FINANCIAL RISK

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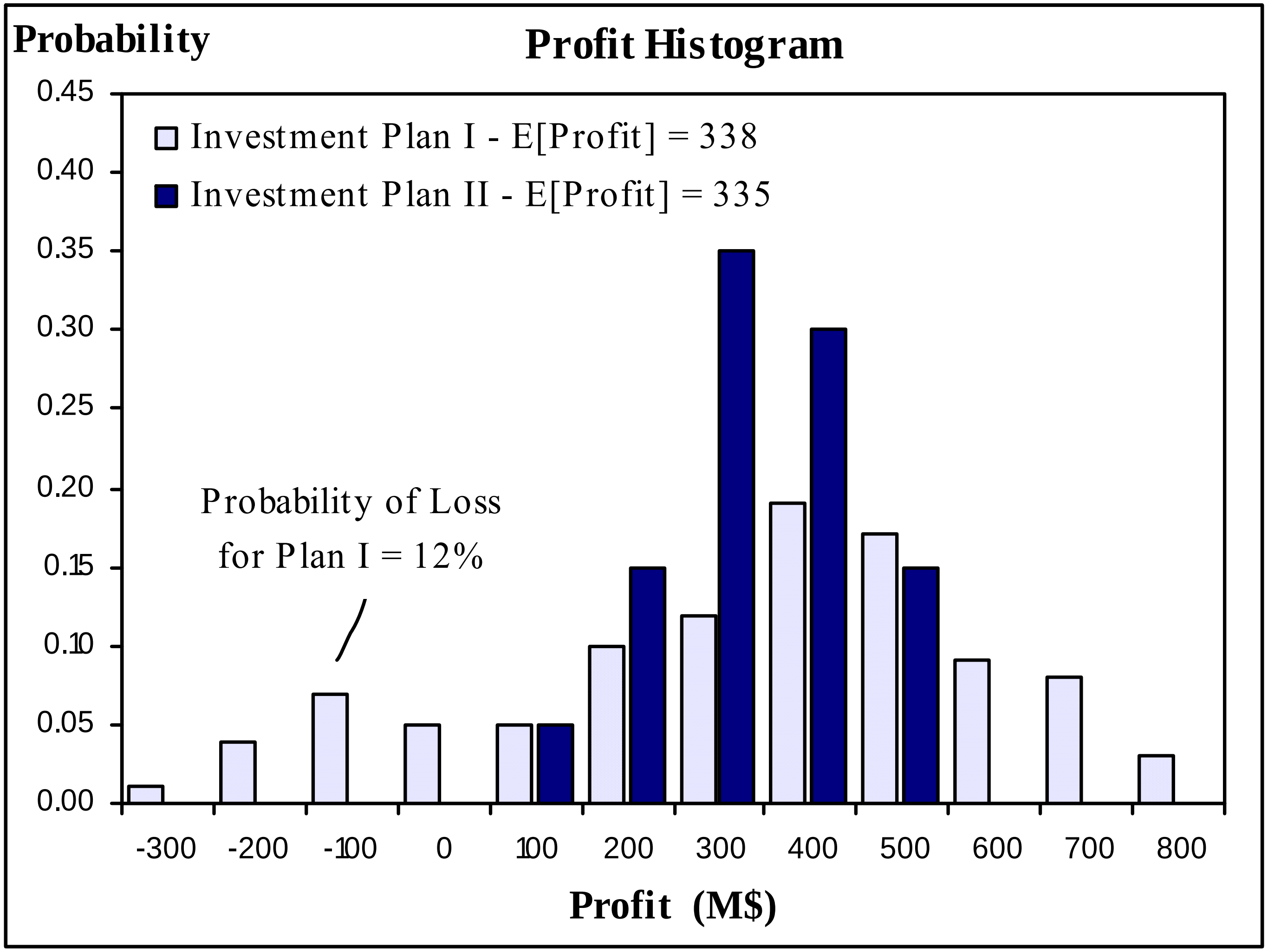
Scope of Discussion

- We will discuss the definition and management of financial risk in in any design process or decision making paradigm, like…
  - Investment Planning
  - Scheduling and more in general, operations planning
  - Supply Chain modeling, scheduling and control
  - Short term scheduling (including cash flow management)
  - Design of process systems
  - Product Design
- Extensions that are emerging are the treatment of other risks in a multiobjective (?) framework, including for example
  - Environmental Risks
  - Accident Risks (other than those than can be expressed as financial risk)

Introduction – Understanding Risk

Consider two investment plans, designs, or operational decisions



Conclusions

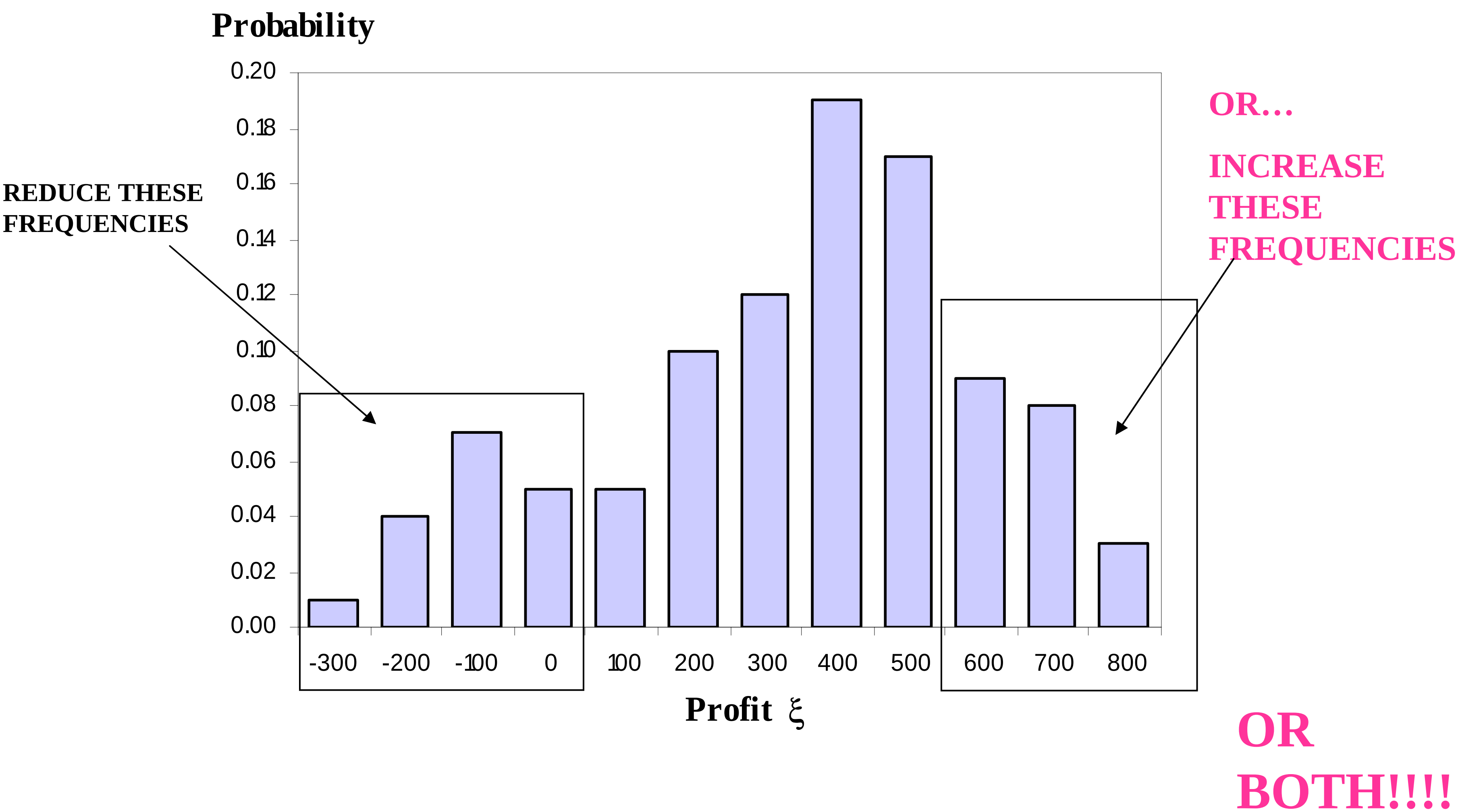
- Risk can only be assessed after a plan has been selected but it cannot be managed during the optimization stage (even when stochastic optimization including uncertainty has been performed).

- There is a need to develop new models that allow not only assessing but managing financial risk.

- The decision maker has two simultaneous objectives:
  - Maximize Expected Profit.
  - Minimize Risk Exposure

What does Risk Management mean?

One wants to modify the profit distribution in order to satisfy the preferences of the decision maker



Characteristics of Two-Stage Stochastic Optimization Models

Philosophy

- Maximize the *Expected Value* of the objective over all possible realizations of uncertain parameters.
- Typically, the objective is *Expected Profit* , usually *Net Present Value*.
- Sometimes the minimization of *Cost* is an alternative objective.

Uncertainty

- Typically, the uncertain parameters are: *market demands, availabilities, prices, process yields, rate of interest, inflation, etc*.
- In Two-Stage Programming, uncertainty is modeled through a finite number of independent *Scenarios*.
- Scenarios are typically formed by *random samples* taken from the probability distributions of the uncertain parameters.

Characteristics of Two-Stage Stochastic Optimization Models

First-Stage Decisions

- Taken before the uncertainty is revealed. They usually correspond to structural decisions (not operational).
- Also called “Here and Now” decisions.
- Represented by “Design” Variables.
- Examples:
  - To build a plant or not. How much capacity should be added, etc.
  - To place an order now.
  - To sign contracts or buy options.
  - To pick a reactor volume, to pick a certain number of trays and size the condenser and the reboiler of a column, etc

Characteristics of Two-Stage Stochastic Optimization Models

Second-Stage Decisions

- Taken in order to adapt the plan or design to the uncertain parameters realization.
- Also called "Recourse" decisions.
- Represented by "Control" Variables.
- Example: the operating level; the production slate of a plant.

- Sometimes first stage decisions can be treated as second stage decisions. In such case the problem is called a multiple stage problem.

Shortcomings

- The model is unable to perform risk management decisions.

Two-Stage Stochastic Formulation

Let us leave it linear because as is it is complex enough.!!!

LINEAR MODEL SP

$$Max \sum_s p_s q_s^T y_s - c^T x$$

Recourse Function — **First-Stage Cost**

$s.t.$

$$Ax = b$$ **First-Stage Constraints**

$$T_s x + W y_s = h_s$$ **Second-Stage Constraints**

$$x \geq 0 \qquad x \in X$$

$$y_s \geq 0$$

Technology matrix

First stage variables

Second Stage Variables

Recourse matrix (Fixed Recourse)

Sometimes not fixed (Interest rates in Portfolio Optimization)

***Complete recourse*:** the recourse cost (or profit) for every possible uncertainty realization remains finite, independently of the first-stage decisions (x).

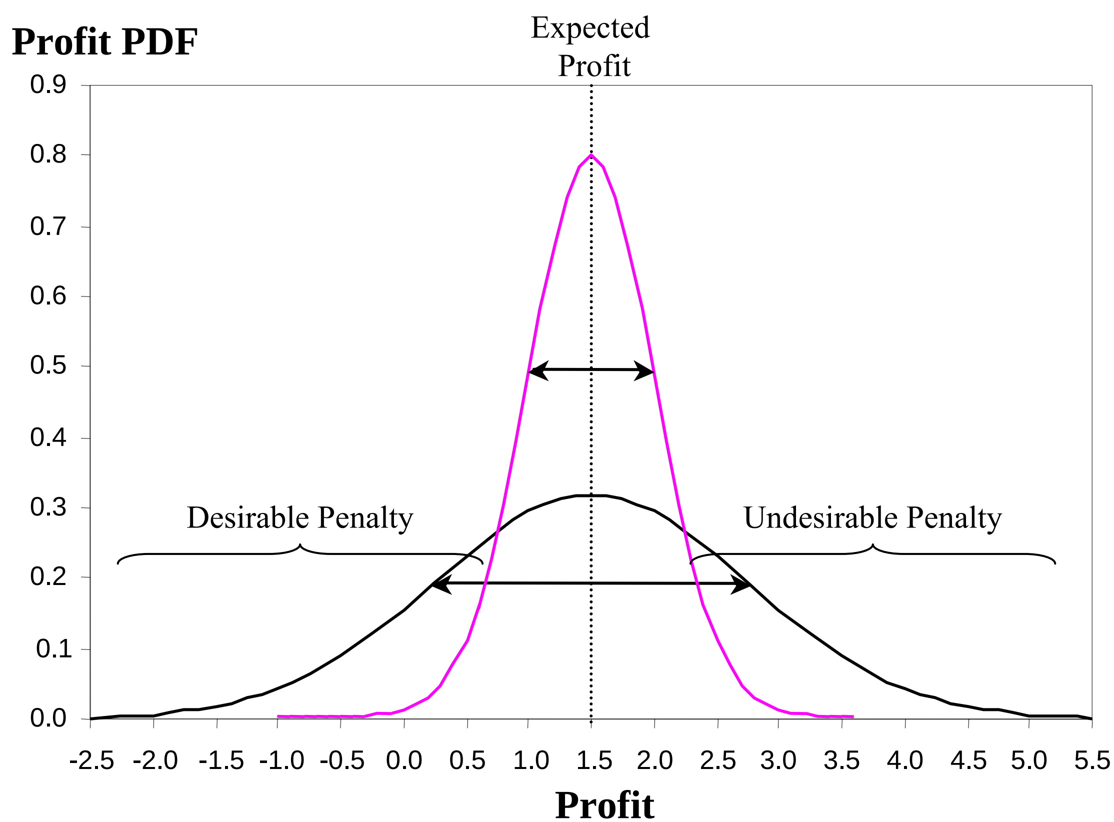
***Relatively complete recourse*:** the recourse cost (or profit) is feasible for the set of feasible first-stage decisions. This condition means that for every feasible first-stage decision, there is a way of adapting the plan to the realization of uncertain parameters.

We also have found that one can sacrifice efficiency for certain scenarios to improve risk management. We do not know how to call this yet.

Previous Approaches to Risk Management

- **Robust Optimization Using Variance (Mulvey et al., 1995)**

Maximize E[Profit] - ρ·V[Profit]



Underlying Assumption: Risk is monotonic with variability

Robust Optimization Using Variance

- **Drawbacks**

  - Variance is a symmetric risk measure: profits both above and below the target level are penalized equally. We only want to penalize profits below the target.

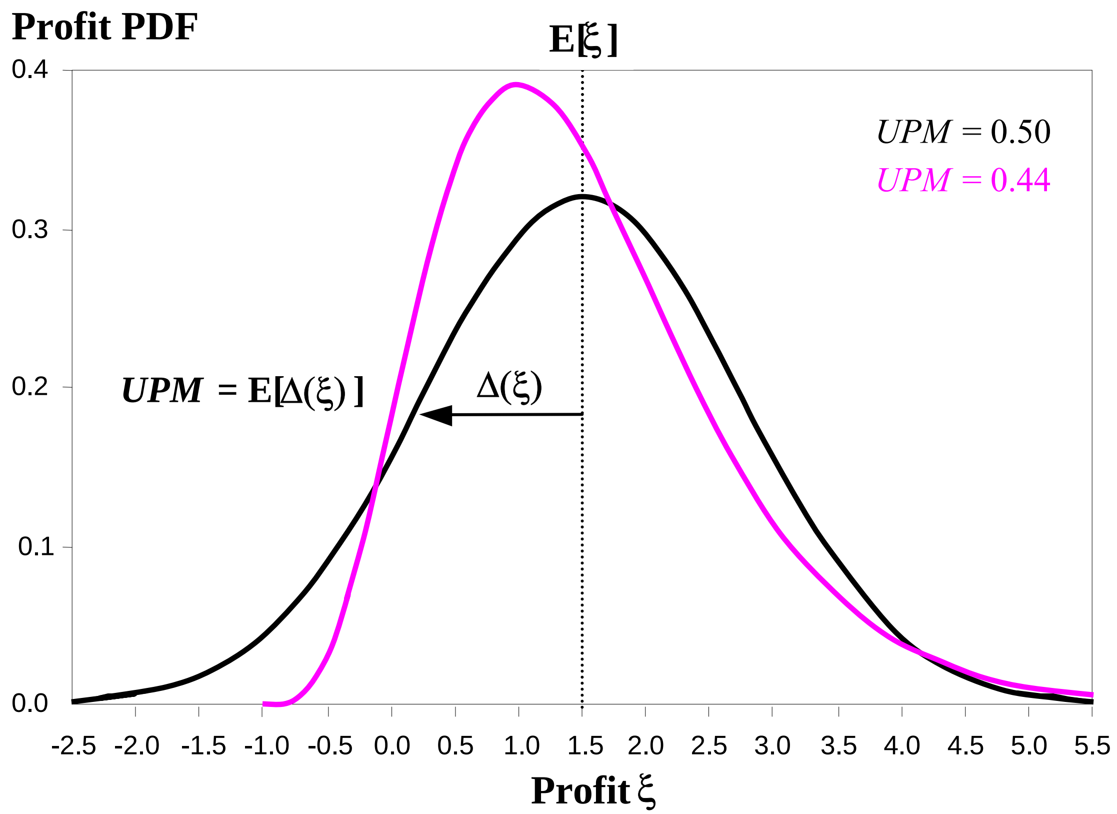
  - Introduces non-linearities in the model, which results in serious computational difficulties, specially in large-scale problems.

