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Multiobjective supply chain design under uncertainty

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Abstract

In this article, the design and retrofit problem of a supply chain (SC) consisting of several production plants, warehouses and markets, and the associated distribution systems, is considered. The first problem formulation modifies and extends other previously presented models, in order to include several essential characteristics for realistically representing the consequences of design decisions on the SC performance. Then, in order to take into account the effects of the uncertainty in the production scenario, a two-stage stochastic model is constructed. The problem objective, i.e., SC performance, is assessed by taking into account not only the profit over the time horizon, but also the resulting demand satisfaction. This approach can be used to obtain different kinds of solutions, that may be valuable at different levels. On one hand, the SC configurations obtained by means of deterministic mathematical programming can be compared with those determined by different stochastic scenarios representing different approaches to face uncertainty. Additionally, this approach enables to consider and manage the financial risk associated to the different design options, resulting in a set of Pareto optimal solutions that can be used for decision-making.

Keywords: Optimisation; Supply chain management; Risk management

1. Introduction

The concept of Supply Chain Management (SCM), which appeared in the early 90s, has recently raised a lot of interest since the opportunity of an integrated management of the supply chain (SC) can reduce the propagation of unexpected/undesirable events through the network and can affect decisively the profitability of all the members. SCM looks for the integration of a plant with its suppliers and its customers to be managed as a whole, and the co-ordination of all the input/output flows (materials, information and finances) so that products are produced and distributed at the right quantities, to the right locations, and at the right time (Simchi-Levi et al., 2000). The main objective is to achieve suitable economic results together with the desired consumer satisfaction levels. The SCM problem may be considered at different levels depending on the planning horizon and the detail of the analysis: strategic, tactical and operational (Fox et al., 2000). In this work, the SC design problem is addressed, thus strategic decisions are considered.

A lot of attempts have been made to model and optimise the SC behaviour, currently existing a big amount of deterministic (Bok et al., 2000; Timpe and Kallrath, 2000; Gjerdrum et al., 2000) and stochastic derived approaches.

Since the nature of most SCs is characterised by numerous sources of technical and commercial uncertainty, the consideration of all the model parameters, such as cost coefficients, production rates, demand, etc., as being known is not realistic. Several works deal with uncertainty in SCM at different levels. One part of the effort has been oriented through control theory in which the uncertainty is modelled as disturbances arriving to a dynamic model of the system. The work by Bose and Pekny (2000) looks for the inventory set points that ensure a desired customer service level with a planning tool, and then track them with a model

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predictive control (MPC) approach. Similarly, Perea-López et al. (2003) determine the optimal variables that maximise the profit of the system, by optimising a multiperiod mixed-integer linear programming (MILP) problem, and using a rolling horizon MPC approach so as to include the disturbance influence. All these approaches work at an operational level.

Other approaches are able to cope with the uncertainty through fuzzy programming (Sakawa et al., 2001) at strategic level. Their limitations are related to the simplicity of the production–distribution models usually used.

A third group, the biggest one, includes statistical analysis-based methods in which it is assumed that the uncertain variable follows a particular probability distribution. As in this article, most works apply an *adapter* strategy in which the SC controls the risk exposure of its assets by constantly adapting its operations to unfolding demand realisations. In the strategy known as *shaper*, in turns, the SC aims to restructure the demand distribution contracting agreements with the customer (Anupindi and Bassok, 1999).

Into the last group, the most popular approach is the two-stage decision process. Applications differ primarily in the selection of the decision variables and the way in which the expected value term, which involves a multi-dimensional integral accounting for the probability distribution of the uncertain parameters, is computed. The difficulty of continuous distributions is avoided by introducing discrete scenarios, or combinations of discrete samples of all the uncertain parameters (Cohen and Lee, 1989; Subrahmanyam, 1996; Iyer and Grossmann, 1998; Tsiakis et al., 2001). Pistikopoulos and co-workers (Acevedo and Pistikopoulos, 1998; Bernardo et al., 1999) have examined alternative strategies for evaluating the integral term, ranging from cubature methods to sampling methods. Maranas and collaborators (Petkov and Maranas, 1997; Gupta and Maranas, 2000, 2003) convert stochastic features of the problem into a chance-constrained programming problem. Finally, a different approach at strategic level is the work of Applequist et al. (2000), who presented a method for evaluating SC projects with the capability of assessing the integral values based upon polytope volumes.

Literature reveals that the most important and extensively studied source of uncertainty has been demand (Gupta and Maranas, 2000, 2003; Petkov and Maranas, 1997; Ierapetritou and Pistikopoulos, 1996; Ahmed and Sahinidis, 1998). The emphasis on incorporating demand uncertainty into the planning decisions is appropriate given the fact that effectively meeting customer demand is what mainly drives most SC planning initiatives.

In traditional SCM, minimising costs or maximising profit as a single objective is often the optimisation focus (Cohen and Lee, 1989; Tsiakis et al., 2001). Moreover, the ability of responding to customer requirements turns out to be one of the most basic functions of the SCM. Thus, customer service should also be taken into consideration when formulating a SC model (Chen et al., 2003) even if it is difficult to quantify as a monetary amount in the objective function. Usually, designs with higher profit will perform better for lower values of customer satisfaction, so they tend to be contradictory objectives. Therefore, it is proposed to set up a multiobjective design problem whose solution will be a set of Pareto optimal possible design alternatives representing the trade-off among the different objectives rather than a unique solution.

Many methodologies have been proposed for treating multiobjective optimisation problems (Miettinen, 1999). Among them, the weighted-sum method, the ε-constraint method, and the goal-programming method, which are based on the conversion of the vector of objectives into a scalar objective (Azapagic and Clift, 1999; Zhou et al., 2000; Chen et al., 2003), are the most widely used in process engineering. Because the optimisation of a multiobjective problem is a procedure looking for a compromise policy, the resulting Pareto-optimal or noninferior solution set consists of an infinite number of options. In order to be able to suggest a specific point of this set, some attempts have been made to compare the objectives between them, for example optimising a Nash-type function (Gjerdrum et al., 2001), defining the objectives as fuzzy sets (Chen et al., 2003) or adding the consideration of the decision-maker input in the problem formulation Rodera et al. (2002).

The present work formulates the SC design problem as a multiobjective stochastic MILP model, which is solved by using the standard ε-constraint method, and branch and bound techniques. This formulation takes into account not only SC profit and customer satisfaction level, but also uncertainty by means of the concept of financial risk, which is defined as the probability of not meeting a certain profit aspiration level (Barbaro and Bagajewicz, 2004).

2. Problem statement

According to the approach previously outlined, the proposed model helps to determine the design of the usual three-echelon SC (production-storage-market) accounting for the maximisation of three objectives (the net present value, the demand satisfaction and the financial risk) and taking into account the decision-maker preferences. Decisions to be decided include the capacity and location of the plants and warehouses, the amount of products to be made at each plant,and the flows of materials between each two nodes of the SC. The structure of the aforementioned SC is depicted in Fig. 1. It includes the following elements:

- a set of plants where products are manufactured prior to be sent to the warehouses;
- a set of warehouses where products are stored before being transported to the final markets;
- a set of final markets where products are available to customers.

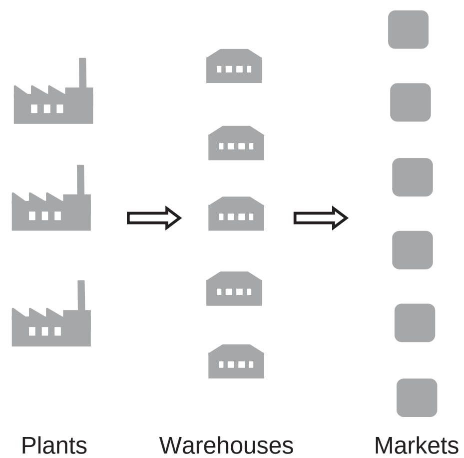

Fig. 1. Supply chain structure.

The overall problem can be formally stated as follows: Given

- number and length of time intervals;
- demand data for each product p, market k, time interval t and scenario s (Dem_{pkts});
- prices of each product at each market, in each time interval (price_{pkt});
- interest rate (ir) and salvage value (SV);
- capacity data of the nodes of the SC such as capacity factors of products in each plant i and each warehouse j (α_{pi} and β_{pj}, respectively), maximum and minimum allowable capacities of plants and warehouses (PCap_i^L, PCap_i^U, WHCap_j^L and WHCap_j^U) and turnover ratios of warehouses (λ_j);
- taxes data such as taxes rate (tr) and number of depreciation time intervals (nd);
- relationship between indirect expenses and capacities of plants and warehouses (IE_{Pi}^L, η_{Pi}, $\mathrm{IE}_{\mathrm{WH}j}^L$ and $\eta_{\mathrm{WH}j}$);
- relationship between fixed capital investment and capacities of plants and warehouses (FCI_{Pi}^L, γ_{Pi}, $\mathrm{FCI}_{\mathrm{WH}j}^L$ and $\gamma_{\mathrm{WH}j}$);
- direct cost parameters such as unit production (VC_{pi}), transport ($\mathrm{TC1}_{pij}$, $\mathrm{TC2}_{pjk}$), handling (HC_{pj}) and inventory (IC_{pj}) costs;

Find

- The configuration of the SC that maximises the net present value (NPV) and the demand satisfaction, and minimises the financial risk:
  - Number, locations and capacities of plants and warehouses to be set;
  - Production rates of each product at each plant, for all the time intervals and scenarios (Q_{pits});
  - Flows of materials between the plants and warehouses (X_{pijts}) and between the warehouses and the markets (Y_{pjkts}).

The result of the model provides a set of Pareto solutions to be used by the decision-maker in order to find the best SC configuration according to her/his preferences.

3. Multiobjective stochastic model

A stochastic programming approach based on a recourse model with two stages is proposed in this work to incorporate the uncertainty associated to the demand within the design process.

In a two-stage stochastic optimisation approach, the uncertain model parameters are considered random variables with an associated probability distribution and the decision variables are classified into two stages. The first-stage variables correspond to those decisions that need to be made here-and-now, prior to the realisation of the uncertainty. The second-stage or recourse variables correspond to those decisions made after the uncertainty is unveiled and are usually referred to as wait-and-see decisions. After the first-stage decisions are taken and the random events realised, the second-stage decisions are made subject to the restrictions imposed by the second-stage problem. Due to the stochastic nature of the performance associated with the second-stage decisions, the objective function consists of the sum of the first-stage performance measure and the expected second-stage performance. More details on stochastic techniques can be found in Birge and Louveaux (1997).

