

# Globally Optimal Design Optimization of Cooling Water Systems

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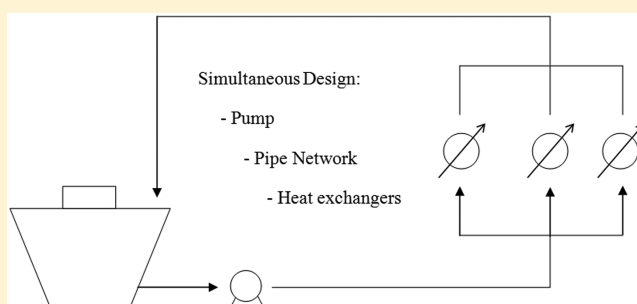
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## Supporting Information

**ABSTRACT:** This study presents a globally optimal linear procedure for the simultaneous design of cooling water systems. The optimization entails obtaining optimal pipe diameters, pump selection, and detailed cooler design. For the latter, the Kern model and the linear formulation proposed by Gonçalves et al. (*Ind. Eng. Chem. Res.* 2017, 56, 5970) were used. The proposed linear model is compared with conventional design procedures, such as optimizing the design of each exchanger first, through area minimization or through area and pressure drop costs. The results illustrate in one of the examples the advantage of using the proposed simultaneous design approach.



## 1. INTRODUCTION

Typical cooling water systems are composed of a cooling tower connected to a pipe network and one or more pumps, which distribute cooling water to a set of coolers, which, in most cases, are aligned in parallel. There are three components to this problem: the cooling water heat exchanger network vis-à-vis its use and reuse structure with a priori known cooling tasks; the associated hydraulic issues; and the design of the cooling water tower. Studies about these structures related to their optimal design and operation started in only the early 2000s.

For the case of the cooling heat exchanger network, Kim and Smith<sup>1,2</sup> proposed the use of the pinch design method to determine what is the appropriate reuse of cooling water. Kim et al.<sup>3</sup> augmented this modeling by analyzing the interactions with the cooling tower. The problem was put in a form of a mathematical model using a water main by Feng et al.<sup>4</sup> Picón-Núñez et al.<sup>5</sup> studied the effect of the network arrangement on the total area of the exchangers upon reuse of cooling water. Later, Ponce-Ortega et al.<sup>6</sup> presented a mixed-integer nonlinear programming (MINLP) formulation based on the Synheat model<sup>7</sup> of cooling water to synthesize the network, thus allowing the reuse of water.

A limited number of follow-up papers addressed the hydraulic analysis of the pipe network associated with cooling water systems. Picón-Núñez et al.<sup>8</sup> investigated the issue by proposing a simplified hydraulic model of the pipe network and the set of coolers. Picón-Núñez et al.<sup>9</sup> investigated the impact of the insertion of new heat exchangers in a cooling water system, including the analysis of the behavior of the

cooling tower heat load and aspects related to fouling. Sun et al.<sup>10</sup> employed simulated annealing for the design optimization of an auxiliary pumping system associated with cooling water networks, although pipe diameters were not optimized. De Souza et al.<sup>11</sup> proposed the hydraulic design of cooling water distribution systems based on the solution of a set of linear programming problems. The solution determines the pump size and diameters of the pipe sections (represented by the lengths of the subsections of commercial diameters). Finally, Souza et al.<sup>12</sup> investigated the optimization of the hydraulic debottlenecking of cooling water systems, by means of substituting pipes and pumps together with the manipulation of valves to redistribute the total flow rate among the different coolers.

Another aspect of the investigation of cooling water systems involves the exploration of the simultaneous analysis of the coolers and the associated piping and pumps. This approach has received some attention in recent years; however, often the focus is directed to the heat exchanger network synthesis, and the details of the design of each cooler are usually ignored (the area evaluation is based on fixed values of film coefficients) and/or the hydraulic design of the pipe network is only partially included in these formulations. Ponce-Ortega et al.<sup>13,14</sup> analyzed the coolers in the system including the equipment design optimization (i.e., the film coefficients are

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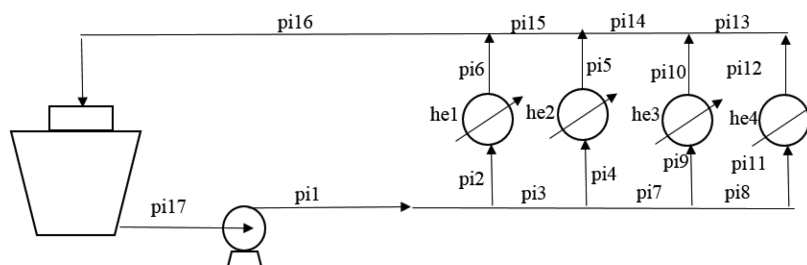


Figure 1. Cooling water system representation.

resultant from the heat exchanger design alternatives selected), but the hydraulic design of the pipe network was not fully explored in these formulations; that is, the dimensioning of pipe diameters and the associated pressure drop were not considered in the model. Sun et al.<sup>15</sup> developed a stepwise optimization method for a cooling water system: the first step uses a model to obtain the optimal cooler network based on parallel branches with several coolers in each branch, and the second step employs a simplified hydraulic model to obtain the optimal pump network with auxiliary pumps installed in parallel branch pipes but with fixed pipe diameters and lengths. Ma et al.<sup>16</sup> also investigated the arrangement of the coolers and the hydraulic aspects, indicating the importance of the simultaneous optimization of the cooler network and the pump network. They also conducted their analysis ignoring the piping pressure drop and allowing the same configuration of several branches where several coolers in series can be installed. Liu et al.<sup>17</sup> investigated the simultaneous synthesis of the heat exchanger network associated with the process streams and the cooling water system. The model considers the pressure drop in the heat exchangers but also ignores the pipe sections, including in the objective function the capital and operating costs associated with the pumps. Note that Polley et al.<sup>18</sup> pointed out that piping may have a considerable impact on the total costs of heat-transfer systems.

In relation to the numerical aspects, the majority of the resultant mathematical formulations in the literature is nonconvex MINLP problems whose solutions, when converged, may be trapped in local optima.

Hitherto, despite the importance of the simultaneous analysis of the hydraulic design and the heat exchanger design in cooling water systems, the design practice has been to solve the problems separately. Each cooler is designed assuming a maximum pressure drop associated with the streams flow, and the design of the pipe network for the selection of the pump and pipe section diameters is conducted using a fixed pressure drop for the coolers. Thus, this practice yields nonoptimal solutions because they do not explore the interrelation between the capital costs of coolers, pipe sections, and pumps and the operating costs of the pumps.

In this article, the piping network, the pump, and the detailed design of the exchangers are considered simultaneously. First, the overall model is discussed, followed by the network model specifics and the heat exchanger linear model. Then, results are presented, where the advantage of the proposed approach in relation to the conventional approach is demonstrated. Finally, the conclusions are presented.

## 2. COOLING SYSTEM MODEL

An optimization solution for the simultaneous design of the coolers and the pipe network of cooling water systems, not

including the cooling tower, is presented. The objective function is the minimization of the total annualized cost, and the constraints refer to pipe network modeling, heat exchanger modeling, and economic equations. The problem solution provides the optimal set of the design variables of each cooler, the diameter of each pipe section, the head losses in the valves, and the pump size to be selected.

Although the literature explores parallel–series arrangements introduced by the pioneering work of Kim and Smith<sup>1</sup> and expanded by several papers afterward, the parallel layout without reuse is still the most common alternative found in practice. This parallel architecture was considered for two reasons:

- The reuse of water implies that a higher temperature of the water is used in downstream units, which prompts higher fouling rates. Until the impact of fouling, the cost of elevating the quality of the water, removing substances that contribute to the fouling, or the cost of frequent cleaning is considered, it is prudent to stay away from reuse.
- Most important, the reuse is only reasonably covered by all the published models that use water-pinch type of design procedures or stages superstructures,<sup>7</sup> when all the coolers are in the same plant in a complex. When the system encompasses serving cooling water to several coolers in different plants, reuse involves piping going from one plant to another or possibly back and forth if the cost of piping is not considered. As a consequence, all the models considering reuse without adding the cost of connections are impractical.

Instead of using continuous variables to describe pipe diameters and pump sizing, the proposed formulation uses discrete commercial alternatives (e.g., the pipe diameters are selected according to a pipe schedule table and the pump sizing is coherent with a set of available pump curves). Therefore, the mathematical optimization problem can be formulated as an integer linear programming (ILP), and its solution is the global optimum. Thus, in contrast with local MINLP approaches, the possibility of being trapped in a poor local optimum is excluded and the need of good initial estimates to promote convergence is eliminated.

## 3. PIPING NETWORK MODEL

The proposed analysis of the cooling water system assumes that the flow rates of the water and process streams in the coolers and the pipe section lengths are previously fixed; that is, the mass and energy balances and the layout of the system is the starting point of the optimization problem. The problem design variables are the pipe diameters of the pipe sections, the pump selection, the head losses of the valves associated with each heat exchanger to guarantee the hydraulic balance

according to the design flow rate, and the heat exchanger geometry (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes).

