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Cost-optimal design of reliable sensor networks

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Abstract

Several papers have been presented in the last years regarding the design of reliable sensor networks. In all these papers, the system reliability was maximized, constrained by a fixed number of sensors. In these models, the cost has played an indirect unclear role. A minimum cost model for the design of reliable sensor networks is presented in this paper. The connections with previous models are established, showing that they are a particular case of the model stated in this work.

Keywords: Sensor network design; Reliability; Data reconciliation; Instrumentation cost

1. Introduction

Sensors are needed in a process plant for a variety of purposes. The most important are control and monitoring, but other non-traditional activities such as safety, fault detection and production accounting have been incorporated as clients of a data processing system. Recently, on-line optimization has added new needs for reliable process data of good quality. Consequently, the selection of sensors in chemical plants to fulfill reliability issues has emerged as a topic of interest.

Ali and Narasimhan (1993) proposed to maximize reliability, which is based on sensor failure probability, observability of variables as well as redundancy. They introduced the concept of reliability of estimation of a variable. In addition they proposed to measure the reliability of the system as the smallest reliability among all variables. While looking at all networks containing the minimum set of sensors to achieve observability they formulated a Max–Min problem using reliability as the objective function. Lately, Ali and Narasimhan (1995) extended their previous work to redundant networks. Their algorithm uses graph theory to build networks with a specified number of sensors and maximum system reliability. In a recent paper, Sen, Narasimhan and Deb (1998) presented a genetic algorithm that can be applied to design non-redundant sensor networks using different objectives functions.

Departing from graph theory and linear algebra approaches, Bagajewicz (1997) formulated a MINLP problem to obtain cost-optimal network structures for linear systems subject to constraints on precision and robustness, that is defined in terms of measures that allow the sensor network to effectively manage gross errors.

This paper concentrates on the connection between the models based on reliability goals and the minimum cost model subject to reliability constraints. The minimum-cost model subject to reliability constraints is presented first. Following, the Maximum Reliability Model presented by Ali and Narasimhan (1993, 1995) is reviewed and its connections to mathematical programming are analyzed. In the next section, a Generalized Maximum Reliability model is presented, that is derived using the minimum cost model as starting point and a duality property of optimization problems. The connections to the model developed by Ali and Narasimhan (1993) are established next. Finally, examples are shown illustrating the power of the minimum cost approach in terms of its ability to handle situations and constraints that the maximum reliability model cannot solve.

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2. Minimum cost model

The design of sensor networks subject to reliability constraints can be written as follows:

$$\left.\begin{aligned} &\mathrm{Min} \sum_{\forall i} c_i q_i \\ &\text{s.t.} \\ &R_k(q) \geq R_k^* \qquad \forall k \in M_R \\ &q_i = 0, 1 \qquad \forall i \in M_1 \end{aligned}\right\} \tag{1}$$

where M_1 is the set of streams where sensors can be placed, M_R is the set of variables whose reliability is to be constrained, q_i are the binary variables determining whether a sensor is located in stream S_i ($q_i = 1$), or not ($q_i = 0$), and c_i are the corresponding costs. The reliability of each variable is calculated using the failure probabilities of all the sensors participating in the cutsets that can be used to calculate it. Ali and Narasimhan (1993) and Ali and Narasimhan (1995) discussed such expressions in detail.

If all sensors have the same cost c, then $\Sigma_{\forall i}\, c_i q_i = cN$, where N is the number of sensors. Therefore, one can rewrite Eq. (1) as a minimum number of sensors model:

$$\left.\begin{aligned} &\mathrm{Min} \sum_{\forall i} q_i \\ &\text{s.t.} \\ &R_k(q) \geq R_k^* \qquad \forall k \in M_R \\ &q_i = 0, 1 \qquad \forall i \in M_1 \end{aligned}\right\} \tag{2}$$

Problems (1) and (2) are mixed integer programming problems. Moreover, the constraints regarding reliability are constructive constraints, in the sense that they involve an algorithm for the evaluation of reliability. However, an explicit mathematical expression to calculate $R_k(q)$ could be derived, but such expression has not been yet presented or used. Thus, we use the tree search methodology developed by Bagajewicz (1997) for this type of problems. This methodology is essentially based on an enumeration of every branch of the tree of alternatives aided by a stopping criterion.

