

# AN MILP Model for Heat Exchanger Networks Retrofit

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This paper addresses the problem of automatically determining the optimal economic retrofit of heat exchanger networks. It is a rigorous MILP approach that considers rearrangement of the existing heat exchanger units, heat transfer area addition and new exchanger installation. We illustrate the method using a crude fractionation unit and we also compare this technique to one existing retrofit approach.

## KEYWORDS

Retrofit, Heat Exchanger Networks, Mixed Integer Linear Programming

## 1. INTRODUCTION

This paper is based on a recently accepted paper for grassroots HEN design. The method consists of a rigorous MILP strategy that is based on a transportation/transshipment strategy. This strategy can handle stream splits and non-isothermal mixing rigorously, without any approximations.

For the case of retrofit, we have added constraints that are able to handle the fact that there is an existing network. The proposed retrofit method as well as the previous HENS design procedure is able to solve complex systems. This is illustrated through 2 application examples.

## 2. OUTLINE APPROACH FOR RETROFIT

The MILP model is based on the transportation-transshipment paradigm which has the following features:

- Counts heat exchangers units and shells
- Determines the area required for each exchanger unit or shell
- Controls the total number of units
- Determines the flow rates in splits
- Handles non-isothermal mixing
- Identifies bypasses in split situations when convenient
- Controls the temperature approximation ( $\Delta T_{\min}$ ) when desired

- Can address areas or temperature zones
- Allows multiple matches between two streams

The model considers a consecutive series of heat exchangers. Heat transfer is accounted using the cumulative heat transferred from intervals up to a specific interval to other counterpart intervals. The key of the model and what differentiates it from other transport/transshipment models is the flow rate consistency equations that allow tracking flows in splits. For retrofit situations, the MILP model is extended by adding constraints as follows:

$$A_{ij}^z \leq A_{ij}^{z^0} + \Delta A_{ij}^{z^0} + A_{ij}^{z^N} \quad (1)$$

$$\Delta A_{ij}^{z^0} \leq \Delta A_{ij \max}^{z^0} \quad (2)$$

$$U_{ij}^z \leq U_{ij \max}^z \quad (3)$$

$$A_{ij}^{z^N} \leq A_{ij \max}^{z^N} \cdot (U_{ij}^z - U_{ij}^{z^0}) \quad (4)$$

where  $A_{ij}^z$  is the new area of an exchanger between streams  $i$  and  $j$ ,  $A_{ij}^{z^0}$  its original area,  $\Delta A_{ij}^{z^0}$  the additional area to the existing shell,  $\Delta A_{ij}^{z^N}$  the new area in new shells, and  $U_{ij}^z$  and  $U_{ij}^{z^0}$  the new and original number of shells, respectively. Maximum values ( $\Delta A_{ij \max}^{z^0}$  and  $U_{ij \max}^z$ ) are also used. When original values are zero, then a new match is added.

The objective function for the retrofit heat exchanger network structure is the total annualized cost which consisted of utility cost, additional area cost and fixed cost for new exchanger installation. All terms of the hot and cold utility cost are the same as in the grassroots design model, but the retrofit programming model has complicated functions for the area cost. In the following, the objective function for the proposed retrofit approach is expressed.

$$\begin{aligned} \text{Min Cost} = & \sum_z \sum_{i \in HU^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_i^H F_i^H \Delta T_i + \sum_z \sum_{j \in CU^z} \sum_{\substack{i \in H^z \\ (i,j) \in P}} c_j^C F_j^C \Delta T_j + \sum_z \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_{ij}^F (U_{ij}^z - U_{ij}^{z^0}) \\ & + \sum_z \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \notin B}} (c_{ij}^{A^0} \Delta A_{ij}^{z^0} + c_{ij}^{A^N} A_{ij}^{z^N}) + \sum_z \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \in B}} \sum_{k=1}^{k_{\max}} (c_{ij}^{A^0} \Delta A_{ij}^{z,k^0} + c_{ij}^{A^N} A_{ij}^{z,k^N}) \end{aligned} \quad (13)$$

Especially constraints are added when more than one exchanger between two streams is allowed. We omit these and concentrate more in showing results.

