential energy change:




$$g\frac{dZ}{dL} = g\sin\phi$$

Friction Losses:

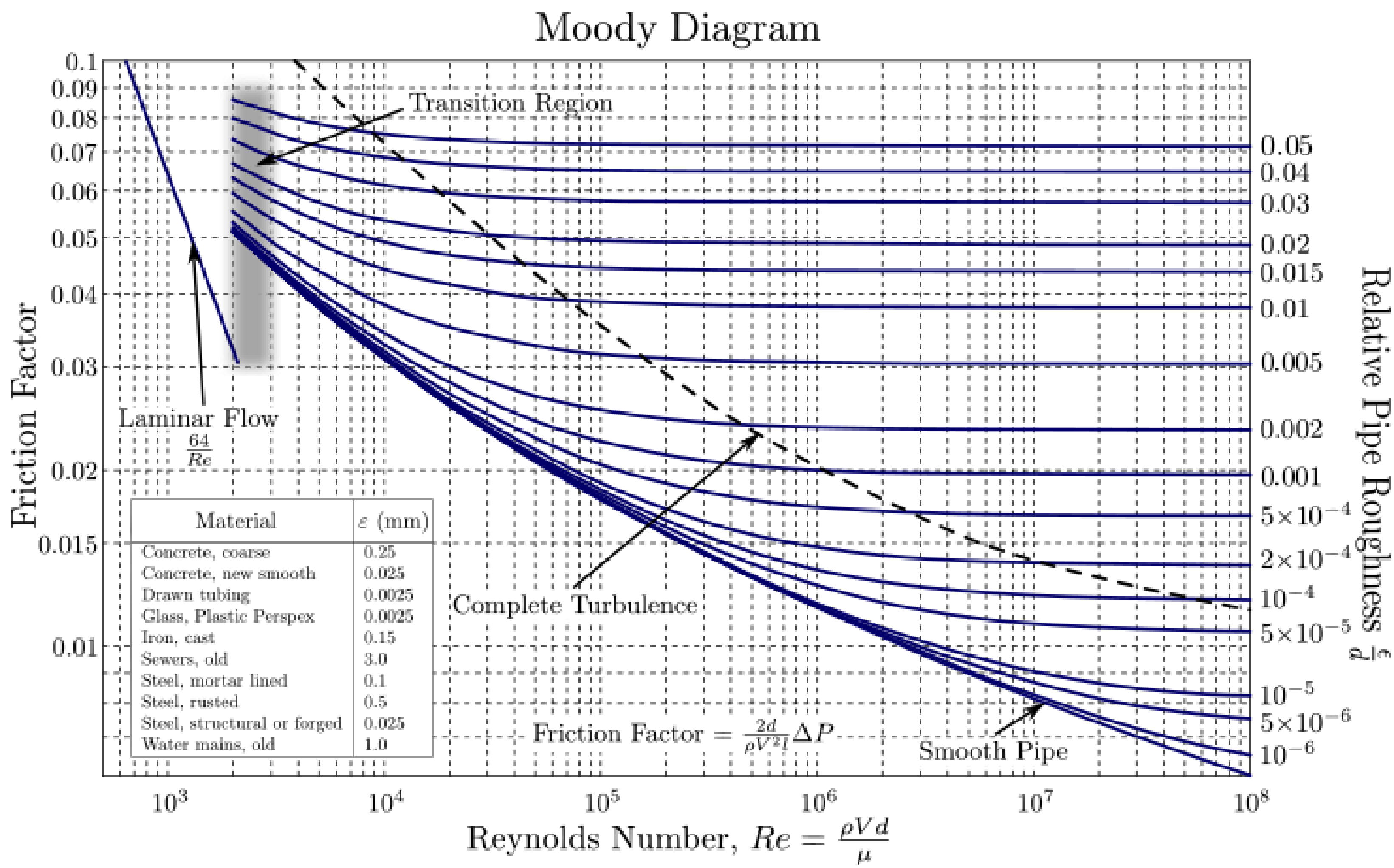
Fanning equation: $$dF = \frac{2V^2 f}{D} dL$$

This equation applies to single phase fluids.

