Risk Management in the Scheduling of Batch Plants under Uncertain Market Demand

Anna Bonfill,[†] Miguel Bagajewicz,[‡] Antonio Espuña,[†] and Luis Puigjaner*[,†]

Department of Chemical Engineering, Universitat Politècnica de Catalunya, ETSEIB, Avenida Diagonal, 647, E-08028 Barcelona, Spain, and School of Chemical Engineering and Materials Science, University of Oklahoma, 100 E. Boyd Street, T-335, Norman, Oklahoma 73019

A stochastic optimization approach is presented to manage risk in the short-term scheduling of multiproduct batch plants with uncertain demand. The problem is modeled using a two-stage stochastic optimization approach accounting for the maximization of the expected profit. The model is also extended to incorporate the availability of option contracts, thus providing significant flexibility within the uncertain environment. Management of risk is explicitly addressed by adding a control measure as a new objective to be considered, thus leading to multiobjective optimization formulations. Three alternative methodologies are assessed and compared. The importance of considering the uncertainty not only in the decision-making process but also in the control of the variability of outcomes is highlighted. Parametric solutions appealing to decision makers with different attitudes toward risk are obtained.

Introduction

The scheduling problem in the chemical industry has been extensively studied, and alternative methodologies and problem statements with different considerations have been proposed in the literature to address the combinatorial character of these problems.[1] However, most of the formulations presented are deterministic, that is, they are based on nominal parameter values and do not consider uncertainty when planning and scheduling operations. In a real environment, various sources of uncertainty in process planning and scheduling can be identified, including not only operating parameter variations, such as equipment breakdowns or processing time fluctuations, but also uncertain market trends. The uncertainty from a real environment entails a risk that initially impacts cost but might eventually lead to an infeasible situation. Some attempts in scheduling under uncertainty have focused mainly on rescheduling algorithms, which are implemented when the uncertainty is actually realized and do not consider the uncertain information prior to scheduling.[2] Moreover, with these approaches, financial risk cannot be controlled. Thus, the incorporation of uncertainty in such parameters at the decision-making level is imperative and not merely a marginal improvement.

Models addressing uncertainty prior to scheduling have recently been developed. Petkov and Maranas[3] treated the multiperiod planning problem for multiproduct plants under demand uncertainty. They maximized the expected profit and used chance constraints to impose a limit on the probability level of demand satisfaction. Lee and Malone[4] proposed a probabilistic approach based on the combination of Monte Carlo simulation and a simulated annealing algorithm to obtain a schedule able to handle uncertainties in parameters of batch process scheduling. Vin and Ierapetritou[5] used a multiperiod mathematical formulation to address the scheduling problem of batch plants under demand uncertainty and proposed several metrics to analyze the schedule robustness, although these were not incorporated into the decision-making process. Furthermore, Engell et al.[6] presented a two-stage stochastic programming approach to address the scheduling of a multiproduct batch plant with uncertain market requirements. Finally, Harjunkoski and Grossmann[7] discussed the use of mixed integer and constraint programming in multistage scheduling, and Balasubramanian and Grossmann[8] proposed the application of fuzzy set theory to solve the problem, thus departing from the use of probabilistic models. Although stochastic models optimize the total expected performance measure, they do not provide any control on their variability over different scenarios, i.e., they assume that the decision maker is risk-neutral. However, different attitudes toward risk might be encountered. In general, most decision makers are risk-averse, implying a major preference for lower variability for a given level of return.

The aim of the present work is to provide a tool to support decision making during the development of a scheduling policy in an uncertain market environment while incorporating the tradeoff between risk and profit at the decision level. For this reason, and for the purpose of incorporating the uncertain parameters, a stochastic approach is pursued.

Most of the mathematical programming approaches in planning and scheduling that are proposed in the operations research literature and that can serve as a basis for the stochastic formulation are based on a continuous- or a discrete-time representation. (An overview is given by Reklaitis.[9]) Discrete-time representations introduce uniform or nonuniform discretizations of time to handle resource constraints, enforcing these constraints at the discrete time points.[10,11] In this case, a considerable number of integer variables is required for the proper representation of the problem. Continuous-time formulations rely on the partitioning of the scheduling horizon into time intervals of unknown

* To whom correspondence should be addressed. E-mail: luis.puigjaner@upc.es. Tel.: +34 934 016 678. Fax: +34 934 010 979.

[†] Universitat Politècnica de Catalunya.

[‡] University of Oklahoma.

10.1021/ie030529f CCC: $27.50
Published on Web 01/08/2004

duration.[12−14]. Despite the limitations of general-purpose optimization algorithms, mathematical programming allows for the easy incorporation of particular operation modes of the production system and is thus considered a standard method for determining optimal production schedules.

In this work, a new two-stage stochastic MILP formulation based on a batch slot concept that does not assume any time representation is first presented. This formulation is further extended to incorporate the availability of option contracts, which should implicitly facilitate the management of risk. Alternate explicit risk measures are finally appended and assessed[15−17] to obtain a spectrum of solutions reflecting different decision-maker attitudes.

This paper is organized as follows: The problem definition is outlined first, along with the assumptions considered. The proposed stochastic formulation accounting for the maximization of the expected profit is detailed in section 3, and in section 4, the formulation is extended to incorporate option contracts. The inclusion of explicit control measures to manage risk is addressed in section 5, and the effectiveness of the proposed approach as a decision-making tool is demonstrated through its application to a scheduling case study in section 6. Finally, concluding remarks are given in section 7.