In our problem, the uncertainty associated to the demand is represented by a set of scenarios with given probability of occurrence. Such scenarios together with their associated probabilities must be provided as input data into the model. In case the demand follows a specific type of probability distribution, this can be discretised using Monte Carlo sampling, thus generating a set of explicit scenarios.

Moreover, decision variables which characterise the network configuration, namely those binary variables which represent the existence of the different nodes of the SC and the continuous ones which are related to the capacities of the sites, are considered as first-stage variables as it is assumed that they have to be taken at the design stage before the demand uncertainty is unveiled.

On the other hand, decision variables related to the amount of products to be produced and stored in the nodes of the SC, the flows of materials transported among the entities of the network and the product sales are considered as second-stage variables.

At the end of the design horizon, a different value of NPV and demand satisfaction is obtained for each particular realisation of demand uncertainty. The proposed model accounts for the maximisation of the expected value of the profit distribution, the target imposed for the customer satisfaction and the financial risk. The mathematical formulation of such model is next described.

3.1. Mass balance constraints

The mass balance must be satisfied in each of the sites embedded in the SC. Therefore, for each plant i the total amount of product p manufactured during time interval t in scenario s (Q_{pits}) must be transported from i to warehouses j as stated by Eq. (1), in which X_{pijts} represents the flow of p sent by i to j in t and s.

$$Q_{pits} = \sum_j X_{pijts} \quad \forall p, i, t, s. \tag{1}$$

With regard to the warehouses, for each time interval t and scenario s, the total amount of product p sent by plant i to warehouse j (X_{pijts}) plus the initial stock of p kept at j at the beginning of t (Inv_{pjt-1s}) must be equal to the amount of p transported from j to final markets k (Y_{pjkts}) plus the final inventory of p in j (Inv_{pjts}), as expressed by constraint (2):

$$\sum_i X_{pijts} + \text{Inv}_{pjt-1s} = \sum_k Y_{pjkts} + \text{Inv}_{pjts} \quad \forall p, j, t, s. \tag{2}$$

Moreover, it is considered that in the first period of time ($t = 1$), when the construction of the different sites of the SC is supposed to take place, the flows of materials between nodes are equal to zero as stated by constraint (3):

$$Q_{pits} = 0, \quad X_{pijts} = 0, \quad Y_{pjkts} = 0 \quad \forall p, i, j, k, t = 1, s. \tag{3}$$

Finally, the sales of product p carried out in market k during time interval t in scenario s (Y_{pjkts}) must be less than or equal to the demand (Dem_{pkts}) of this product at this market (4):

$$\sum_j Y_{pjkts} \leqslant \text{Dem}_{pkts} \quad \forall p, k, t, s. \tag{4}$$

3.2. Capacity constraints

The capacity of each plant i is represented by a continuous variable (PCap_i), which must be higher than the total amount of products p manufactured at i for every time interval t and scenario s as stated by Eq. (5). The parameter α_{pi} used in this expression weighs the amount of resources consumed by each product p at i.

$$\sum_p Q_{pits}\alpha_{pi} \leqslant \text{PCap}_i \quad \forall i, t, s. \tag{5}$$

Furthermore, PCap_i is constrained by upper and lower bounds (PCap_i^L and PCap_i^U) in case the plant is finally set as stated by Eq. (6). In this equation, A_i is a binary variable which takes the value of 1 in case the plant is opened and 0 otherwise. As can be observed, if the plant is not set, it cannot manufacture any product as its capacity is forced to take a value equal to zero:

$$\text{PCap}_i^L A_i \leqslant \text{PCap}_i \leqslant \text{PCap}_i^U A_i \quad \forall i. \tag{6}$$

As occurs with the plants, it is defined a continuous variable in order to represent the capacity of the warehouses (WHCap_j). Therefore, the total inventory of p kept at warehouse j during time interval t in scenario s (Inv_{pjts}) must be lower than the capacity of the warehouse as stated by Eq. (7), where β_{pj} is a coefficient that weighs the storage resources consumed by each product p at j:

$$\sum_p \text{Inv}_{pjts}\beta_{pj} \leqslant \text{WHCap}_j \quad \forall j, t, s. \tag{7}$$

Moreover, the total flow of materials sent by warehouse j to the final market k is also constrained by the variable WHCap_j as expressed by Eq. (8). In such expression, it is supposed that for each time interval t, scenario s and warehouse j, the capacity needed to handled a given amount of products, assuming regular shipment and delivery schedule, is twice the summation of the average inventory levels of products p (AIL_{pjts}) kept at j weighed by the storage coefficients β_{pj} (Simchi-Levi et al., 2000):

$$2\sum_p \text{AIL}_{pjts}\beta_{pj} \leqslant \text{WHCap}_j \quad \forall p, j, t, s. \tag{8}$$

AIL_{pjts} is computed by means of Eq. (9), in which λ_j represents the turnover ratio of the warehouse j, i.e., the number of times that the stock is completely replaced per time interval:

$$\text{AIL}_{pjts} = \frac{\sum_k Y_{pjkts}}{\lambda_j} \quad \forall p, j, t, s. \tag{9}$$

Finally, the capacity of the warehouse j is constrained by lower and upper bounds (WHCap_j^L and WHCap_j^U) in case the warehouse is finally set, as stated by Eq. (10). The binary variable B_j used in this expression represents the existence of a warehouse j and takes the value of 1 in case j is opened and 0 otherwise.

$$\text{WHCap}_j^L B_j \leqslant \text{WHCap}_j \leqslant \text{WHCap}_j^U B_j \quad \forall j. \tag{10}$$

3.3. Objective function

The production/distribution system whose model has been described before must attain three targets:

- maximise the NPV;
- maximise the demand satisfaction, which in turn may bring future sales;
- minimise the financial risk.

3.3.1. Net present value

As it has been mentioned before, different NPV values are obtained for each scenario under study (NPV_s) once the uncertainty is unveiled. The model described before must account for the maximisation of the expected value ($E[\text{NPV}]$) of the resulting NPV distribution, which can be computed by performing an average of the aforementioned NPV_s as stated by Eq. (11):

$$E[\mathrm{NPV}] = \sum_s \mathrm{prob}_s\, \mathrm{NPV}_s. \tag{11}$$

The values of NPV_s must be determined according to applicable rules (depreciation), legislation (taxes), etc., so this may lead to different formulations. The following equations intend to reflect a general case:

Having in mind such purpose, NPV_s is calculated for each scenario s as the summation of the discounted cash flows (CF_{ts}) generated in each of the time intervals t in which the time horizon is divided as expressed by Eq. (12):

$$\mathrm{NPV}_s = \sum_t \frac{\mathrm{CF}_{ts}}{(1+ir)^{t-1}} \quad \forall s. \tag{12}$$

With regard to the cash flows, these are computed for each scenario s and time interval t as the difference between the revenues (Rev_{ts}) and the total costs. Such costs include the direct (DE_{ts}) and indirect expenses (IE_t) as well as the taxes (Tax_{ts}) originated by all the production, distribution and storage activities performed in the day to day SC operation. Moreover, it is supposed that the necessary capital investment (CI) for carrying out the construction of the SC takes place in the first period of time, while the working capital (WC), which is part of this initial investment, and the salvage value of the SC (SV) are recovered in the last time interval as indicated by Eqs. (13)–(15):

$$\mathrm{CF}_{ts} = -\mathrm{CI} = -(\mathrm{FCI} + \mathrm{WC}) \quad \forall s, t = 1, \tag{13}$$

$$\mathrm{CF}_{ts} = -\mathrm{Rev}_{ts} - \mathrm{DE}_{ts} - \mathrm{IE}_t - \mathrm{Tax}_{ts} \quad \forall s, 1 < t < T, \tag{14}$$

$$\mathrm{CF}_{ts} = -\mathrm{Rev}_{ts} - \mathrm{DE}_{ts} - \mathrm{IE}_t - \mathrm{Tax}_{ts} + \mathrm{WC} + \mathrm{SV} \quad \forall s, t = T. \tag{15}$$

The total capital investment (CI) is calculated by adding the fixed capital investment (FCI) and the working capital (WC). The first term includes the cost of setting the plants and warehouses embedded in the SC and is a linear function of their capacities (16).

$$\mathrm{FCI} = \sum_i (\mathrm{FCI}^L_{Pi} A_i + \mathrm{PCap}_i \gamma_{Pi}) + \sum_j (\mathrm{FCI}^L_{WHj} B_j + \mathrm{WHCap}_j \gamma_{WHj}). \tag{16}$$

On the other hand, the working capital (WC), which represents the initial amount of money that is necessary in order to start the production and distribution activities in the network, is supposed to be directly proportional to FCI as expressed by constraint (17). In such equation, the proportionality constant μ is considered to be equal to $\mu = 0.194$ (Biegler et al., 1997):

$$\mathrm{WC} = \mu \mathrm{FCI}. \tag{17}$$

The revenues obtained in each time interval t and scenario s are proportional to the sales of products p at markets k (Sales_{pkts}) and their associated prices (Price_{pkt}) as stated by Eqs. (18) and (19):

$$\mathrm{Rev}_{ts} = \sum_{pk} \mathrm{Sales}_{pkts}\, \mathrm{Price}_{pkt} \quad \forall t, s, \tag{18}$$

$$\mathrm{Sales}_{pkts} = \sum_j Y_{pjkts} \quad \forall p, k, t, s. \tag{19}$$

Regarding the indirect expenses generated in each time interval t (IE_t), it is considered that these are proportional to the capacities of the plants/warehouses, as expressed by constraint (20).

$$\mathrm{IE}_t = \sum_i (\mathrm{IE}^L_{Pi} A_i + \mathrm{PCap}_i \eta_{Pi}) + \sum_j (\mathrm{IE}^L_{WHj} B_j + \mathrm{WHCap}_j \eta_{WHj}) \quad \forall t. \tag{20}$$

It should be mentioned that this term is computed directly from the first-stage variables (SC configuration) and without evaluating second-stage decisions, and therefore remains constant for all the scenarios under study once the uncertainty is unveiled.