The elements that compose the system are represented using graph theory.<sup>19</sup> The edges ( $k \in STR$ ) represent the heat exchangers ( $HE \subset STR$ ), pipe sections ( $PI \subset STR$ ), and pumps ( $PU \subset STR$ ). The nodes ( $t \in VET$ ) represent the cooling tower basin ( $PS \subset VET$ ), the cooling tower top ( $PD \subset VET$ ), and the interconnections among the edges ( $INT \subset VET$ ). The cooling tower basin node represents the cooling water supply to the network, and the cooling tower top represents the cooling water return to the tower. Figure 1 contains an illustration of a typical cooling water system with its interconnected elements.

The network edges are organized according to a set of independent hydraulic circuits ( $l \in HY$ ). A hydraulic circuit is a path along the digraph starting at the cooling water supply node and ending at the cooling water return. All edges belong to at least one hydraulic circuit. Each heat exchanger is associated with a unique circuit. It is assumed that each hydraulic circuit can be associated with a valve/restriction plate. The network circuits are described by the matrix  $\widehat{\Lambda}_{l,k}$ , such that, if  $\widehat{\Lambda}_{l,k} = 1$ , then the edge  $k$  belongs to the circuit  $l$ , otherwise,  $\widehat{\Lambda}_{l,k} = 0$ . For example, Table 1 contains the representation of the hydraulic circuits of the cooling water system depicted in Figure 1.

**Table 1. Hydraulic Circuits of the Cooling Water System Present in Figure 1**

circuit	edges
1	pi17, pi1, pi2, he1, pi6, pi16
2	pi17, pi1, pi3, pi4, he2, pi5, pi15, pi16
3	pi17, pi1, pi3, pi7, pi9, he3, pi10, pi14, pi15, pi16
4	pi17, pi1, pi3, pi7, pi8, pi9, he4, pi12, pi13, pi14, pi15, pi16

Because the layout of the system and the necessary volumetric flow rate in each cooler are already established ( $\widehat{qhe}_i$ ), the flow rates along each network element ( $\widehat{q}_k$ ) can be calculated, prior to the optimization:

$$\widehat{q}_k = \sum_{l \in HY} \widehat{qhe}_i \widehat{\Lambda}_{l,k} \quad k \in STR \quad (1)$$

The diameters of the pipe sections must be chosen according to a set of available commercial options ( $n \in SD$ ). The values of the standard inner and nominal diameters are represented by  $\widehat{D}_n^{int}$  and  $\widehat{D}_n^{nom}$  (the nominal diameter is identified in inches, according to the industrial practice), respectively. In turn, pump selection is based on a set of available commercial options ( $s \in SPU$ ). Each available pump option  $s$  is represented by the corresponding head at the design flow rate. In the presentation of the model, the problem parameters, which are fixed prior to the optimization, are represented with the symbol “ $\wedge$ ” on top.

The hydraulic behavior of the pipe network is represented by a mechanical energy balance along each hydraulic circuit (the kinetic head at the cooling water return is dismissed here because of its relatively small value):

$$\sum_{k \in PU} \widehat{\Lambda}_{l,k} fpu_k - \widehat{\Delta z} - \sum_{l \in HY} \widehat{\Lambda}_{l,k} fhe_l - \sum_{k \in PI} \widehat{\Lambda}_{l,k} \widehat{L}_k fpi_k - f_{v_l} = 0 \quad l \in HY \quad (2)$$

where  $fpu_k$  is the head of the pump  $k$ ,  $\widehat{\Delta z}$  the elevation difference between the top and the bottom of the cooling tower,  $fhe_l$  the head loss along the heat exchanger present in the hydraulic circuit  $l$ ,  $\widehat{L}_k$  the pipe section length,  $fpi_k$  the unitary head loss along the pipe section  $k$ , and  $f_{v_l}$  the head loss in the valve/restriction plate associated with the circuit  $l$ . Meanwhile, the head loss in the heat exchangers is calculated as follows:

$$fhe_l = \frac{\Delta P_t \widehat{yT}_{c,l} + \Delta P_s \widehat{yT}_{h,l}}{\widehat{\rho}_w \widehat{g}} \quad l \in HY \quad (3)$$

where  $\widehat{\rho}_w$  is the cooling water density and  $\widehat{g}$  is the gravity acceleration;  $\Delta P_t$  and  $\Delta P_s$  are the pressure drops in the heat exchanger tube-side and shell-side, respectively, and  $\widehat{yT}_{c,l}$  and  $\widehat{yT}_{h,l}$  are parameters that indicate if the cooling water is in the tube-side ( $\widehat{yT}_{c,l} = 1$ ) or if the hot process stream is in the tube-side ( $\widehat{yT}_{h,l} = 1$ ). Next, the head loss of pipe section  $k$  is calculated using the Hazen–Williams equation:

$$f_k = \frac{10.67}{\widehat{c}^{1.85}} \frac{\widehat{q}_k^{1.852}}{D_k^{4.8704}} \quad k \in PI \quad (4)$$

where  $f_k$  is the unitary head loss,  $D_k$  the inner diameter, and  $\widehat{c}$  the Hazen–Williams constant. In turn, pipe diameter is a discrete variable, according to the commercially available values. Therefore, it can be represented by binary variables:

$$D_k = \sum_{n \in SD} \widehat{D}_n^{int} y_{k,n}^{pi} \quad k \in PI \quad (5)$$

where  $y_{k,n}^{pi}$  is a binary variable that it is equal to 1 if the pipe section  $k$  is composed of a commercial diameter  $n$ ; otherwise, it is equal to 0. Because the problem solution must present a unique diameter for each pipe section

$$\sum_{n \in SD} y_{k,n}^{pi} = 1 \quad k \in PI \quad (6)$$

The substitution of eq 5 into eq 4 of unitary head loss yields a linear constraint:

$$f_k = \sum_{n \in SD} \widehat{pf}_{k,n} y_{k,n}^{pi} \quad k \in PI \quad (7)$$

where

$$\widehat{pf}_{k,n} = 10.67 \frac{\widehat{q}_k^{1.852}}{\widehat{c}^{1.85} (\widehat{D}_n^{int})^{4.8704}} \quad k \in PI, n \in SD \quad (8)$$

The head of a pump  $k$  is also represented by a linear relation:

$$fpu_k = \sum_{s \in SPU} \widehat{pfpu}_{k,s} y_{k,s}^{pu} \quad k \in PU \quad (9)$$

where  $\widehat{pfpu}_{k,s}$  is the head of the pump model  $s$  at the design flow rate of the edge  $k$ .  $y_{k,s}^{pu}$  is a binary variable that it is equal to

1, if the pump  $k$  corresponds to the available option  $s$ ; otherwise, it is equal to 0.

Because only one pump option must be chosen

$$\sum_{s \in \text{SPU}} y_{k,s}^{pu} = 1 \quad k \in \text{PU} \quad (10)$$

In order to avoid erosion and fouling, maximum and minimum flow velocity bounds are imposed:

$$\widehat{vmax} - \frac{4\hat{q}_k}{\pi D_k^2} \geq 0 \quad k \in \text{PI} \quad (11)$$

$$\widehat{vmin} - \frac{4\hat{q}_k}{\pi D_k^2} \leq 0 \quad k \in \text{PI} \quad (12)$$

where  $\frac{4\hat{q}_k}{\pi D_k^2}$  is the velocity. Substitution of eq 5 into eqs 11 and 12 yields the following linear relations:

$$\widehat{vmax} - \frac{4}{\pi} \sum_{n \in \text{SD}} \frac{\hat{q}_k}{(\hat{D}_n^{int})^2} y_{k,n}^{pi} \geq 0 \quad k \in \text{PI} \quad (13)$$

$$\widehat{vmin} - \frac{4}{\pi} \sum_{n \in \text{SD}} \frac{\hat{q}_k}{(\hat{D}_n^{int})^2} y_{k,n}^{pi} \leq 0 \quad k \in \text{PI} \quad (14)$$

According to engineering practice, the pipe diameter at the pump suction must be equal or higher than the diameter at the pump discharge:

$$\sum_{n \in \text{SD}} y_{kps,n}^{pi} \hat{D}_n^{int} \geq \sum_{n \in \text{SD}} y_{kpd,n}^{pi} \hat{D}_n^{int} \quad (15)$$

where  $kps$  and  $kpd$  are indices associated with the pump suction and discharge, respectively.

#### 4. COOLER DETAILED MODEL

The heat exchanger model is based on Gonçalves et al.,<sup>20</sup> where the design problem of E-type shell-and-tube heat exchangers without phase change is formulated as an integer linear programming (ILP). The allocation of the process and cooling water streams in the tube-side or in the shell-side is considered a designer decision established prior to the optimization. The design variables are the tube inner and outer diameters ( $d_{ti}$  and  $d_{te}$ ), tube length ( $L$ ), tube layout ( $lay$ ), tube pitch ratio ( $rp$ ), number of passes in the tube-side ( $N_{tp}$ ), shell diameter ( $D_s$ ), and number of baffles ( $N_b$ ).

The presentation of the constraints related to the heat exchanger model is organized here in two parts. First, the original nonlinear thermofluid dynamic model and design equations are presented. For the sake of simplicity, only the main equations of the original nonlinear model are presented here; further details can be found in Gonçalves et al.<sup>20</sup> After this, a reformulation is implemented to obtain a linear model.