  - The model may render solutions that are stochastically dominated by others. This is known in the literature as not showing Pareto-Optimality. In other words there is a better solution (y_s, x^*) than the one obtained (y_s^*, x^*).

Previous Approaches to Risk Management

- Robust Optimization using Upper Partial Mean (Ahmed and Sahinidis, 1998)

Maximize E[Profit] - ρ·UPM



Underlying Assumption: Risk is monotonic with lower variability

Robust Optimization using the UPM

- Robust Optimization using the UPM

Advantages

- Linear measure

Disadvantages

- The UPM may misleadingly favor non-optimal second-stage decisions.
- Consequently, financial risk is not managed properly and solutions with higher risk than the one obtained using the traditional two-stage formulation may be obtained.
- The model losses its scenario-decomposable structure and stochastic decomposition methods can no longer be used to solve it.

Robust Optimization using the UPM

Objective Function: Maximize E[Profit] - ρ·UPM

$$UPM = \sum_{s \in S} p_s \Delta_s \qquad \Delta_s = Max\left\{0\,;\sum_{k \in S} p_k Profit_k - Profit_s\right\}$$

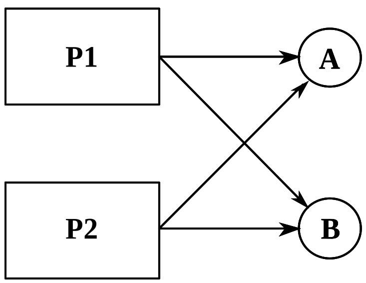
ρ = 3	$Profit_s$		Δ_s	
	Case I	Case II	Case I	Case II
S1	150	100	0	0
S2	125	100	0	0
S3	75	75	25	6.25
S4	50	50	50	31.25
E[Profit]	100.00	81.25		
UPM	18.75	9.38		
Objective	43.75	53.13		

Downside scenarios are the same, but the UPM is affected by the change in expected profit due to a different upside distribution.

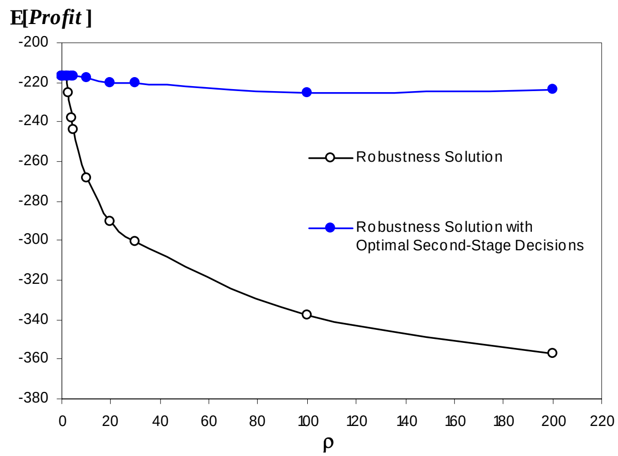
As a result a wrong choice is made.

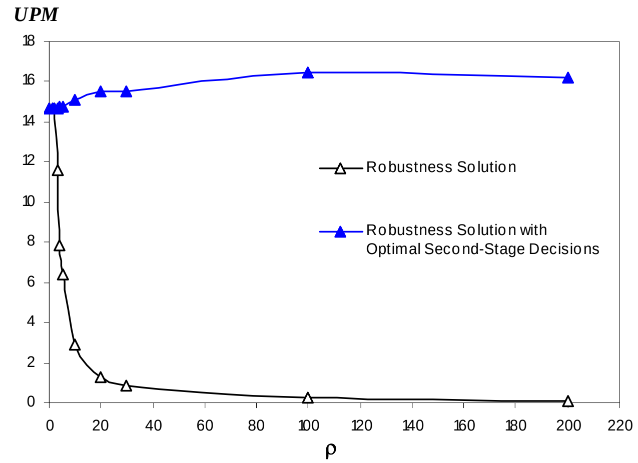
Robust Optimization using the UPM

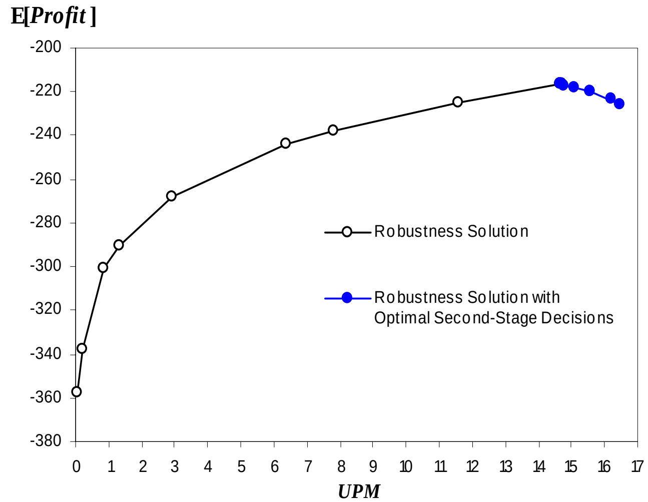
Effect of Non-Optimal Second-Stage Decisions



Both technologies are able to produce two products with different production cost and at different yield per unit of installed capacity







OTHER APPROACHES

- Cheng, Subrahmanian and Westerberg (2002, unpublished)
  - Multiobjective Approach: Considers Downside Risk, ENPV and Process Life Cycle as alternative Objectives.
  - Multiperiod Decision process modeled as a Markov decision process with recourse.
  - The problem is sometimes amenable to be reformulated as a sequence of single-period sub-problems, each being a two-stage stochastic program with recourse. These can often be solved backwards in time to obtain Pareto Optimal solutions.

This paper proposes a new design paradigm of which risk is just one component. We will revisit this issue later in the talk.

OTHER APPROACHES

- **Risk Premium (Applequist, Pekny and Reklaitis, 2000)**
  - Observation: Rate of return varies linearly with variability. The of such dependance is called Risk Premium.
  - They suggest to benchmark new investments against the historical
  - risk premium by using a two objective (risk premium and profit)
  - problem.
  - The technique relies on using variance as a measure of variability.

Previous Approaches to Risk Management

Conclusions

- The minimization of Variance penalizes both sides of the mean.
- The Robust Optimization Approach using Variance or UPM is not suitable for risk management.
- The Risk Premium Approach (Applequist et al.) has the same problems as the penalization of variance.

THUS,

- Risk should be properly defined and *directly* incorporated in the models to manage it.
- The multiobjective Markov decision process (Applequist et al, 2000) is very closely related to ours and can be considered complementary. In fact (Westerberg dixit) it can be extended to match ours in the definition of risk and its multilevel parametrization.

Probabilistic Definition of Risk

Financial Risk = Probability that a plan or design does not meet a certain profit target

$$Risk(x,\Omega) = P(Profit < \Omega)$$

$$\Downarrow$$

Scenarios are independent events

$$Risk(x,\Omega) = \sum_s p_s \, P(Profit_s \leq \Omega)$$

$$\Downarrow$$

For each scenario the profit is either greater/equal or smaller than the target

$$P(Profit_s < \Omega) = \begin{cases} 1 & \text{If } Profit_s < \Omega \\ 0 & \text{else} \end{cases}$$