The direct expenses obtained in time interval t and scenario s (DE_{ts}) are proportional to the amount of products manufactured, stored and transported through the different nodes of the SC, as stated by Eq. (21). Such costs include, therefore, the following terms:

- Variable production costs at the plants, which are assumed to be equal to the production rates of products p manufactured at plants i in each time interval t and scenario s (Q_{pits}) multiplied by the unit production costs (VC_{pi}).
- Handling costs at the warehouses, which in this case are assumed to be equal to the flows of products p sent by warehouses j to markets k in time interval t and scenario s (Y_{pjkts}) multiplied by the unit handling costs (HC_{pj}).
- Transport costs, which are supposed to be equal to the flows of materials transported between plants and warehouses (X_{pijts}) and warehouses and markets (Y_{pjkts}) in each scenario s and time interval t multiplied by the unit transport costs ($\mathrm{TC1}_{pij}$ and $\mathrm{TC2}_{pjk}$, respectively).
- Inventory-holding costs at the warehouses, which are supposed to be equal to the average inventory levels of product p kept at warehouse j in each scenario s and time interval t (AIL_{pjts}) multiplied by the unit inventory costs (IC_{pj}).

$$\mathrm{DE}_{ts} = \sum_{pi} Q_{pits} \mathrm{VC}_{pi} + \sum_{pjk} Y_{pjkts} \mathrm{HC}_{pj} + \sum_{pij} X_{pijts} \mathrm{TC1}_{pij} + \sum_{pjk} Y_{pjkts} \mathrm{TC2}_{pjk} + \sum_{pjk} \frac{Y_{pjkts}}{\lambda_j} \mathrm{IC}_{pj} \quad \forall t, s. \tag{21}$$

Finally, the taxes to be paid in each time interval t and scenario s (Tax_{ts}) are computed assuming a linear depreciation policy as stated by constraints (22)–(24). In these equations, tr represents the taxes rate to be applied on the gross benefit, and nd the number of time periods through which the depreciation will be carried out:

$$\mathrm{Tax}_{ts} = (\mathrm{Rev}_{ts} - \mathrm{Dep}_t)\mathrm{tr} \quad \forall 1 < t \leqslant \mathrm{nd} + 1, s, \tag{22}$$

$$\mathrm{Tax}_{ts} = \mathrm{Rev}_{ts}\mathrm{tr} \quad \forall \mathrm{nd} + 1 < t \leqslant T, s, \tag{23}$$

$$\mathrm{Dep}_t = \frac{\mathrm{FCI} - \mathrm{SV}}{\mathrm{nd}} \quad \forall t. \tag{24}$$

3.3.2. Demand satisfaction

Customer demand satisfaction for each time interval t and scenario s (DSat_{ts}) is measured as the average of the not covered demand (25):

$$\mathrm{DSat}_{ts} = \frac{\sum_{pk} \mathrm{Sales}_{pkts}}{\sum_{pk} \mathrm{Dem}_{pkts}} \quad \forall t > 1, s. \tag{25}$$

As occurred with the NPV, it is possible to compute the expected value of the demand satisfaction by performing an average of the values of DSat_{ts} for all the scenarios s and time intervals t as expressed by constraint (26):

$$E[\mathrm{DSat}] = \frac{\sum_{t=2}^{T} \sum_s \mathrm{prob}_s \mathrm{Dsat}_{ts}}{T - 1}. \tag{26}$$

The SC design problem, without considering the financial risk, would be therefore mathematically formulated as follows:

$$\begin{aligned} &\text{maximise}\{E[\mathrm{NPV}];\ \ E[\mathrm{DSat}]\} \\ &\text{subject to} \\ &\text{Eqs.}(1)\text{–}(26). \end{aligned}$$

The main drawback of such mathematical formulation lies on the fact that it does not reflect a realistic operational policy in terms of the demand satisfaction level to be achieved by the SC. If the expected value of the demand satisfaction is pursued as objective, the resulting Pareto optimal SC configurations may exhibit in some scenarios and time intervals demand satisfaction levels under the average while in others may exceed it. This means that the operational strategy of the SC towards the demand satisfaction will depend on the scenario that finally materialises, as well as the time interval considered, which does not seem desirable from the decision-maker's perspective.

In order to overcome such difficulty and explicitly take into account the demand satisfaction strategy of the enterprise, a *minimum target* for the demand satisfaction (MDSat), which must be attained in all the time intervals and scenarios, is incorporated as an objective within the existing formulation, thus avoiding the use of the E[DSat]. This new issue leads to the following model:

$$\begin{aligned} &\text{maximise } \{E[\mathrm{NPV}];\ \mathrm{MDSat}\} \\ &\text{subject to} \\ &\text{Eqs. } (1)\text{–}(25), \end{aligned}$$

where

$$\mathrm{MDSat} \leqslant \frac{\sum_{pk} \mathrm{Sales}_{pkts}}{\sum_{pk} \mathrm{Dem}_{pkts}} \quad \forall t > 1, s. \tag{27}$$

Therefore, by selecting a certain value for MDSat, it is guaranteed that for each scenario s and time interval $t > 1$ at least the minimum desired demand satisfaction level is reached. The formulation described above is thus able to reflect the operational policy that consists in obtaining the actual maximum profit at each scenario and time interval while ensuring that the demand satisfaction previously fixed is also achieved in all of them.

Each SC configuration leads to two histograms, one for the NPV and another one for the demand satisfaction, and therefore exhibits an E[NPV] for a given operational policy (MDSat). This value of E[NPV] may change depending on the demand satisfaction target imposed to the SC operation, as will be discussed in the case study.

3.3.3. Financial risk

The financial risk associated with a design project under uncertainty is defined as the probability of not meeting a certain target profit (maximisation) or cost (minimisation) level referred to as Ω (Barbaro and Bagajewicz, 2004). For the two-stage stochastic problem, the financial risk associated with a design x and target profit Ω is therefore expressed by the following probability:

$$\mathrm{Risk}(x, \Omega) = P(\mathrm{NPV}(x) < \Omega), \tag{28}$$

where $\mathrm{NPV}(x)$ is the NPV after the uncertainty has been unveiled and a scenario realised. The definition of $\mathrm{Risk}(x, \Omega)$ can be rewritten with the help of binary variables as follows:

$$\mathrm{Risk}(x, \Omega) = \sum_s \mathrm{prob}_s z_s(x, \Omega), \tag{29}$$

where z_s is a new binary variable defined for each scenario as follows:

$$z_s(x, \Omega) = \begin{cases} 1 & \text{if } \mathrm{NPV}_s < \Omega, \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

In a discrete scenario case, financial risk is given by the cumulative frequency obtained from the NPV histogram as depicted in Fig. 2. A more straightforward way of assessing and understanding the trade-offs between risk and profit is to use the cumulative risk curve as depicted in Fig. 3.

A possible way of avoiding the use of binary variables to determine the risk consists of reformulating the problem without explicitly using this definition. For this purpose, the use of the concept of downside risk, in the way introduced

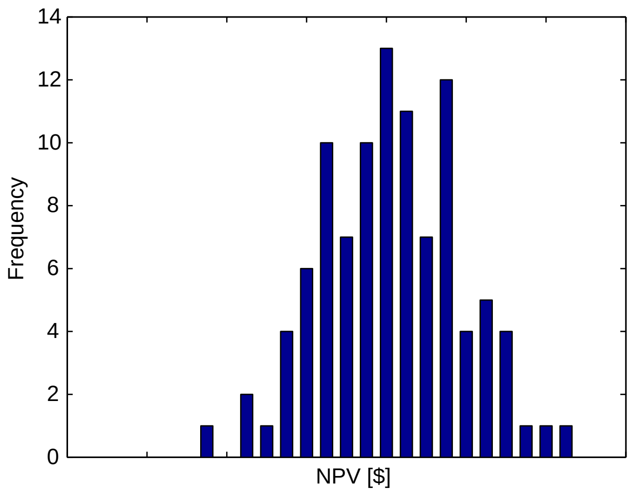

Fig. 2. Histogram of frequencies.

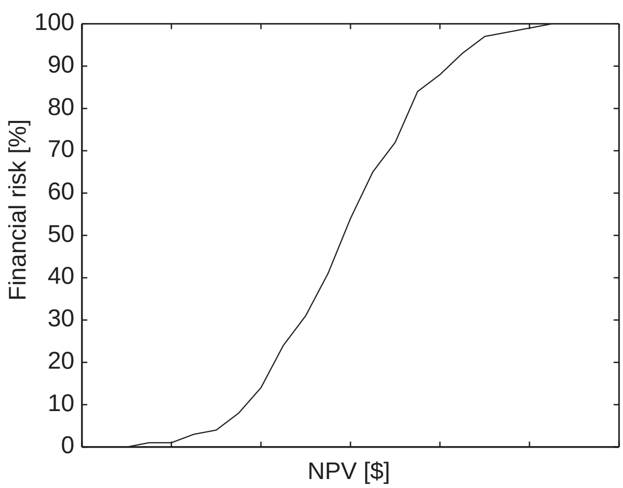

Fig. 3. Cumulative risk curves.

by Eppen et al. (1989), is applied. The $\mathrm{DRisk}(x, \Omega)$ is calculated with the help of the following constraints:

$$\mathrm{DRisk}(x, \Omega) = \sum_s \mathrm{prob}_s \delta_s(x, \Omega),$$
$$\delta_s(x, \Omega) \geqslant \Omega - \mathrm{NPV}_s \quad \forall s,$$
$$\delta_s(x, \Omega) \geqslant 0 \quad \forall s, \tag{31}$$

where δ_s is a continuous variable. The $\mathrm{DRisk}(x, \Omega)$ can be utilised to control financial risk at different NPV targets by varying Ω from small values up to higher values and obtaining a full spectrum of solutions to be used by the decision-maker as a decision support tool.

4. Multiobjective problem

The resulting objective function which includes the three objectives (NPV, demand satisfaction and financial risk) can be finally expressed as follows:

$$\text{maximise } \{E[\mathrm{NPV}];\ \mathrm{MDSat};\ -\mathrm{DRisk}\}. \tag{32}$$

The solution of this problem consists of a set of Pareto optimal SC configurations. These are obtained in this work by applying the ε-constraint method, first introduced by Haimes et al. (1971). Such method is based on the maximisation of one objective function, and considering the other objectives as constraints bounded by some allowable levels ε_0. Then, the levels ε_0 may be altered to generate the entire Pareto-optimal set. Therefore, the following single MILP optimisation formulation is applied to obtain the Pareto solutions:

$$\text{maximise } E[\mathrm{NPV}]$$
$$\text{subject to}$$
$$\text{Eqs. (1)–(24), (27) and (31),}$$

$$\mathrm{MDSat} \geqslant \varepsilon_1,$$

$$\mathrm{DRisk}(x, \Omega) \leqslant \varepsilon_2.$$

Therefore, by changing the values of the bound levels ε_1 and ε_2, as well as the target Ω, a set of results can be obtained. Each of these results implies an SC configuration. The resulting Pareto solutions might be represented in a three-dimensional chart (E[NPV], MDSat and $\mathrm{DRisk}(x, \Omega)$).

