CHEMICAL ENGINEERING DESIGN & SAFETY

CHE 4253

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Process Engineering Economics

3-Interest and Profitability

INTEREST

- **Interest –**
- *Simple Interest:*

$$I = P * i * n$$

I : Total interest paid
P : Principal or Capital borrowed
i : Interest rate for one period of time
n : Number of periods.

Repayment is

$$S = P + I = P * (1 + i * n)$$

Usually : 1 period = 1 year . For less than 1 year we have:

Ordinary Simple Interest = $P * i * d/360$

Exact Simple Interest = $P * i * d/365$

INTEREST

- *Compound Interest:*

At the end of each interest period the interest is added to the principal.

Period	Principal at start of period	Interest earned	Compound amount S at the end of period
1	P	P i	P(1+i)
2	P(1+i)	P(1+i)i	$P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2 i$	$P(1+i)^3$
.....			
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} i$	$P(1+i)^n$

Repayment is $S = P*(1 + i)^n$

$(1 + i)^n$: Discrete single payment compound-amount factor

INTEREST

- *Nominal Interest:*

General Formula:

$$S_{\text{after 1 year}} = P*(1 + r/m)^m$$

r : Nominal annual Interest
m: Number of periods of compounding per year.

$$S_{\text{after n years}} = P*(1 + r/m)^{m*n}$$

- *Effective Annual Interest Rate:*

Simple interest that will produce the same total interest at the end of one year.

$$S_{\text{after 1 year}} = P*(1 + r/m)^m \quad \text{Nominal}$$
$$= P*(1 + i_{eff}) \quad \text{Effective}$$

Then $i_{eff} = (1+r/m)^m - 1$

INTEREST

- *Nominal Interest:*

Interest rate for 1-year period but compounded for periods different than one year.

Example : P = 1000, at 6% compounded every 6 months.
At the end of six months the interest is:

$$I_{\text{after 1/2 year}} = P\ 0.06 / 2 = 30$$
$$S_{\text{after 1/2 year}} = P\ (1 + 0.06/2) = 1030$$

Then

$$I_{\text{after 1 year}} = P\ (1 + 0.06/2)\ 0.06 / 2 = 30.90$$
$$S_{\text{after 1 year}} = P(1 + 0.06/2)(1 + 0.06/2) = 1060.9$$

INTEREST

Example :

P = $1000
Interest = 2% monthly
Total Time = 2 years.

- *Simple :* $S = 1000\ (1+0.02*24) = \$1480$
- *Compounded :* $S = 1000(1+0.02)^{24} = \$1608$
- *Nominal Interest Rate:* 2 x 12 = 24 % (annual)
- *Effective Rate:* $i_{eff}=(1+0.02)^{12}-1=0.268$ (26.8%)

INTEREST

Continuous Compound Interest

At time n $\quad S = P + i*P*n$

At time $n+dn$ $\quad S+dS = S+i*(P + i*P*n)dn$

Then $\quad dS = i*S*dn$

Integrate from time zero ($S(0)=P$) to time n to get

$\ln(S_n/P) = i*n$ and $S_n = P*\exp(i*n)$

Compare to $\quad S_n = P*(1 + i_{eff})^n$

Effective Annual Interest Rate: $i_{eff} = e^i - 1$

INTEREST

	Repayment ($S_n = P + I$)
Simple Interest:	$S_n = P * (1 + i * n)$
Compound Interest	$S_n = P * (1 + i)^n$
Nominal Interest	$S_n = P * (1 + r/m)^{m*n}$
Continuous Compound Interest	$S_n = P * \exp(i*n)$
Effective Interest Rate	$S_n = P * (1 + i_{eff})^n$

Present Worth: Solve for P (Note: P is S_0)