$$\Downarrow$$

z_s is a new *binary* variable

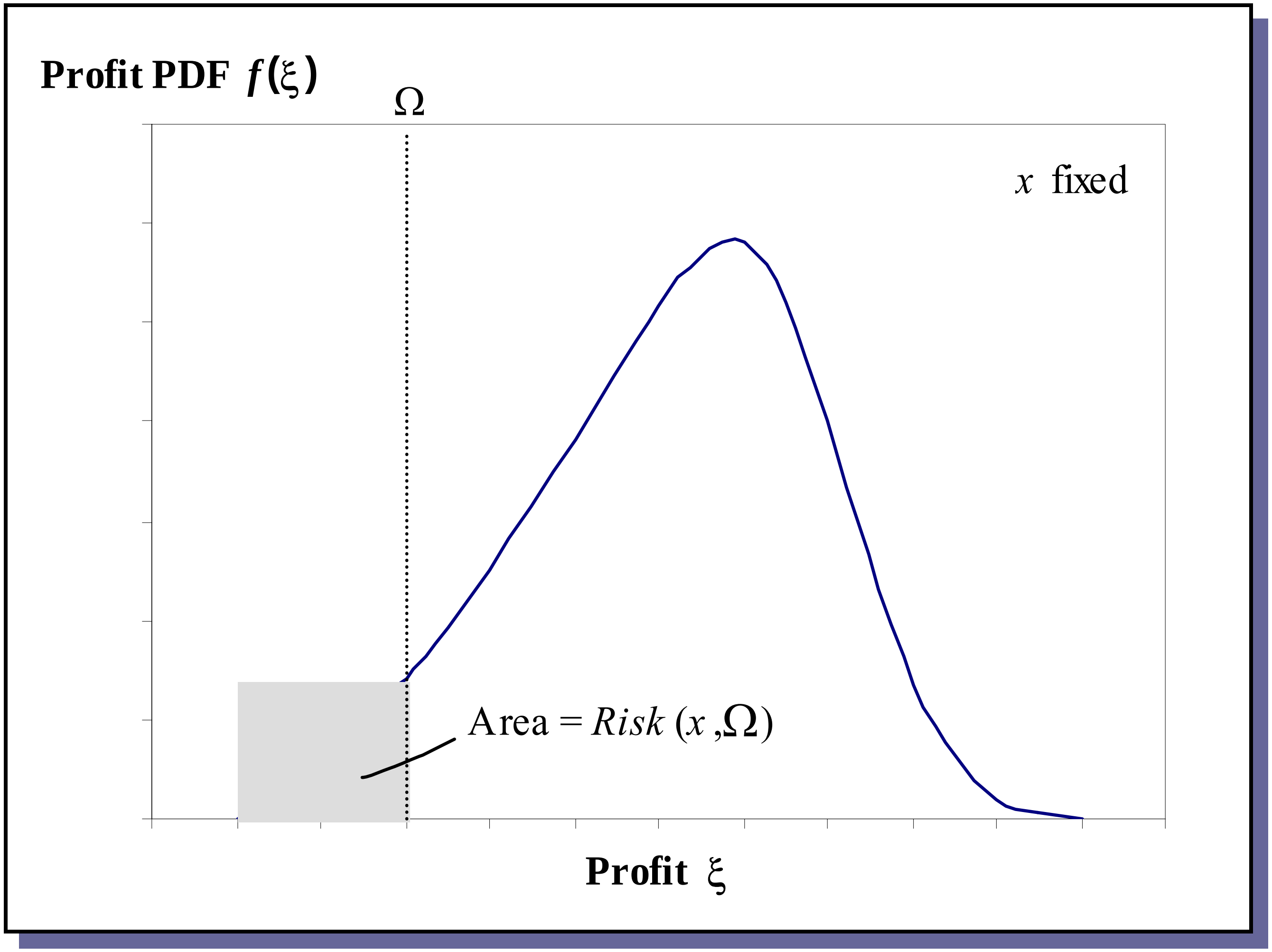
$$P(Profit_s > \Omega) = z_s$$

$$\Downarrow$$

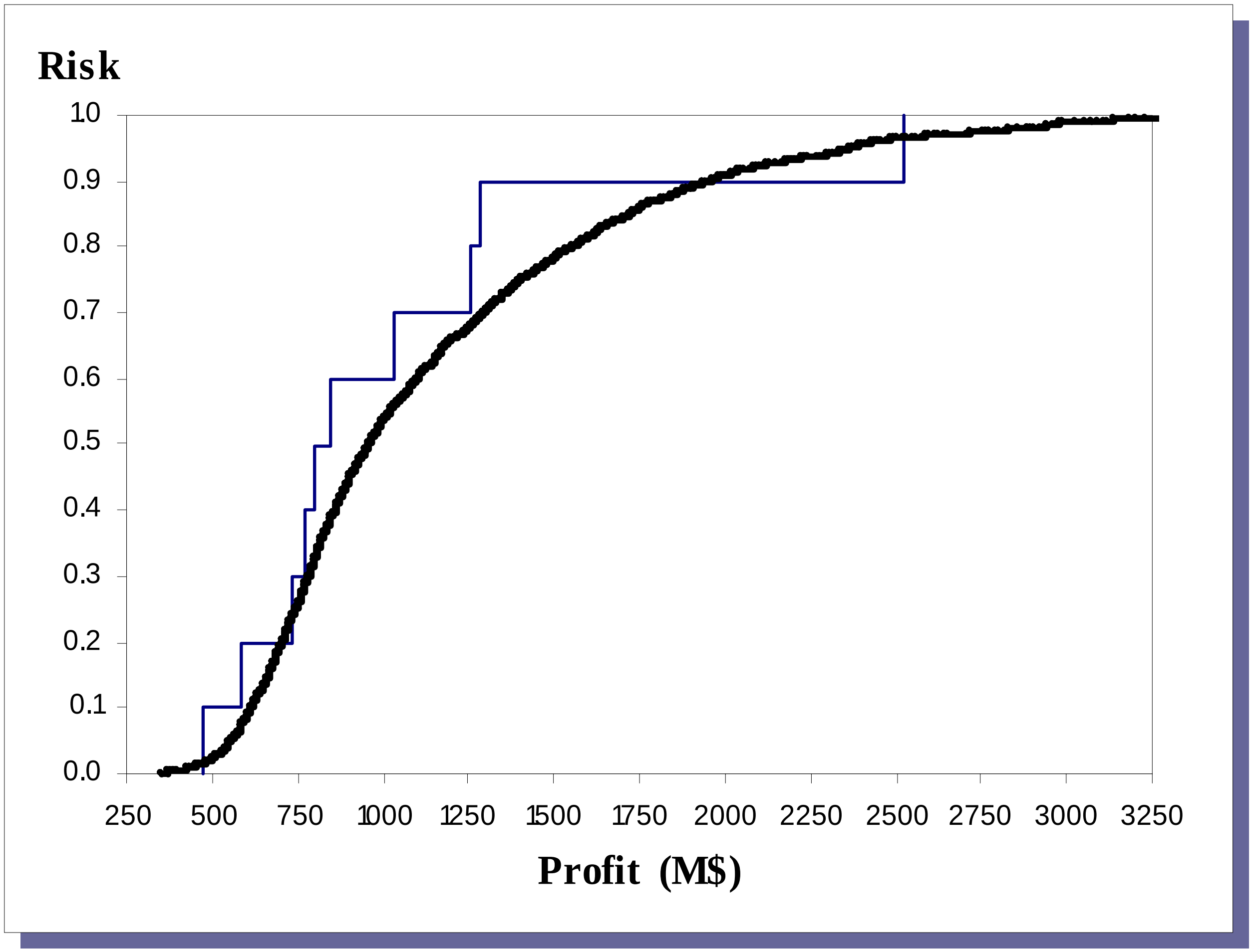
Formal Definition of Financial Risk

$$Risk(x,\Omega) = \sum_s p_s z_s$$

Financial Risk Interpretation

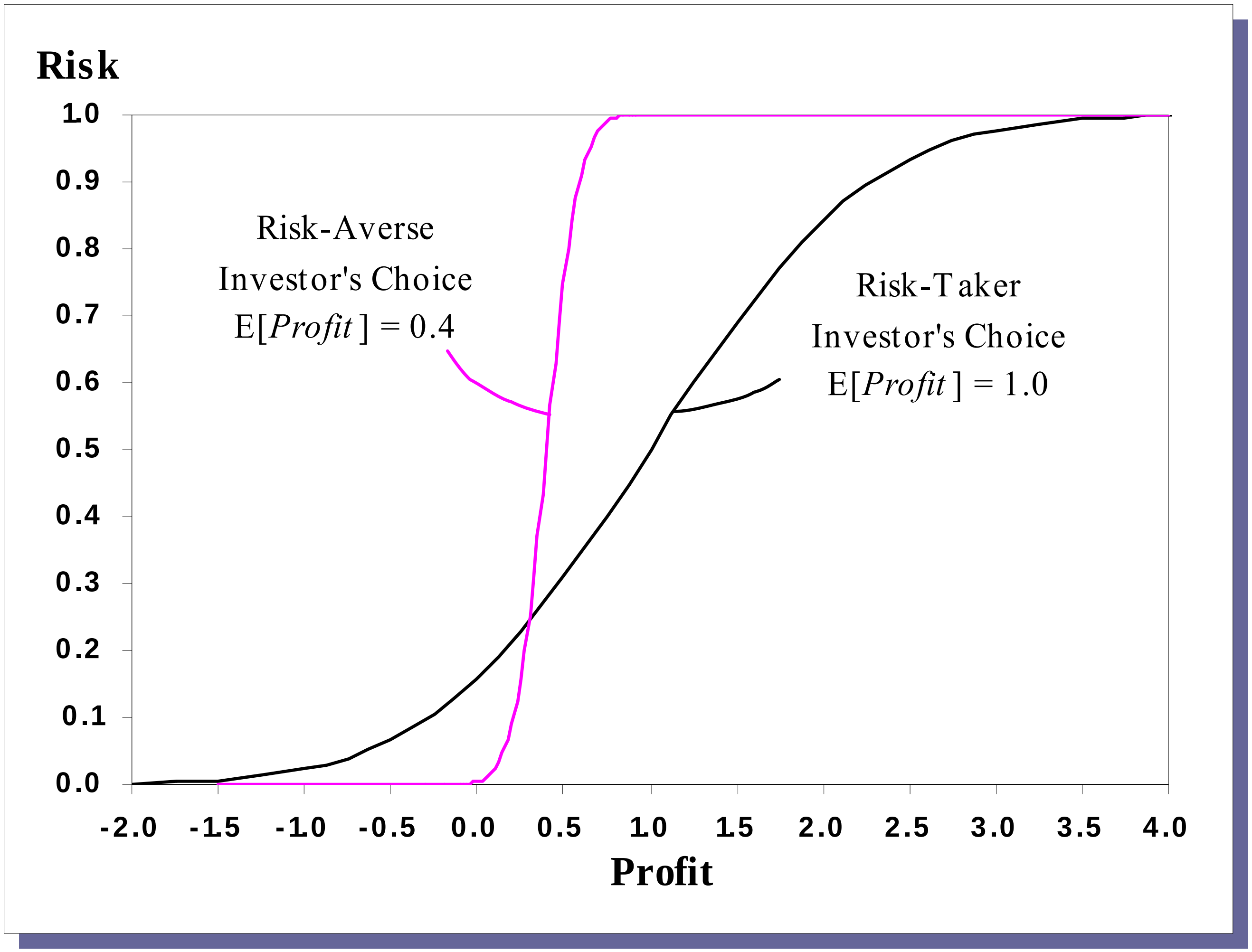


Cumulative Risk Curve



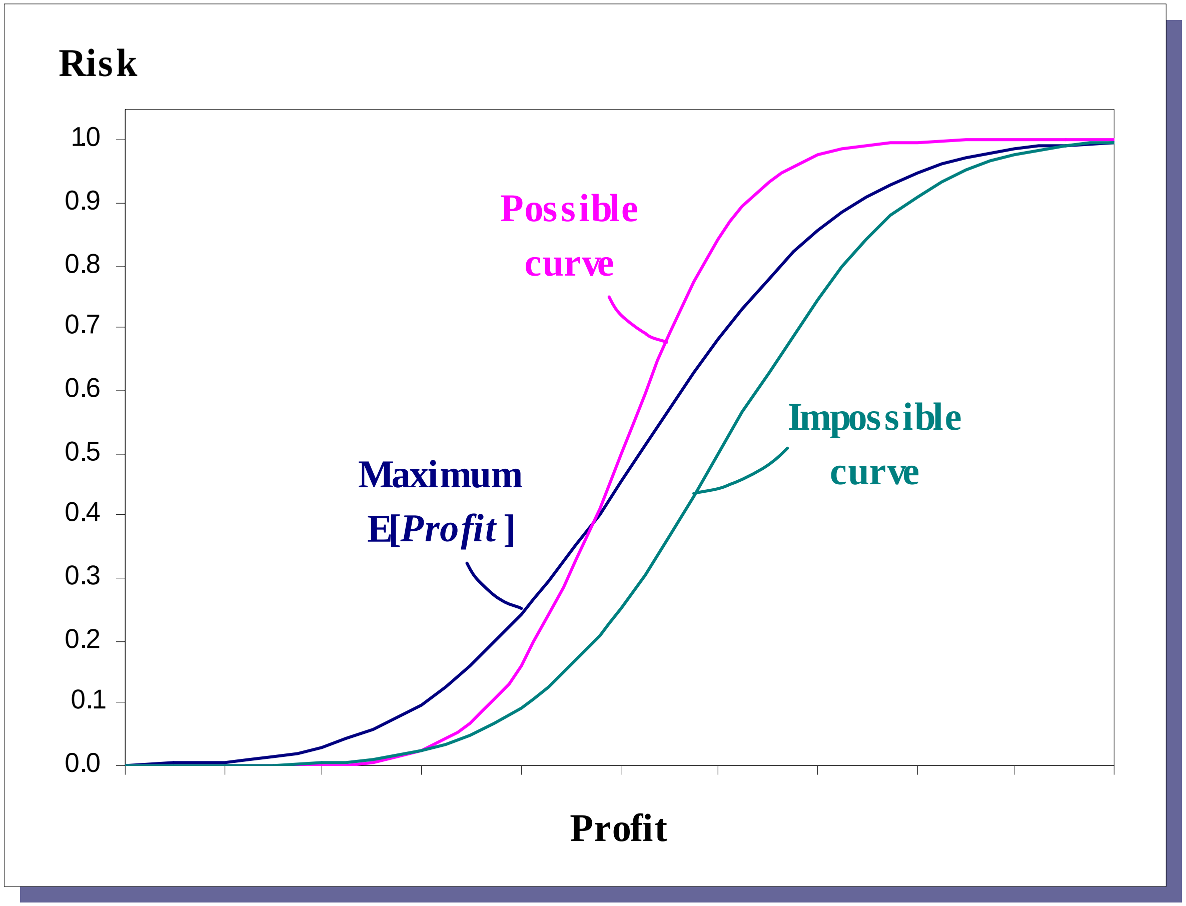
Our intention is to modify the shape and location of this curve according to the attitude towards risk of the decision maker

Risk Preferences and Risk Curves

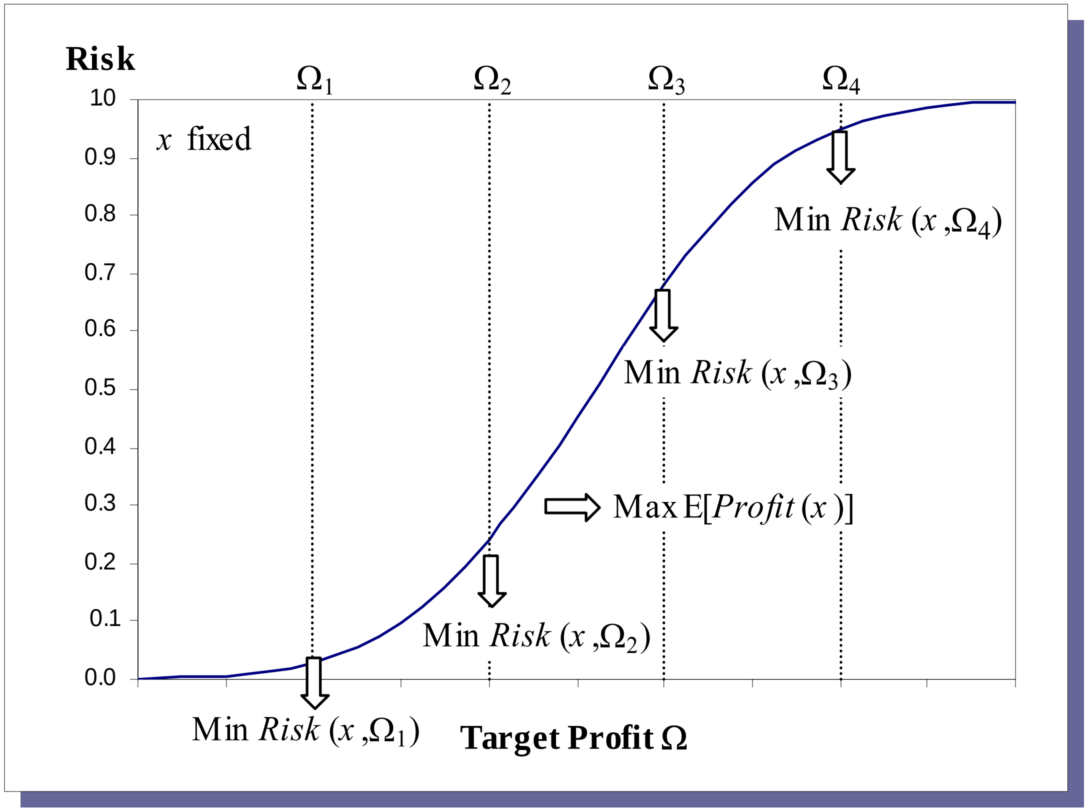


Risk Curve Properties

A plan or design with Maximum E[*Profit*] (i.e. optimal in Model SP) sets a theoretical limit for financial risk: it is impossible to find a feasible plan/design having a risk curve entirely beneath this curve.



Minimizing Risk: a Multi-Objective Problem



$$Max \;\; E[Profit] = \sum_s p_s q_s y_s - c^T x$$

$$Min \;\; Risk[\Omega_1] = \sum_s p_s z_{s1}$$

$$\vdots$$

$$Min \;\; Risk[\Omega_i] = \sum_s p_s z_{si}$$

s.t.

$$Ax = b$$

$$T_s x + W y_s = h_s$$

$$x \geq 0 \qquad x \in X$$

$$y_s \geq 0$$

$$q_s^T y_s - c^T x \geq \Omega - U_s z_s$$

$$q_s^T y_s - c^T x \leq \Omega + U_s (1 - z_s)$$

$$z_s \in (0,1)$$

Multiple Objectives:

- At each profit we want minimize the associated risk
- We also want to maximize the expected profit

Parametric Representations of the Multi-Objective Model – Restricted Risk

Restricted Risk MODEL

$$Max \sum_s p_s q_s y_s - c^T x$$

$s.t.$

$$Ax = b$$

$$T_s x + W y_s = h_s$$

$$x \geq 0 \qquad x \in X$$

$$y_s \geq 0$$

$$\sum_s p_s z_{si} \leq \varepsilon_i$$

Forces Risk to be lower than a specified level

$$q_s^T y_s - c^T x \geq \Omega_i - U_s z_{si}$$

$$q_s^T y_s - c^T x \leq \Omega_i + U_s (1 - z_{si})$$

$$z_s \in (0,1)$$

Risk Management Constraints

Parametric Representations of the Multi-Objective Model – Penalty for Risk

Risk Penalty MODEL

$$Max \sum_s p_s q_s^T y_s - c^T x - \sum_i \rho_i \sum_s p_s z_{si}$$

Penalty Term

$s.t.$

$$Ax = b$$

$$T_s x + W y_s = h_s$$

$$x \geq 0 \qquad x \in X$$

$$y_s \geq 0$$

$$\left.\begin{aligned} & q_s^T y_s - c^T x \geq \Omega_i - U_s z_{si} \\ & q_s^T y_s - c^T x \leq \Omega_i + U_s (1 - z_{si}) \\ & z_s \in (0,1) \end{aligned}\right\}$$

Risk Management Constraints

STRATEGY

Define several profit Targets and penalty weights to solve the model using a multi-parametric approach

Risk Management using the New Models

Advantages

- Risk can be effectively managed according to the decision maker's criteria.
- The models can adapt to risk-averse or risk-taker decision makers, and their risk preferences are easily matched using the risk curves.
- A full spectrum of solutions is obtained. These solutions always have optimal second-stage decisions.
- Model Risk Penalty conserves all the properties of the standard two-stage stochastic formulation.

Disadvantages

- The use of binary variables is required, which increases the computational time to get a solution. This is a major limitation for large-scale problems.

Risk Management using the New Models

Computational Issues

- The most efficient methods to solve stochastic optimization problems reported in the literature exploit the decomposable structure of the model.
- This property means that each scenario defines an independent second-stage problem that can be solved separately from the other scenarios once the first-stage variables are fixed.
- The Risk Penalty Model is decomposable whereas Model Restricted Risk is not. Thus, the first one is model is preferable.
- Even using decomposition methods, the presence of binary variables in both models constitutes a major computational limitation to solve large-scale problems.
- It would be more convenient to measure risk indirectly such that binary variables in the second stage are avoided.

Downside Risk

Downside Risk (Eppen et al, 1989) = Expected Value of the Positive Profit Deviation from the target

$$DRisk(x,\Omega) = E[\delta(x,\Omega)]$$

⇩

Positive Profit Deviation from Target Ω

$$\delta(x,\Omega) = \begin{cases} \Omega - Profit(x) & \text{If } Profit(x) < \Omega \\ 0 & \text{Otherwise} \end{cases}$$

⇩

The Positive Profit Deviation is also defined for each scenario

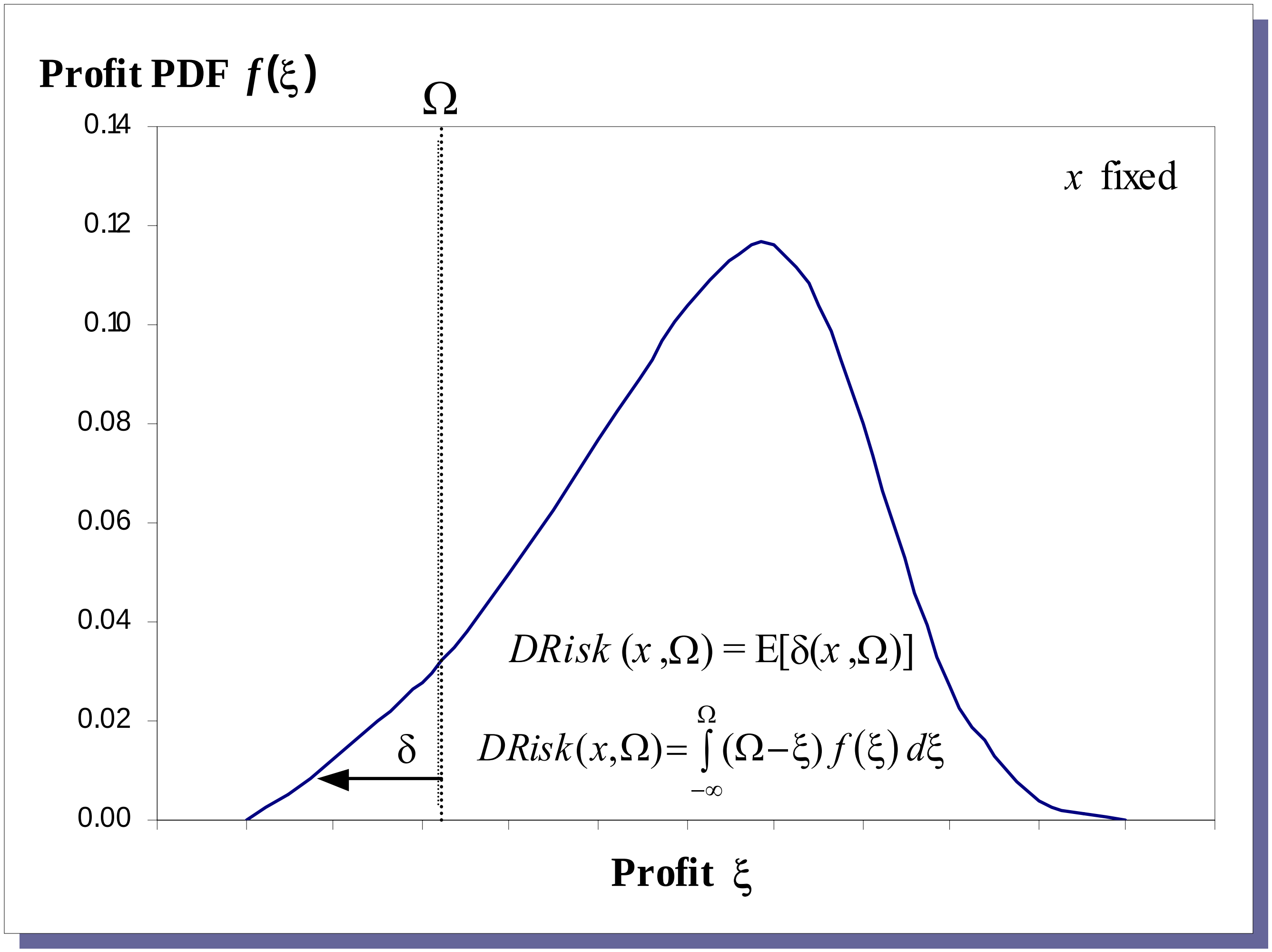
$$\delta_s = \begin{cases} \Omega - Profit_s & \text{If } Profit_s < \Omega \\ 0 & \text{Otherwise} \end{cases}$$