The methodology to solve the proposed problem is as follows:

(1) Select a target for the downside risk calculation (Ω).
(2) Set initial targets for each objective (ε_1 and ε_2).
(3) Solve the proposed model.
(4) Obtain the corresponding Pareto solution.
(5) Choose a configuration from the Pareto set.

If the decision-maker is satisfied with the design then stop. Otherwise go to step 1.

The proposed strategy should lead to a final SC design which would represent the desired compromise among the different objectives from the decision-maker's perspective. Regarding financial risk, it is also important to point out that this term can be managed by changing both, the target associated to the downside risk itself (ε_2) and the aspiration level for which such term is computed (Ω).

5. A motivating example

In order to illustrate the capabilities of the proposed model, a hypothetical case study has been studied. The problem consists of finding the optimal retrofit of an existing SC established in Europe in terms of economic (NPV), demand satisfaction (MDSat) and risk ($\mathrm{DRisk}(x, \Omega)$) performance. The information available to carry out such task includes the cost data concerning the production and distribution activities of the network and the probability distribution of the uncertain demand. The optimal redesign must include

Fig. 4. Case study.

the number, location and capacities of the new plants and warehouses to be established, as well as the new capacities of the existing sites in case these should be modified.

The structure of the case study is indicated in Fig. 4. It is assumed that the existing plants and warehouses, which are located in Barcelona and Milan, are forced to remain opened in the future and only their capacities can be increased in case this would be necessary (the model would therefore provide the additional capacities to be added to these new nodes with respect to the original one). The original capacities of the plants in Barcelona and Milan are equal to 200,000 and 80,000 kg, respectively, and 160,000 and 60,000 kg for the warehouses, in the same order.

Four potential location candidates distributed among East Europe countries are provided for plants and five for warehouses. All the required information related to the plants is shown in Tables 1–4, while all the data concerning the warehouses are given in Tables 5–9. The original network manufactures three different products (P1, P2 and P3) which are delivered to 11 final markets. Tables 10–15. present the transport costs. The mean demand (MDem_{pkt}) as well as the prices of the products for the first time interval are listed in Table 16 With regard to the demand of future time intervals, it is assumed that it remains constant in Valencia (V), increases by 2.5% per year in Barcelona (Ba), Bristol (B), Manchester (M), London (L), Milan (Mi) and Berlin (Be) and by 10% per year in Bratislava (Br), Warsaw (W) Bucharest (Bu), and Moscow (Mo). On the other hand, it is also considered that prices of products remain constant for the whole time horizon. The tax rate and the discount rate are assumed to be equal to 30% and 10%, respectively, and the salvage value of the SC is supposed to be a 10% of the FCI. The time horizon is divided into ten time intervals and depreciation takes place in the first seven periods of time. Finally, it is assumed that the flows of materials associated to the existing sites (plants and warehouses located in B and Mi) are not forced to be equal to zero for the first time interval. This issue is considered in the determination of the

Table 1
Lower and upper capacity bounds of plants i (PCap_i^L and PCap_i^U)

Plant location	PCap_i^L (kg)	PCap_i^U (kg)
Ba	0	50,000,000
Mi	0	50,000,000
Br	0	50,000,000
Mo	0	50,000,000
Bu	0	50,000,000
W	0	50,000,000

Table 2
Production capacity factors of products p at plants i (α_{pi} (adim))

Product	Plant location					
	Ba	Mi	Br	Mo	Bu	W
P1	1.5	1.5	1.5	1.5	1.5	1.5
P2	1.3	1.3	1.3	1.3	1.3	1.3
P3	1.0	1.0	1.0	1.0	1.0	1.0

Table 3
Variable production costs of products p at plants i (VC_{pi} (m.u./kg))

Product	Plant location					
	Ba	Mi	Br	Mo	Bu	W
P1	30	40	25	20	15	25
P2	22	30	19	15	11	19
P3	15	20	13	10	8	13

Table 4
Fixed capital investment and indirect expenses parameters of plants i (FCI^L_{Pi}, γ_{Pi}, IE^L_{Pi} and η_{Pi})

Plant location	FCI^L_{Pi} (m.u.)	γ_{Pi} (m.u./kg)	IE^L_{Pi} (m.u.)	η_{Pi} (m.u./kg)
Ba	200,000	10	600,000	10
Mi	100,000	10	800,000	10
Br	500,000	10	500,000	10
Mo	800,000	10	400,000	10
Bu	300,000	10	300,000	10
W	500,000	10	500,000	10

Table 5
Lower and upper capacity bounds and turnover ratios of warehouses j ($WHCap^L_j$, $WHCap^U_j$ and λ_j)

Warehouse location	$WHCap^L_j$ (kg)	$WHCap^U_j$ (kg)	λ_j (adim)
Ba	0	50,000,000	4
D	0	50,000,000	4
Mi	0	50,000,000	4
Br	0	50,000,000	4
Mo	0	50,000,000	4
Bu	0	50,000,000	4
W	0	50,000,000	4

cash flow of the first time interval, where the production, storage and distribution activities carried out by the existing nodes are also considered together with the capital investment term. Finally, it is assumed that the indirect expenses of the existing SC are equal to 3,500,000 monetary units (m.u.).

5.1. The deterministic Pareto optimal solutions

In first place, the model is solved as a deterministic case, which means that the behavior of the demand for each time interval and market is assumed to be perfectly known and therefore only one scenario with mean demand values is considered (Table 16).

The resulting mathematical formulation has 1128 single equations, 4010 continuous variables and 13 binary variables, and it is implemented in GAMS (Brooke et al., 1988) and solved using the MILP solver of CPLEX 7.0. The time required to obtain solutions with 0% integrality gap on an AMD Athlon 3000 computer ranges from 0.3 to 0.6 s (depending on the target imposed to the demand satisfaction).

When the deterministic case is solved without constraining the value of demand satisfaction, the result of the model leads to a solution with MDSat = 36.3%, i.e., the best economic performance is reached, satisfying customer demand up to a certain level. In other words, only above such value of demand satisfaction level some trade-off between the objectives exists and below it the solution is the same as that of the model without constraining demand satisfaction.

The Pareto deterministic optimal curve is next obtained by maximising the NPV and progressively constraining the MDSat. Therefore, each point of this Pareto curve implies a SC design operating under a demand satisfaction policy represented by the target ε_1 imposed to the optimisation problem. Fig. 5 shows the aforementioned Pareto curve while in Fig. 6 three SC configurations (locations and capacities expressed in kg) which correspond to different points of the curve (MDSat = 40%, 60% and 80%) are given. It is interesting to notice how the number of plants and warehouses established as well as their capacities increase as more demand satisfaction is requested. For instance, the solution with a MDSat = 40% implies the set-up of two new nodes

Table 6
Storage capacity factors of products p at warehouses j (β_{pj} (adim))

Product	Warehouse location						
	Ba	D	Mi	Br	Mo	Bu	W
P1	1.5	1.5	1.5	1.5	1.5	1.5	1.5
P2	1.3	1.3	1.3	1.3	1.3	1.3	1.3
P3	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 7
Handling costs of products p at warehouses j (HC_{pj} (m.u./kg))

Product	Warehouse location						
	Ba	D	Mi	Br	Mo	Bu	W
P1	3.0	5.0	4.0	2.5	2.0	1.5	2.5
P2	2.5	3.8	3.0	1.9	1.5	1.1	1.9
P3	1.5	2.5	2.0	1.3	1.0	0.8	1.3

Table 8
Inventory costs of products p at warehouses j (IC_{pj} (m.u./kg year)

Product	Warehouse location						
	Ba	D	Mi	Br	Mo	Bu	W
P1	3.0	5.0	4.0	2.5	2.0	1.5	2.5
P2	2.5	3.8	3.0	1.9	1.5	1.1	1.9
P3	1.5	2.5	2.0	1.3	1.0	0.8	1.3

Table 9
Fixed capital investment and indirect expenses parameters of warehouses j (FCI^L_{WHj}, γ_{WHj}, IE^L_{WHj} and η_{WHj})

Warehouse location	FCI^L_{WHj} (m.u.)	γ_{WHj} (m.u./kg)	IE^L_{WHj} (m.u)	η_{WHj} (m.u./kg)
Ba	20,000.0	10.0	60,000.0	2.5
D	400,000.0	10.0	90,000.0	2.5
Mi	10,000.0	10.0	80,000.0	2.5
Br	50,000.0	10.0	50,000.0	2.5
Mo	80,000.0	10.0	40,000.0	2.5
Bu	30,000.0	10.0	30,000.0	2.5
W	50,000.0	10.0	50,000.0	2.5

Table 10
Transport cost between plants i and warehouses j for product P1 ($\mathrm{TC1}_{pij}$ (m.u./kg))

Plant location	Warehouse location						
	Ba	D	Mi	Br	Mo	Bu	W
Ba	0.0	12.9	9.7	18.3	35.9	25.9	23.3
Mi	9.8	10.7	0.0	8.7	27.6	16.3	15.1
Br	18.3	13.1	8.7	0.0	19.0	10.0	6.4
Mo	35.9	26.7	27.6	19.0	0.0	17.8	12.6
Bu	25.9	22.8	16.3	10	17.8	0.0	11.6
W	23.3	14.1	15.1	6.4	12.6	11.6	0.0

Table 11
Transport cost between plants i and warehouses j for product P2 ($TC1_{pij}$ (m.u./kg))

Plant location	Warehouse location						
	Ba	D	Mi	Br	Mo	Bu	W
Ba	0.0	16.1	12.2	22.9	44.8	32.3	29.2
Mi	12.2	13.4	0.0	10.9	34.5	20.4	18.9
Br	22.9	16.3	10.9	0.0	23.7	12.4	8.1
Mo	44.8	33.3	34.5	23.7	0.0	22.3	15.8
Bu	32.3	28.5	20.4	12.4	22.3	0.0	14.5
W	29.2	17.6	18.9	8.1	15.8	14.5	0.0

Table 12
Transport cost between plants i and warehouses j for product P3 ($TC1_{pij}$ (m.u./kg))

