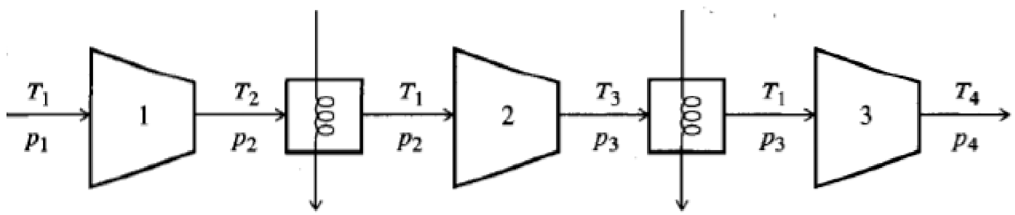
ASSIGNMENT 1 CHE 5480

DUE: January 25: Send through e-mail. Include the GAMS and Excel files and a narrative explaining what was done and how.

#Problem 1

In this example we describe the calculation of the minimum work for ideal compressible adiabatic flow using two different optimization techniques, (a) analytical, and (b) numerical. Most real flows lie somewhere between adiabatic and isothermal flow. For adiabatic flow, the case examined here, you cannot establish a priori the relationship between pressure and density of the gas because the temperature is unknown as a function of pressure or density, hence the relation between pressure and



density is derived using the mechanical energy balance. If the gas is assumed to be ideal, and $k = C_p/C_v$ is assumed to be constant in the range of interest from p_1 to p_2, you can make use of the well-known relation

$$pV^k = \text{Constant} \qquad (a)$$

in getting the theoretical work per mole (or mass) of gas compressed for a single-stage compressor (McCabe and colleagues, 1993)

$$W = \frac{kRT_1}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right] \qquad (b)$$

where T_1 is the inlet gas temperature and R the ideal gas constant ($p_1\hat{V}_1 = RT_1$). For a three-stage compressor with intercooling back to T_1 between stages as shown in Figure E13.2, the work of compression from p_1 to p_4 is

$$\hat{W} = \frac{kRT_1}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} + \left(\frac{p_3}{p_2}\right)^{(k-1)/k} + \left(\frac{p_4}{p_3}\right)^{(k-1)/k} - 3\right] \qquad (c)$$

We want to determine the optimal interstage pressures p_2 and p_3 to minimize $\hat{W}$ keeping p_1 and p_4 fixed.

- Solve the above problem analytically.
- Is the function convex?
- Use GAMS/CONOPT to solve the problem.

#Problem 2

What is the feasible region for **x** given the following constraints? Sketch the feasible region for the two-dimensional problems.

(a) $h_1(\mathbf{x}) = x_1 + x_2 - 3 = 0$

$h_2(\mathbf{x}) = 2x_1 - x_2 + 1 = 0$

(b) $h_1(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 = 0$

$h_2(\mathbf{x}) = x_1 + x_2 + x_3 = 0$

(c) $g_1(\mathbf{x}) = x_1 - x_2^2 - 2 \geq 0$

$g_2(\mathbf{x}) = x_1 - x_2 + 4 \geq 0$

(d) $h_1(\mathbf{x}) = x_1^2 + x_2^2 + 3$

$g_1(\mathbf{x}) = x_1 - x_2 + 2 \geq 0$

$g_2(\mathbf{x}) = x_1 \geq 0$

$g_3(\mathbf{x}) = x_2 \geq 0$

#Problem 3

The following function has several minima.

$$F(X)=\sin^2(3\pi x_1) + (x_6 - 1)^2 [1 + \sin^2 (2 \pi x_6)]+\sum_{i=1}^{5} (x_i -1)^2 [1 + \sin^2(3 \pi x_{i+1})]$$

$(-10 < x_i \sim< 10)$

a) Is the function convex?
b) Find as many as you can using different starting points using GAMS/CONOPT.
c) Pick 2 of the starting points that lead to different minima when using GAMS and repeat using Excel implementations of

- Steepest Descent
- Conjugate gradients
- Newton Steps.
- BFGS