⇩

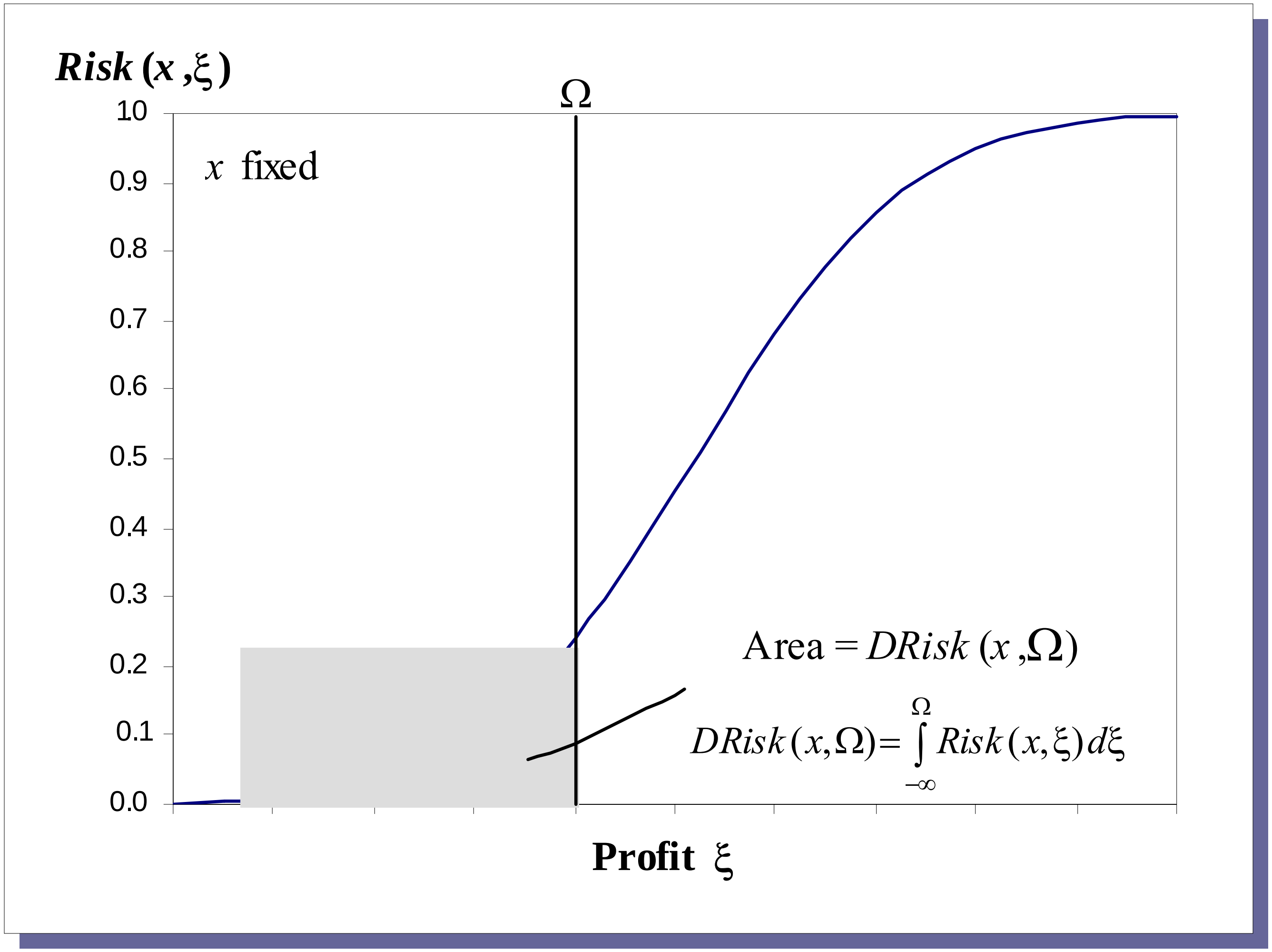
Formal definition of Downside Risk

$$DRisk(x,\Omega) = \sum_s p_s \delta_s$$

Downside Risk Interpretation



Downside Risk & Probabilistic Risk



Two-Stage Model using Downside Risk

MODEL DRisk

$$Max \;\; \mu\left(\sum_s p_s q_s^T y_s \; - c^T x\right) - \sum_s p_s \delta_s$$

$s.t.$ **Penalty Term**

$$Ax = b$$

$$T_s x + W y_s = h_s$$

$$x \geq 0 \qquad x \in X$$

$$y_s \geq 0$$

$$\delta_s \geq \Omega - (q_s^T y_s - c^T x)$$

$$\delta_s \geq 0$$

Downside Risk Constraints

Advantages

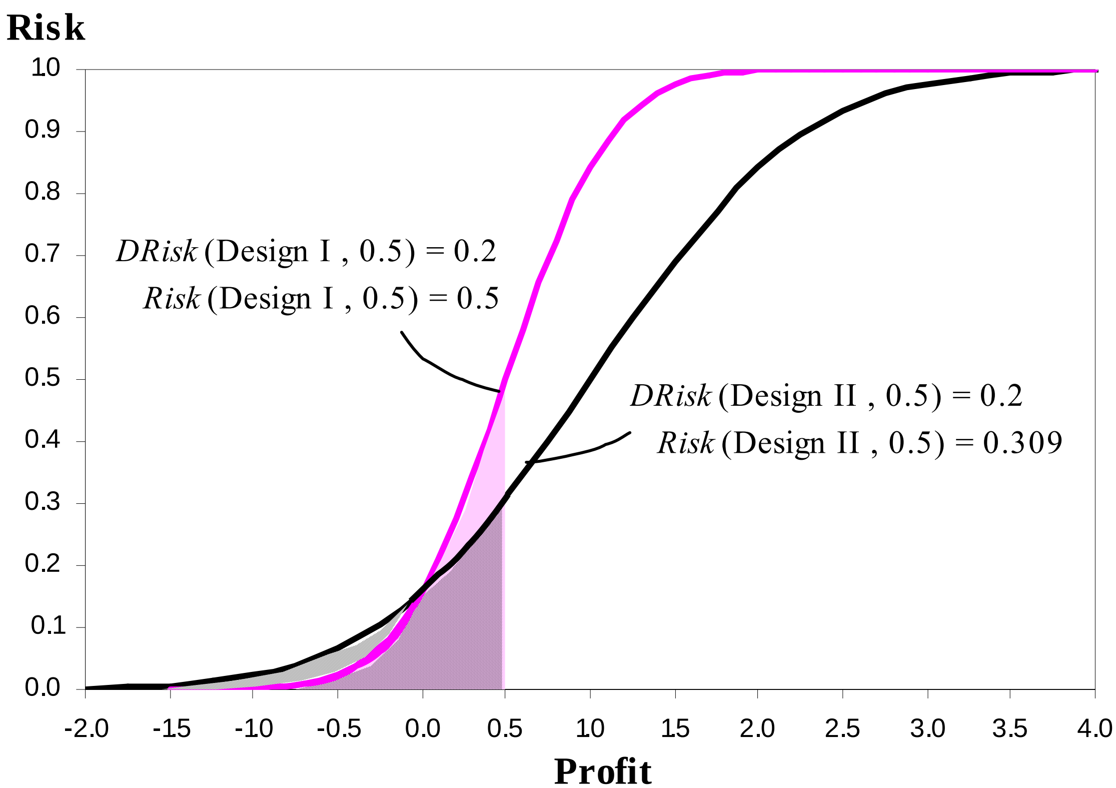
- **Same as models using Risk**
- **Does not require the use of binary variables**
- **Potential benefits from the use of decomposition methods**

Strategy

Solve the model using different profit targets to get a full spectrum of solutions. Use the risk curves to select the solution that better suits the decision maker's preference

Two-Stage Model using Downside Risk

Warning: **The same risk may imply different Downside Risks.**



Immediate Consequence:

Minimizing downside risk does not guarantee minimizing risk.

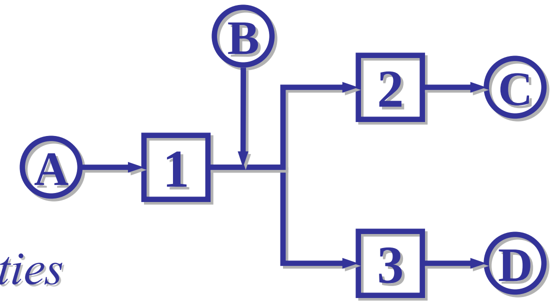
Commercial Software

- Riskoptimizer (Palisades) and CrystalBall (Decisioneering)
  - Use excell models
  - Allow uncertainty in a form of distribution
  - Perform Montecarlo Simulations or use genetic algorithms to optimize (Maximize ENPV, Minimize Variance, etc.)
- Financial Software. Large variety
  - Some use the concept of downside risk
  - In most of these software, Risk is mentioned but not manipulated directly.

Process Planning Under Uncertainty

GIVEN:

- **Process Network**
  - *Set of Processes*
  - *Set of Chemicals*



- **Forecasted Data**
  - *Demands & Availabilities*
  - *Costs & Prices*
  - *Capital Budget*

DETERMINE:

- **Network Expansions**
  - *Timing*
  - *Sizing*
  - *Location*
- **Production Levels**

OBJECTIVES:

- **Maximize Expected Net Present Value**
- **Minimize Financial Risk**

Process Planning Under Uncertainty

Design Variables: to be decided before the uncertainty reveals

$$x = \{Y_{it}, E_{it}, Q_{it}\}$$

Y: Decision of building process *i* in period *t*
E: Capacity expansion of process *i* in period *t*
Q: Total capacity of process *i* in period *t*

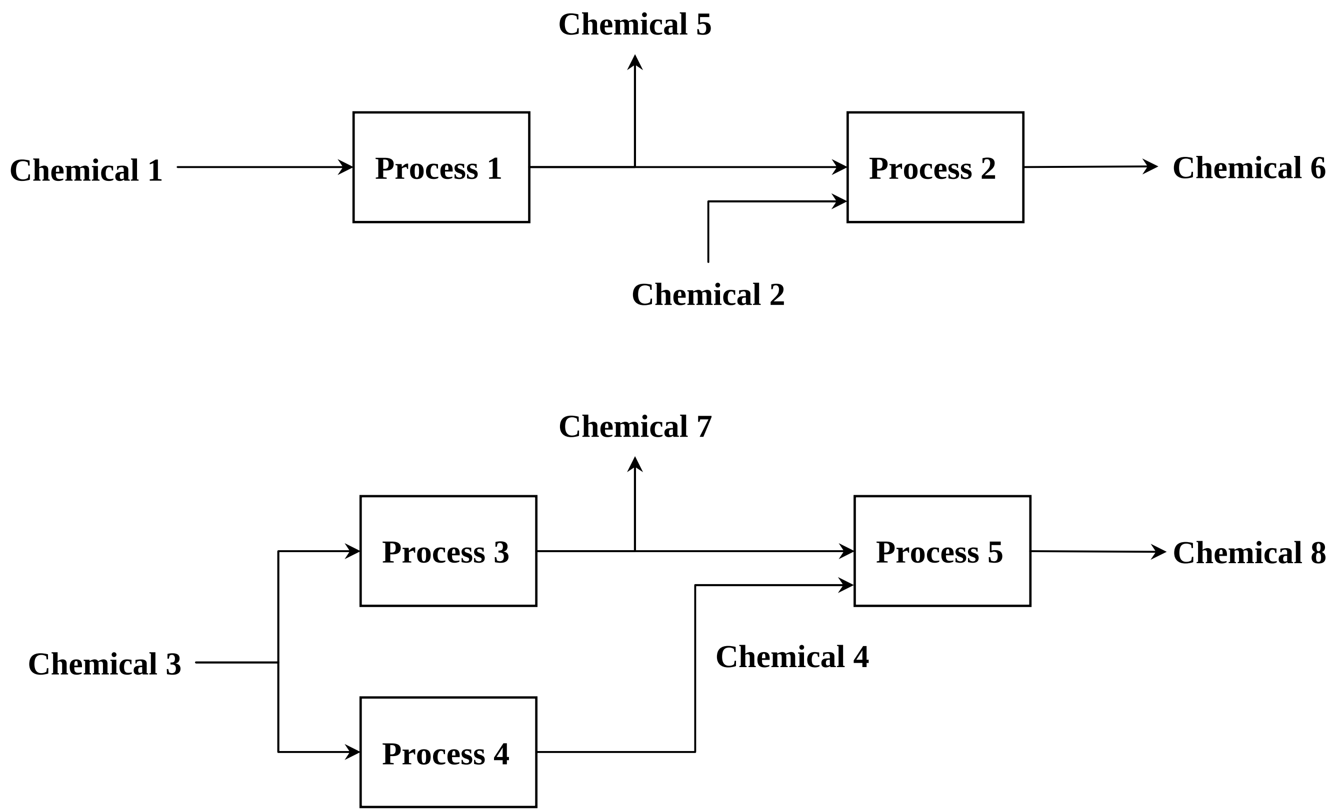
Control Variables: selected after the uncertain parameters become known

$$y_s = \{S_{jlts}, P_{jlts}, W_{its}\}$$

S: Sales of product *j* in market *l* at time *t* and scenario *s*
P: Purchase of raw mat. *j* in market *l* at time *t* and scenario *s*
W: Operating level of of process *i* in period *t* and scenario *s*

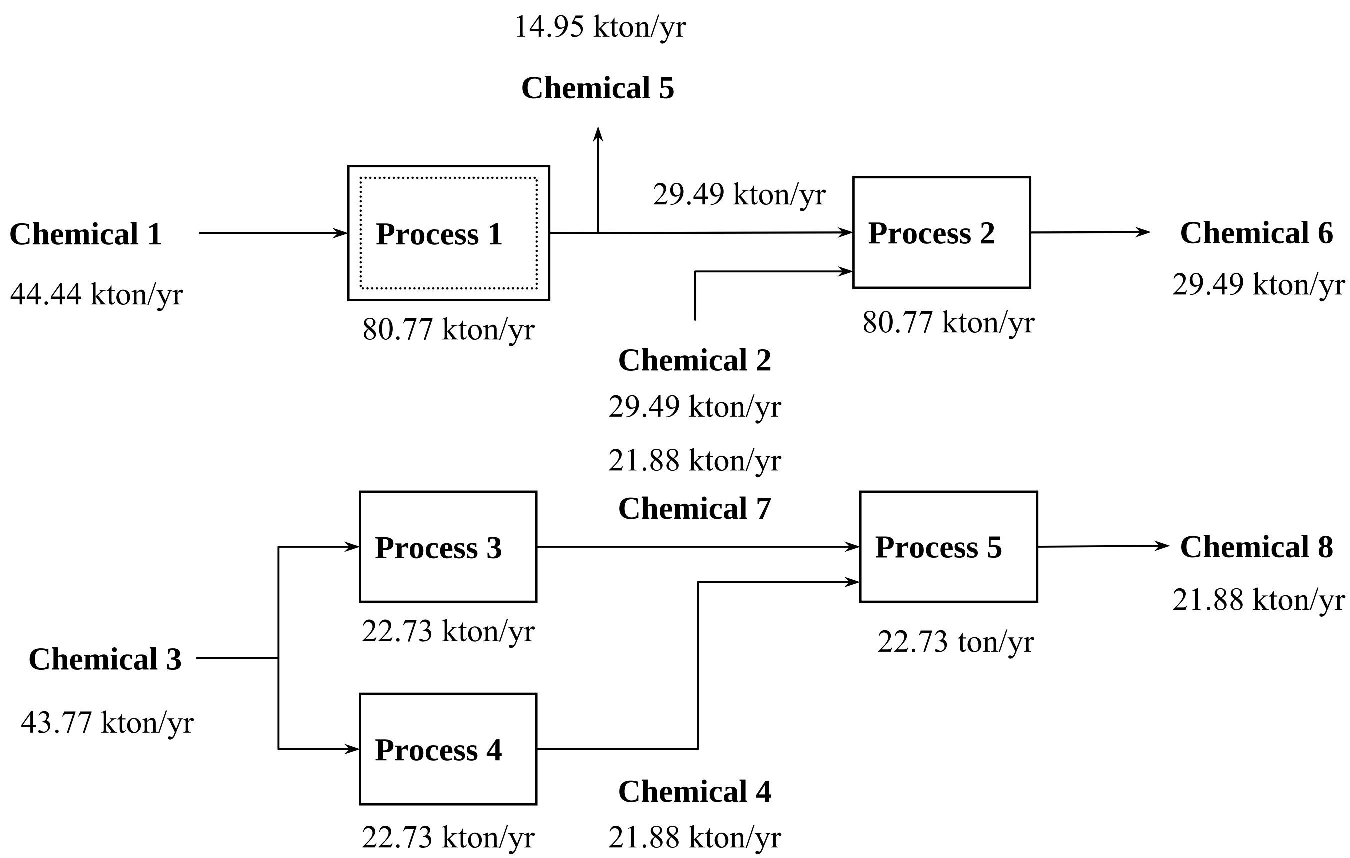
Example

- Uncertain Parameters: Demands, Availabilities, Sales Price, Purchase Price
- Total of 400 Scenarios
- Project Staged in 3 Time Periods of 2, 2.5, 3.5 years