Plant location	Warehouse location						
	Ba	D	Mi	Br	Mo	Bu	W
Ba	0.0	19.3	14.6	27.5	53.8	38.8	35.0
Mi	14.6	16.1	0.0	13.1	41.5	24.4	22.7
Br	27.5	19.6	13.1	0.0	28.5	14.9	9.7
Mo	53.8	40.0	41.5	28.5	0.0	26.8	18.9
Bu	38.8	34.2	24.4	14.9	26.8	0.0	17.4
W	35.0	21.1	22.7	9.7	18.9	17.4	0.0

Table 13
Transport cost between warehouses j and markets k for product P1 ($TC2_{pjk}$ (m.u./kg))

Ware	Market location										
	V	Ba	B	M	L	Mi	Be	Br	W	Mo	Bu
Ba	3.4	0.0	13.2	15.8	14	10	18.5	18.3	23.3	35.9	25.9
D	16	12.9	3.2	4.6	1.2	10.7	8.5	13.1	14.1	26.7	22.8
Mi	13.1	9.7	13.8	15.2	11.8	0.0	10.3	8.7	15.1	27.6	16.3
Br	21.7	18.3	17.0	18.4	15.0	8.7	6.6	0.0	6.4	19.0	9.95
Mo	39.2	35.9	30.6	26.9	28.7	27.6	18.6	19.0	12.6	0.0	17.8
Bu	29.2	25.9	26.7	28.1	24.7	16.3	16.5	10.0	11.6	17.8	0.0
W	26.7	23.3	18.0	14.3	16.1	15.1	5.9	6.4	0.0	12.6	11.6

Table 14
Transport cost between warehouses j and markets k for product P2 ($TC2_{pjk}$ (m.u./kg))

Ware	Market location										
	V	Ba	B	M	L	Mi	Be	Br	W	Mo	Bu
Ba	4.3	0.0	16.5	19.7	17.5	12.2	23.2	22.9	29.2	44.8	32.3
D	19.9	16.1	4.0	5.8	1.5	13.4	10.6	16.3	17.6	33.3	28.4
Mi	16.4	12.2	17.2	19.0	14.8	0.0	13	10.9	18.9	34.5	20.4
Br	27.1	22.9	21.3	23.0	18.8	10.9	8.2	0.0	8.1	23.7	12.4
Mo	49.0	44.8	38.3	33.6	35.8	34.5	23.2	23.7	15.8	0.0	22.3
Bu	36.6	32.3	33.4	35.1	30.9	20.4	20.6	12.4	14.5	22.3	0.0
W	33.4	29.2	22.5	17.9	20.1	18.9	7.4	8.1	0.0	15.8	14.5

Table 15
Transport cost between warehouses j and markets k for product P3 ($TC2_{pjk}$ (m.u./kg))

Ware	Market location										
	V	Ba	B	M	L	Mi	Be	Br	W	Mo	Bu
Ba	5.2	0.0	19.8	23.7	21.0	14.6	27.8	27.5	35.0	53.8	38.8
D	23.9	19.3	4.8	6.9	1.9	16.1	12.7	19.6	21.1	40.0	34.1
Mi	19.6	14.6	20.7	22.8	17.7	0.0	15.5	13.1	22.7	41.5	24.4
Br	32.5	27.5	25.5	27.6	22.6	13.1	9.8	0.0	9.7	28.5	14.9
Mo	58.5	53.8	45.9	40.4	43.0	41.5	27.8	28.5	18.9	0.0	26.8
Bu	43.9	38.8	40.1	42.2	37.1	24.4	24.7	14.9	17.4	26.8	0.0
W	40.1	35.0	27.0	21.5	24.1	22.7	8.9	9.7	0.0	18.9	17.4

Table 16
Demand and prices of products p at markets k in the first time interval

Market	Dem_{pkt} (kg)			$Price_{pkt}$ (m.u.)		
	P1	P2	P3	P1	P2	P3
V	10,000.0	5,000.0	5,000.0	73.8	65.0	53.1
Ba	60,000.0	20,000.0	20,000.0	73.8	65.0	53.1
B	5,000.0	7,500.0	2,500.0	57.5	50.6	41.4
M	10,000.0	15,000.0	5,000.0	57.5	50.6	41.4
L	10,000.0	50,000.0	15,000.0	57.5	50.6	41.4
Mi	75,000.0	25,000.0	25,000.0	47.5	41.8	34.2
Be	25,000.0	10,000.0	15,000.0	40	35.2	28.8
Br	20,000.0	2,500.0	2,500.0	27.5	24.2	19.8
W	50,000.0	30,000.0	10,000.0	28.8	25.3	20.7
Mo	75,000.0	25,000.0	50,000.0	47.5	41.8	34.2
Bu	50,000.0	10,000.0	10,000.0	21.3	18.7	15.3

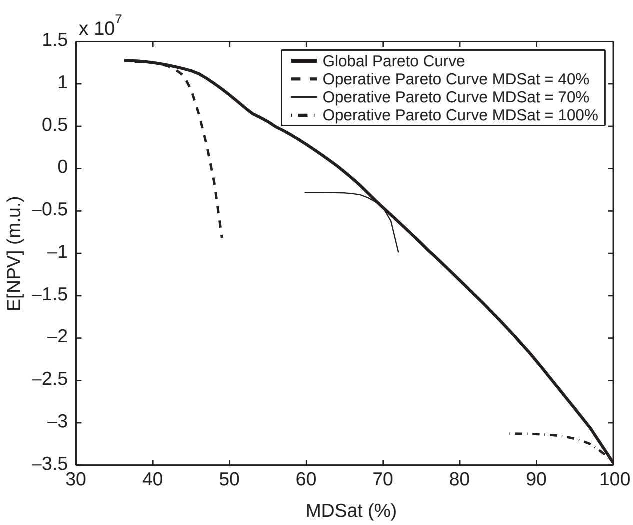

Fig. 5. Deterministic Pareto curve.

(one plant and one warehouse) while four and six new sites are opened when MDSat is forced to be higher than 60% and 80%, respectively.

It should also be mentioned that although each of the obtained SC configurations is Pareto optimal for a certain value of MDSat, they can operate under different demand satisfaction policies, thus leading to dissimilar NPVs. Therefore, it is possible to obtain for each SC design another *operative* Pareto curve by fixing the variables which represent the configuration of the SC and solving the multiobjective problem which accounts for the maximisation of the NPV and the DSat. In Fig. 5, the Pareto operative curves associated to the same SC designs (MDSat = 40%, 60% and 80%) are also depicted. As it can be observed, there is a trade-off between both objectives above a certain demand satisfaction level as occurred with the general Pareto curve. Moreover, each operative curve ends at a point that corresponds to the MDSat level above which the SC design is not able to operate due to the capacity constraints. It may occur that some SC designs are able to operate for higher demand satisfaction levels than those for which they were originally designed. This can be achieved by manufacturing products with lower unit capacity coefficients. It is worthwhile to mention that the Pareto curve envelops all the particular operative Pareto curves and touches them only at a certain demand satisfaction level. At this level, the configuration is Pareto solution of the global multiobjective optimisation problem.

5.2. *Stochastic case*

The same case study, with the same input parameters but taking into account demand uncertainty, is next solved. In

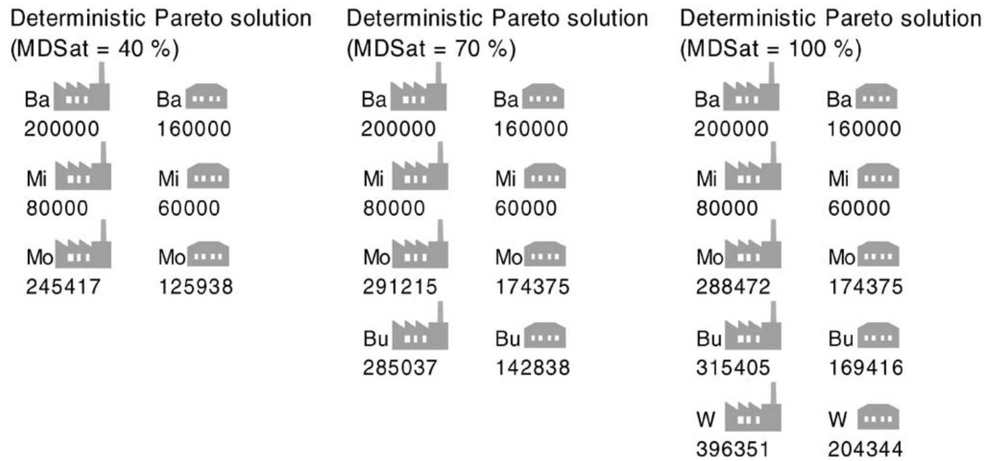

Fig. 6. Deterministic Pareto designs.

this work, the uncertainty associated to such parameter is represented by 100 equiprobable scenarios. The scenarios are generated as follows. In the first place, a Monte Carlo sampling is performed over a set of normal probability distributions which describe the demand associated to product P1 in each market and time interval. In the second place, the demands of P2 and P3 are computed, assuming that the rates between the demands of the different products equal their corresponding deterministic counterparts for each market and time interval. The standard deviations of all the probability distributions are supposed to be equal to 10% for all the markets except for Mo, for which it is assumed to be equal to 30%. In all the cases, the standard deviations increase by 1% per time interval.

The resulting mathematical formulation has 109,946 single equations, 398,129 continuous variables and 13 binary variables, and is also implemented in GAMS (Brooke et al., 1988) and solved using CPLEX 7.0. The time required to obtain solutions with 0% integrality gap on a AMD Athlon 3000 computer ranges from 7200 to 28,800 s depending on the target imposed to the customer satisfaction (note, however, that the major purpose of the work is to propose a SC design framework rather than develop the best efficient solution algorithm). It is important to mention that the number of binary variables in this case is the same as in the deterministic formulation, since they represent first-stage decisions (SC configuration) which are not scenario dependant.

5.2.1. The stochastic Pareto optimal solutions

Fig. 7 shows the Pareto curve obtained for the stochastic problem while in Fig. 8 three SC configurations (location and capacities of the different nodes expressed in kg) which correspond to the different depicted points of the curve. As occurred in the deterministic case, satisfying the demand is profitable until a certain level MDSat = 27.8%. Above this level, there is a trade-off between both objectives, since an increment in the value of MDSat implies a decrease in the associated E[NPV] as shown in the figure. As can be also observed, solutions with higher MDSat imply networks with larger capacities due to the need of covering more demand. For instance, the design which corresponds to MDSat = 40% involves the establishment of two new nodes while four and six new sites are set up when the demand satisfaction level imposed is equal to 60% and 80%, respectively.