INTEREST

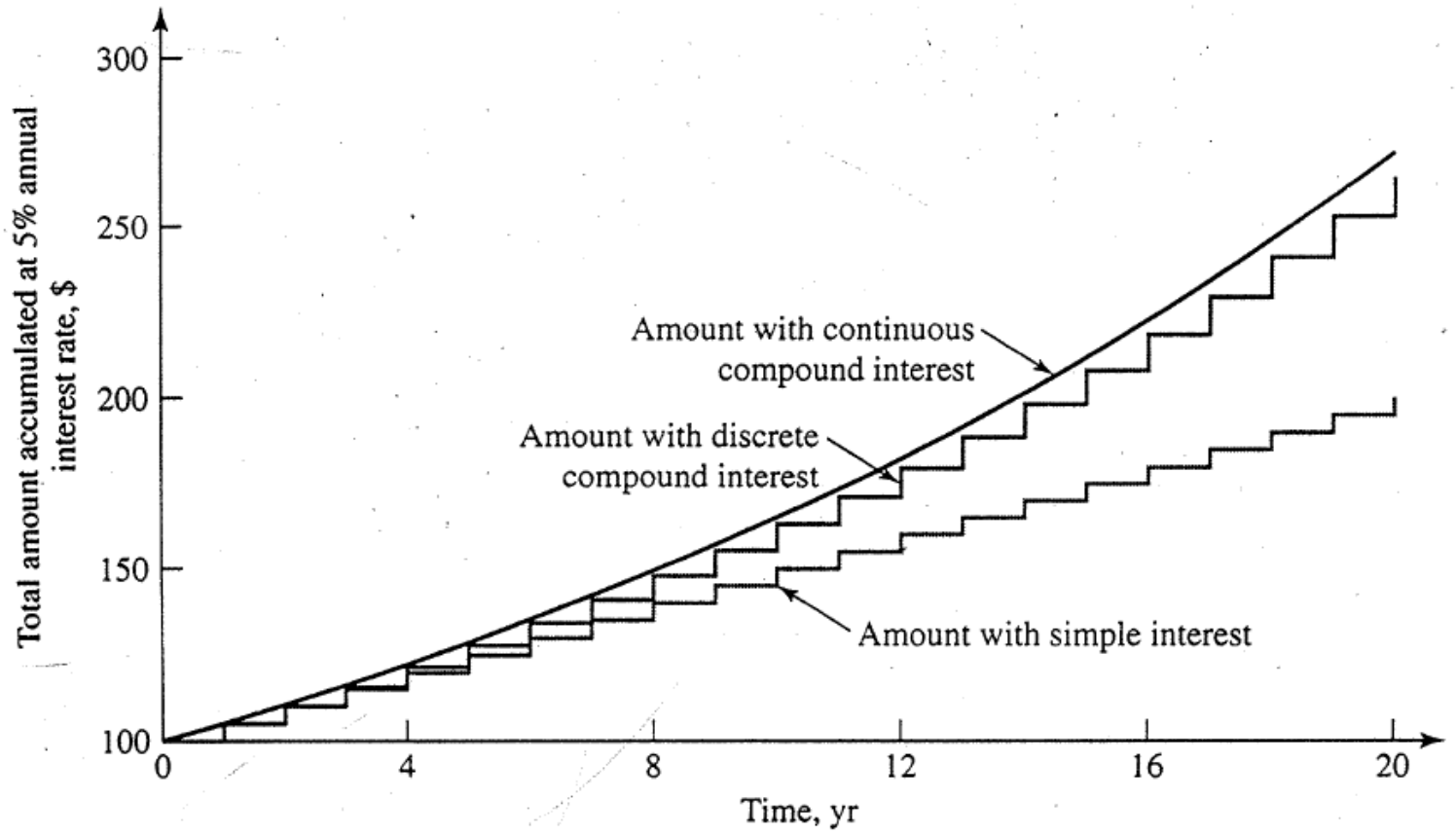

Figure 7-1
Comparison among total amounts accumulated with simple annual interest, discrete

PRESENT WORTH

Present principal that will yield a desired amount in the future.

Continuous compounding $S_n = P*\exp(i*n)$

$S_o = P = S_n *\exp(- i*n)$

Discrete compounding $S_n = P*(1 + i)^n$

$S_o = P = \dfrac{S_n}{(1+i)^n}$

$\dfrac{1}{(1+i)^n}$: Discrete single-payment present-worth factor.

Discount (used in bonds): $S_n - S_o$

PROFITABILITY

Term used to measure the amount of profit from a certain investment.

Total Profit cannot be used as means of comparing investments.

Example:

Investment	Profit
$ 100,000	10,000$/yr
$ 1,000,000	25,000$/yr

The second investment has a larger profit. However, when the profit is compared to the investment, the first investment looks better.

METHODS OF PROFITABILITY EVALUATION

1) *Return On Investment, ROI*. (Annual rate)

$$ROI = \frac{Net\ Profit}{Total\ \ Capital\ \ Investment}$$

Usually:

$$ROI = \frac{R - D - (R - d\,FCI)t}{FCI + WC}$$

Recall: *R=Sales-Cost=S-C*

- It is a "point" value in time.
- Does not take into account time value of money.
- Profits and costs may vary throughout the project.

METHODS OF PROFITABILITY EVALUATION

2) *Net Present Worth*

Physical meaning:
(Present value of annual cash flows) - (initial investment)

Example : Same data as before.
Assume the capital of the company is put at 15% interest.

Year	Cash Flow	Present Value	
1	$ 30,000	$ 26,087	(=30,000/(1+i))
2	$ 31,000	$ 23,440	(=31,000/(1+i)^2)
3	$ 36,000	$ 23,670	
4	$ 40,000	$ 22,870	
5	$ 43,000		
	$ 20,000	+ $ 31,332	
	TOTAL =	$ 127,399	

Net Present Worth = $127,399 - $110,000 = $17,399

METHODS OF PROFITABILITY EVALUATION

3) *Net Present Worth*

The formula is:

$$NPW = \sum_{k=1}^{n-1} \frac{CF_k}{(1+i)^k} + \frac{CF_n + V_S + WC}{(1+i)^n} - TCI$$

How does the rate of return for discount cash flow correlate to NPW?

Well, it is the interest rate when *NPW=0*

Excel: NPV(rate,value1,value2……,valuen)

METHODS OF PROFITABILITY EVALUATION

3) *Pay Out Time, POT*

Minimum time needed to recover the investment.

$$POT = \frac{FCI - V_S}{Average \quad Cash \quad Flow}$$

Other names: Payback time, Cash Recovery Period.

ALTERNATIVE INVESTMENTS

Acceptable Returns

Need to compare with other investments and their risks

Investment	*Return*	*Risk*
Government Bonds	5-7 %	Almost none
Preferred Stock	7-9 %	Some
Common Stock	7-9 %	Higher

ALTERNATIVE INVESTMENTS

Example

Investment	*Profit*	*ROI*	*NPW(5%, 15 yr)*
$ 1,200,000	$ 240,000	20%	$ 1,175,674
$ 2,000,000	$ 300,000	15%	$ 969,592

If the amount of investment is not an issue, we are faced with two options:

1) Invest 1,200,000 and put $800,000 in some other place
2) Invest $2,000,000

Incremental investment=$800,000.
Incremental profit=$ 60,000.
This corresponds to a ROI of 7.5%.
Bad deal !! Put the $800,000 in some other investment.

What if we use NPW? Same result!!! NPW 800,000 with 60,000 profit is=$ -206,081)