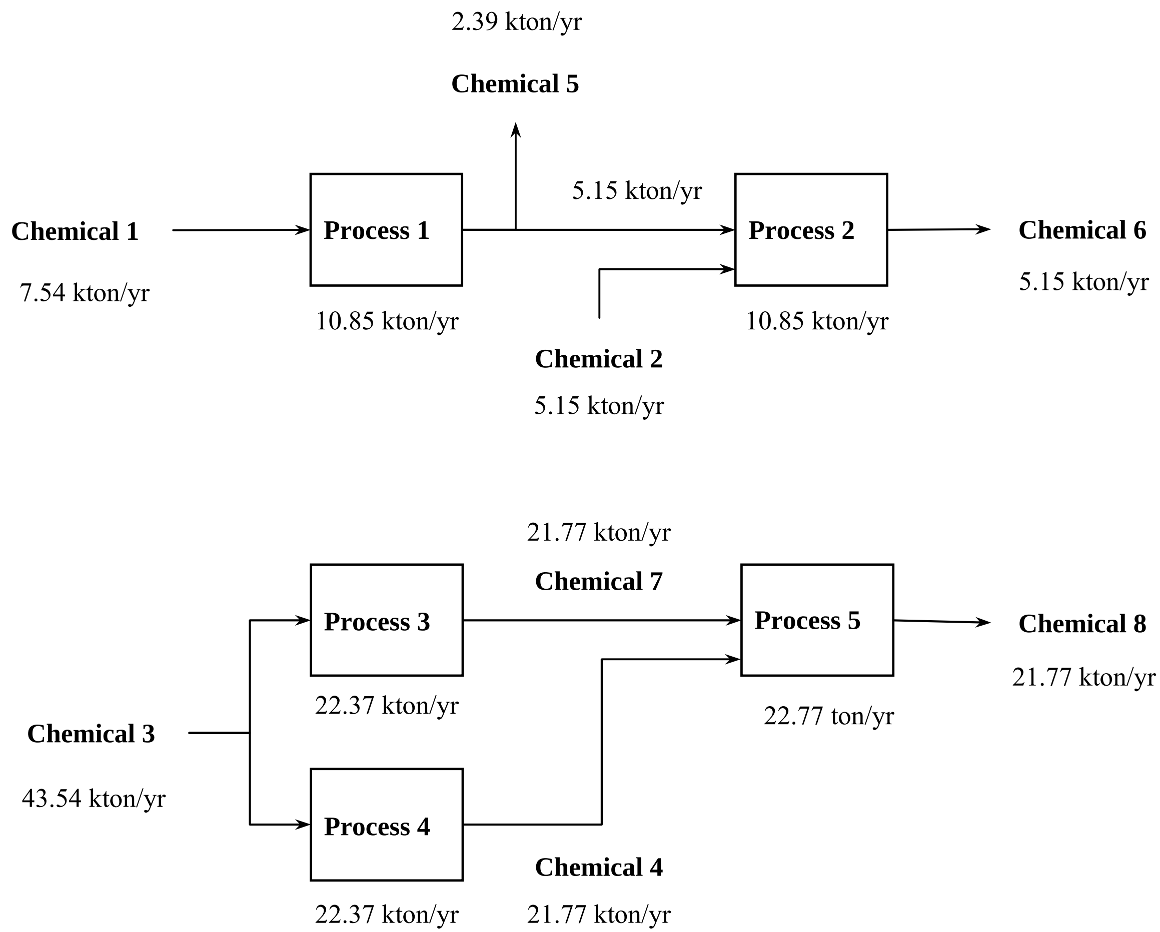
Example – Solution with Max ENPV

Period 3
3.5 years



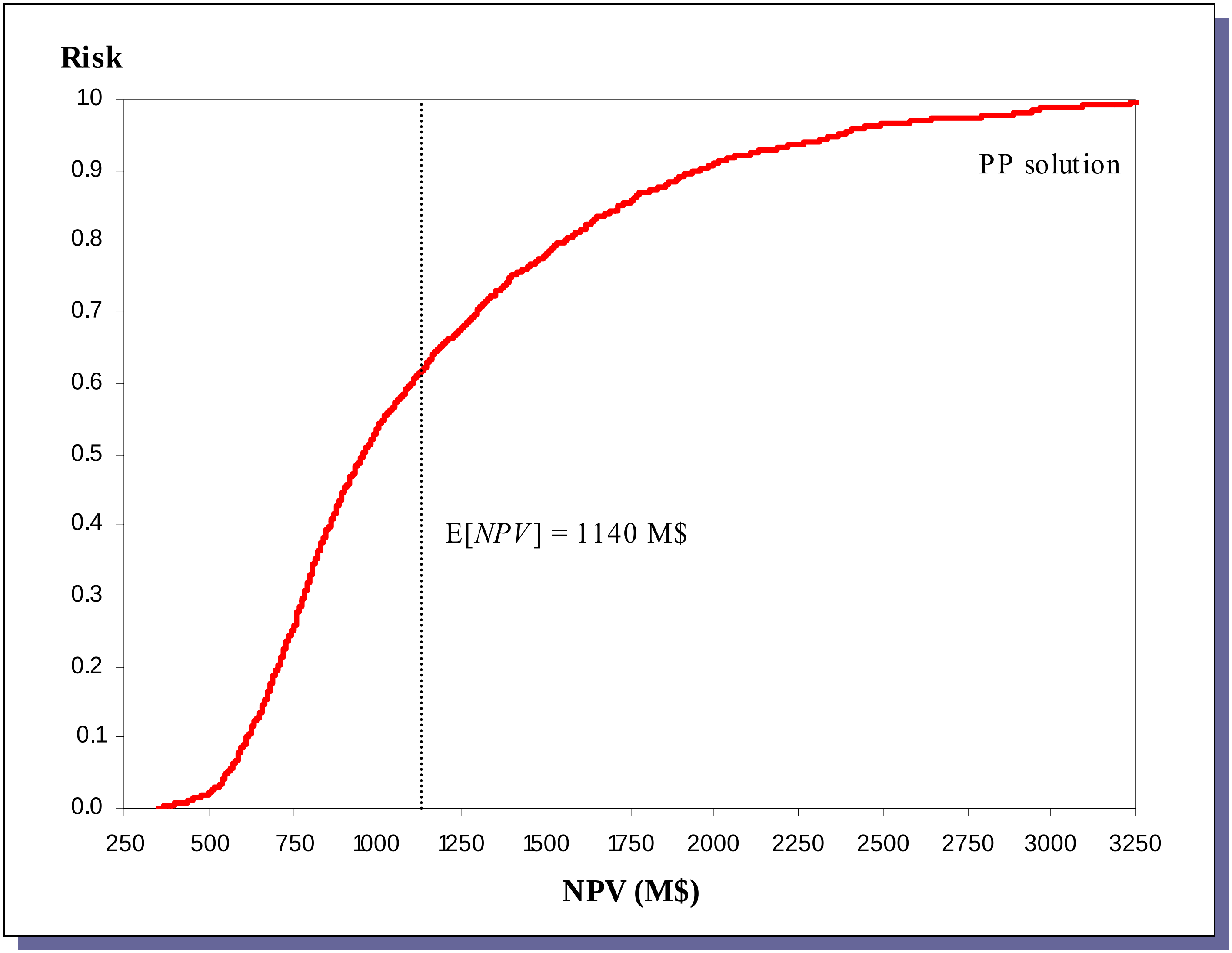
Example – Solution with Min DRisk(Ω=900)

Period 3
3.5 years

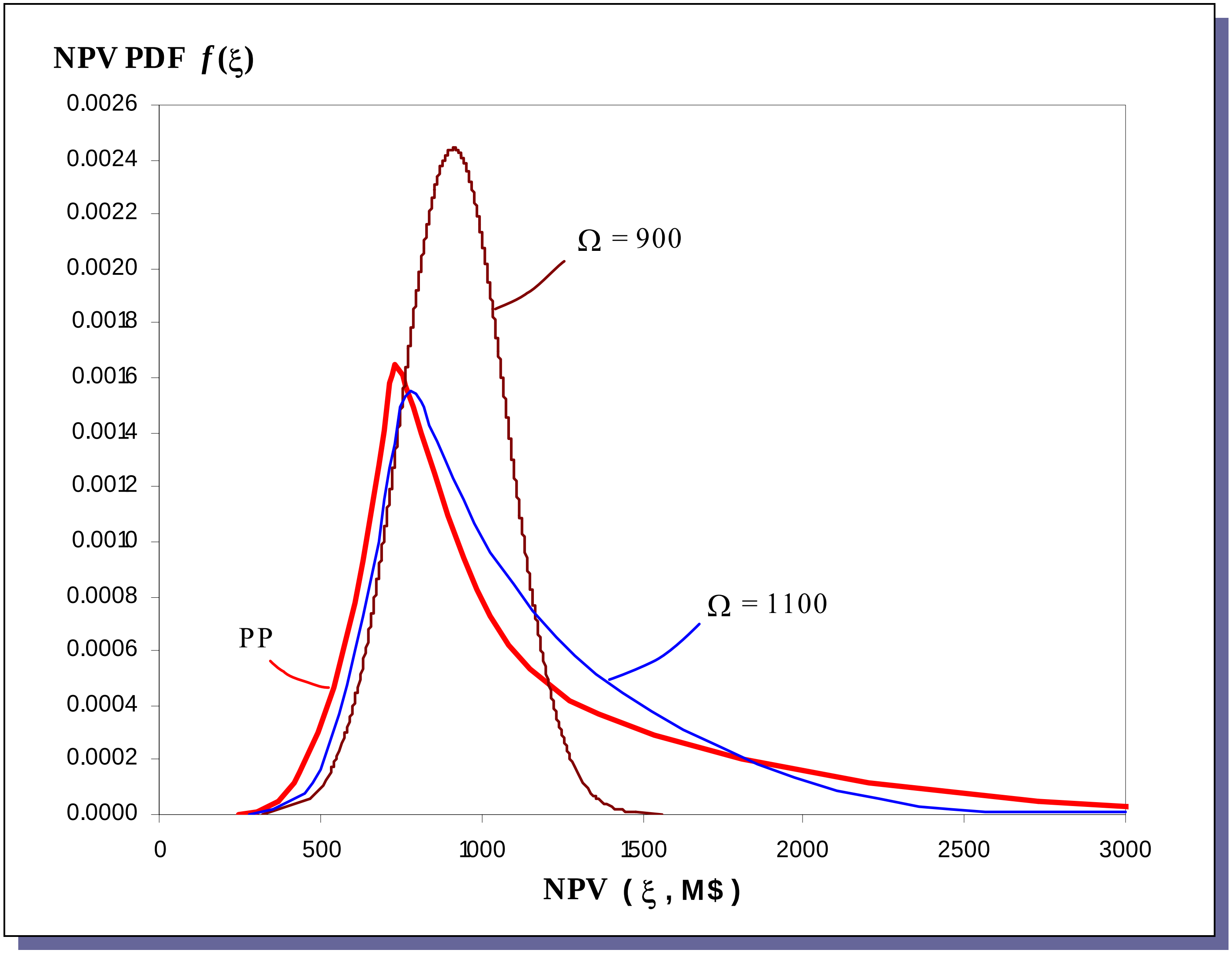


- **Same final structure, different production capacities.**

Example – Solution with Max ENPV

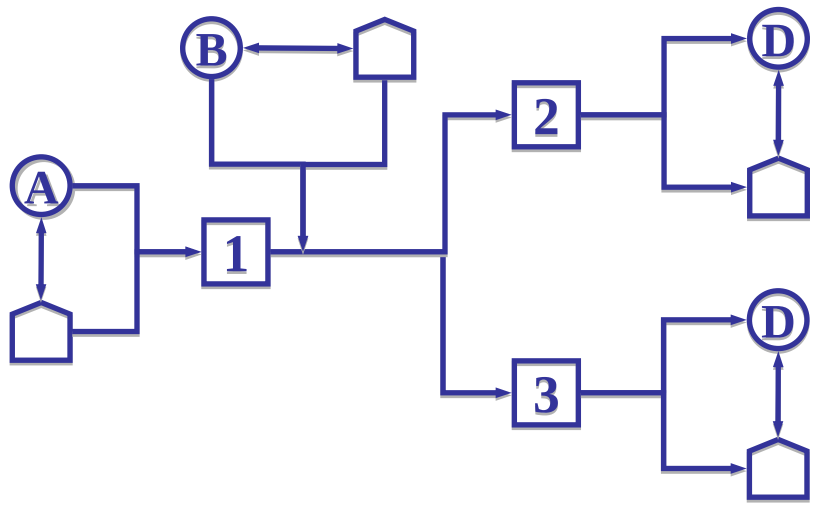


Example – Risk Management Solutions



Process Planning with Inventory

PROBLEM DESCRIPTION:

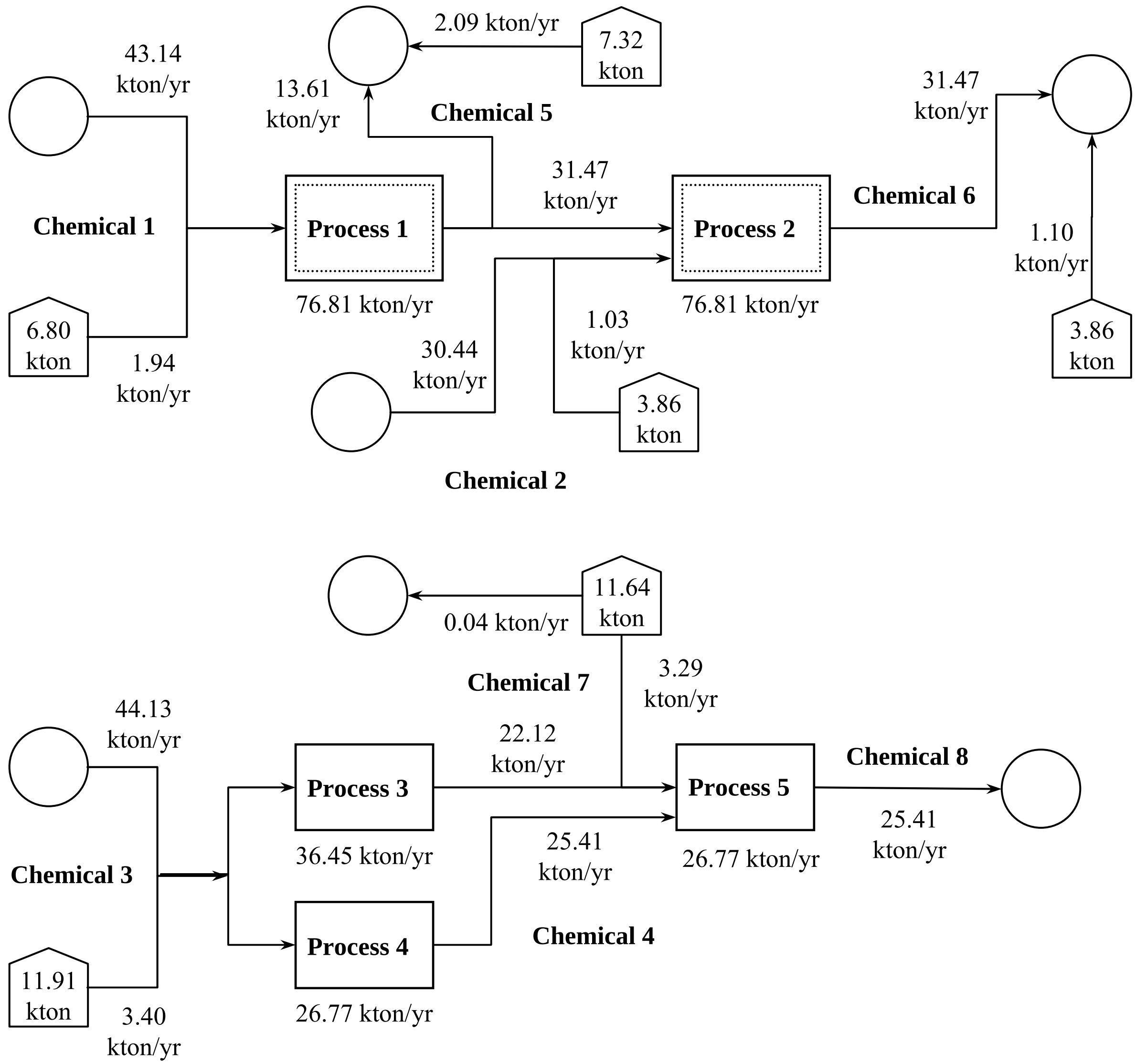


MODEL:
- The mass balance is modified such that now a certain level of inventory for raw materials and products is allowed
- A storage cost is included in the objective