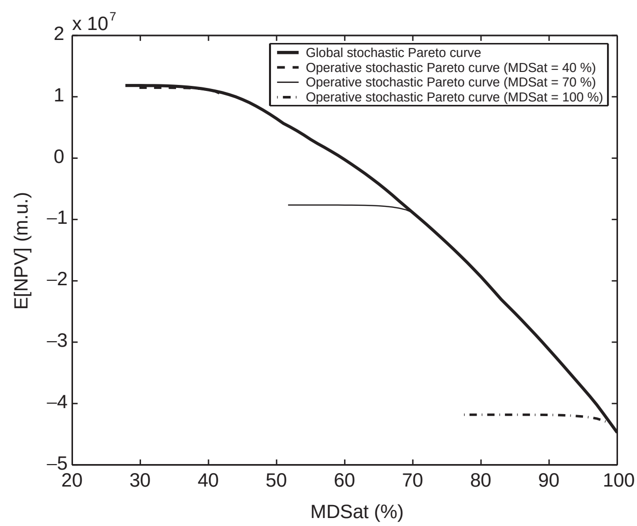

Fig. 7. Stochastic Pareto curve.

As occurred before in the deterministic case, the SC configurations can operate under different demand satisfaction policies once the uncertainty is unveiled thus leading to different E[NPV]s. Therefore, an operative stochastic Pareto curve can be generated for each of these configurations by fixing the design variables in the stochastic formulation and maximising the E[NPV] for different targets of MDSat. The operative Pareto curves corresponding to the configurations with MDSat = 40%, 60% and 80% have been depicted together with the global Pareto stochastic curve in Fig. 7. It can be observed how the last one wraps all the operative

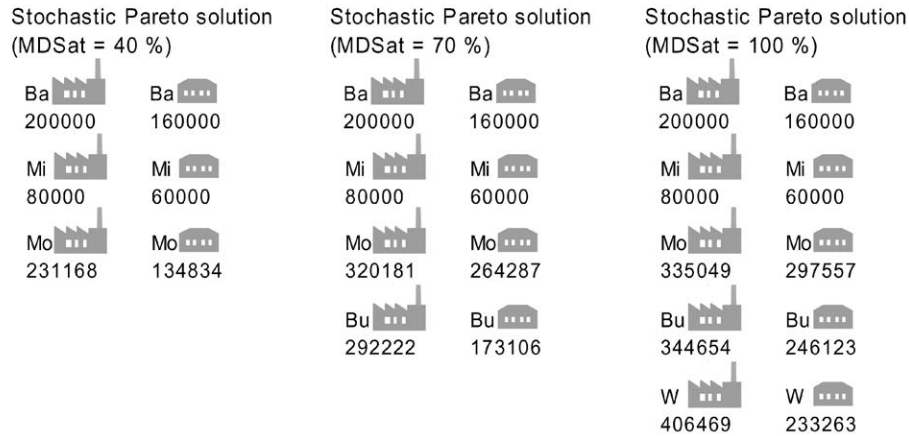

Fig. 8. Stochastic Pareto designs.

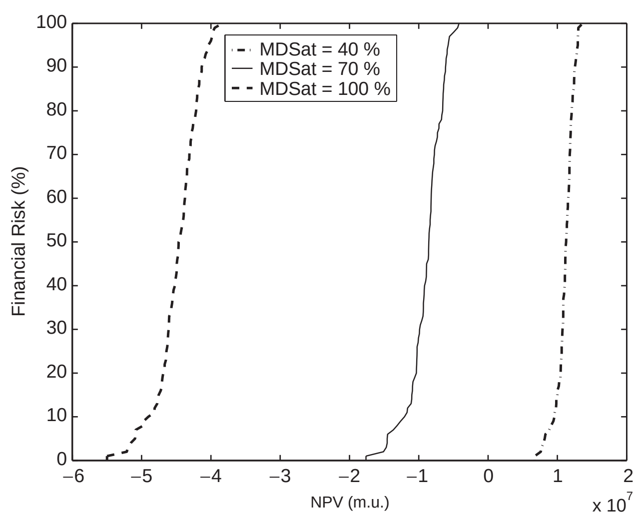

Fig. 9. Risk curves of stochastic Pareto solutions.

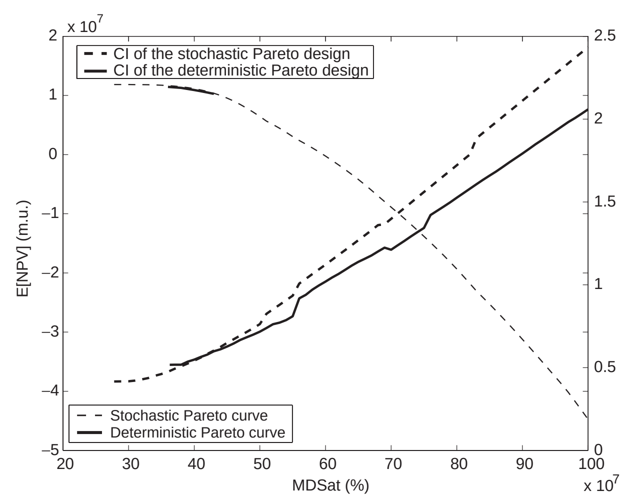

Fig. 10. Deterministic vs. stochastic Pareto solutions.

stochastic Pareto curves of the Pareto designs and touches them at one point for which they are Pareto optimal solutions of the overall problem.

Moreover, Fig. 9 depicts the financial risk curves associated to the same points of the Pareto optimal curve (MDSat = 40%, 60% and 80%). It can be observed in such a figure how the risk curves of the Pareto solutions move to the left as they are forced to fulfil more demand. In other words, SC designs with larger MDSat values exhibit lower E[NPV]s and higher probabilities of lower profits. For instance, for the curve with an MDSat = 40%, the probability of obtaining an NPV lower than 0 m.u. is equal to zero, while such probability increases up to 100% for configurations with MDSat = 70% and 100%.

5.3. *Deterministic vs stochastic solutions*

A comparison between the deterministic and the stochastic solution is performed next, with the aim of measuring the effect of not considering uncertainty in those situations where the environment is actually uncertain. An example of this kind of mistakes may appear when a deterministic model is used to solve problems which are affected by uncertain parameters considering just mean values for such parameters.

In order to perform the aforementioned comparison, the SC designs obtained by solving the proposed multiobjective formulation for the mean scenario are evaluated against the uncertain environment. Such evaluation is carried out as follows.

In the first place, a demand satisfaction level is selected. In the second place, for the selected MDSat, the corresponding values of the design variables (location and capacities of the different nodes of the SC) of the deterministic Pareto solution computed for the mean value scenario are taken. Finally, these variables are fixed as first-stage variables in the stochastic model considering the set of 100 scenarios, and the multiobjective stochastic formulation is next solved accounting for the maximisation of the E[NPV] and constraining the MDSat to be higher than the MDSat target for which the deterministic design was obtained. Such procedure allows to compute the values of the second-stage

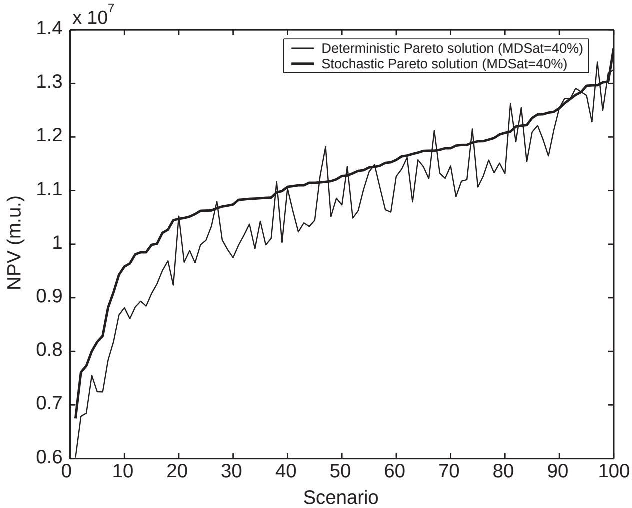

Fig. 11. Comparison of the deterministic and stochastic Pareto designs for MDSat = 40% for all the scenarios.

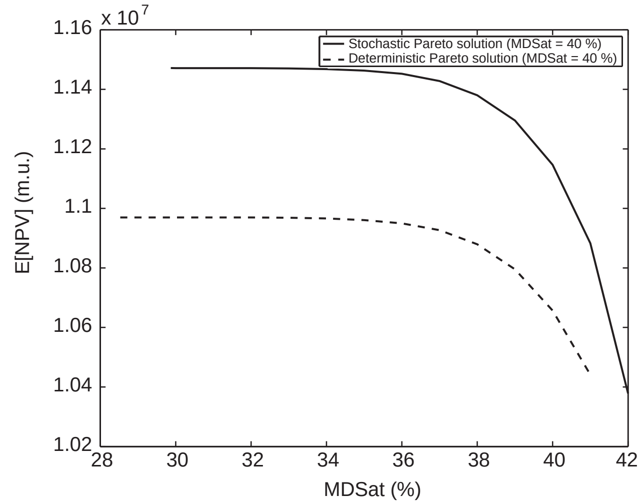

Fig. 13. Comparison of the deterministic and stochastic Pareto designs for MDSat = 40% under different operative policies.

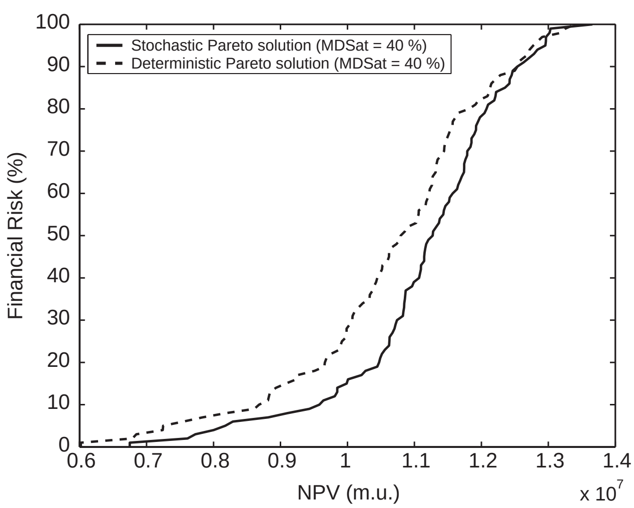

Fig. 12. Risk curves of the deterministic and stochastic Pareto designs for MDSat = 40%.