3. Maximum reliability models

Ali and Narasimhan (1993) first defined the reliability of a system, R, as the smallest reliability among all the variables

$$R = \underset{\forall k}{\mathrm{Min}} R_k(q) \tag{3}$$

They proposed to search for a set of sensors that satisfies the property that all the variables of the system are observable while R is maximized. In fact the observability issue can be satisfied by choosing the sensors on the chords of any spanning tree of the graph (Madron, 1992). In order to design non-redundant sensor networks, these authors presented a strategy where one sensor is removed at a time and is being replaced by another such that: (a) a spanning tree is obtained with the new sensor: (b) the reliability of the new spanning tree is higher.

Ali and Narasimhan (1995) extended their model to consider a fixed number of sensors larger than the minimum so that redundant systems are obtained. The solution procedure relies on choosing a spanning tree first. The remaining variables are added to some of its branches. With this starting point the same strategy of choosing a leaving variable and an entering variable is attempted. Once the leaving variable is chosen, the entering variable is selected in such a way that: (a) the system reliability is increased; and (b) the new variable belongs to a cutset that was not among the cutsets estimating the variable of smallest reliability. This solution procedure guarantees observability of all variables because the number of measurements is larger than the minimum needed for such purpose and the cutsets are chosen appropriately.

The model presented by Ali and Narasimhan (1993) can be put in the following form

$$\left.\begin{aligned} &\mathrm{Max\ Min}\ R_k(q) \\ &\text{s.t.} \\ &\sum_{\forall i} q_i = N^* \\ &E_i(q) = 1 \qquad \forall i \\ &q_i = 0, 1 \qquad \forall i \end{aligned}\right\} \tag{4}$$

The first constraint fixes the number of sensors. In this case N^* is chosen to be the number of chords of a spanning tree, which is equal to the number of streams minus the number of units of the process flowsheet. The observability requirement is mathematically expressed by asking that the degree of estimability of all variables to be equal to one. In turn, the degree of estimability concept has been recently introduced by Bagajewicz and Sánchez (1999a), it defines in how many ways a measured or unmeasured variable can be calculated.

For the case of redundant networks the model presented by Ali and Narasimhan (1995) can be represented introducing a slight modification of the model given by Eq. (4). The number of sensors N^* is now larger than the minimum number. In addition the estimability constraint is relaxed to allow redundancy and at the same time ensure observability of all variables, as shown next:

$$\left.\begin{array}{l}\text{Max}\ \underset{\forall k}{\text{Min}}\ R_k(q)\\ \text{s.t.}\\ \sum_{\forall i} q_i = N^*\\ E_i(q) \geq 1 \qquad \forall i\\ q_i = 0, 1 \qquad \forall i\end{array}\right\} \tag{5}$$

4. Limitations of previous models

Even though the models given by Eqs. (4) and (5) are successful in identifying reliable sensor networks, they have some limitations:

1. Cost is not explicitly considered so that the solutions may not be cost effective. Cost is only controlled by N^*.
2. The set of variables for which reliability is requested (M_R) is equal to the whole set of variables. Thus models do not guarantee desired reliability levels on specific variables, as they rely on maximizing only the smallest reliability.
3. They cannot be used in conjunction with other goals such as accuracy, error detectability, etc.
4. In the case of the design of redundant sensor networks, there is no control over which variable is redundant. Moreover, there is no control on the different ways a particular variable can be estimated.
5. The set of variables where sensors can be placed (M_1) is also the whole set of variables. Although some modifications can be made to address this problem in the context of the methodology based on the choice of leaving and entering variables, the effect of such restrictions on the ability to find optimal solutions has not been investigated.

To address these limitations we propose a general maximum reliability model. In the next section we present such model, and we establish its connections with the minimum cost model given by Eq. (1).