### 3. EXAMPLES

In this section, two examples are solved to illustrate the rigorous MILP method. The optimization model was constructed in GAMS and run in a PC with a 2.4 GHz processor and 1 Gb of ram memory.

#### Example 1

Example 1 is the retrofit problem of crude distillation unit that composed of 18 streams and 18 existing exchangers. Streams properties are shown in Table 1 and Table 2 while the results of retrofit network are given in Table 3 and 4. Cost comparisons are given in Table 5. The retrofit solution achieves 23.21% annual cost savings with two new exchanger units and three shells addition. The original and retrofit networks are shown in Figure 1 and Figure 2.

We report a solution that consumes 9577.781 sec. to reach 0.00% Gap. The original and retrofit networks are shown in Figure 1 and Figure 2. If solution time is an issue one can use several other solutions with smaller gap. One in particular has 22.4% annual cost savings with also two new exchanger units and three new shells that is obtained in 977 seconds.

Table 1 Stream properties for Example1

Stream	F Ton/hr	Cp KJ/kg-C	Tin C	Tout C	h MJ/h-m2-C
I1	155.1	3.161	319.4	244.1	4.653
I2	5.695	4.325	73.24	30	18.211
I4	151.2	2.93	263.5	180.2	4.894
I7	91.81	2.262	73.24	40	4.605
I3	251.2	3.111	347.3	202.7	3.21
		2.573	202.7	45	2.278
I5	26.03	3.041	45	203.2	4.674
		2.689	203.2	110	3.952
I6	86.14	2.831	110	147.3	4.835
		2.442	147.3	50	3.8
I8	63.99	2.854	50	176	5.023
		2.606	176	120	4.846
I9	239.1	2.595	167.1	116.1	4.995
		2.372	116.1	69.55	4.88
I10	133.8	6.074	146.7	126.7	1.807
		4.745	126.7	99.94	3.373
		9.464	99.94	73.24	6.878
J1	519	2.314	30	108.1	1.858
		2.645	108.1	211.3	2.356
		3.34	211.3	232.2	2.212
J2	496.4	3.54	232.2	343.3	2.835
J3	96.87	13.076	226.2	228.7	11.971
		15.808	228.7	231.8	11.075
I11			250	249	21.6
I12			1000	500	0.4
J4			20	25	13.5
J5			124	125	21.6
J6			174	175	21.6

Table 2 Cost data for Example1

Utilities	Cost \$/(MJ/hr-yr)
I11	19.75
I12	37.222
J4	1.861
J5	-6.494
J6	-12.747
Heat Exchanger Cost	5291.9+77.788A \$/yr

Table 3 Model statistics for Example1

Model Statistics	
Single Variables	3024
Discrete Variables	459
Single Equations	5930
Non Zero Elements	29046
Time to reach a feasible solution	9577.781 sec
Optimality Gap	0.00%