The LMTD method is used and, considering a design margin, “excess area” ( $\widehat{Aexc}$ ), the heat-transfer rate equation is represented by the following relation:

$$UA \geq \left(1 + \frac{\widehat{Aexc}}{100}\right) \frac{\hat{Q}}{\Delta T_{lm} F} \quad (16)$$

where  $U$  is the overall heat-transfer coefficient,  $A$  the heat-transfer area,  $\hat{Q}$  the heat load,  $\Delta T_{lm}$  the logarithmic mean temperature difference, and  $F$  the LMTD correction factor. In turn, the heat-transfer area is given by

$$A = N_{tt} \pi d_{te} L \quad (17)$$

where  $N_{tt}$  is the total number of tubes (simplified expressions to represent the relation between the total number of tubes and the design variables can be found in Kakaç and Liu<sup>21</sup>). Finally, the overall heat-transfer coefficient is given by

$$U = \frac{1}{\frac{d_{te}}{d_{ti} h_t} + \frac{\widehat{Rft} d_{te}}{d_{ti}} + \frac{d_{te} \ln\left(\frac{d_{te}}{d_{ti}}\right)}{2k_{tube}} + \widehat{Rfs} + \frac{1}{h_s}} \quad (18)$$

where  $k_{tube}$  is the thermal conductivity of the tube wall;  $\widehat{Rft}$  and  $\widehat{Rfs}$  are the tube-side and shell-side fouling factors, and  $h_t$  and  $h_s$  are the convective heat-transfer coefficients of the tube-side and shell-side, respectively. The tube-side and shell-side heat-transfer coefficients are calculated using the Dittus–Boelter correlation for the tube-side<sup>22</sup> and the Kern model for the shell-side.<sup>23</sup>

Bounds on pressure drops are represented by

$$\Delta P_s \leq \Delta \widehat{Psdisp} \quad (19)$$

$$\Delta P_t \leq \Delta \widehat{Ptdisp} \quad (20)$$

where  $\Delta P_s$  and  $\Delta P_t$  are the pressure drops on the shell-side and tube-side, and  $\Delta \widehat{Psdisp}$  and  $\Delta \widehat{Ptdisp}$  are the corresponding admissible values, respectively. The pressure drop in the tube-side is calculated using the Darcy–Weisbach equation<sup>24</sup> and the Kern model for the shell-side.<sup>23</sup>

Additional bounds are applied to velocities and Reynolds numbers:

$$v_s \geq \widehat{vsmin} \quad (21)$$

$$v_s \leq \widehat{vsmax} \quad (22)$$

$$v_t \geq \widehat{vtmin} \quad (23)$$

$$v_t \leq \widehat{vtmax} \quad (24)$$

$$Re_s \geq 2 \times 10^3 \quad (25)$$

$$Re_t \geq 10^4 \quad (26)$$

Design standards impose the following geometric bounds:<sup>25</sup>

$$lbc \geq 0.2D_s \quad (27)$$

$$lbc \leq 1.0D_s \quad (28)$$

$$L \geq 3D_s \quad (29)$$

$$L \leq 15D_s \quad (30)$$

Design variables have a discrete nature, according to their physical nature and/or standard commercially available options. Instead of using separate sets of binaries for each variable, a single set of binaries was used. Each individual binary variable ( $y_{row\ srow}$ ) identifies a heat exchanger candidate, which represents a combination of values of the design variables (identified by the multi-index  $srow$ ). Previous results indicate that this approach provides considerable reduction of the computational effort.<sup>20</sup>

The final structure of the set of linear constraints as in Gonçalves et al.<sup>20</sup> is represented below, adding an extra index for each individual heat exchanger ( $l \in \text{HY}$ ).

Because only one option of heat exchanger must be selected



$$\sum_{srow} yrow_{srow,l} = 1 \quad l \in HY \quad (31)$$

After substitution of discrete representation and reformulation, the heat-transfer rate equation (eq 16), becomes

$$\begin{aligned} Q_l & \left( \sum_{srow} \frac{\widehat{Pdt}_{srow}}{\widehat{Pht}_{srow,l} \widehat{Pdt}_{srow}} yrow_{srow,l} + \widehat{Rft}_l \sum_{srow} \frac{\widehat{Pdt}_{srow}}{\widehat{Pdt}_{srow}} yrow_{srow,l} \right. \\ & \quad \left. + \sum_{srow} \frac{\widehat{Pdt}_{srow} \ln \left( \frac{\widehat{Pdt}_{srow}}{\widehat{Pdt}_{srow}} \right) yrow_{srow}}{2ktube_l} + \widehat{Rfs}_l + \sum_{srow} \frac{yrow_{srow,l}}{\widehat{Phs}_{srow,l}} \right) \\ & \leq \left( \frac{100\pi}{100 + A_{exc}} \right) \left( \sum_{srow} \widehat{PNtt}_{srow} \widehat{Pdt}_{srow} \widehat{PL}_{srow} yrow_{srow,l} \right) \Delta Tlm_l \widehat{F}_{srow,l} \\ & \quad l \in HY \end{aligned} \quad (32)$$

where  $\widehat{Pdt}_{srow}$ ,  $\widehat{Pdt}_{srow}$ , and  $\widehat{PL}_{srow}$  are values of the outer and inner tube diameters and tube length and  $\widehat{PNtt}_{srow}$  is the total number of tubes calculated prior to the optimization for each solution alternative. The other parameters in eq 32 are expressed by

$$\widehat{Pht}_{srow,l} = \frac{\widehat{kt}_l 0.023 \left( \frac{4\widehat{mt}_l}{\pi \widehat{\mu}_l} \right)^{0.8} \widehat{Pr}_l^n \left( \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}} \right)^{0.8}}{\widehat{Pdt}_{srow}^{1.8}} \quad (33)$$

$$\begin{aligned} \widehat{Phs}_{srow,l} &= \frac{\widehat{ks}_l 0.36 \left( \frac{\widehat{ms}_l}{\widehat{\mu}_s} \right)^{0.55} \widehat{Pr}_s^{1/3}}{\widehat{PDeq}_{srow}^{0.45}} \\ & \quad \left( \frac{(\widehat{PNb}_{srow} + 1)}{\widehat{PDs}_{srow} \widehat{PFAR}_{srow} \widehat{PL}_{srow}} \right)^{0.55} \end{aligned} \quad (34)$$

$$\widehat{PFAR}_{srow} = 1 - \frac{1}{\widehat{Prp}_{srow}} \quad (35)$$

$$\widehat{pDeq}_{srow} = \frac{\widehat{aDeq}_{srow} \widehat{Prp}_{srow}^2 \widehat{Pdt}_{srow}^2}{\pi \widehat{Pdt}_{srow}} - \widehat{Pdt}_{srow} \quad (36)$$

$$\widehat{aDeq}_{srow} = \begin{cases} 4 & \text{if } \widehat{Play}_{srow} = 1 \\ 3.46 & \text{if } \widehat{Play}_{srow} = 2 \end{cases} \quad (37)$$

$$\widehat{F}_{srow,l} = \begin{cases} \frac{(\widehat{R}_l^2 + 1)^{0.5} \ln \left( \frac{(1 - \widehat{P}_l)}{(1 - \widehat{R}_l \widehat{P}_l)} \right)}{(\widehat{R}_l - 1) \ln \left( \frac{2 - \widehat{P}_l(\widehat{R}_l + 1 - (\widehat{R}_l^2 + 1)^{0.5})}{2 - \widehat{P}_l(\widehat{R}_l + 1 + (\widehat{R}_l^2 + 1)^{0.5})} \right)} & \text{if } \widehat{PNpt}_{srow} \neq 1 \\ 1 & \text{if } \widehat{PNpt}_{srow} = 1 \end{cases} \quad (38)$$

where  $\widehat{mt}_l$  and  $\widehat{ms}_l$  are the mass flow rates in the tube-side and shell-side, respectively;  $\widehat{kt}_l$  and  $\widehat{ks}_l$  are the thermal conductivities;  $\widehat{\mu}_l$  and  $\widehat{\mu}_s$  are the viscosities;  $\widehat{Pr}_l$  and  $\widehat{Pr}_s$  are the Prandtl Numbers;  $\widehat{PNpt}_{srow}$  is the value of the number of tube passes;  $\widehat{PNb}_{srow}$  is the value of the number of baffles;  $\widehat{Prp}_{srow}$  is the value

of the tube pitch ratio;  $\widehat{Play}_{srow}$  is the indication of the tube layout arrangement; and  $\widehat{R}_l$  and  $\widehat{P}_l$  are parameters for the evaluation of the correction factor of the LMTD.<sup>22</sup>

The constraints related to bounds on pressure drops, velocities, Reynolds numbers, and geometrical are represented in an alternative way, where the violation of these constraints for each solution candidate can be identified prior to the optimization, and for these options, an exclusion constraint is added, as presented below. According to Souza et al.,<sup>26</sup> this procedure allows a better computational performance.