The friction factor is obtained from the “Moody Diagram”

# Mechanical Energy Balance

## Moody Chart



## Mechanical Energy Balance

Friction factor equations. (Useful for computers and Excel)

$$f = \frac{16}{\mathrm{Re}}$$

Laminar Flow

$$f = \frac{0.046}{\mathrm{Re}^a}$$

Smooth pipes: *a* = 0.2
Iron or steel pipes: *a* = 0.16

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Colebrook equation for turbulent flow.

Equivalent length of valves and fittings.

**Pressure drop for valves and fittings is accounted for as equivalent length of pipe.**

# Mechanical Energy Balance

Equivalent length of valves and fittings.

| Equivalent Length of Straight Pipe for Valves and Fittings (feet) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Flanged Fittings | | Pipe Size | | | | | | | | | | | | |
| | | 1/2 | 3/4 | 1 | 1 1/4 | 1 1/2 | 2 | 2 1/2 | 3 | 4 | 5 | 6 | 8 | 10 |
| Elbows | Regular 90 deg | 0.9 | 1.2 | 1.6 | 2.1 | 2.4 | 3.1 | 3.6 | 4.4 | 5.9 | 7.3 | 8.9 | 12 | 14 |
| | Long radius 90 deg | 1.1 | 1.3 | 1.6 | 2.0 | 2.3 | 2.7 | 2.9 | 3.4 | 4.2 | 5 | 5.7 | 7 | 8 |
| | Regular 45 deg | 0.5 | 0.6 | 0.8 | 1.1 | 1.3 | 1.7 | 2.0 | 2.6 | 3.5 | 4.5 | 5.6 | 7.7 | 9 |
| Tees | Line flow | 0.7 | 0.8 | 1.0 | 1.3 | 1.5 | 1.8 | 1.9 | 2.2 | 2.8 | 3.3 | 3.8 | 4.7 | 5.2 |
| | Branch flow | 2.0 | 2.6 | 3.3 | 4.4 | 5.2 | 6.6 | 7.5 | 9.4 | 12.0 | 15 | 18 | 24 | 30 |
| Return Bends | Regular 180 deg | 0.9 | 1.2 | 1.6 | 2.1 | 2.4 | 3.1 | 3.6 | 4.4 | 5.9 | 7.3 | 8.9 | 12 | 14 |
| | Long radius 180 deg | 1.1 | 1.3 | 1.6 | 2.0 | 2.3 | 2.7 | 2.9 | 3.4 | 4.2 | 5 | 5.7 | 7 | 8 |
| Valves | Globe | 38.0 | 40.0 | 45.0 | 54.0 | 59.0 | 70.0 | 77.0 | 94.0 | 120.0 | 150 | 190 | 260 | 310 |
| | Gate | | | | | | 2.6 | 2.7 | 2.8 | 2.9 | 3.1 | 3.2 | 3.2 | 3.2 |
| | Angle | 15.0 | 15.0 | 17.0 | 18.0 | 18.0 | 21.0 | 22.0 | 28.0 | 38.0 | 50 | 63 | 90 | 120 |

# Mechanical Energy Balance - Fluid Flow

Scenario I

Need pressure drop in known pipes (pump or compressor is not present.)

**Incompressible Flow**

a) Isothermal ($\rho$ is constant)

$$\left.\frac{dp}{dL}\right|_{Tot} = -\rho\left(g\cdot\frac{dZ}{dL} + V\cdot\frac{dV}{dL} + \frac{\delta F}{\delta L}\right)$$

for a fixed $\rho$ $\Rightarrow$ V constant $\Rightarrow$ $dV = 0$

Integral form:

$$\boxed{\Delta p = -\rho\left[g\cdot\Delta Z + 2V^2\cdot f\cdot\frac{L+L_e}{D}\right]}$$

b) Nonisothermal

Liquids: It will not have a big error if you use $\rho(T_{average})$

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow**

a) Relatively small change in T (known)

For small pressure drop (something you can check after you are done) can use Bernoulli and fanning equation as follows

$$g \cdot dz + v \cdot dp + d\left(\frac{V^2}{2}\right) = -\delta F$$

$$\frac{g}{v^2} \cdot dz + \frac{1}{v} \cdot dp + \frac{V}{v^2} \cdot dV = -\frac{\delta F}{v^2}$$