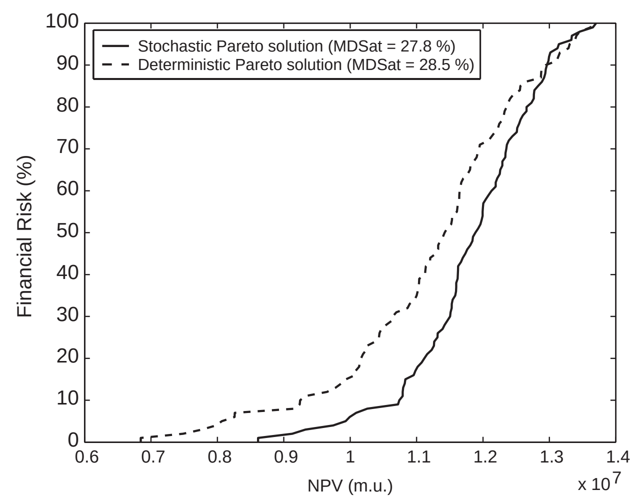

Fig. 14. Risk curves of the deterministic and stochastic Pareto designs without constraining MDSat.

variables for the given first-stage ones. The procedure is finally repeated until the entire set of deterministic Pareto solutions is evaluated through the stochastic formulation. By applying this procedure, it can be seen how almost all the deterministic SC configurations are not able to reach the MDSat for which they were originally designed once the uncertainty is unveiled. Moreover, in Fig. 10, a curve that represents the deterministic SC configurations which manage to keep the MDSats under the uncertain environment has been plotted together with the Pareto stochastic curve. It can be observed how although such designs guarantee the demand satisfaction level previously imposed in the deterministic formulation under all the scenarios, their E[NPV]s are approximately 4–5% lower than those achieved by their corresponding stochastic counterparts, i.e., the SC configurations computed by means of the stochastic formulation considering the 100 scenarios and the same MDSat target. The CI associated to the stochastic and deterministic designs are also depicted in Fig. 10. As can be observed, the CI values associated to the SC configurations computed by means of the stochastic formulation are higher than those corresponding to their deterministic counterparts above MDSat = 41%. The stochastic formulation forces to chose SC configurations with higher capacities which are able to reach the MDSat for which they are designed under all the scenarios and not only for the mean demand scenario that finally leads to SC configurations with higher CI values.

In Fig. 11, the SC configuration calculated by means of the stochastic formulation for MDSat = 40% is compared with its deterministic analogue under all the scenarios. The figure shows that there are many scenarios in which the deterministic configuration yields higher benefits than the stochastic one, although others result in larger losses, which finally leads to a lower E[NPV] (10,656,700 m.u. for

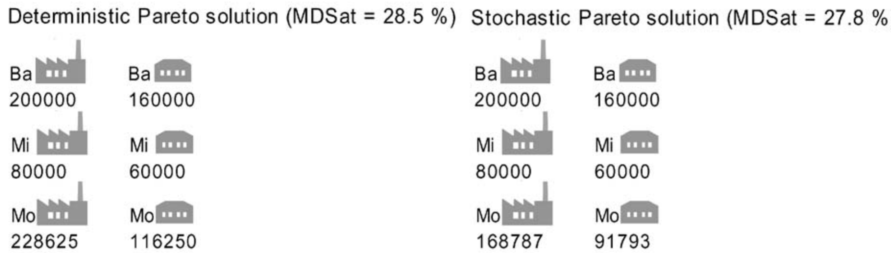

Fig. 15. Designs of the deterministic and stochastic Pareto designs without constraining MDSat.

the deterministic case and 11,146,620 m.u. for the stochastic one). This can be also observed in the NPV histograms corresponding to both SC configurations (Fig. 12). Not only does the deterministic design exhibit less E[NPV] but it also presents higher probabilities of low profits as can be seen in the figure. For instance, the deterministic design exhibits a 14% probability of profits below 6×10^6 m.u., while the stochastic configuration yields a 7%. In addition, in Fig. 13 both designs are compared under different operational policies (MDSat). The associated operative Pareto curves show how the stochastic design performs better than the deterministic one for the entire range of demand satisfaction levels. It is also interesting to point out that the stochastic design is able to operate at most for a demand satisfaction level equal to 42%, while the deterministic one is only able to reach a MDSat = 41%.

Finally, the most profitable deterministic and stochastic SC configurations, i.e., those computed by maximising NPV and without constraining the MDSat for the mean scenario and considering the set of 100 scenarios, have been compared. In Fig. 14, the histograms associated to both designs are depicted. It can be observed how the stochastic design implies less risk at lower profits. For instance, the probability of an NPV lower than 9×10^6 m.u. is equal to 7% for the deterministic design and 1% for the stochastic one. With regard to the SC configurations, which are also given in Fig. 15, it can be observed that the stochastic design exhibits lower capacities and it is thus able to cover less demand than the deterministic one. Such conservative design, whose consequences can be observed in the risk curves, tries to reduce the impact of the high uncertainty associated to the market of Moscow, which finally leads to a higher E[NPV] with respect to its deterministic counterpart (11,242,919 m.u. for the deterministic SC and 11,836,501 m.u. for the stochastic one).

5.4. *Risk management*

In this section, the financial risk has been considered. The resulting multiobjective problem (maximise E[NPV], maximise MDSat and minimise DRisk) has been solved by following the methodology described before.

Therefore, for a MDSat = 37%, it is aimed to reduce the downside risk for a target $\Omega = 9 \times 10^6$ m.u. while maximising the E[NPV]. To achieve such goal, the model is solved for different values of ε_2 (∞, 6, 500, 2, 000 and 250 m.u.). In all these cases holding $\varepsilon_1 = 37\%$. The resulting mathematical formulation has 110,046 single equations, 398,230 continuous variables and 13 binary variables, and is also implemented in GAMS (Brooke et al., 1988) and solved using CPLEX 7.0. The time required to obtain solutions with 0% integrality gap on a AMD Athlon 3000 computer ranges from 7200 to 28,800 s depending on the target imposed to the downside risk calculation. By applying the proposed methodology, different Pareto solutions are obtained. The corresponding risk curves of such solutions are depicted in Fig. 16 together with the deterministic Pareto solution, i.e., the one obtained for the mean scenario with MDSat = 37%.

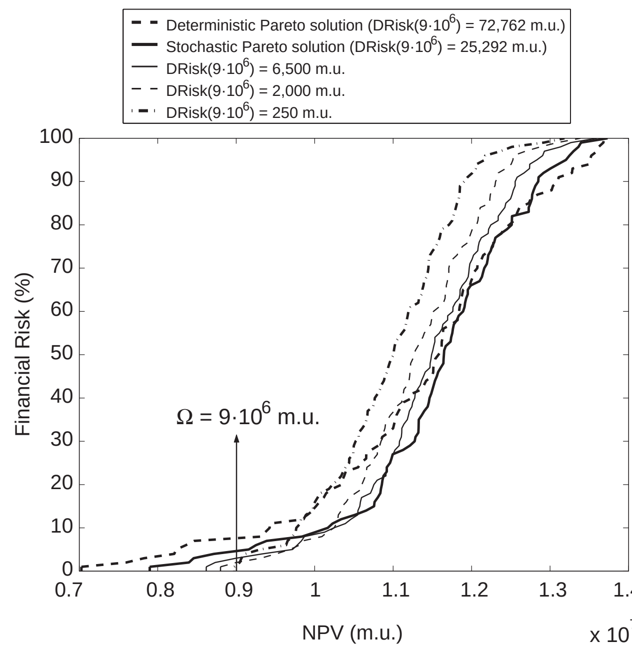

Fig. 16. Risk curves for MDSat = 37%.

As it can be observed, when the downside risk is constrained, E[NPV] is reduced (Fig. 16), thus moving the financial risk curves to the left at the top of the curve and decreasing the probability of low NPVs. For instance, there is a 7% probability of profits bellow 9×10^6 m.u. in the

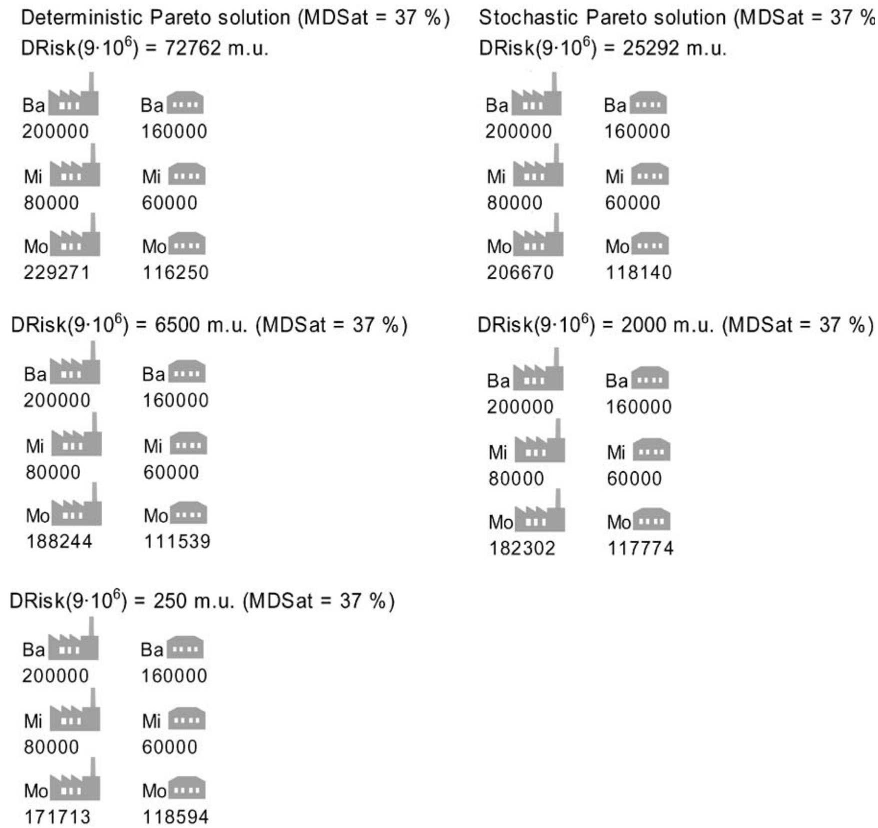

Fig. 17. Designs of the risk Pareto solutions for MDSat = 37%.

deterministic solution (DRisk $= 72,762$ m.u.) and a 4% in the stochastic one (DRisk $= 25,292$ m.u.), while such probability is reduced to 3%, 2%, and 1% in the solutions with DRisk $= 6500, 2000$ and 250 m.u., respectively. These cumulative probability curves clearly intersect, at least in one point, the stochastic optimal curve computed without constraining the downside risk.