Pressure drop bounds (equivalent to eqs 19 and 20):

$$yrow_{srow,l} = 0 \quad \text{for } srow \in SDP_{smaxout} \text{ and } l \in HY \quad (39)$$

$$yrow_{srow,l} = 0 \quad \text{for } srow \in SDP_{tmaxout} \text{ and } l \in HY \quad (40)$$

The sets  $SDP_{smaxout}_l$  and  $SDP_{tmaxout}_l$  are given by (where  $\hat{\epsilon}$  is a small positive number)

$$SDP_{smaxout}_l = \{srow / \widehat{P\Delta P}_{srow,l} \geq \Delta P_{sdisp} + \hat{\epsilon}\} \quad (41)$$

$$\begin{aligned} SDP_{tmaxout}_l &= \{srow / P\Delta P_{tturb1_{srow,l}} + P\Delta P_{tturb2_{srow,l}} \\ & \quad + P\Delta P_{tcab_{srow,l}} \hat{K}_{srow} \geq \Delta P_{tdisp} - \hat{\epsilon}\} \end{aligned} \quad (42)$$

where

$$\begin{aligned} \widehat{P\Delta P}_{srow,l} &= 0.864 \frac{\widehat{ms}_l^{1.812} \widehat{\mu}_s^{0.188}}{\widehat{\rho}_s} \\ & \quad \left( \frac{(\widehat{PNb}_{srow} + 1)^{2.812}}{\widehat{PDs}_{srow}^{0.812} (\widehat{PFAR}_{srow} \widehat{PL}_{srow})^{1.812} \widehat{PDeq}_{srow}^{1.188}} \right) \end{aligned} \quad (43)$$

$$P\Delta P_{tturb1_{srow,l}} = \left( \frac{0.112 \widehat{mt}_l^2}{\pi^2 \widehat{\rho}_l} \right) \left( \frac{\widehat{PNpt}_{srow}^3 \widehat{PL}_{srow}}{\widehat{PNtt}_{srow}^2 \widehat{Pdt}_{srow}^5} \right) \quad (44)$$

$$P\Delta P_{tturb2_{srow,l}} = 0.528 \left( \frac{4^{1.58} \widehat{mt}_l^{1.58} \widehat{\mu}_l^{0.42}}{\pi^{1.58} \widehat{\rho}_l} \right) \left( \frac{\widehat{PNpt}_{srow}^{2.58} \widehat{PL}_{srow}}{\widehat{PNtt}_{srow}^{1.58} \widehat{Pdt}_{srow}^{4.58}} \right) \quad (45)$$

$$P\Delta P_{tcab_{srow,l}} = \left( \frac{8 \widehat{mt}_l^2}{\pi^2 \widehat{\rho}_l} \right) \frac{\widehat{PNpt}_{srow}^3}{\widehat{PNtt}_{srow}^2 \widehat{Pdt}_{srow}^4} \quad (46)$$

It is important to note that eqs 39 and 40 are not included in the formulation for the cooling water streams. In this case, the optimal pressure drop will be obtained through the trade-off between capital and operating costs in the context of the entire system solution.

Flow velocity bounds (equivalent to eqs 21–24):

$$yrow_{srow,l} = 0 \quad \text{for } srow \in (Svsminout \cup Svsmaxout) \text{ and } l \in HY \quad (47)$$

$$yrow_{srow,l} = 0 \quad \text{for } srow \in (Svtminout \cup Svtmaxout) \text{ and } l \in HY \quad (48)$$

where the sets  $Svsminout$ ,  $Svsmaxout$ ,  $Svtminout$ , and  $Svtmaxout$  are given by

$$Svsm_{inout_l} = \left\{ srow / \frac{\widehat{ms}_l}{\widehat{\rho s}} \frac{(\widehat{PNb}_{srow} + 1)}{\widehat{PDs}_{srow} \widehat{PFAR}_{srow} \widehat{PL}_{srow}} \leq \widehat{vsmin} - \widehat{\varepsilon} \right\} \quad (49)$$

$$Svsm_{out_l} = \left\{ srow / \frac{\widehat{ms}_l}{\widehat{\rho s}} \frac{(\widehat{PNb}_{srow} + 1)}{\widehat{PDs}_{srow} \widehat{PFAR}_{srow} \widehat{PL}_{srow}} \geq \widehat{vsmax} + \widehat{\varepsilon} \right\} \quad (50)$$

$$Svtm_{inout_l} = \left\{ srow / \frac{4\widehat{mt}_l}{\pi\widehat{\rho t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow} \widehat{Pdti}_{srow}^2} \leq \widehat{vtmin} - \widehat{\varepsilon} \right\} \quad (51)$$

$$Svtm_{out_l} = \left\{ srow / \frac{4\widehat{mt}_l}{\pi\widehat{\rho t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow} \widehat{Pdti}_{srow}^2} \geq \widehat{vtmax} + \widehat{\varepsilon} \right\} \quad (52)$$

Reynolds number bounds (equivalent to eqs 25 and 26):

$$yrow_{srow,l} = 0 \quad \text{for } srow \in Resminout \text{ and } l \in HY \quad (53)$$

$$yrow_{srow,l} = 0 \quad \text{for } srow \in Retminout \text{ and } l \in HY \quad (54)$$

where the sets  $Resminout$  and  $Retminout$  are given by

$$Resminout = \left\{ srow / \frac{\widehat{ms}_l}{\widehat{\mu s}} \frac{\widehat{PDeq}_{srow} (\widehat{PNb}_{srow} + 1)}{\widehat{PDs}_{srow} \widehat{PFAR}_{srow} \widehat{PL}_{srow}} \leq 2 \times 10^3 - \widehat{\varepsilon} \right\} \quad (55)$$

$$Retminout = \left\{ srow / \frac{4\widehat{mt}_l}{\pi\widehat{\mu t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow} \widehat{Pdti}_{srow}^2} \leq 10^4 - \widehat{\varepsilon} \right\} \quad (56)$$

Baffle spacing bounds (equivalent to eqs 27 and 28):

$$yrow_{srow,l} = 0 \quad \text{for } srow \in (SLNbminout \cup SLNbmaxout) \quad (57)$$

where the sets  $SLNbminout$  and  $SLNbmaxout$  are given by

$$SLNbminout = \left\{ srow / \frac{\widehat{PL}_{srow}}{\widehat{PNb}_{srow} + 1} \leq 0.2\widehat{PDs}_{srow} - \varepsilon \right\} \quad (58)$$

$$SLNbmaxout = \left\{ srow / \frac{\widehat{PL}_{srow}}{\widehat{PNb}_{srow} + 1} \geq 1.0\widehat{PDs}_{srow} + \varepsilon \right\} \quad (59)$$

Tube length/shell diameter ratio bounds (equivalent to eqs 29 and 30):

$$yrow_{srow,l} = 0 \quad \text{for } srow \in (SLDminout \cup SLDmaxout) \quad (60)$$

where the sets  $SLDminout_l$  and  $SLDmaxout_l$  are given by

$$SLDminout = \{srow / \widehat{PL}_{srow} \leq 3\widehat{PDs}_{srow} - \varepsilon\} \quad (61)$$

$$SLDmaxout = \{srow / \widehat{PL}_{srow} \geq 15\widehat{PDs}_{srow} + \varepsilon\} \quad (62)$$

Additionally, it is possible to include a constraint associated with a lower bound on the heat-transfer area, which can accelerate the solution convergence:

$$yrow_{srow,l} = 0 \quad \text{for } srow \in SAminout \quad (63)$$

where the set of heat exchangers with area lower than the minimum possible is

$$SAminout_l = \{srow / \pi \widehat{PNtt}_{srow} \widehat{Pdti}_{srow} \widehat{PL}_{srow} \leq \widehat{Amin}_l - \varepsilon\} \quad (64)$$

The lower bound on the heat-transfer area can be determined by

$$\widehat{Amin}_l = \frac{\widehat{Q}_l}{\widehat{Umax}_l \Delta Tlm_l} \quad l \in HY \quad (65)$$

$$\widehat{Umax}_l = 1 / \left[ \frac{1}{htmax_l} d\widehat{rmin} + \widehat{Rf}_l d\widehat{rmin} + \frac{\min(\widehat{Pdti}_{srow}) \ln(d\widehat{rmin})}{2ktube} + \widehat{Rf}_l s_l + \frac{1}{hsmax_l} \right] \quad (66)$$

$$htmax_l = \max(\widehat{Ph}_{srow,l}) \quad (67)$$

$$hsmax_l = \max(\widehat{Ph}_{srow,l}) \quad (68)$$

$$d\widehat{rmin} = \min(\widehat{Pdti}_{srow} / \widehat{Pdti}_{srow}) \quad (69)$$

## 5. ECONOMIC MODEL

The annualized capital cost of the pipe sections is given by

$$C_{pipe} = Fm\widehat{spip} \sum_{k \in PI} \sum_{n \in SD} \widehat{Cpi}_n \widehat{f}_{k,n} y_{k,n}^{pi} \quad (70)$$

where  $Fm\widehat{spip}$  is a Marshall-swift index correction factor and the unitary cost of a standard pipe  $n$  ( $\widehat{Cpi}_n$ ) is calculated using the proposal of Narang et al.;<sup>27</sup>  $\widehat{C}_1$  and  $\widehat{m}$  are correlation parameters:

$$\widehat{Cpi}_n = (\widehat{C}_1 / 0.3048) (\widehat{D}_n^{nom} / 12)^{\widehat{m}} \quad n \in SD \quad (71)$$

The annualized capital cost of the pump is given by

$$C_{pump} = \sum_{k \in PU} \sum_{s \in SPU} \widehat{Cpu}_s y_{k,s}^{pu} \quad (72)$$

where the parameter that expresses the capital cost of each pump ( $\widehat{Cpu}_s$ ) is calculated by<sup>28</sup>

$$\widehat{Cpu}_s = \widehat{r} Fm\widehat{s} Fmp\widehat{Ft}_s (1.39 \exp(8.833 - 0.6019 \ln(\widehat{z}_s) + 0.0519 (\ln(\widehat{z}_s))^2)) \quad s \in SPU \quad (73)$$

$$\widehat{z}_s = 28710 \widehat{qpu}_s^{design} \sqrt{\widehat{fpu}_s^{design}} \quad s \in SPU \quad (74)$$

where  $\widehat{r}$  is the annualization factor,  $Fm\widehat{s}$  the Marshall-swift index correction factor,  $Fmp$  a cost factor related to the pump

material,  $\widehat{Ft}_s$  a cost factor related to the pump type,  $\widehat{qpu}_s^{design}$  the design flow rate of the pump  $s$ , and  $\widehat{fpu}_s^{design}$  the corresponding head at the design flow rate.