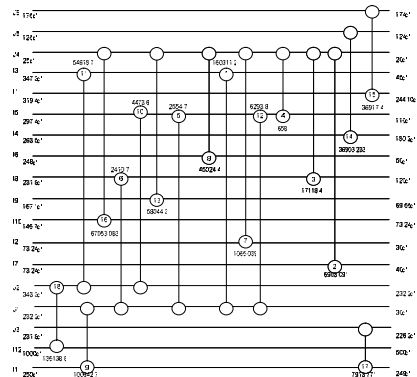


Figure 1 Original HEN for Example 1

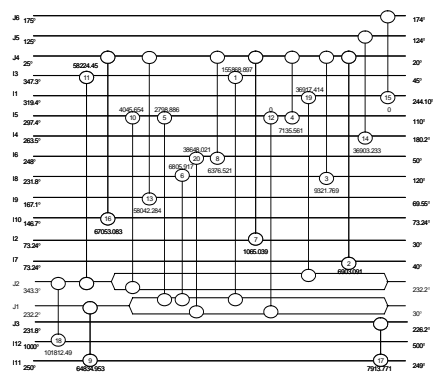


Figure 2 Retrofit HEN for Example 1

Table 4 Resulting of retrofit heat exchanger for Example 1

HE	Original Load MJ/hr	Retrofit Load MJ/hr	Original Area m <sup>2</sup>	Retrofit Area m <sup>2</sup>	Area addition m <sup>2</sup>	Shell Addition	Cost \$
1	160311.2	155868.897	3211.829	3753.624	541.795		42145.16826
2	6903.091	6903.091	59.329	63.797	4.468		347.5274998
3	17118.4	9321.769	33.164	21.017			
4	658	7135.561	2.297	17.67	15.373	YES	6487.764154
5	2554.7	2798.886	40.019	48.022	8.003		622.5638119
6	2410.7	10208.875	22.047	418.476	396.429	YES	36129.30553
7	1065.039	1065.039	5.483	5.869	0.386		29.9928395
8	45024.4	6376.521	142.591	43.955			
9	100642.7	64163.082	1067.823	966.626			
10	4473.6	4045.654	109.632	109.632	0.000		
11	54618.7	59060.593	653.792	1239.901	586.109	YES	50884.1824
12	6293.8	0	40.019	0			
13	58044.3	58042.284	184.221	182.391			
14	36903.2	36903.233	101.363	101.472	0.109		8.501893586
15	36917.4	0	93.793	0			
16	67053.083	67053.083	258.814	288.967	30.153		2345.510728
17	7913.771	7913.771	53.044	52.239			
18	136138.8	95207.481	947.807	708.996			
19				719.75	NEW		61279.813
20				629.645	NEW		54270.72526
			7027.066	9372.049	33.37%	3	254551.0554

Table 5 Annual cost comparison between original and retrofit network

Cost \$/yr	Existing	Retrofit
Total utility cost	6,865,616.51	5,017,806.074
Total fixed and area cost	-	254,551.055
Total cost	6,865,616.51	5,272,357.129
Cost saving	1,593,259.380	(23.21%)

## Example 2

We now compare our method with Hypertargets (Briones and Kokossis, 1999). Table 6 shows the stream and cost data for crude distillation unit which consisted of 12 streams and 11 existing units. Figure 3 shows the original network and Figure 4 shows the

retrofit structure generated by our MILP strategy. Hypertargets established two retrofit designs (B1 and B2) with the same utility cost and one new unit in each case. They are shown in Figure 5 and 6. Our MILP approach suggests using two new smaller exchangers and more utility. The results are shown in Tables 7 and 8 and the total annual cost in Table 9. The retrofit has a 4.17% saving over the original structure.

Table 6 Stream and cost data for Example 2

Stream	FCp kW/C	Tin C	Tout C	h kW/m2-C
I1	470.00	140.00	40.00	0.8
I2	825.00	160.00	120.00	0.8
I3	42.42	210.00	45.00	0.8
I4	100.00	260.00	60.00	0.8
I5	357.14	280.00	210.00	0.8
I6	50.00	350.00	170.00	0.8
I7	136.36	380.00	160.00	0.8
J1	826.09	270.00	385.00	0.8
J2	500.00	130.00	270.00	0.8
J3	363.64	20.00	130.00	0.8
I8		500.00	499.00	0.8
J4		20.00	40.00	0.8

Note: Exchanger cost=300xArea; stream cost=60\$/kW yr;  
cooling water cost=5\$/kW yr

Table 7 Model statistics for Example 2

Model Statistics	
Single Variables	3120
Discrete Variables	382
Single Equations	6347
Non Zero Elements	23949
Time to reach a feasible solution	411.41 sec
Optimality Gap	0.00%