5. Generalized maximum reliability model

Assume now that a new constraint is added to Eq. (1) in which the minimum reliability of all the variables of interest is used. In such case one can rewrite Eq. (1) as follows:

$$\left.\begin{array}{l}\text{Min} \sum_{\forall i} c_i q_i\\ \text{s.t.}\\ \underset{\forall k \in M_R}{\text{Min}}\ R_k(q) \geq \underset{\forall k \in M_R}{\text{Min}}\ R_k^*\\ R_k(q) \geq R_k^* \qquad \forall k \in M_R\\ q_i = 0, 1 \qquad \forall i \in M_1\end{array}\right\} \tag{6}$$

Note that the addition of the constraint on the system reliability is trivial, as it is guaranteed by the other constraints on reliability of individual streams. We now use the concept of duality, according to Tuy (1987) and used previously by Bagajewicz and Sánchez (1999b) in the context of sensor network design, to obtain the result presented in Eq. (7) that allows to state the maximum reliability model constrained by cost, as is indicated by Eq. (8):

$$\left\{\begin{array}{l}c_T = \alpha \leq \text{Min}\, f(x) = \text{Min} \sum_{i \in M_1} c_i q_i\\ \text{s.t.}\\ g(x) = \underset{\forall k \in M_R}{\text{Min}}\ R_k \geq \underset{\forall k \in M_R}{\text{Min}}\ R_k^* = \beta\\ R_k \geq R_k^* \qquad \forall k \in M_R\\ q_i = 0, 1 \qquad \forall i \in M_1\end{array}\right\}$$

$$\Leftrightarrow \left\{\begin{array}{l}\beta = \underset{\forall k \in M_R}{\text{Min}}\ R_k^* \geq \text{Max}\, g(x) = \text{Max} \underset{\forall k \in M_R}{\text{Min}}\ R_k\\ \text{s.t.}\\ f(x) = \sum_{i \in M_1} c_i q_i \leq c_T = \alpha\\ R_k \geq R_k^* \qquad \forall k \in M_R\\ q_i = 0, 1 \qquad \forall i \in M_1\end{array}\right\} \tag{7}$$

$$\left.\begin{array}{l}\text{Max}\ \underset{\forall k \in M_R}{\text{Min}}\ R_k\\ \text{s.t.}\\ \sum_{i \in M_1} c_i q_i \leq c_T\\ R_k \geq R_k^* \qquad \forall k \in M_R\\ q_i = 0, 1 \qquad \forall i \in M_1\end{array}\right\} \tag{8}$$

The objective function of this problem is the same as the one proposed by Ali and Narasimhan (1993). Furthermore, a few simplifying assumptions will produce a problem constrained by a minimum number of sensors, which is the model they presented. Indeed, consider all the sensors having the same cost, so that the cost constraint becomes a constraint on the total number of sensors.

$$\sum_{i=M_1} c_i q_i \leq c_T \Rightarrow \sum_{i \in M_1} q_i \leq N^* \tag{9}$$

Thus, when the total number of sensors is set to a minimum and the constraints on the reliability of individual variables are dropped, the problem becomes:

$$\left.\begin{array}{l} \text{Max} \underset{\forall k \in M_R}{\text{Min}} R_k \\ \text{s.t.} \\ \sum_{i \in M_1} q_i \leq N^* \\ q_i = 0, 1 \qquad \forall i \in M_1 \end{array}\right\} \tag{10}$$

Consider now the following problem, where the constraint is made an equality

$$\left.\begin{array}{l} \text{Max} \underset{\forall k \in M_R}{\text{Min}} R_k \\ \text{s.t.} \\ \sum_{i \in M_1} q_i = N^* \\ q_i = 0, 1 \qquad \forall i \in M_1 \end{array}\right\} \tag{11}$$

Problem (11) is a more stringent version of problem (4) as it has a restricted set M_1 where the sensors can be placed, and a restricted set of variables of interest M_R. We now show that problems (10) and (11) have the same reliability, but can have different solutions.

Property. The solution to problem (10) has the same objective function value as the solution of problem (11), or equivalently, problem (10) can have a binding constraint at the optimum without altering the optimum system reliability.