The expression for evaluation of  $\widehat{Ft}_s$  is

$$\widehat{Ft}_s = \exp \left( \sum_j \hat{b}_j (\ln(\hat{z}_s))^{(j-1)} \right) \quad s \in SPU \quad (75)$$

where  $\hat{b}_j$  are correlation parameters.

The pump curve relates  $\widehat{qpu}_s^{design}$  and  $\widehat{fpu}_s^{design}$ :

$$\widehat{fpu}_s^{design} = \sum_i \hat{a}_{i,s} (\widehat{qpu}_s^{design})^i \quad s \in SPU \quad (76)$$

The expression of the annualization factor is

$$\hat{r} = \frac{\hat{i}(1 + \hat{i})^{\hat{n}_y}}{(1 + \hat{i})^{\hat{n}_y} - 1} \quad (77)$$

where  $\hat{i}$  is the interest rate and  $\hat{n}_y$  is the number of years of the project life.

The operational costs associated with the pipe network operation are given by

$$C_{oper} = \sum_{k \in PU} cop_k \quad (78)$$

$$cop_k = \left( \frac{\hat{q}_k \hat{\rho}_w \hat{g} \hat{fpu}_k}{\hat{\eta}} \right) \frac{\hat{N}_{Op} \hat{p}_c}{10^3} \quad (79)$$

where  $\hat{N}_{Op}$  is the number of operating hours per year,  $\hat{p}_c$  the energy price, and  $\hat{\eta}$  the pump and driver efficiency.

The annualized total capital cost of the heat exchangers is given by the sum of the individual equipment costs:

$$C_{heatexc} = \sum_{srow} \sum_{l \in HY} Cheatexc_{srow,l} \quad (80)$$

where the evaluation of the cost of each unit is based on the equations presented by<sup>28</sup>

$$\begin{aligned} Cheatexc_{srow,l} = & 1.218 \hat{Fm} \hat{h}_l \hat{Fph}_l \hat{Fmseq} \{ 8.821 \\ & - 0.30863 [\ln(10.76 \hat{pA}_{srow})] + 0.0681 [\ln(10.76 \hat{pA}_{srow})]^2 \\ & - 1.1156 + 0.0906 (\ln 10.76 \hat{pA}_{srow}) \} \end{aligned} \quad (81)$$

where  $\hat{Fm} \hat{h}_l$  is a cost factor related to the heat exchanger (shell/tube) material,  $\hat{Fph}_l$  a cost factor related to pressure range,  $\hat{Fmseq}$  the Marshall-swift index correction factor, and  $\hat{pA}_{srow,l}$  the area ( $m^2$ ) of the corresponding heat exchanger:

$$\hat{pA}_{srow} = \pi \hat{PN} \hat{t}_{srow} \hat{Pdt}_{srow} \hat{PL}_{srow} \quad (82)$$

## 6. OBJECTIVE FUNCTION

The objective function is the minimization of the total annualized costs, including the capital costs of the pipe sections, pumps, and heat exchangers and the operational costs associated with the pumps:

$$\min C = C_{pipe} + C_{pump} + C_{heatexc} + C_{oper} \quad (83)$$

## 7. RESULTS

The performance of the proposed approach, where the pipe network and the coolers are designed simultaneously, is illustrated through its comparison with conventional procedures, where the design steps of the coolers and the pipe network are conducted separately.

The procedure to design the heat exchangers separately from the pipe network is tested here using two different alternatives:

- (A) The coolers are designed seeking to minimize the heat-transfer surface according to maximum pressure drop constraints, and then the pipe network is optimized considering the total annualized cost of the pipe sections and pumps (this approach will be called here: two-step design A).
- (B) The coolers are first optimized using the total annualized cost associated with the corresponding capital and operational costs, and then the pipe network is designed through the minimization of the total annualized cost of the pipe sections and pumps (this approach will be called here: two-step design B).

For the heat exchanger design, the formulation proposed by Gonçalves et al.<sup>20</sup> was used. Then, the set of values of the cooling water pressure drops obtained in the optimization of each cooler is employed in the optimization of the pipe network. These pressure drops are considered fixed in the hydraulic optimization of the pipe network using the formulation presented above to minimize the corresponding annualized cost of the pipe network.

The entire network is considered in the same elevation, with the exception of the cooling tower top that is 2 m higher. The pipes are made of carbon steel, with STD Schedule, A106 pipe. The pipe commercial diameters are based on the standard ASME/ANSI B.36.10/19. The capital costs of the pipes are calculated using the coefficients  $C_1$  and  $m$  in eq 71 equal to 7.0386 and 1.4393. The Hazen–Williams parameter ( $\hat{c}$ ) is considered equal to 100 for all pipe sections. The parameters of the pump, heat exchangers, and pipes capital cost correlations are shown in Table 2. The set of standard values

**Table 2. Pump, Heat Exchangers, and Pipes Capital Cost Correlation Parameters**

factor	value
$Fph$	1
$Fms$	1.308
$Fmseq$	1.308
$Fmspip$	1.144
$Fmh$	1
$Fmp$	1.35
$b_1$	5.1029
$b_2$	−1.2217
$b_3$	0.0771

of the discrete variables for the design of the heat exchangers is shown in Table 3. The thermal conductivity of the tubes of the heat exchangers is 50 W/(m·K). The flow velocity in the tube-side of the heat exchangers must be between 1 and 3 m/s, and the corresponding bounds in the shell-side are 0.5 and 2 m/s. Physical properties of the cooling water are shown in Table 4. The rest of the problem parameters are shown in Table 5.

Table 3. Standard Values of the Discrete Design Variables of the coolers

variable	values
outer tube diameter $\widehat{pdt}_{sd}$ (m)	0.019, 0.025, 0.032, 0.038, 0.051
tube length, $\widehat{pL}_{sl}$ (m)	1.220, 1.829, 2.439, 3.049, 3.659, 4.877, 6.098
number of baffles, $\widehat{pNb}_{sNb}$	1, 2, ..., 20
number of tube passes, $\widehat{pNpt}_{sNpt}$	1, 2, 4, 6
tube pitch ratio, $\widehat{pTp}_{sp}$	1.25, 1.33, 1.50
shell diameter, $\widehat{pDs}_{ds}$ (m)	0.787, 0.838, 0.889, 0.940, 0.991, 1.067, 1.143, 1.219, 1.372, 1.524
tube layout, $\widehat{play}_{slay}$	1 = square; 2 = triangular

Table 4. Physical Properties of the Cooling Water

	cold stream
density (kg/m <sup>3</sup> )	995
heat capacity (J/(kg·K))	4187
viscosity (mPa·s)	0.72
thermal conductivity (W/m·K)	0.59

Table 5. Problem Parameters

parameter	value
maximum flow velocity in the pipes (m/s)	3.0
minimum flow velocity in the pipes (m/s)	1.0
pump efficiency	0.80
number of operating hours per year (h/y)	8760
energy cost (USD/kWh)	0.1308
interest rate	0.05
project horizon (y)	10

**7.1. Example 1: Cooling Water System with One Heat Exchanger.** Figure 2 presents the cooling water system of example 1.

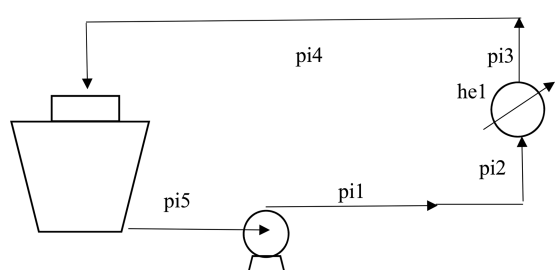


Figure 2. Example 1: water cooling system.

Table 6 presents the set of hydraulic heads associated with the available pumps; Table 7 presents the physical properties of the hot stream that flows in the shell-side of the cooler; Table 8 presents the features of the thermal task of the cooler, and Table 9 presents the lengths of the pipe sections of each case.