$V$ = Velocity

$G$ = Mass flow (Kg/hr)

$v$ = Specific volume ($m^3$/Kg)=1/$\rho$

$A$ = Cross sectional area

Note: $$V = v \cdot \frac{G}{A}$$

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow.** Relatively small change in T (known)

$$\frac{g}{v^2}\cdot dz+\frac{1}{v}\cdot dp+\left(\frac{G}{A}\right)\cdot\frac{dV}{v}=-\frac{\delta F}{v^2}=-2\cdot f\cdot\left(\frac{G}{A}\right)^2\frac{dL}{D}$$

Now put in integral form

$$g\int\frac{dz}{v^2}+\int\frac{dp}{v}+\left(\frac{G}{A}\right)^2\int\frac{dV}{V}=-2\cdot\left(\frac{G}{A}\right)^2\cdot\frac{1}{D}\cdot\int f\,dL$$

Assume:

$$T_{av}=\frac{T_{in}+T_{out}}{2} \qquad f_{av}=\frac{f_{in}+f_{out}}{2}$$

$$\rho_{av}=\frac{\rho(T_{in},P_{in})+\rho(T_{out},P_{out})}{2} \qquad f_{av}=\frac{f(T_{in},P_{in})+f(T_{out},P_{out})}{2}$$

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow.** Relatively small change in T (known)

The integral form will be:

$$\rho_{av}^2 g\Delta z + \int_{in}^{out} \frac{dp}{v} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{V_{out}}{V_{in}}\right) = -2\left(\frac{G}{A}\right)^2 f_{av}\frac{L}{D}$$

Recall: $p\,v = \dfrac{Z\,R\,T}{M}$ M: Molecular weight

Then: $v \cong Z_{av}\dfrac{RT_{av}}{pM}$

and

$$\int \frac{dp}{v} = \frac{M}{Z_{av}RT_{av}}\int p\cdot dp = \frac{M}{2\cdot Z_{av}RT_{av}}\left(p_{out}^2 - p_{in}^2\right)$$

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow.** Relatively small change in T (known)

Substitute in the integral form:

$$\rho_{av}^2 g \cdot \Delta z + \frac{M}{2 \cdot Z_{av} R T_{av}} \left( p_{out}^2 - p_{in}^2 \right) + \left( \frac{G}{A} \right)^2 \ln\left( \frac{V_{out}}{V_{in}} \right) = -2 \left( \frac{G}{A} \right)^2 f_{av} \frac{L}{D}$$

Since: $$\frac{V_{out}}{V_{in}} = \left( \frac{Z_{out} \cdot T_{out}}{Z_{in} \cdot T_{in}} \right) \cdot \frac{p_{in}}{p_{out}}$$

we get

$$\boxed{p_{out} = \left[ p_{in}^2 - 2 \frac{Z_{av} R T_{av}}{M} \left\{ 2 \left( \frac{G}{A} \right)^2 \cdot f_{av} \cdot \frac{L}{D} + \left( \frac{G}{A} \right)^2 \ln\left( \frac{Z_{out} T_{out} p_{in}}{Z_{in} T_{in} p_{out}} \right) + \rho_{av}^2 \cdot g \cdot \Delta z \right\} \right]^{\frac{1}{2}}}$$

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow.** Relatively small change in T (known)

This is an equation of the form: $p_{out} = F(p_{out})$

Algorithm:

a) Assume $p_{out}^{(1)}$

b) Use formula to get a new value $p_{out}^{(2)} = F\left(p_{out}^{(1)}\right)$

c) Continue using $p_{out}^{(i+1)} = F\left(p_{out}^{(i)}\right)$

until $$\frac{p_{out}^{(i+1)} - p_{out}^{(i)}}{p_{out}^{(i)}} \le \varepsilon$$

OR BETTER: Use Solver in EXCEL

# Mechanical Energy Balance - Fluid Flow

## **Compressible Flow.** Old Formulas

$$Q = 77.54\frac{T_b}{P_b}\left(\frac{P_1^2 - e^s P_2^2}{GT_f L_e Z f}\right)^{0.5} D^{2.5}$$ General Flow Equation