Proof: assume that the constraint is not binding. Then, if $\tilde{q}$ is the vector corresponding to the optimal solution

$$\sum_{i \in M_1} \tilde{q}_i < N^* \tag{12}$$

Therefore, it is possible to add a sensor to the system without violating Eq. (12). Such an addition can leave the reliability of the system unaltered or increase it. Assume without loss of generality that the optimum of the problem corresponds to the reliability of estimation of the flowrate of stream S_1. That is

$$\underset{\forall k \in M_R}{\text{Min}} \tilde{R}_k = \tilde{R}_1 \tag{13}$$

A new sensor can be located in a stream where a new balance equation involving the flowrate of S_1 cannot be written. In such case the reliability of S_1 is not altered. On the contrary, if a new balance equation can be written, then the reliability of S_1 will increase, because reliability is a monotone function of the number of sensors involved. Q.E.D.

Although problems (10) and (11) have the same optimal objective function value, problem (10), however, will render either the same set of sensors or a set of sensors that is a proper subset of the solution of problem (11).

The above generalized model addresses most of the concerns raised when analyzing the procedure presented by Ali and Narasimhan (1993). It explicitly considers the cost, it can impose lower bounds on the reliability of specific variables, and it can restrict the set where sensors can be located. We have therefore established that the model given by Ali and Narasimhan (1993, 1995) is just one simplified version of the generalized maximum reliability model. In addition, the generalized maximum reliability model is equivalent to the minimum cost model.

6. Reliable sensor networks with estimability and precision constraints

Other constraints, such as bounds on the degree of estimability or precision for key variables, can be easily added to the minimum cost model. These constraints restrict the feasible region and will remain as constraints if the maximum reliability model is obtained using the Tuy duality. Let us consider the addition of estimability constraints to the minimum cost model (1)

$$\left.\begin{array}{ll} \text{Min} \sum_{\forall M_1} c_i q_i & \\ \text{s.t.} & \\ R_k(q) \geq R_k^* & \forall k \in M_R \\ E_k(q) \geq E_k^* & \forall k \in M_E \\ q_i = (0, 1) & \forall i \end{array}\right\} \tag{14}$$

Using the duality according to Tuy, one can convert this model into maximum reliability models. Therefore, the model given by (14) is a generalized version of (4) and (5). One must recall that (4) and (5) are restricted to the case in which $M_{1,}$ M_R and M_E are the whole set of sensors and $E_k^* = 1$. In the absence of specific targets for the reliability of the variables, one may find convenient to solve (5) or (6) or variants of it.

If precision constraints are considered, the following minimum cost model results,

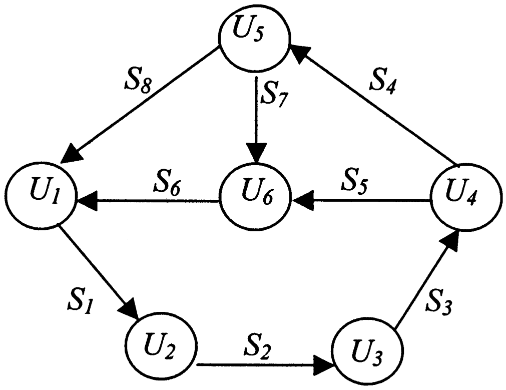

Fig. 1. Simplified ammonia plant network.

$$\left.\begin{aligned} &\text{Min} \sum_{\forall M_1} c_i q_i \\ &\text{s.t.}\\ &R_k(q)\ge R_k^* \qquad \forall k\in M_R\\ &\sigma_k(q)\ge\sigma_k^* \qquad \forall k \in M_P \\ &q_i=(0,1) \qquad \forall i \end{aligned}\right\} \tag{15}$$

Using the duality according to Tuy, one can convert this model into a maximum reliability model.