OBJECTIVES:
- Maximize Expected Net Present Value
- Minimize Financial Risk

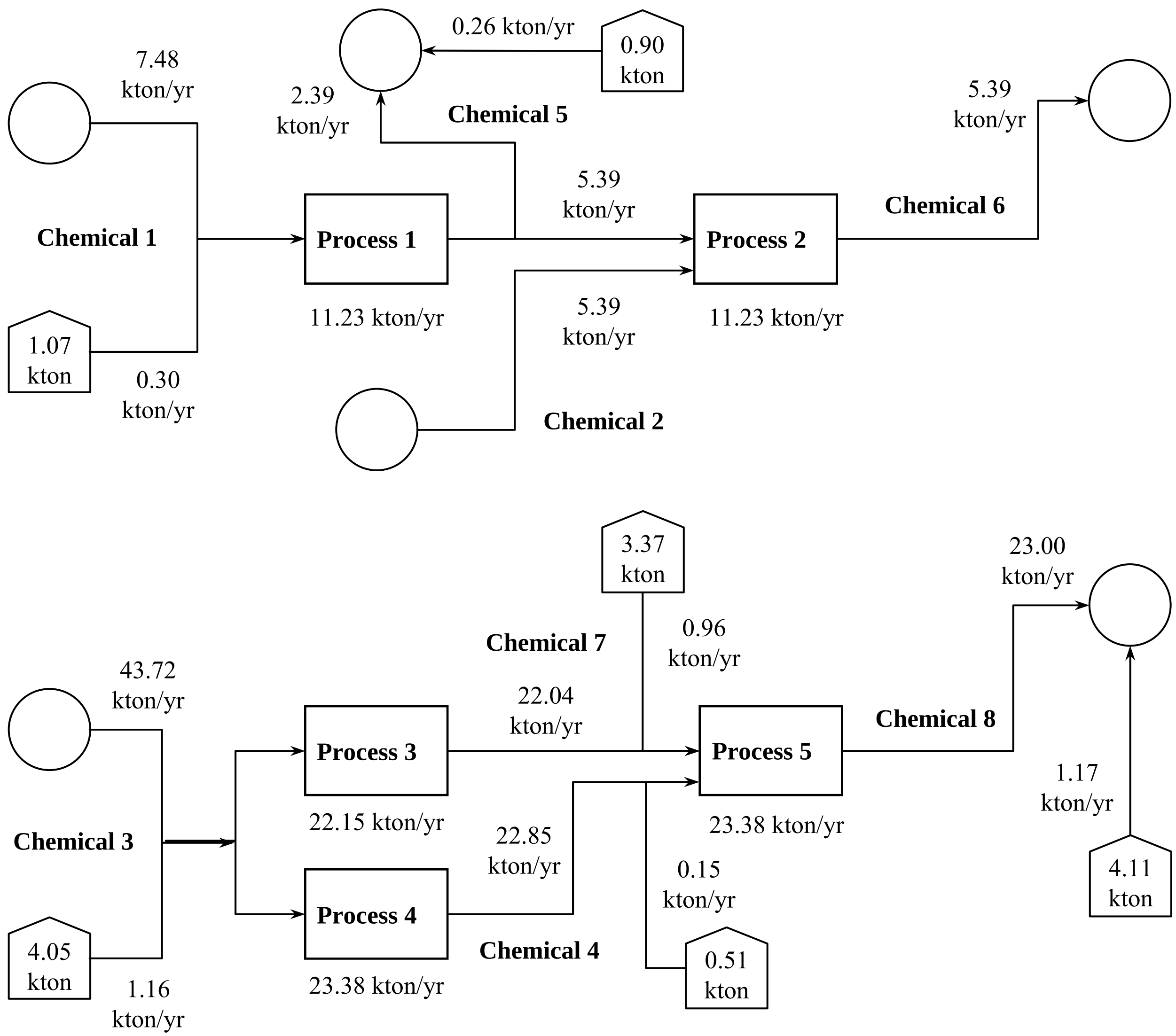
Example with Inventory – SP Solution

Period 3
3.5 years

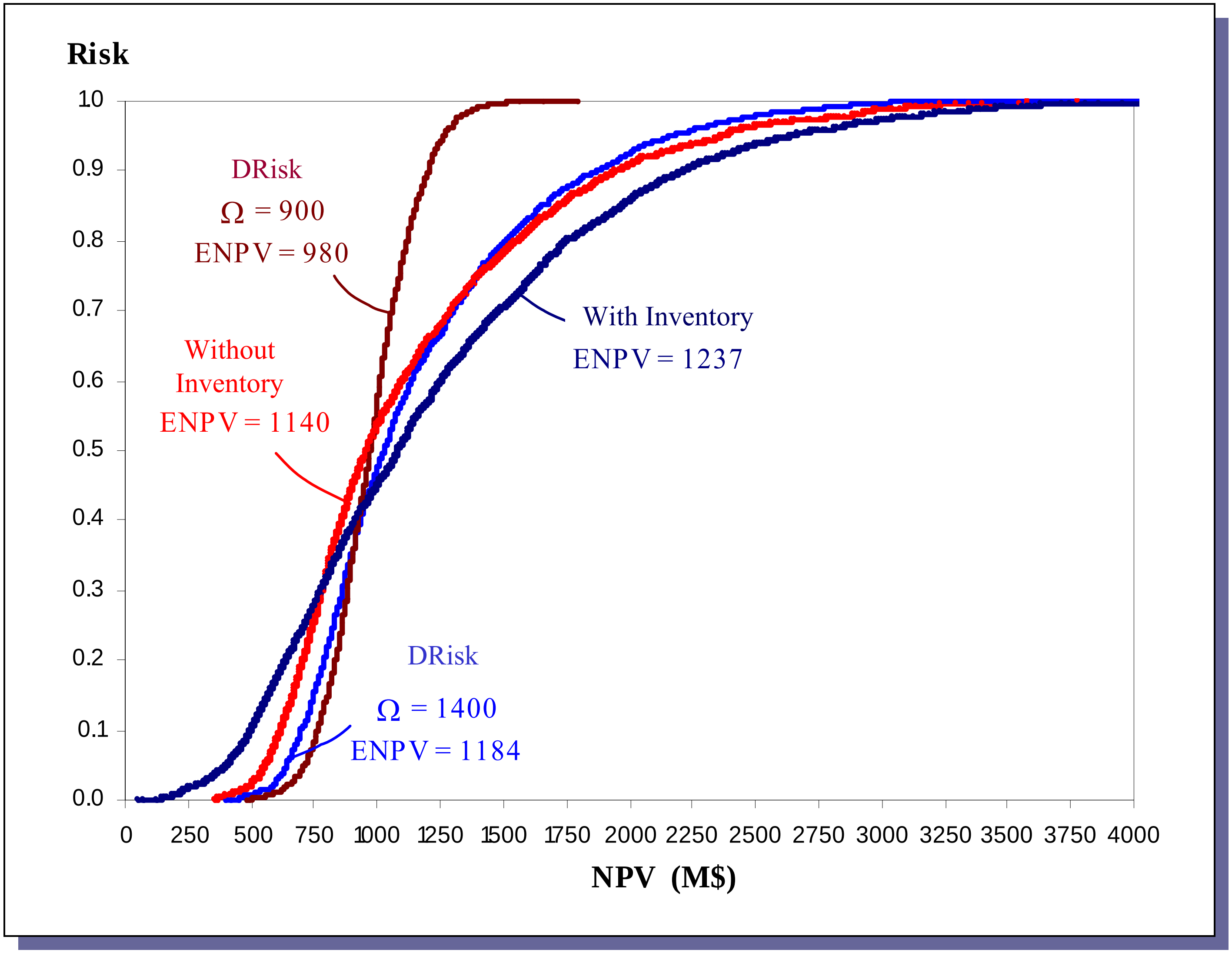


Example with Inventory
Solution with Min DRisk (Ω=900)

Period 3
3.5 years

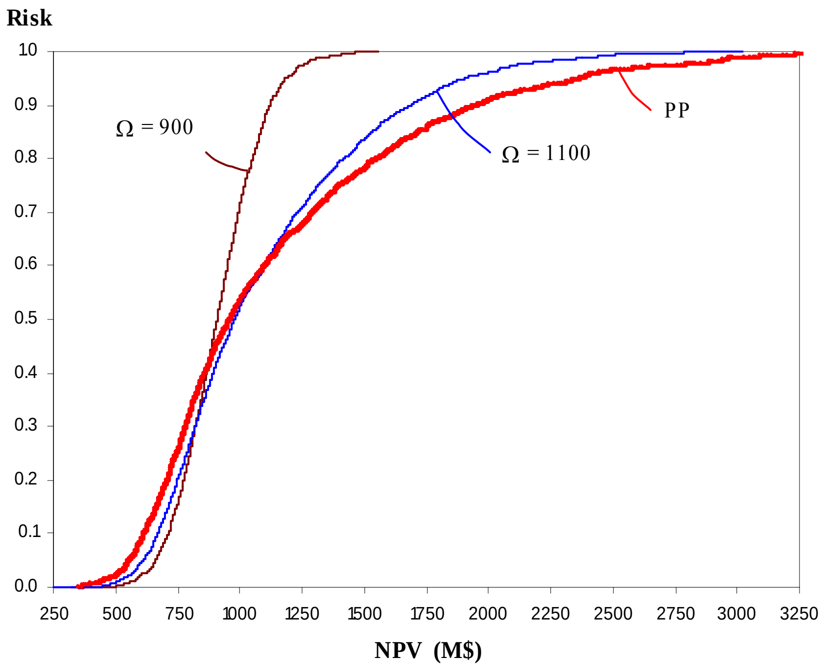


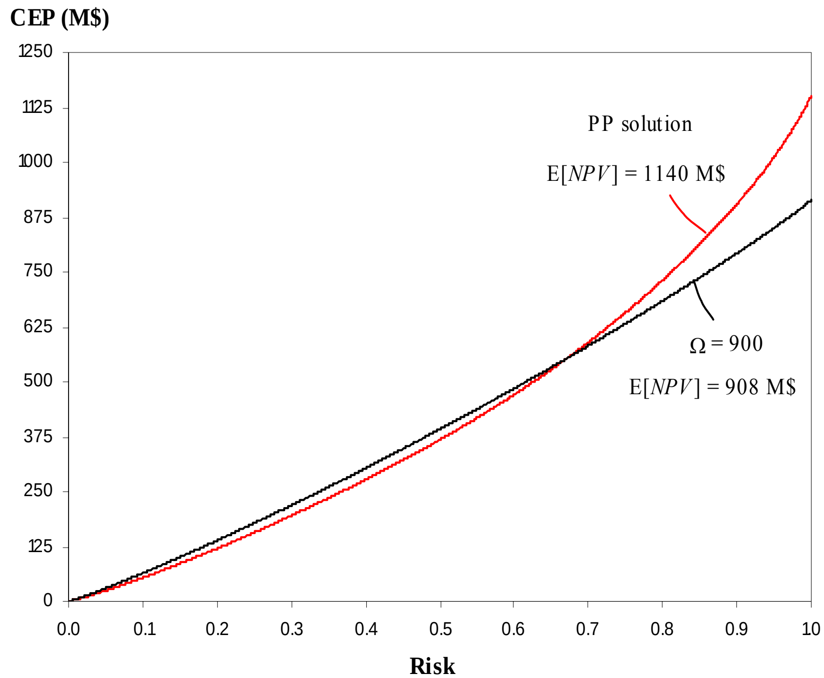
Example with Inventory - Solutions



Downside Expected Profit

- **Definition:** $DENPV(x, p_\Omega) = \int_{-\infty}^{\Omega} \xi \, f(x,\xi) \, d\xi = \Omega \; Risk(x,\Omega) - DRisk(x,\Omega)$

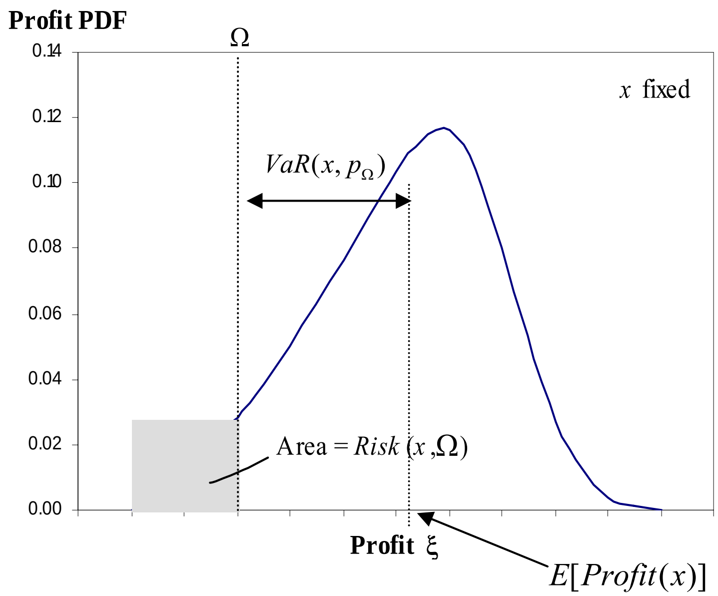




- **Up to 50% of risk (confidence?) the lower ENPV solution has higher profit expectations.**

Value at Risk

- **Definition:** ***VaR*** **is given by the difference between the mean value of the profit and the profit value corresponding to the *p*-quantile.**



$$VaR(x, p_\Omega) = E[Profit(x)] - Risk^{-1}(x, \Omega)$$

- **VaR=$z_p\sigma$ for symmetric distributions (Portfolio optimization)**

COMPUTATIONAL APPROACHES

- **Sampling Average Approximation Method:**
  - Solve M times the problem using only N scenarios.
  - If multiple solutions are obtained, use the first stage variables to solve the problem with a large number of scenarios N’>>N to determine the optimum.

- **Generalized Benders Decomposition Algorithm**
  - First Stage variables are complicating variables.
  - This leaves a primal over second stage variables, which is decomposable.

Example

- Generate a model to:
  - ***Evaluate*** a large network of natural gas supplier-to-market transportation alternatives
  - ***Identify*** the most profitable alternative(s)
  - ***Manage*** financial risk

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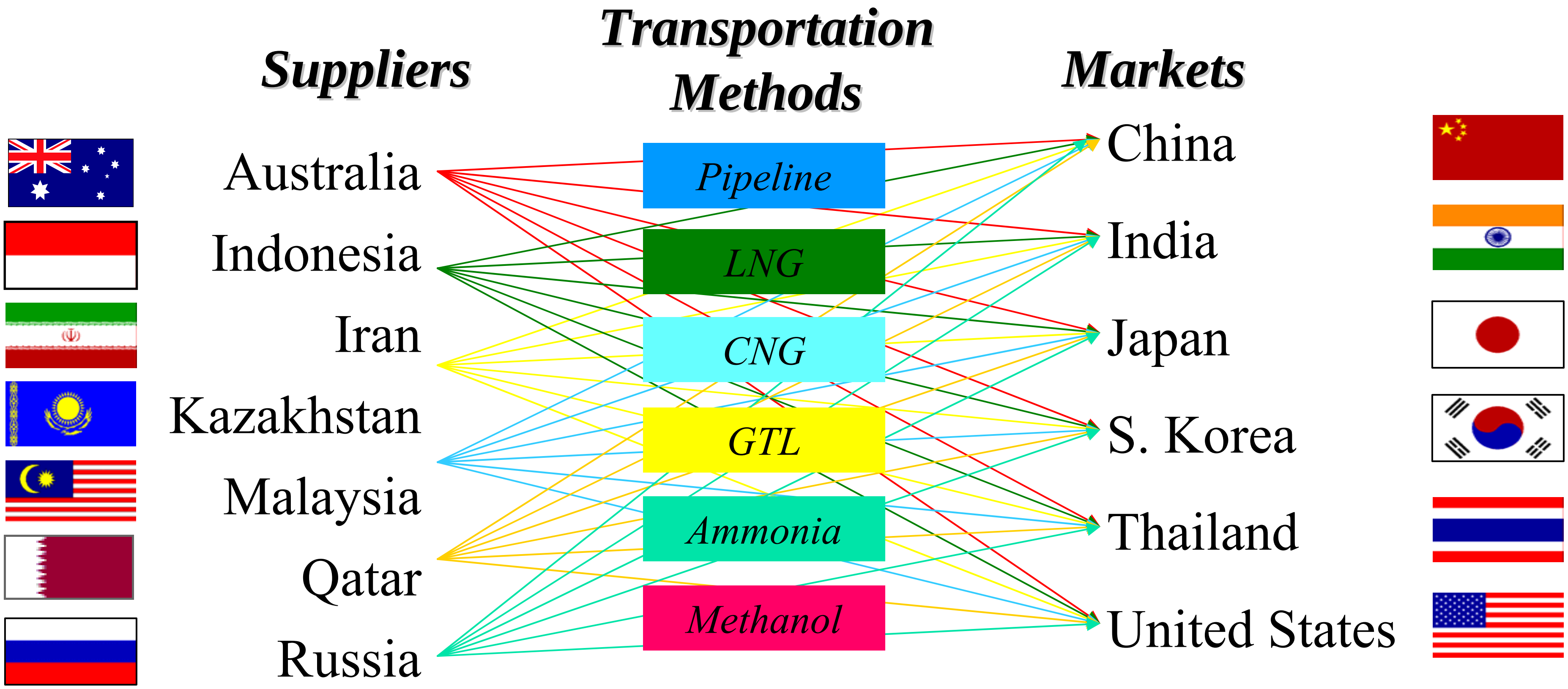
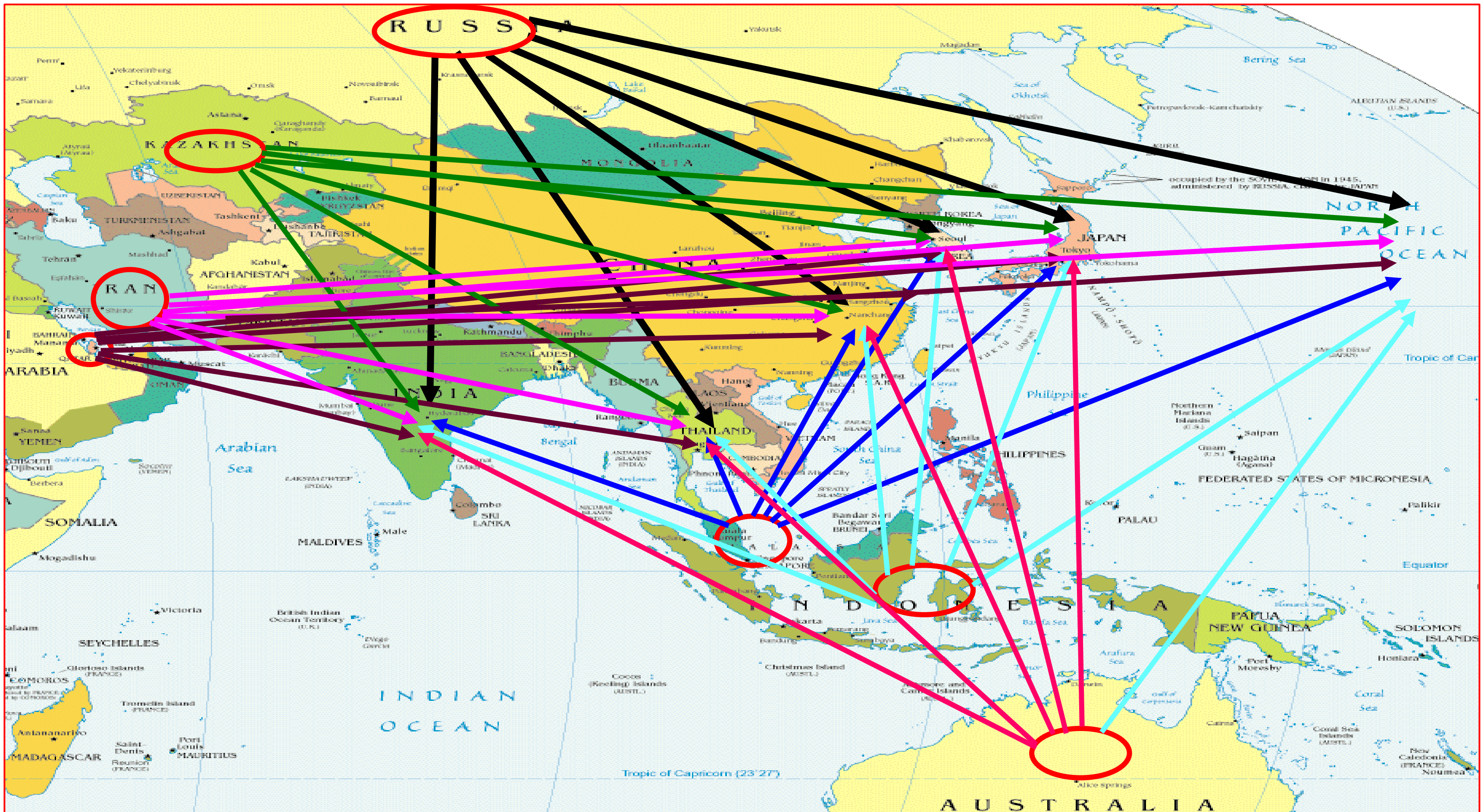
Network of Alternatives