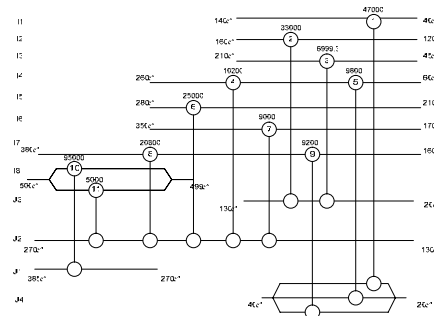


Figure 3 Original HEN for Example 2

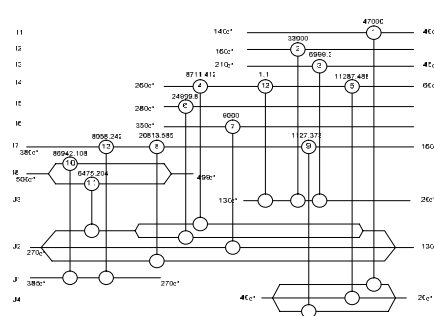


Figure 4 Retrofit HEN for Example 2

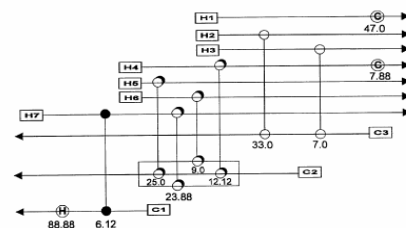


Figure 5 Hypertarget retrofit designs B1

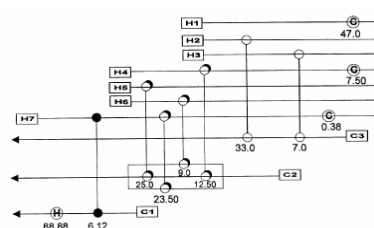


Figure 6 Hypertarget retrofit designs B2

Table 8 Resulting of retrofit heat exchanger network for Example2

HE	Original Load MJ/hr	Retrofit Load MJ/hr	Original Area m <sup>2</sup>	Retrofit Area m <sup>2</sup>	Area addition m <sup>2</sup>	Shell Addition	Cost \$
1	47000	47000	2363.861934	2402.06	38.198066		
2	33000	33000	1609.620648	1613.931	4.310352	YES	1293.1056
3	7000	6999.3	230.6907841	242.319	11.6282159	YES	3488.46477
4	10200	8711.412	692.1394308	692.139			
5	9800	11287.488	339.7977779	366.255	26.4572221		
6	25000	24999.8	1226.755323	1286.336	59.580677	YES	17874.2031
7	9000	9000	224.9150338	396.584	171.6689662	YES	51500.68986
8	20800	20813.585	1211.000328	1211			
9	9200	1127.373	141.4713555	20.484			
10	95000	86942.108	1434.978907	1344.353			
11	5000	6475.204	53.31144862	66.928	13.61655138		
12		1.1		0.051	NEW		15.3
13		8058.242		298.33	NEW		89499
			9528.542971	9940.77	4.33%	4	163670.7633

Table 9 Annual cost comparison between Hypertargets and MILP algorithm

Cost	Existing \$/yr	Hypertarget B1 \$/yr	Hypertarget B2 \$/yr	MILP \$/yr
Total utility cost	6,330,000	5,607,200	5,607,200	5,902,113
Total fixed and area cost		531,900	576,720	163,671
Total cost	6,330,000	6,139,100	6,183,920	6,065,784
MILP more saving (\$/yr)	264,216	73,316	118,136	
	4.17%	1.19%	1.91%	

## 4. CONCLUSION

A new MILP formulation for the retrofit of heat exchanger networks, which takes into account the retrofit options involving modification of the existing structure and new exchanger placement, was presented. The model is very robust and capable of handling *rigorously* large networks such as those of crude distillation units.

## 5. REFERENCES

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