7. Examples

Consider the ammonia network example presented by Ali and Narasimhan (1993) (Fig. 1). In case 1 of Table 1, the problem presented by these authors is solved using the minimum cost model for a sensor cost of \$2000. The same eight solutions are found when the imposed constraints are equal to the solution obtained by Ali and Narasimhan (1993). Different sensor costs are considered then, $c = [1500\ 2000\ 2300\ 2800\ 1700\ 2000\ 1500\ 2800]$. Thus only one optimal solution is obtained for case 2.

Reliability constraints on key variables are imposed in cases 3 and 4, these types of solutions can not be obtained using previous models presented in the literature. In case 5, reliability and estimability constraints are considered, in case 6, we include tight constraints with reliability constraints in different variables than the estimability constraints. To show how reliability and accuracy constraints interact, case 7 was prepared.

8. Conclusions

A new generalized model for maximizing reliability in sensor networks has been presented. Connections to the minimum cost model constrained by reliability have been established. It was shown that the new model has superior capabilities to previous models presented in the literature.

Table 1
Results for the minimum cost model

#	Constraints bounds	Solution	Cost	Constraints at the solution
1	$R^* = 0.81$*on [11111111]	$S_1\ S_4\ S_8$	6000	[0.9-0.9-0.9-0.9-0.81-0.81-0.81-0.9]
		$S_1\ S_5\ S_6$		[0.9-0.9-0.9-0.81-0.9-0.9-0.81-0.81]
		$S_2\ S_4\ S_8$		[0.9-0.9-0.9-0.9-0.81-0.81-0.81-0.9]
		$S_2\ S_5\ S_6$		[0.9-0.9-0.9-0.81-0.9-0.9-0.81-0.81]
		$S_3\ S_4\ S_8$		[0.9-0.9-0.9-0.9-0.81-0.81-0.81-0.9]
		$S_3\ S_5\ S_6$		[0.9-0.9-0.9-0.81-0.9-0.9-0.81-0.81]
		$S_4\ S_5\ S_7$		[0.81-0.81-0.81-0.9-0.9-0.81-0.9-0.81]
		$S_6\ S_7\ S_8$		[0.81-0.81-0.81-0.81-0.81-0.9-0.9-0.9]
2	$R^* = 0.81$*[11111111]	$S_1\ S_5\ S_6$	5200	[0.9 0.9 0.9 0.81 0.9 0.9 0.81 0.81]
3	$R^* = [0.9\ 0\ 0.81\ 0\ 0\ 0\ 0.729\ 0]$	$S_1\ S_7$	3000	$R_1 = 0.9$ $R_3 = 0.9$ $R_7 = 0.9$
4	$R^* = [0.95\ 0\ 0.95\ 0\ 0\ 0\ 0.95\ 0]$	$S_1\ S_5\ S_7\ S_8$	7500	$R_1 = 0.9729$ $R_3 = 0.9729$ $R_7 = 0.9729$
5	$R^* = 0.81[1\ 0\ 1\ 0\ 0\ 0\ 1\ 0]$ $E^* = [2\ 0\ 3\ 0\ 0\ 0\ 2\ 0]$	$S_1\ S_2\ S_5\ S_7\ S_8$	9500	$R_1 = 0.9973$ $R_3 = 0.9973$ $R_7 = 0.9802$ $E_1 = 3$ $E_3 = 3$ $E_7 = 2$
6	$R^* = 0.9[1\ 0\ 0\ 1\ 0\ 0\ 1\ 0]$ $E^* = [0\ 3\ 0\ 0\ 0\ 3\ 0\ 3]$	$S_1\ S_2\ S_5\ S_6\ S_7\ S_8$	11500	$R_1 = 0.998$ $R_3 = 0.988$ $R_7 = 0.989$ $E_2 = 3$ $E_6 = 3$ $E_8 = 3$
7	$R^* = 0.9$*[1 0 0 1 0 0 1 0] $\sigma^* = [0\ 3\ 0\ 0\ 0\ 2\ 0\ 2]$	$S_2\ S_5\ S_7\ S_8$	8000	$R_1 = 0.9729$ $R_3 = 0.9639$ $R_7 = 0.9729$ $\sigma_2 = 1.33$ $\sigma_6 = 1.991$ $\sigma_8 = 1.925$

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