Network of Alternatives



Results

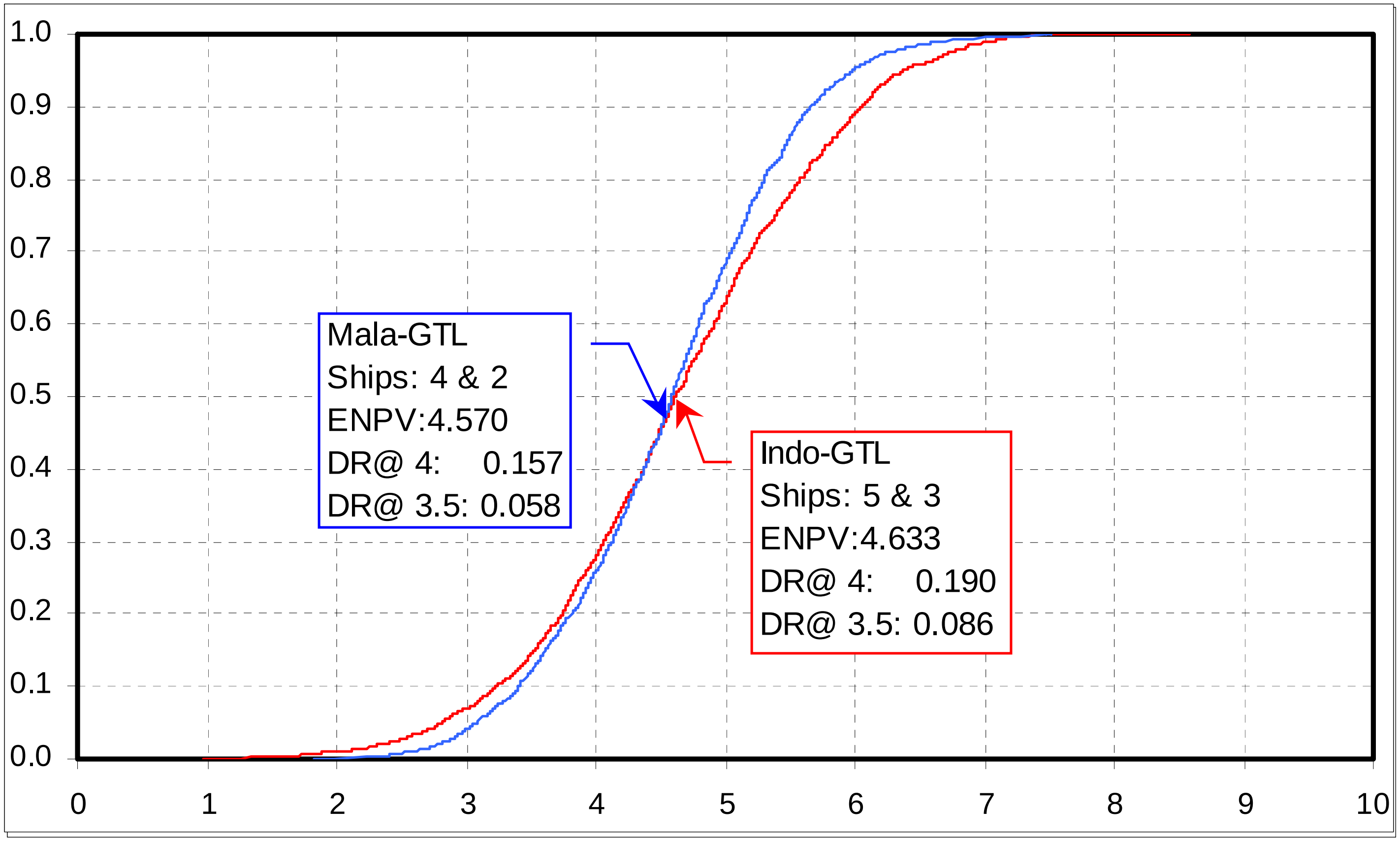
Risk Management (Downside Risk):

Malaysia

GTL

Thailand

China



Value at Risk (VaR):

VaR is the expected loss for a certain confidence level usually set at 5%

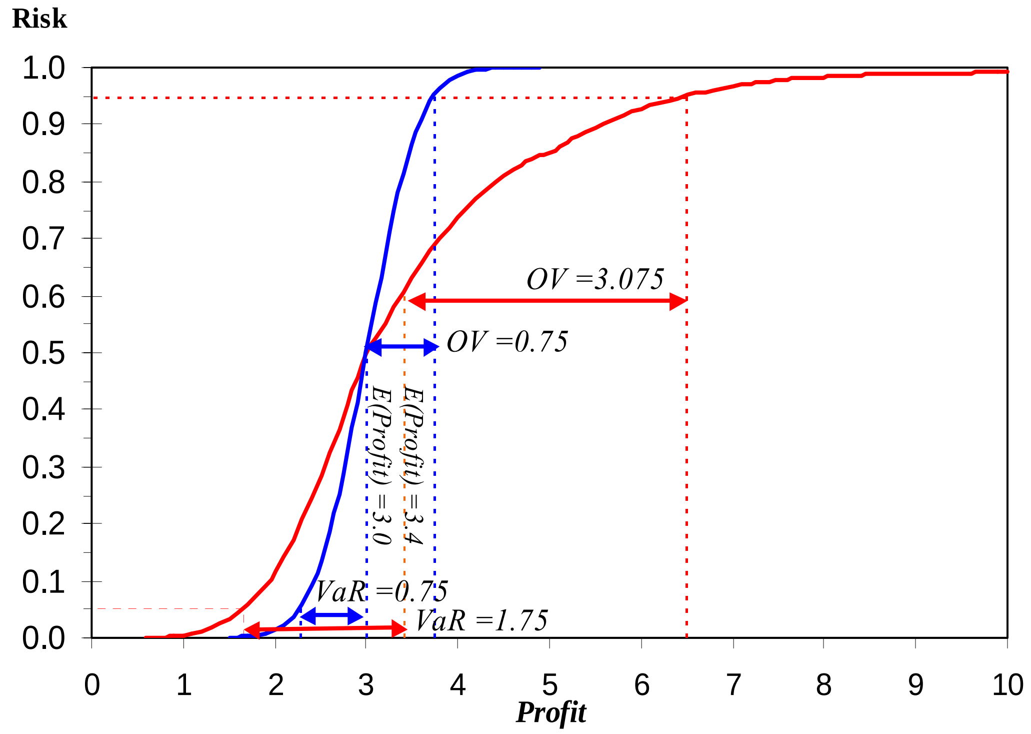
VaR =ENPV – NPV @ p-quantile

Opportunity Value (OV) or Upper Potential (UP):

OV = NPV @ (1-p)-quantile – ENPV

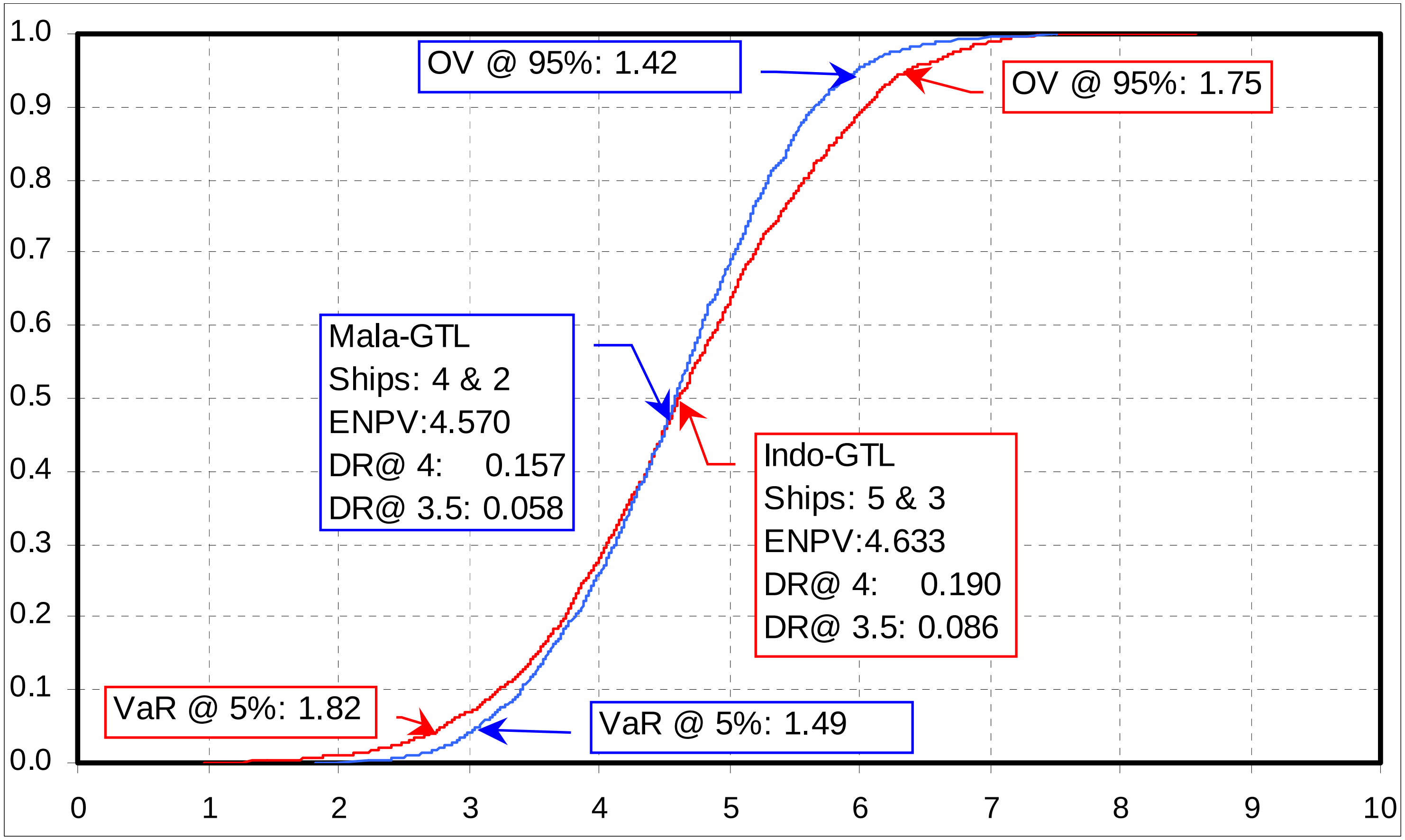
UPSIDE POTENTIAL

Point measure for the upside



Results

Value at Risk (VaR) and Opportunity Value (OV):

