S&T HEX Kern Design Method

The analysis is focused on shell and tube heat exchanger without phase change. We use an E-shell type and the service must be executed in a single shell without loss of generality. The flow regime considered is a turbulent one, as it is common in industrial equipment. The physical properties are assumed constant, according to average values. Because we are focusing on the design procedure and not on the model, we chose the simpler Kern formulation for the shell-side equations[1] and the Dittus-Boelter as well as the Darcy-Weisbach for the tube-side.

The problem parameters (fixed prior the design), are represented with the symbol “^”.

Fluid Allocation: The selection of the tube-side and shell-side streams depends on several factors, e.g., fouling, temperature, pressure, flow rate, etc. Therefore, it will be considered that the stream allocation is established prior the optimization. Thus, the values of the physical properties in the tube-side and shell-side streams are fixed parameters.

Shell-Side Thermal and Hydraulic Equations: The flow velocity in the shell-side (*vs*) depends on the flow area between adjacent baffles (*Ar*):

$$vs = \frac{\widehat{ms}}{\widehat{\rho s}\, Ar} \tag{1}$$

where $\widehat{ms}$ and $\widehat{\rho s}$ are the shell-side stream flow rate and density, respectively.

This flow area corresponds to the area delimited by the shell diameter (*Ds*) and baffle spacing (*lbc*) multiplied by the free area ratio (*FAR*):

$$Ar = Ds\ FAR\ lbc \tag{2}$$

The free area ratio between baffles is given by:

$$FAR = \frac{(ltp - dte)}{ltp} = 1 - \frac{dte}{ltp} = 1 - \frac{1}{rp} \tag{3}$$

where *ltp* is the tube pitch, *dte* is the outer tube diameter, and *rp* is the ratio between the tube pitch and the tube diameter.

The Reynolds number associated to the shell-side velocity (*Res*) is given by:

$$Res = \frac{Deq\ vs\ \widehat{\rho s}}{\widehat{\mu s}} \tag{4}$$

where *Deq* is the equivalent diameter, and $\widehat{\mu s}$ is the shell-side stream viscosity.

The equivalent diameter present in the Reynolds number depends on the tube layout. For a square and triangular pattern, respectively:

$$Deq = \frac{4\ ltp^2}{\pi\ dte} - dte \qquad \text{(Square pattern)} \tag{5}$$

$$Deq = \frac{3.46\ ltp^2}{\pi\ dte} - dte \qquad \text{(Triangular pattern)} \tag{6}$$

The Nusselt number for the shell-side flow (*Nus*) is a function of the Reynolds and Prandtl numbers (*Res* and $\widehat{Prs}$):[1]

$$Nus = 0.36\ Res^{0.55} \widehat{Prs}^{1/3} \tag{7}$$

where the dimensionless groups Nusselt and Prandtl are defined by:

$$Nus = \frac{hs\ Deq}{\widehat{ks}} \tag{8}$$

$$\widehat{Prs} = \frac{\widehat{Cps}\ \widehat{\mu s}}{\widehat{ks}} \tag{9}$$

where *hs* is the shell-side convective heat transfer coefficient, $\widehat{ks}$ is the thermal conductivity, and $\widehat{Cps}$ is the heat capacity.

The head loss in the shell-side flow, dismissing nozzle pressure drop, can be calculated by:[1]

$$\frac{\Delta Ps}{\widehat{\rho s}\,\hat{g}} = fs\,\frac{Ds(Nb+1)}{Deq}\left(\frac{vs^2}{2\,\hat{g}}\right) \tag{10}$$

where *ΔPs* is the shell-side stream pressure drop, *fs* is the shell-side friction factor and *Nb* is the number of baffles.

The expression for evaluation of the shell-side friction factor is:

$$fs = 1.728\,Res^{-0.188} \tag{11}$$

The number of baffles is directly related to the baffle spacing and tube length:

$$Nb = \frac{L}{lbc} - 1 \tag{12}$$

Tube-Side Thermal and Hydraulic Equations: The flow velocity in the tube-side (*vt*) depends on the number of tubes per pass (*Ntp*) and the inner tube diameter (*dti*):

$$vt = \frac{4\,\widehat{mt}}{Ntp\,\pi\,\widehat{\rho t}\,dti^2} \tag{13}$$

where $\widehat{mt}$ and $\widehat{\rho t}$ are the tube-side stream flow rate and density, respectively.

The equation of the Reynolds number related to the tube-side flow rate (*Ret*) is:

$$Ret = \frac{dti\,vt\,\widehat{\rho t}}{\widehat{\mu t}} \tag{14}$$

where *dti* is the inner tube diameter, and $\widehat{\mu t}$ is the tube-side stream viscosity.

The Prandtl number for the tube-side stream ($\widehat{Prt}$) is:

$$\widehat{Prt} = \frac{\widehat{Cpt}\,\widehat{\mu t}}{\widehat{kt}} \tag{15}$$

where $\widehat{kt}$ and $\widehat{Cpt}$ are the tube-side stream thermal conductivity, and heat capacity, respectively.

The Reynolds and Prandtl numbers allow the evaluation of the tube-side Nusselt number (*Nut*) through the Dittus-Boelter correlation:[24]

$$Nut = 0.023\,Ret^{0.8}\widehat{Prt}^{n} \tag{16}$$

where the parameter *n* is equal to 0.4 for heating services and 0.3 for cooling services.

The definition of the Nusselt number is:

$$Nut = \frac{ht\,dti}{\widehat{kt}} \tag{17}$$

where *ht* is the tube-side convective heat transfer coefficient.