Table 6. Example 1: Pump Head Options

	options
pump head (m)	3, 4, 6, 7, 10, 12, 14, 18, 20, 22, 30, 33, 35, 37, 40

Table 7. Example 1: Physical Properties of the Streams

	hot stream
density (kg/m <sup>3</sup> )	1080
heat capacity (J/(kg·K))	3601
viscosity (mPa·s)	1.30
thermal conductivity (W/m·K)	0.58

Table 8. Example 1: Cooling Task Data

	he1 (hot/cold side)
mass flow rate (kg/s)	11.00/37.84
inlet temperature (°C)	90.0/30.0
outlet temperature (°C)	50.0/40.0
fouling factor (m <sup>2</sup> ·K/W)	0.0001/0.0004
allowable pressure drop (kPa)	100/100

Table 9. Example 1: Pipe lengths

pipe	length (m)
pi1	198
pi2	15
pi3	15
pi4	200
pi5	2

The results of the optimization in each approach are displayed in Tables 10–14. Table 10 contains the design

Table 10. Example 1: Heat Exchanger Design Results (he1)

	simultaneous design	two-step design A	two-step design B
area (m <sup>2</sup> )	62.7	57.4	62.7
tube diameter (m)	0.019	0.025	0.019
tube length (m)	3.049	3.049	3.049
number of baffles	19	20	19
number of tube passes	2	6	2
tube pitch ratio	1.25	1.25	1.25
shell diameter (m)	0.489	0.54	0.489
tube layout	2	2	2
total number of tubes	344	236	344
baffle spacing (m)	0.152	0.145	0.152

Table 11. Example 1: Thermo-fluid Dynamic Results (he1)

	simultaneous design	two-step design A	two-step design B
shell-side flow velocity (m/s)	0.683	0.650	0.683
tube-side flow velocity (m/s)	1.135	2.522	1.135
shell-side coefficient (W/m <sup>2</sup> ·K)	4212.6	3600.6	4212.6
tube-side coefficient (W/m <sup>2</sup> ·K)	5407.5	9567.7	5407.5
overall coefficient (W/m <sup>2</sup> ·K)	925.0	1007.1	925.0
shell-side pressure drop (Pa)	57458	43217	57458
tube-side pressure drop (Pa)	9275	91550	9275



Table 12. Example 1: Diameters and Head Losses of the Pipes

pipe	simultaneous design		two-step design A		two-step design B	
	diameter (in)	head loss (m)	diameter(in)	head loss (m)	diameter(in)	head loss (m)
pi1	8	2.329	8	2.329	8	2.329
pi2	5	1.643	6	0.670	5	1.643
pi3	6	0.670	6	0.670	6	0.670
pi4	8	2.352	8	2.352	8	2.352
pi5	8	0.024	8	0.024	8	0.024

variables of heat exchanger he1; Table 11 displays the thermofluid dynamic behavior of heat exchanger he1; Table 12 displays the diameters and head losses in the pipe sections; Table 13 shows the pump head and head loss in the valve, and Table 14 presents the objective function and its corresponding components.

Table 13. Example 1: Pump Head and Valve Head Loss

design	pump head (m)	valve head loss (m)
simultaneous design	10	0.032
two-step design A	18	0.575
two-step design B	10	0.032

Table 14. Example 1: Annualized Costs

cost (\$/year)	simultaneous design	two-step design A	two-step design B
pump cost	662.10	726.58	662.10
heat exchanger cost	6080.93	5731.90	6080.93
pipe cost	6154.28	6188.01	6154.28
operation cost	5313.13	9563.64	5313.13
total cost	18210.44	22210.13	18210.44

Table 14 indicates that the two-step design presented a higher value of the objective function in relation to the simultaneous design and the two-step design B, which obtained the same results.

According to Table 11, the design solution of the heat exchanger he1 in the two-step design A is associated with a pressure drop near the maximum available value ( $91.55 \text{ kPa} \times 100 \text{ kPa}$ ), i.e., the optimization sought to minimize the heat-transfer area through the exploration of the available pressure drop. In fact, Table 10 indicates that the two-step design A presented a heat exchanger area that is 8.4% lower than the other approaches. However, the increase of the pressure drop in heat exchanger he1 penalized the total pressure drop of the system. Consequently, the operational cost of two-step design A is 22% higher than that of the other approaches.

The simultaneous design and the two-step design B yielded equivalent solutions; that is, there was no difference to explore the trade-off between capital and operational costs simultaneously or in two steps. The two separated trade-offs involving capital and operational costs for the heat exchanger and the pipe network superposed without affecting the optimal point; therefore, there was no difference between the results of the simultaneous design and the two-step design B.

**7.2. Example 2: Cooling Water System with Four Heat Exchangers.** Figure 1 presents the network architecture. Table 15 presents the set of hydraulic heads associated with the available pump alternatives; Table 16 presents the physical properties of the hot streams that flow in the shell-side with the exception of heat exchanger he2 where the hot stream flows in

Table 15. Example 2: Pump Head Options

options	
pump head (m)	6, 7, 10, 12, 14, 18, 20, 22, 25, 28, 30, 33, 35, 37, 40

Table 16. Example 2: Physical Properties of the Streams

	hot stream			
	he1	he2	he3	he4
density ( $\text{kg}/\text{m}^3$ )	1080	750	786	1080
heat capacity ( $\text{J}/\text{kg}\cdot\text{K}$ )	3601	2840	2177	3601
viscosity ( $\text{mPa}\cdot\text{s}$ )	1.3	0.34	1.89	1.30
thermal conductivity ( $\text{W}/\text{m}\cdot\text{K}$ )	0.58	0.19	0.12	0.58

the tube-side; Table 17 presents the features of the thermal task of each cooler, where in this example, no maximum pressure drops constraints were imposed for any stream in the simultaneous and two-step design B approaches, and Table 18 presents the lengths of the pipe sections.

The results of the optimization in each approach are displayed in Tables 19–25. Tables 19–22 contain the design variables of the heat exchangers; Table 23 displays the thermofluid dynamic behavior of heat exchanger he2 (the corresponding results of the other heat exchangers are omitted to save space); Table 24 displays the optimal pump heads and valve head losses, and Table 25 presents the objective function and its corresponding components. The complete set of results of example 2 is available in the Supporting Information.

Table 25 indicates a behavior of the two-step design A approach similar to the previous example. The heat exchanger cost is minimized according to the available pressure drop at the expense of the operational costs, which results in a higher total annualized cost.

Unlike in the case of one exchanger, the two-step design B approach yielded a solution associated with a higher total annualized cost when compared with the simultaneous solution. The optimization using the simultaneous approach and the two-step design problem B yielded heat exchangers with the same area, with exception of heat exchanger he2, as is displayed in Tables 19–22. In relation of this heat exchanger, the simultaneous design selected an option with a larger area (Table 20,  $246.6 \text{ m}^2 \times 211.5 \text{ m}^2$ ), but lower shell-side pressure drop, where the cooling water flows (Table 23,  $16.1 \text{ kPa} \times 36.6 \text{ kPa}$ ). Despite the penalty of a more expensive heat exchanger, the analysis of the selected pumps for each case in Table 24 indicates that this selection allowed the identification of an optimal solution associated with a pump with lower power, which yielded a reduction of the operational costs, as can be observed in Table 25. The optimization of the total annualized cost of the heat exchangers separately in the two-step design problem B penalized the operational costs, which implies a higher value of the total annualized cost of the entire system.

Table 17. Example 2: Cooling Tasks Data

	he1 hot/cold	he2 hot/cold	he3 hot/cold	he4 hot/cold
mass flow rate (kg/s)	21.94/75.48	27.8/56.57	30/77.99	14/36.12
inlet temperature (°C)	90/30	70/30	100/30	80/30
outlet temperature (°C)	50/40	40/40	50/40	50/40
fouling factor (m <sup>2</sup> ·K/W)	0.0001/0.0004	0.0002/0.0004	0.0002/0.0004	0.0001/0.0004
allowable pressure drop (kPa)	100/100	70/50	60/100	100/50

Table 18. Example 2: Pipe lengths

pipe	length (m)	pipe	length (m)
pi1	98	pi10	8
pi2	12	pi11	10
pi3	20	pi12	10
pi4	15	pi13	18
pi5	15	pi14	25
pi6	12	pi15	20
pi7	25	pi16	100
pi8	18	pi17	2
pi9	8		

Table 19. Example 2: Heat Exchanger Design Results (he1)

	simultaneous design	two-step design A	two-step design B
area (m <sup>2</sup> )	123.5	113.9	123.5
tube diameter (m)	0.019	0.019	0.019
tube length (m)	3.049	2.439	3.049
number of baffles	15	14	18
number of tube passes	2	4	2
tube pitch ratio	1.25	1.25	1.25
shell diameter (m)	0.686	0.737	0.686
tube layout	2	2	2
total number of tubes	677	781	677
baffle spacing (m)	0.191	0.163	0.160

Table 20. Example 2: Heat Exchanger Design Results (he2)

	simultaneous design	two-step design A	two-step design B
area (m <sup>2</sup> )	246.6	227.8	211.5
tube diameter (m)	0.019	0.019	0.019
tube length (m)	6.098	4.877	6.098
number of baffles	8	8	9
number of tube passes	6	6	6
tube pitch ratio	1.25	1.25	1.25
shell diameter (m)	0.737	0.737	0.635
tube layout	1	2	2
total number of tubes	676	781	580
baffle spacing (m)	0.678	0.542	0.610

## 8. CONCLUSIONS

This paper presented a new approach for the design of cooling water systems encompassing the dimensioning of coolers, pipes, and pumps. This approach allows the full exploration of the trade-off between capital and operational costs, including all system elements simultaneously.