$$Q = 435.87\,E\left(\frac{T_b}{P_b}\right)^{1.0788}\left(\frac{P_1^2 - e^s P_2^2}{G^{0.8539} T_f L_e Z}\right)^{0.5394} D^{2.6182}$$ Panhandle A

$$Q = 737\,E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1^2 - e^s P_2^2}{G^{0.961} T_f L_e Z}\right)^{0.51} D^{2.53}$$ Panhandle B

$$Q = 433.5\,E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^s P_2^2}{GT_f L_e Z}\right)^{0.5} D^{2.667}$$ Weymouth

$$Q = 38.77F\frac{T_b}{P_b}\left(\frac{P_1^2 - e^s P_2^2}{GT_f L_e Z}\right)^{0.5} D^{2.5}$$ AGA

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow.**

Generalized Metamodel

$$L\frac{Q^2}{D^5} = A(p_{in}^2 - p_{out}^2) - p_{av}^2 B\Delta z$$

obtained by detailed simulation (Aspen- ProII) and then regressed.

These simulators actually integrate the original ODE numerically.

$$\frac{g}{v^2}\cdot dz + \frac{1}{v}\cdot dp + \left(\frac{G}{A}\right)\cdot\frac{dV}{v} = -\frac{\delta F}{v^2} = -2\cdot f\cdot\left(\frac{G}{A}\right)^2\frac{dL}{D}$$

# Mechanical Energy Balance - Fluid Flow

**Compressible Flow.** Relatively small change in T (known)

The above algorithm can be applied for cases where

$$\frac{p_{out} - p_{in}}{p_{in}} \leq 0.2 - 0.3$$

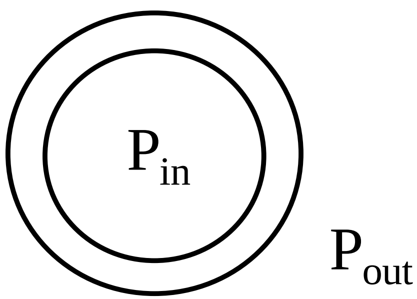
For longer pipes, break the pipe into smaller sections

# PIPING STRENGTH

## Bursting pressure of a pipe

$$P_b = 2S_T \frac{t_m}{D_m}$$

$D_m$= Mean Diameter
$t_m$= Wall Thickness
$S_T$= Tensile Strength (properties of material and fabricate)
$P_b$= Bursting pressure




$$P_b = P_{in} - P_{out}$$

# PIPING STRENGTH

## Safe Working Pressure

$$P_S = 2S_S \frac{t_m}{D_m}$$

We substitute with a safe working stress, $S_s < S_T$

Range of $S_s$ = 6500-9000 psi (T < 250 $^o$F)

(Low end) butt-welded

lap-welded (High end)

# PIPING STANDARD SIZES

***Schedule of a Pipe (American Standard Association)***

$$Sch = 100 * (P/S)$$

P = working pressure
S = allowable stress (design pressure)

- There are 10 Sch numbers:

10, 20, 30, **40**, 60, 80, 100, 120, 140, 160

# PIPING STANDARD SIZES

- Old Standards (before 1939), like "standard" (STD), "extra strong" (XS), "extra heavy" (XH) , "double extra strong" (XXS) or double extra-heavy" (XXH), are still in use.

- Pipes with thinner walls also exist (5S and 10S).

Do not get confused with BWG (Birmingham Wire Gauge) standards, used for Heat Exchanger tubes as given by the Tubular Exchanger Manufacturers Association (TEMA) Standards.

# PIPING STRENGTH

## *Schedule of a Pipe (American Standard Association)*

You specify a pipe by giving the diameter and the Schedule

- Get pressure inside , $P_{in}$ (psia)
- $P_S = P_{in} - 14.696$
- $\alpha = 1000 \frac{P_S}{S_S}$ ; $S_S$ ==> Characteristic of pipe (6500 – 9000 psi)
- Pick lower possible Sch standard.

$$Sch > a$$