The head loss in the tube-side flow, dismissing nozzle pressure drop and the variation of the physical properties, is given by:

$$\frac{\Delta Pt}{\widehat{\rho t}\,\hat{g}} = \frac{ft\;Npt\;L\;vt^2}{2\,\hat{g}\,dti} + \frac{K\;Npt\;vt^2}{2\,\hat{g}} \tag{18}$$

where *ΔPt* is the tube-side stream pressure drop, and *ft* is the tube-side friction factor. The first term in the RHS corresponds to the head loss in the tube bundle and the second corresponds to the head loss in the front and rear headers. The parameter *K* is equal to 0.9 for one tube pass and 1.6 for two or more tube passes.

The expression for the Darcy friction factor for turbulent flow can be expressed by:

$$ft = 0.014 + \frac{1.056}{Ret^{0.42}} \tag{19}$$

Overall Heat Transfer Coefficient: The expression of the overall heat transfer coefficient (*U*) is:

$$U = \frac{1}{\frac{dte}{dti\,ht} + \frac{\widehat{Rft}\,dte}{dti} + \frac{dte\ln\left(\frac{dte}{dti}\right)}{2\,\widehat{ktube}} + \widehat{Rfs} + \frac{1}{hs}} \tag{20}$$

where the $\widehat{ktube}$ is the thermal conductivity of the tube wall, and $\widehat{Rft}$ and $\widehat{Rfs}$ are the fouling factors of the tube-side and shell-side streams, respectively.

Heat Transfer Rate Equation: According to the LMTD method, the heat transfer rate expression is:

$$\hat{Q} = UAreq\ \widehat{\Delta Tlm}\ F \tag{21}$$

where $\hat{Q}$ is the heat load, *Areq* is the required area, $\widehat{\Delta Tlm}$ is logarithmic mean temperature difference (LMTD), and *F* is the LMTD correction factor.

The LMTD is given by:

$$\widehat{\Delta Tlm} = \frac{(\widehat{Thi}-\widehat{Tco})-(\widehat{Tho}-\widehat{Tci})}{\ln\left(\frac{(\widehat{Thi}-\widehat{Tco})}{(\widehat{Tho}-\widehat{Tci})}\right)} \tag{22}$$

The LMTD correction factor is equal to 1, for one tube pass and is equal to the following expression for an even number of tube passes:

$$F = \frac{(\hat{R}^2+1)^{0.5}\ \ln\left(\frac{(1-\hat{P})}{(1-\hat{R}\,\hat{P})}\right)}{(\hat{R}-1)\ln\left(\frac{2-\hat{P}\left(\hat{R}+1-(\hat{R}^2+1)^{0.5}\right)}{2-\hat{P}\left(\hat{R}+1+(\hat{R}^2+1)^{0.5}\right)}\right)} \tag{23}$$

where:

$$\hat{R} = \frac{\widehat{Thi}-\widehat{Tho}}{\widehat{Tco}-\widehat{Tci}} \tag{24}$$

$$\hat{P} = \frac{\widehat{Tco}-\widehat{Tci}}{\widehat{Thi}-\widehat{Tci}} \tag{25}$$

The heat transfer area (*A*) is represented by the sum of the area of the surface of each tube:

$$A = Ntt\ \pi\ dte\ L \tag{26}$$

where *Ntt* is the total number of tubes.

In order to guarantee an adequate design margin, the exchanger area must be higher than the required area according to a certain “excess area” ($\widehat{Aexc}$), specified by the design engineer:

$$A \geq \left(1+\frac{\widehat{Aexc}}{100}\right) * Areq \tag{27}$$

Therefore, the heat transfer rate equation is reorganized using actual heat exchanger area:

$$UA \geq \left(1+\frac{\widehat{Aexc}}{100}\right)\frac{\hat{Q}}{\widehat{\Delta Tlm}\ F} \tag{28}$$

Bounds on Pressure Drops, Flow Velocities and Reynolds Numbers: During the process design, allowable pressure drops are imposed according to the pressure profile of the unit. These parameters are related to a trade-off between capital and operating costs. The corresponding constraints are:

$$\Delta Ps \leq \widehat{\Delta Psdisp} \tag{29}$$

$$\Delta Pt \leq \widehat{\Delta Ptdisp} \tag{30}$$

Additionally, lower and upper bounds on flow velocities are also established:

$$vs \geq \widehat{vsmin} \tag{31}$$

$$vs \leq \widehat{vsmax} \tag{32}$$

$$vt \geq \widehat{vtmin} \tag{33}$$

$$vt \leq \widehat{vtmax} \tag{34}$$

Flow velocity lower bounds seek to avoid fouling susceptible conditions. Corresponding upper bounds aims to avoid erosional conditions that could damage the heat exchanger during its operational life.

According to the application range of the convective heat transfer coefficient correlations, there are bounds on the Reynolds numbers in the shell-side and tube-side:

$$Res \geq 2 \cdot 10^3 \tag{35}$$

$$Ret \geq 10^4 \tag{36}$$

Geometric Constraints: The baffle spacing must be limited between 20% and 100% of the shell diameter

$$lbc \geq 0.2\ Ds \tag{37}$$

$$lbc \leq 1.0\ Ds \tag{38}$$

The ratio between the tube length and shell diameter must be between 3 and 15: [26]

$$L \geq 3\,Ds \qquad (39)$$

$$L \leq 15\,Ds \qquad (40)$$

Objective function: The optimization problem seeks to minimize the heat transfer area, which has a direct impact in the exchanger cost:

$$\min \quad A \qquad (41)$$