Two examples were explored to analyze the comparison of the proposed approach in relation to alternative schemes based

Table 21. Example 2: Heat Exchanger Design Results (he3)

	simultaneous design	two-step design A	two-step design B
area (m <sup>2</sup> )	197.5	207.3	197.5
tube diameter (m)	0.019	0.019	0.019
tube length (m)	4.877	3.049	4.877
number of baffles	17	9	17
number of tube passes	2	4	2
tube pitch ratio	1.25	1.25	1.25
shell diameter (m)	0.686	0.889	0.686
tube layout	2	2	2
total number of tubes	677	1137	677
baffle spacing (m)	0.271	0.305	0.271

Table 22. Example 2: Heat Exchanger Design Results (he4)

	simultaneous design	two-step design A	two-step design B
area (m <sup>2</sup> )	75.3	61.1	75.3
tube diameter (m)	0.019	0.019	0.019
tube length (m)	3.659	2.439	3.659
number of baffles	15	17	17
number of tube passes	2	4	2
tube pitch ratio	1.25	1.25	1.25
shell diameter (m)	0.489	0.540	0.489
tube layout	2	2	2
total number of tubes	344	419	677
baffle spacing (m)	0.229	0.136	0.261

Table 23. Example 2: Thermofluid Dynamic Results (he2)

	simultaneous design	two-step design A	two-step design B
shell-side flow velocity (m/s)	0.570	0.712	0.734
tube-side flow velocity (m/s)	1.689	1.462	1.969
shell-side coefficient (W/m <sup>2</sup> K)	3819.9	4980.4	5064.3
tube-side coefficient (W/m <sup>2</sup> K)	3471.5	3092.8	3924.0
overall coefficient (W/m <sup>2</sup> K)	776.2	787.4	844.7
shell-side pressure drop (Pa)	16106	35163	35587
tube-side pressure drop (Pa)	71172	45165	94475

on the separation of the design of the coolers and the pipe network (two-step designs A and B).

In the first example, the proposed approach yielded a better result than the two-step approach based on the design of the coolers through the area minimization constrained by pressure drop bounds (two-step design A). However, the proposed

Table 24. Example 2: Pump Head and Valve Head Losses

design	pump head (m)	head loss-he1 (m)	head loss-he2 (m)	head loss-he3 (m)	head loss-he4 (m)
simultaneous design	6	0.450	0.017	0.024	0.027
two-step design A	10	0.599	0.588	0.726	0.008
two-step design B	10	2.522	0.102	0.259	0.033

Table 25. Example 2: Annualized Costs

cost (\$/year)	simultaneous design	two-step design A	two-step design B
pump cost	1455.21	1752.81	1752.81
heat exchanger cost	48350.33	46232.95	46253.56
pipe cost	15606.02	13591.52	12354.19
operational cost	20739.07	34565.12	34565.12
total cost	86150.64	96142.40	94925.69

approach brought no additional gain in relation to the two-step design based on the minimization of the total annualized cost of the coolers followed by the minimization of the total annualized cost of the pipe network (two-step design B).

However, in the second example, the proposed approach yielded better results than both two-step design approaches, illustrating potential gains that could be achieved when the design problem is addressed considering all elements simultaneously. It seems that in the simultaneous approach, a heat exchanger could be chosen by the optimization to be bigger so that the corresponding pressure drop is lower, thus reducing the cost of pumping. The sequential approach cannot handle this trade-off.

Because of the complexity of the chemical process design problem, the traditional approach involves its solution in "layers", where the solution of one layer is employed for the resolution of the next. If this conventional approach allows the analysis of complex systems through a set of simpler problems, it may yield suboptimal solutions. In fact, this paper presents one kind of system where a holistic design approach may achieve better results, therefore reinforcing the need to use optimization tools for the solution of chemical process design problems.

Finally, the design of the cooling water tower will certainly include a new trade-off. For it to be considered, one also needs to take into account modifications in the return and supply temperatures, the inclusion of the capital cost of the cooling tower, blow-down issues, and makeup water associated with its operational characteristics. This is the objective of future work.

## ■ ASSOCIATED CONTENT

### ● Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.iecr.8b06478.

Complete set of results of example 2 (PDF)

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## Notes

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## ■ NOMENCLATURE (PARAMETERS)

$\hat{A}_{exc}$  = excess area (%)

$\hat{c}$  = Hazen–Williams parameter

$\widehat{CPI}_n$  = annualized cost of a pipe section with unitary length (USD/y·m)

$\widehat{Cpu}_s$  = annualized capital cost of the pump alternative  $s$  (USD/y)

$\hat{C}$  = correlation parameter in eq 72,

$\hat{D}_n^{nom}$  = nominal diameter of the commercial pipe  $n$  (in)

$\hat{D}_n^{int}$  = inner diameter of the commercial pipe  $n$  (m)

$\hat{F}_{srow,l}$  = correction factor of the LMTD for a configuration 1 –  $N$ ,  $N$  even

$\widehat{Fmp}$  = cost factor related to the pump material

$\widehat{Fmh}_l$  = cost factor related to the heat exchanger (shell/tube) material

$\widehat{Fms}$  = Marshall–Swift index correction factor, for the pump cost

$\widehat{Fmse}_q$  = Marshall–Swift index correction factor, for the heat exchangers cost

$\widehat{Fms}_{pip}$  = Marshall–Swift index correction factor, for the tubes cost

$\widehat{Fph}_l$  = cost factor related to pressure range of the heat exchanger

$\widehat{Ft}_s$  = cost factor related to the pump type

$\hat{g}$  = gravity acceleration (m/s<sup>2</sup>)

$\hat{i}$  = interest rate

$\hat{ks}_l$  = thermal conductivity of the shell-side stream in the cooler  $l$  (W/(m K))

$\hat{kt}_l$  = thermal conductivity of the tube-side stream in the cooler  $l$  (W/(m K))

$\widehat{ktube}$  = tube wall thermal conductivity (W/(m K))

$\widehat{L}_k$  = length of the pipe section  $k$  (m)

$\hat{m}$  = correlation parameter in eq 72

$\widehat{ms}_l$  = shell-side mass flow rate in the cooler  $l$  (kg/s)

$\widehat{mt}_l$  = tube-side mass flow rate in the cooler  $l$  (kg/s)

$\widehat{Nop}$  = number of operating hours per year

$\widehat{ny}$  = number of years of the project life

$\hat{P}_l$  = LMTD correction factor parameter

$\widehat{pc}$  = power cost (USD/kWh)

$\widehat{PDeq}_{srow}$  = equivalent diameter of the cooler alternative  $srow$  (m)

$\widehat{PDs}_{srow}$  = shell diameter of the cooler alternative  $srow$  (m)

$\widehat{Pdt}_{srow}$  = outer tube diameter of the cooler alternative  $srow$  (m)

$\widehat{Pdt}_{irow}$  = inner tube diameter of the cooler alternative  $srow$  (m)

$\widehat{pf}_{kn}$  = unitary head loss for a pipe diameter  $n$  in a pipe section  $k$  (m/m)

$\widehat{PL}_{srow}$  = tube length of the cooler alternative  $srow$  (m)

$\widehat{Play}_{srow}$  = tube layout of the cooler alternative  $srow$

$\widehat{PNb}_{srow}$  = number of baffles of the cooler alternative  $srow$

$\widehat{PNpt}_{srow}$  = number of tube passes of the cooler alternative  $srow$

$\widehat{PNtt}_{srow}$  = total number of tubes of the cooler alternative  $srow$

$\widehat{Prp}_{srow}$  = tube pitch ratio of the cooler alternative  $srow$

$\widehat{q}_k$  = volumetric flow rate in the edge  $k$  (m<sup>3</sup>/s)

$\widehat{qhe}_l$  = volumetric flow rate at the cooler related to the hydraulic circuit  $l$  (m<sup>3</sup>/s)

$\widehat{qpu}_s^{design}$  = volumetric flow rate at the design condition of the pump  $s$  (m<sup>3</sup>/s)

$\widehat{Q}_l$  = heat duty of the cooler  $l$  (W)

$\hat{r}$  = annualization factor

$\widehat{R}_l$  = LMTD correction factor parameter

$\widehat{Rfs}$  = shell-side fouling factor (m<sup>2</sup> K/W)

$\widehat{Rft}$  = tube-side fouling factor (m<sup>2</sup> K/W)

$\widehat{vmax}$  = maximum flow velocity in the pipe sections

$\widehat{vmin}$  = minimum flow velocity in the pipe sections

$\widehat{vsmax}$  = maximum flow velocity in the heat exchanger shell-side

$\widehat{vsmin}$  = minimum flow velocity in the heat exchanger shell-side

$\widehat{vtmax}$  = maximum flow velocity in the heat exchanger tube-side

$\widehat{vtmin}$  = minimum flow velocity in the heat exchanger tube-side

$\widehat{yT}_{c,l}$  = binary parameter, if the cooling water is in the tube-side ( $\widehat{yT}_{c,l} = 1$ )

$\widehat{yT}_{h,l}$  = binary parameter, if the hot process stream is in the tube-side ( $\widehat{yT}_{h,l} = 1$ )

$\widehat{z}_s$  = parameter for the calculation of pump capital cost in eq 74

$\widehat{\Delta Psdisp}_l$  = shell-side available pressure drop in the cooler  $l$  (Pa)

$\widehat{\Delta Ptdisp}_l$  = tube-side available pressure drop in the cooler  $l$  (Pa)

$\widehat{\Delta Tlm}_l$  = log-mean temperature difference of the cooler  $l$  (°C)

$\widehat{\Delta z}$  = elevation difference between top and bottom of the cooling tower (m)