The SC configurations associated to each of the aforementioned risk curves are also depicted in Fig. 17. Such designs show how in order to reduce the financial risk, the formulation is forced to reduce the capacity of the plant and warehouse located in the most uncertain market (Mo). This strategy avoids low earnings but on the other hand it leads to poorer chances of high benefits.

6. Conclusions

Determining the optimal SC configuration is a difficult problem since a lot of factors and objectives must be taken into account when designing the network. Therefore, the multiobjective approach developed in this paper seems to be the best way of capturing the high complexity of this problem.

By using this methodology, the trade-off between the considered objectives (Pareto curve) can be obtained not only for the nominal case, but also when there is uncertainty about some of the parameters defining the production/distribution scenario.

In this case, a Pareto stochastic curve can be obtained and the comparison with the equivalent deterministic one has demonstrated the convenience of using the stochastic formulation. The effects of these uncertainty can be accounted as a risk associated to the NPV of the investment, which has been introduced as an additional objective into the model. Then, this risk can be managed to reduce the probability of having low earnings derived from the investment.

The interaction between the design objectives has been shown. This way of generating different possible configurations will help the decision-maker determine the best design according to the selected objectives.

Notation

A_i binary variable ($A_i = 1$ if i is opened, $A_i = 0$ otherwise)
AIL_{pjts} average inventory level of p at j during t in s
B_j binary variable ($B_j = 1$ if j is opened, $B_j = 0$ otherwise)
CF_{ts} cash flow during t in s
CI capital investment

DE_{ts}	direct expenses during t in s
Dem_{pkts}	demand of p at k during t in s
Dep_t	amount depreciated in t
$DRisk(x, \Omega)$	downside risk associated with design x at target level Ω
$DSat_{ts}$	total demand satisfaction during t in s
$E[DSat]$	expected demand satisfaction
$E[NPV]$	expected net present value
FCI	fixed capital investment
FCI^L_{Pi}	fixed cost parameter at i
FCI^L_{WHj}	fixed cost parameter at j
HC_{pj}	unit handling cost of p at j
ir	interest rate
IC_{pj}	unit inventory cost of p at j
IE^L_{Pi}	indirect expenses parameter at i
IE^L_{WHj}	indirect expenses parameter at j
IE_t	indirect expenses during t
Inv_{pjts}	amount of p kept at j during t in s
$MDem_{pkt}$	mean demand of p at k during t
MDSat	demand satisfaction
nd	number of time intervals for depreciation
NPV	net present value
NPV_s	net present value in s
$PCap_i$	capacity of i
$PCap^L_i$	lower capacity bound of i
$PCap^U_i$	upper capacity bound of i
$Price_{pkt}$	price of p at k during t
$prob_s$	probability of s
Q_{pits}	amount of p produced at i during t in s
Rev_{ts}	revenues in s during t
$Risk(x, \Omega)$	risk associated with design x at target level Ω
$Sales_{pkts}$	sales of product p at k in s during t
SV	salvage value
tr	taxes rate
T	number of time intervals in the time horizon
Tax_{ts}	taxes in s during t
$TC1_{pij}$	unit transport cost of p between i and j
$TC2_{pjk}$	unit transport cost of p between j and k
VC_{pi}	unit production cost of p at i
WC	working capital
$WHCap_j$	capacity of j
$WHCap^L_j$	lower capacity bound of j
$WHCap^U_j$	upper capacity bound of j
x	generic design variable
X_{pijts}	amount of p transported from i to j during t in s
Y_{pjkts}	amount of p transported from j to k during t in s
z_s	binary variable ($z_s = 1$ if $NPV_s < \Omega$, $z_s = 0$ otherwise)

Greek letters

α_{pi}	production capacity factor of p at i
β_{pj}	storage capacity factor of p at j
γ_{Pi}	fixed cost coefficient at i
γ_{WHj}	fixed cost coefficient at j
δ_s	auxiliar variable for downside risk definition
ε_0	bound level for 0
η_{Pi}	indirect expenses coefficient at i
η_{WHj}	indirect expenses coefficient at j
λ_j	turnover inventory rate at j
μ	working capitalfactor
Ω	aspiration target level of profit

Subscripts

i	plants
j	warehouses
k	markets
o	objectives
p	products
s	scenarios
t	time intervals

Superscripts

L	lower bound of the variables
U	upper bound of the variables

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References

Acevedo, J., Pistikopoulos, E.N., 1998. Stochastic optimisation based algorithms for process synthesis under uncertainty. Computer and Chemical Engineering 22, 647–671.

Ahmed, S., Sahinidis, N.V., 1998. Robust process planning under uncertainty. Industrial and Engineering Chemistry Research 37, 1883–1892.

Anupindi, R., Bassok, Y., 1999. Supply Chain Contracts with Quantity Commitments and Stochastic Demand. Quantitative Models for Supply Chain Management, Kluwer Academic Publishers, Norwell, MA.

Applequist, G.E., Pekny, J.F., Reklaitis, G.V., 2000. Risk and uncertainty in managing manufacturing supply chains. Computer and Chemical Engineering 24, 2211–2222.

Azapagic, A., Clift, R., 1999. The application of life cycle assessment to process optimisation. Computer and Chemical Engineering 10, 1509–1526.

Barbaro, A.F., Bagajewicz, M.J., 2004. Managing financial risk in planning under uncertainty. A.I.Ch.E. Journal 50 (5), 963–989.

Bernardo, F., Pistikopoulos, E., Saraiva, P., 1999. Integration and computational issues in stochastic design and planning optimization problems. Industrial and Engineering Chemistry Research 38, 3056–3068.

Biegler, L.T., Grossmann, I.E., Westerberg, A.W., 1997. Systematic Methods of Chemical Process Design, Prentice-Hall PTR, New Jersey.

Birge, J.R., Louveaux, F., 1997. Introduction to Stochastic Programming, Springer, New York.

Bok, J.K., Grossmann, I.E., Park, S., 2000. Supply chain optimization in continuous flexible process networks. Industrial and Engineering Chemistry Research 39, 1279–1290.

Bose, S., Pekny, J.F., 2000. A model predictive framework for planning and scheduling problems: a case study of consumer goods supply chain. Computer and Chemical Engineering 24, 329–335.

Brooke, A., Kendrick, D., Meeraus, A., 1988. GAMS—A User's Guide.

Chen, C.L., Wang, B.W., Lee, W.C., 2003. Multi-objective optimization for a multi-enterprise supply chain network. Industrial and Engineering Chemistry Research 42, 1879–1889.

Cohen, M.A., Lee, H.L., 1989. Resource deployment analysis of global manufacturing and distribution networks. Journal of Manufacturing and Operations Management 2, 81–104.

Eppen, G.D., Martin, R.K., Schrage, L., 1989. A scenario approach to capacity planning. Operations Research 37, 517–527.

Fox, M., Barbuceanu, M., Teigen, R., 2000. Agent-oriented supply chain management. The International Journal of Flexible Manufacturing Systems 12, 165–188.

Gjerdrum, J., Shah, N., Papageorgiou, L.G., 2000. A combined optimisation and agent-based approach for supply chain modelling and performance assessment. Production Planning and Control 12, 81–88.

Gjerdrum, J., Shah, N., Papageorgiou, L.G., 2001. Transfer prices for multienterprise supply chain optimisation. Industrial and Engineering Chemistry Research 40, 1650–1660.

Gupta, A., Maranas, C.D., 2000. A two-stage modelling and solution framework for multisite midterm planning under demand uncertainty. Industrial and Engineering Chemistry Research 39, 3799–3813.

Gupta, A., Maranas, C.D., 2003. Managing demand uncertainty in supply chain planning. Computer and Chemical Engineering 27, 1219–1227.

Haimes, Y.Y., Lasdon, L.S., Wismer, D.A., 1971. On a bicriterion formulation of the problems of integrated system identification and system optimization. IEEE Transactions on Systems, Man and Cybernetics 1, 296–297.

Ierapetritou, M.G., Pistikopoulos, E.N., 1996. Batch plant design and operations under uncertainty. Industrial and Engineering Chemistry Research 35, 772–787.

Iyer, R.R., Grossmann, I.E., 1998. A bilevel decomposition algorithm for long-range planning of process networks. Industrial and Engineering Chemistry Research 37, 474–481.

Miettinen, K.M., 1999. Nonlinear Multiobjective Optimization, Kluwer Academic Publishers, Boston.

Perea-López, E., Ydstie, B.E., Grossmann, I.E., 2003. A model predictive control strategy for supply chain optimization. Computer and Chemical Engineering 27, 1201–1218.

Petkov, S.B., Maranas, C.D., 1997. Multiperiod planning and scheduling of multipurpose batch plants under demand uncertainty. Industrial and Engineering Chemistry Research 36, 4864–4881.

Rodera, H., Bagajewicz, M.J., Trafalis, T.B., 2002. Mixed-integer multiobjective process planning under uncertainty. Industrial and Engineering Chemistry Research 41, 4075–4084.

Sakawa, M., Nishizaki, I., Uemura, Y., 2001. Fuzzy programming and profit and cost allocation for a production and transportation problem. European Journal of Operational Research 131, 1–15.

Simchi-Levi, D., Kamisky, P., Simchi-Levi, E., 2000. Designing and Managing the Supply Chain. Concepts, Strategies, and Case Studies, Irwin McGraw-Hill,

Subrahmanyam, S., 1996. Issues in the design and planning of batch chemical plants. Ph.D. Thesis, Purdue University, West Lafayette, USA.

Timpe, C.H., Kallrath, J., 2000. Optimal planning in large multi-site production networks. European Journal of Operational Research 126, 422–435.

Tsiakis, P., Shah, N., Pantelides, C.C., 2001. Design of multi-echelon supply chain networks under demand uncertainty. Industrial and Engineering Chemistry Research 40, 3585–3604.

Zhou, Z., Cheng, S., Hua, B., 2000. Supply chain optimization of continuous process industries with sustainability considerations. Computer and Chemical Engineering 24, 1151–1158.