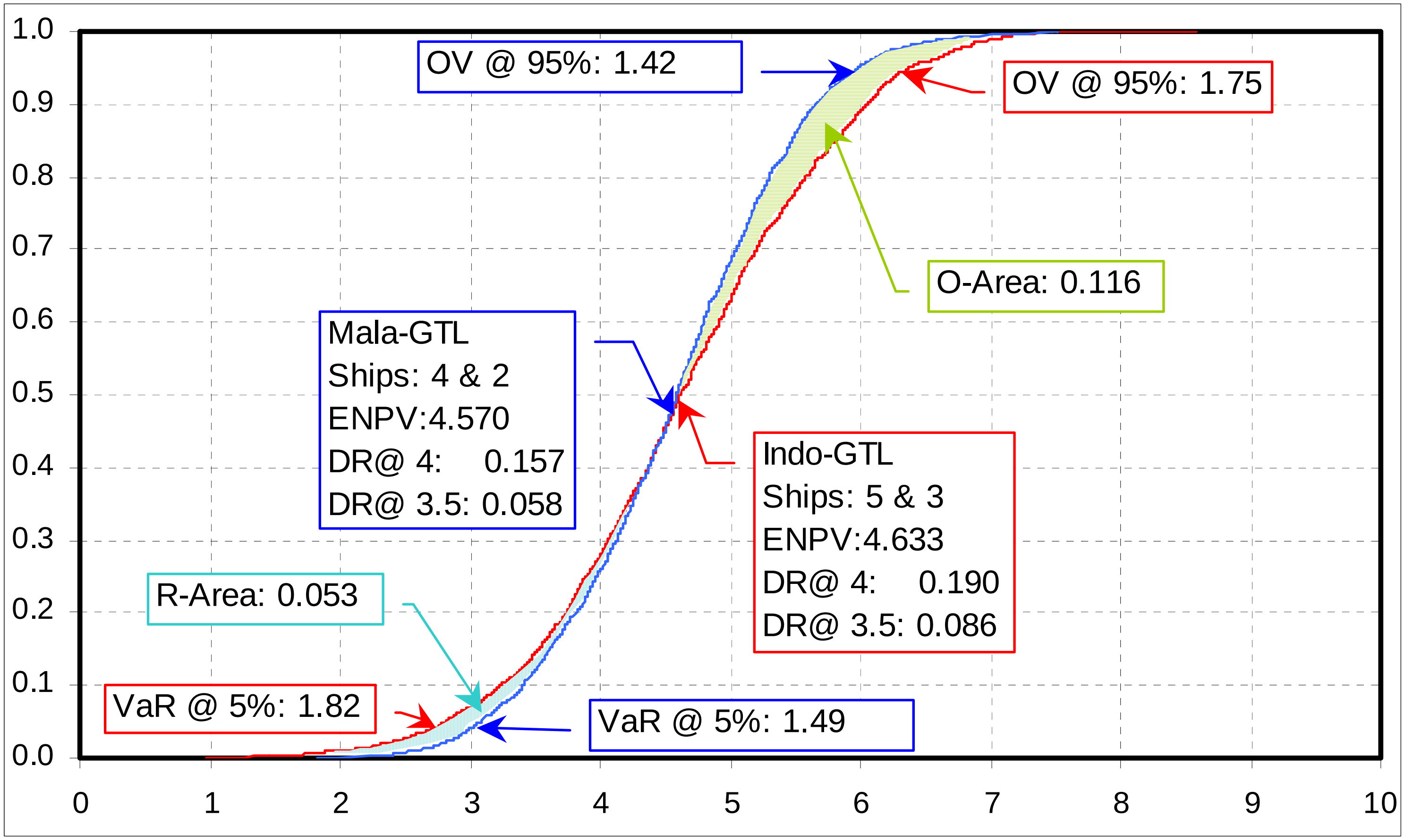


Reduction in VaR: 18.1%

Reduction in OV: 18.9%

Results

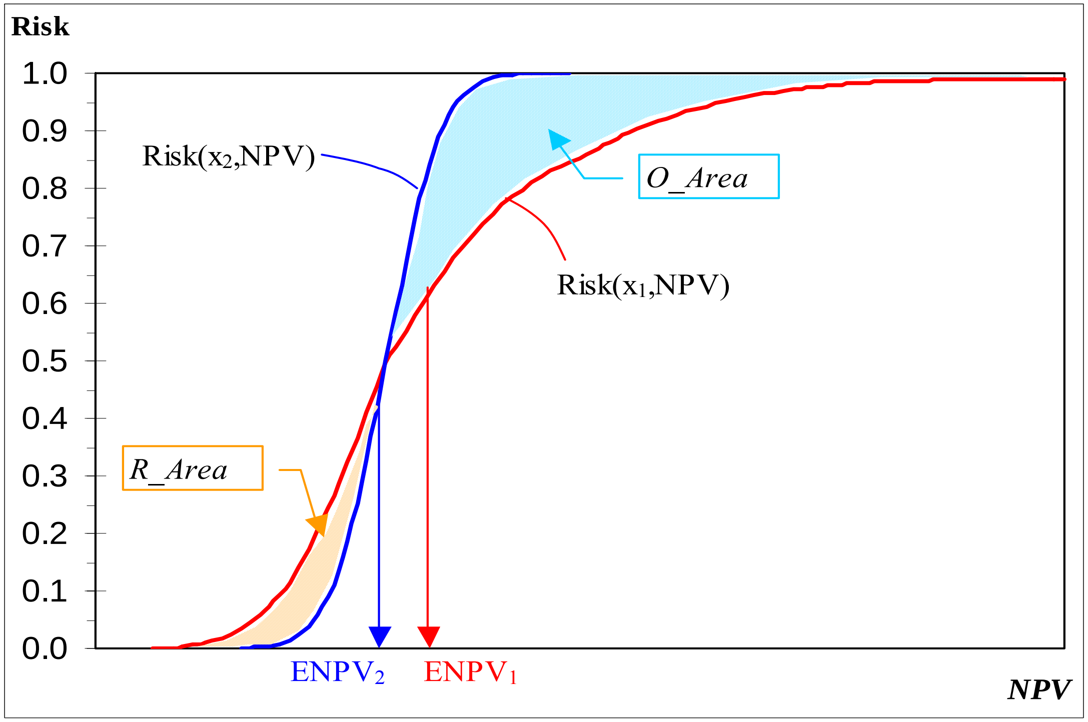
Risk /Upside Potential Loss Ratio



Risk /Upside Potential Loss Ratio: 2.2

Risk /Upside Potential Loss Ratio

$$\text{Risk /Upside Potential Loss Ratio} = \frac{O\_Area}{R\_Area} = \frac{\int_{-\infty}^{+\infty} \psi^{-} dNPV}{\int_{-\infty}^{+\infty} \psi^{+} dNPV}$$



where:

$$\psi^{+} = \begin{cases} \psi & if \quad \psi \geq 0 \\ 0 & otherwise \end{cases}$$

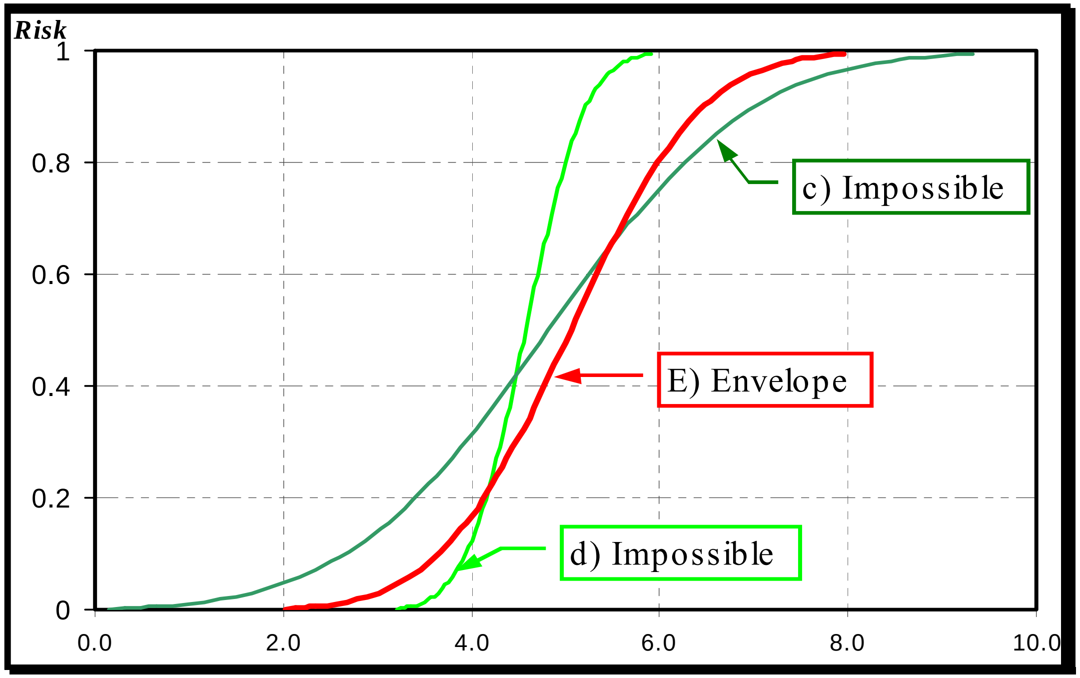
$$\psi^{-} = \begin{cases} -\psi & if \quad \psi < 0 \\ 0 & otherwise \end{cases}$$

$$\psi = Risk_{NGC}(NPV) - Risk_{NGC-DR}(NPV)$$

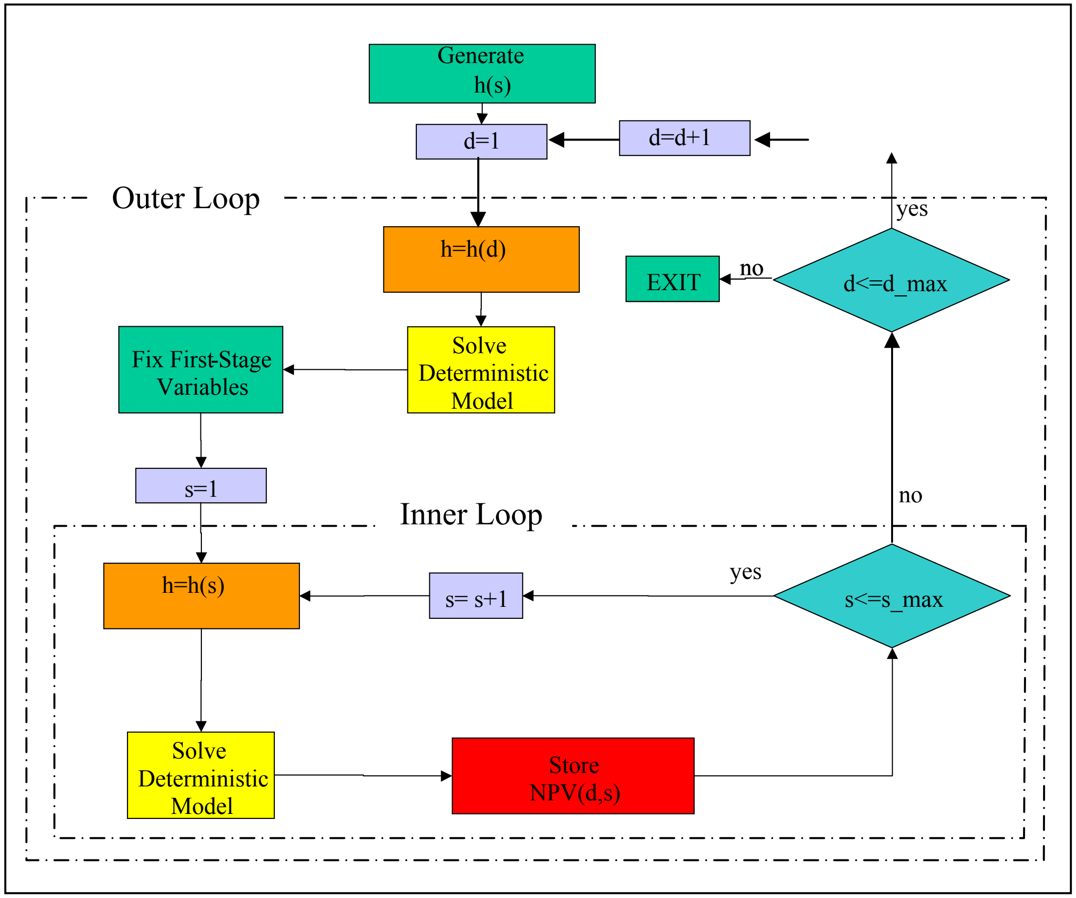
Upper and Lower Bound Risk Curve

Upper Bound Risk Curve (Envelope):

The curve constructed by plotting the set of net present values (NPV) for the best design under each scenario.



SAMPLING ALGORITHM



This generates several solutions

Upper and Lower Bound Risk Curve

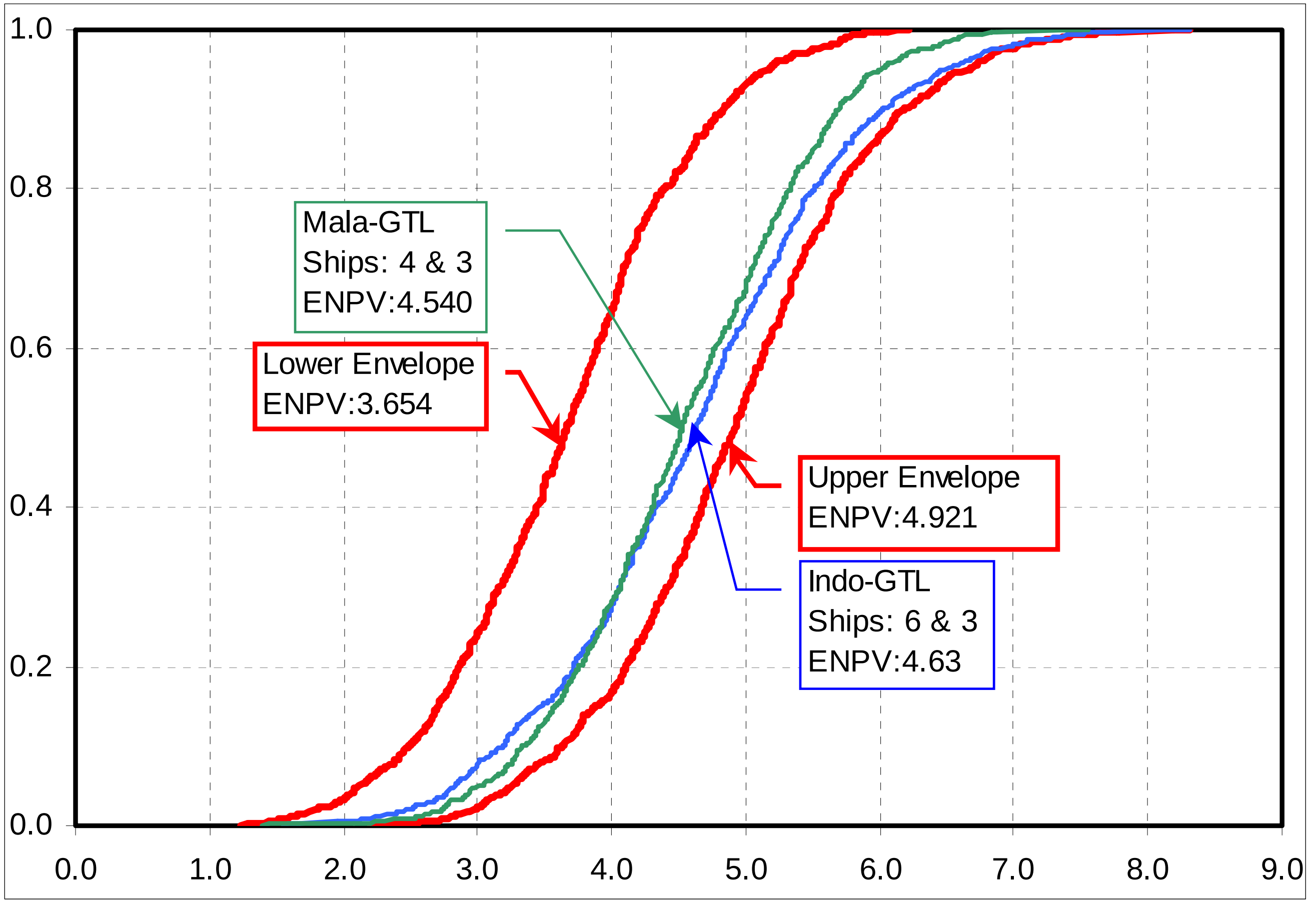
Lower Bound Risk Curve:

The curve constructed by plotting the set of net present values (NPV) for the worst (of the set of best designs) under each scenario.

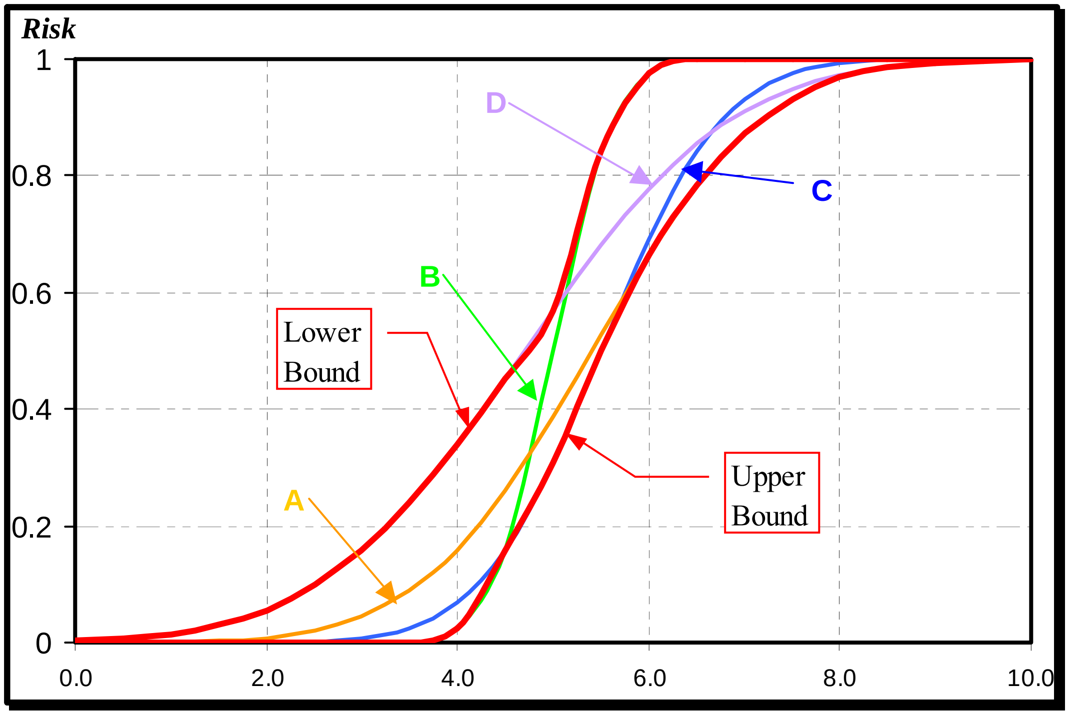
$s \backslash d$	s_1	s_2	s_3	...	s_n
d_1	$NPV_{d1,s1}$	$NPV_{d2,s1}$	$NPV_{d3,s1}$	...	$NPV_{dn,s1}$
d_2	$NPV_{d1,s2}$	$NPV_{d2,s2}$	$NPV_{d3,s2}$	...	$NPV_{dn,s2}$
d_3	$NPV_{d1,s3}$	$NPV_{d2,s3}$	$NPV_{d3,s3}$	...	$NPV_{dn,s3}$
⋮	⋮	⋮	⋮		⋮
d_n	$NPV_{d1,sn}$	$NPV_{d2,sn}$	$NPV_{d3,sn}$	...	$NPV_{dn,sn}$
Min	NPV_{s1}^{Min}	NPV_{s2}^{Min}	NPV_{s3}^{Min}	...	NPV_{sn}^{Min}

Results

Upper and Lower Bound Risk Curve



UPPER AND LOWER BOUNDS



This is very useful because it allows nice decomposition,

That is, there is no need to solve the full stochastic problem

Conclusions

- A probabilistic definition of Financial Risk has been introduced in the framework of two-stage stochastic programming. Theoretical properties of related to this definition were explored.
- New formulations capable of managing financial risk have been introduced. The multi-objective nature of the models allows the decision maker to choose solutions according to his risk policy. The cumulative risk curve is used as a tool for this purpose.
- The models using the risk definition explicitly require second-stage binary variables. This is a major limitation from a computational standpoint.
- To overcome the mentioned computational difficulties, the concept of Downside Risk was examined, finding that there is a close relationship between this measure and the probabilistic definition of risk.
- Using downside risk leads to a model that is decomposable in scenarios and that allows the use of efficient solution algorithms. For this reason, it is suggested that this model be used to manage financial risk.
- An example illustrated the performance of the models, showing how the risk curves can be changed in relation to the solution with maximum expected profit.