$\widehat{\eta}$  = pump efficiency

$\widehat{\Lambda}_{lk}$  = circuit matrix

$\widehat{\mu}_{s_l}$  = viscosity of the shell-side stream in the cooler  $l$  (Pa·s)

$\widehat{\mu}_{t_l}$  = viscosity of the tube-side stream in the cooler  $l$  (Pa·s)

$\widehat{\rho}_{s_l}$  = density of the shell-side stream in the cooler  $l$  (kg/m<sup>3</sup>)

$\widehat{\rho}_{t_l}$  = density (kg/m<sup>3</sup>)

$\widehat{\rho}_w$  = water density (kg/m<sup>3</sup>)

## BINARY VARIABLES

$y_{k,n}^{pi}$  = binary variable, if the commercial diameter  $n$  is in the edge  $k$ , so  $y_{k,n}^{pi} = 1$

$y_{k,s}^{pu}$  = binary variable, if the pump  $s$  is in the edge  $k$ , so  $y_{k,s}^{pu} = 1$

$yrow_{srow,l}$  = variable representing the set of heat exchanger design variables

## CONTINUOUS AND DISCRETE VARIABLES

$A$  = heat-transfer area (m<sup>2</sup>)

$C$  = objective function (USD/y)

$cop_k$  = operating cost of pump  $k$  (USD/y)

$C_{heatexch}$  = annualized capital cost of the heat exchangers (USD/y)

$C_{oper}$  = pipe network operating cost (USD/y)

$C_{pipe}$  = annualized capital cost of the pipe sections (USD/y)

$C_{pump}$  = annualized capital cost of the pump (USD/y)

$D_k$  = inner diameter of the pipe section  $k$  (m)

$dte$  = outer tube diameter (m)

$dti$  = outer tube diameter (m)

$Ds$  = shell diameter (m)

$f_k$  = unitary head loss along pipe sections  $k$  (dimensionless)

$fhe_l$  = head loss in the cooler  $l$  (m)

$fpi_k$  = head loss in the pipe section  $k$  (m)

$fpu_k$  = head of the pump  $k$  (m)

$fv_l$  = head loss in the valve  $l$  (m)

$hs$  = shell-side convective heat-transfer coefficient (W/m<sup>2</sup> K)

$ht$  = tube-side convective heat-transfer coefficient (W/m<sup>2</sup> K)

$L$  = heat exchanger tube length (m)

$L_k$  = tube length in the pipe section  $k$  (m)

$lay$  = tube layout (1 = square, 2 = triangular)

$lbc$  = baffle spacing (m)

$Nb$  = number of baffles

$Npt$  = number of passes in the tube-side

$Ntt$  = total number of tubes

$Res$  = shell-side Reynolds number

$Ret$  = tube-side Reynolds number

$rp$  = tube pitch ratio

$U$  = overall heat-transfer coefficient (W/m<sup>2</sup> K)

$v_k$  = flow velocity in pipe section  $k$  (m/s)

$vs$  = shell-side flow velocity (m/s)

$vt$  = tube-side flow velocity (m/s)

$\Delta Pt$  = tube-side pressure drop (Pa)

$\Delta Ps$  = shell-side pressure drop (Pa)

## SETS

$HE$  = subset of heat exchangers

$HY$  = set of hydraulic circuits

$INT$  = node subset of interconnections

$PD$  = water return node subset

$PI$  = subset of pipe sections

$PS$  = water supply node subset

$PU$  = subset of the pumps

$SD$  = set of commercial diameters

$SPU$  = set of available pumps

$STR$  = set of edges

$VET$  = set of nodes



## SUBSCRIPTS

$k$  = index of the edges  
 $l$  = index of the hydraulic circuits  
 $n$  = index of commercial diameters  
 $s$  = index of the available pump alternatives  
 $t$  = index of the nodes

## SUPERSCRIPTS

*design* = design condition  
*int* = internal  
*nom* = nominal  
*pi* = pipe  
*pu* = pump

## REFERENCES

- (1) Kim, J.; Smith, R. Cooling water system design. *Chem. Eng. Sci.* **2001**, *56*, 3641.
- (2) Kim, J.; Smith, R. Automated retrofit design of cooling-water systems. *AIChE J.* **2003**, *49*, 1712.
- (3) Kim, J.; Lee, G. C.; Zhu, F. X. X.; Smith, R. Cooling system design. *Heat Transfer Eng.* **2002**, *23*, 49.
- (4) Feng, X.; Shen, R. J.; Wang, B. Recirculating cooling-water network with an intermediate cooling-water main. *Energy Fuels* **2005**, *19*, 1723.
- (5) Picón-Núñez, M.; Morales-Fuentes, A.; Vázquez-Ramírez, E. E. Effect of network arrangement on the heat transfer area of cooling networks. *Appl. Therm. Eng.* **2007**, *27*, 2650.
- (6) Ponce-Ortega, J. M.; Serna-González, M.; Jiménez-Gutiérrez, A. MINLP synthesis of optimal cooling networks. *Chem. Eng. Sci.* **2007**, *62*, 5728.
- (7) Yee, T. F.; Grossmann, I. E. Simultaneous optimization models for heat integration – II. Heat exchanger network synthesis. *Comput. Chem. Eng.* **1990**, *14*, 1165.
- (8) Picón-Núñez, M.; Polley, G. T.; Canizalez-Dávalos, L.; Medina-Flores, J. M. Short cut performance method for the design of flexible cooling systems. *Energy* **2011**, *36*, 4646.
- (9) Picón-Núñez, M.; Polley, G. T.; Canizalez-Dávalos, L.; Tamakloe, E. K. Design of coolers for use in an existing cooling water network. *Appl. Therm. Eng.* **2012**, *43*, 51.
- (10) Sun, J.; Feng, X.; Wang, Y.; Deng, C.; Chu, K. H. Pump network optimization for a cooling water system. *Energy* **2014**, *67*, 506.
- (11) De Souza, J. N. M.; Ventin, F. F.; Tavares, V. B. G.; Costa, A. L. H. A matrix approach for optimization of pipenetworks in cooling water systems. *Chem. Eng. Commun.* **2014**, *201*, 1054.
- (12) Souza, J. N. M.; Levy, A. L. L.; Costa, A. L. H. Optimization of cooling water system hydraulic debottlenecking. *Appl. Therm. Eng.* **2018**, *128*, 1531.
- (13) Ponce-Ortega, J. M.; Serna-González, M.; Jiménez-Gutiérrez, A. A Disjunctive Programming Model for Simultaneous Synthesis and Detailed Design of Cooling Networks. *Ind. Eng. Chem. Res.* **2009**, *48*, 2991.
- (14) Ponce-Ortega, J. M.; Serna-González, M.; Jiménez-Gutiérrez, A. Optimization model for re-circulating cooling water systems. *Comput. Chem. Eng.* **2010**, *34*, 177.
- (15) Sun, J.; Feng, X.; Wang, Y. Cooling-water system optimization with a novel two-step sequential method. *Appl. Therm. Eng.* **2015**, *89*, 1006.
- (16) Ma, J.; Wang, Y.; Feng, X. Simultaneous optimization of pump and cooler networks in a cooling water system. *Appl. Therm. Eng.* **2017**, *125*, 377.
- (17) Liu, F.; Ma, J.; Feng, X.; Wang, Y. Simultaneous integrated design for heat exchanger network and cooling water system. *Appl. Therm. Eng.* **2018**, *128*, 1510.
- (18) Polley, G. T.; Picón-Núñez, M.; López-Maciél, J. J. Design of water and heat recovery networks for the simultaneous minimization of water and energy consumption. *Appl. Therm. Eng.* **2010**, *30*, 2290.
- (19) Mah, R. S. H. *Chemical Process Structures and Information Flows*; Butterworth Publishers: Markham, 1990.
- (20) Goncalves, C. O.; Costa, A. L. H.; Bagajewicz, M. J. Alternative mixed-integer linear programming formulations for shell and tube heat exchanger optimal design. *Ind. Eng. Chem. Res.* **2017**, *56*, 5970.
- (21) Kakaç, S.; Liu, H. *Heat Exchangers Selection, Rating, and Thermal Design*; CRC Press: Boca Raton, FL, 2002.
- (22) Incropera, F. P.; De Witt, D. P. *Fundamentals of Heat and Mass Transfer*; John Wiley & Sons: New York, 2002.
- (23) Kern, D. Q. *Process Heat Transfer*; McGraw-Hill: New York, 1950.
- (24) Saunders, E. A. D. *Heat Exchangers: Selection, Design, and Construction*; John Wiley & Sons: New York, 1988.
- (25) Taborek, J. Input Data and Recommended Practices. In *Heat Exchanger Design Handbook*; Hewitt, G. F., Ed.; Begell House: New York, 2008.
- (26) Souza, P. A.; Costa, A. L. H.; Bagajewicz, M. J. Globally optimal linear approach for the design of process equipment: The case of air coolers. *AIChE J.* **2018**, *64*, 886.
- (27) Narang, R.; Sarin, S.; George, A. A. De-emphasize capital costs for pipe size selection. *Chem. Eng.* **2009**, *116*, 41.
- (28) Couper, J. R.; Penney, W. R.; Fair, J. R.; Walas, S. M. *Chemical Process Equipment: Selection and Design*; Gulf Professional Publishing: MA, 2005.