Problem Statement

The scheduling problem of a multiproduct batch plant incorporating demand uncertainty is addressed to obtain alternate scheduling policies reflecting different attitudes toward risk. The proposed stochastic model accounts for the maximization of the expected profit. The production lines, a set of products to be produced with their given recipes, the time horizon, the economic data, and the probability distributions associated with the uncertain parameters are given. The scheduling decisions involve the number of batches of each product to be produced, the detailed production sequence, and the start and end times of each operation performed. The following assumptions are made:

(1) The uncertainty associated with product demands can be represented indifferently by discrete or continuous probability distributions. For the case study presented in this paper, a normal distribution is assumed, which can be justified on the basis of the central limit theorem considering that product demands are affected by a large number of stochastic events.[3]

(2) One production line with a fixed assignment of equipment units to tasks and fixed batch sizes for each product is considered. This assumption can be easily relaxed with slight modifications in some constraints.

(3) The zero-wait transfer policy is adopted. Under this policy, an intermediate product must be transferred immediately to the next processing step just after its production. Neither intermediate storage nor waiting time in the processing units is available. This assumption could easily be modified to consider unlimited-intermediate-storage (UIS) or no-intermediate-storage (NIS) transfer policies.

(4) Scheduling is addressed for a time horizon of 1 week. It is assumed that products have to be delivered at the end of the week under a just-in-time (JIT) policy, but to meet customer demands, scheduling decisions must be made beforehand.

(5) Fixed costs for final inventory and shortage are adopted for each product. To consider the difficulties that arise with the change of products and to avoid excessive shifts between them, penalties for product changeovers are also taken into account.

(6) All cost parameters are assumed to be in monetary units (mu's), and demand and product levels are in weight units (wu's).

Scheduling Model

A stochastic programming approach based on a recourse model with two stages is proposed in this work to incorporate demand uncertainties into scheduling.

In a two-stage stochastic optimization approach, each uncertain model parameter is considered as a random variable with an associated probability distribution, and the decision variables are classified into two stages. The first-stage variables correspond to those decisions that need to be made "here and now", prior to the realization of the uncertainty. The second-stage or recourse variables correspond to those decisions made after the uncertainty is revealed and are usually referred to as wait-and-see decision variables. After the first-stage decisions are made and the random events realized, the second-stage decisions are made subject to the restrictions given by the second-stage problem. Because the performance associated with the second-stage decisions is uncertain at the first stage, the objective function consists of the sum of the first-stage performance measure and the expected second-stage performance. (For an overview of stochastic techniques, see ref 18).

Product demands represented by continuous probability distributions can be discretized using Monte Carlo sampling, thus generating a set of explicit scenarios. When discrete probability distributions are used to characterize the uncertainty, Monte Carlo sampling can be also performed if a reduction of the state space is desired. The scenario-based representation of the uncertainty avoids high-dimensional numerical integration in the solution of the problem, because the expectations can be written as finite sums and each constraint can be duplicated for each scenario, i.e., a second-stage variable can be associated with each realization of the random data.

An MILP formulation is derived according to a batch slot concept. With this formulation, the time horizon is viewed as a sequence of batches, each of which will be assigned to one particular product. Decision variables related to the number of batches of each product to be produced and the detailed schedule, that is, the product sequence and the starting and finishing times of each operation, are considered as first-stage decisions as it is assumed that these decisions have to be made at the scheduling stage before the uncertainty is revealed. In the second stage, the sales, inventory, and unsatisfied orders are evaluated for each possible scenario. At the end of the scheduling horizon, a different profit value is realized for each particular realization of the demand uncertainty. The proposed model (SCHED) accounts for the maximization of the expected value of this profit distribution, as described next.

SCHED

$$\max \text{EPV} = \sum_k (\omega_k \{\sum_j [\gamma_j Q^{S}_{jk} - c^{I}_j\, Q^{I}_{jk} - c^{U}(\gamma_j - c^{P}_j) Q^{U}_{jk}]\}) - \sum_j c^{P}_j\, Q^{P}_j - \sum_{b,j,j'} c^{c}_{jj'} \text{XM}_{bjj'} - \alpha \sum_{i,b} \text{Tin}_{ib} \quad (1)$$

subject to

$$H \geq \text{Tfn}_{ib} \qquad \forall i, b \quad (2)$$

$$T_{ib} = \sum_j (X_{bj} \text{Top}_{ij}) \qquad \forall i, b \quad (3)$$

$$\text{Tfn}_{ib} = \text{Tin}_{ib} + T_{ib} \qquad \forall i, b \quad (4)$$

$$\text{Tin}_{ib'} \geq \text{Tfn}_{ib} \qquad \forall i, b < B, b' = b + 1 \quad (5)$$

$$\text{Tfn}_{ib} = \text{Tin}_{i'b} \qquad \forall i, i' = i + 1, b \quad (6)$$

$$\sum_j X_{bj} \leq 1 \qquad \forall b \quad (7)$$

$$\sum_b X_{bj} = n_j \qquad \forall j \quad (8)$$

$$X_{bj} + X_{b'j'} - 1 \leq \text{XM}_{bjj'} \qquad \forall b < B, b' = b + 1, j, j' \quad (9)$$

$$\sum_j n_j \leq B \quad (10)$$

$$Q^{P}_j = n_j \text{BS}_j \qquad \forall j \quad (11)$$

$$Q^{S}_{jk} = \min(\theta_{jk}, Q^{P}_j) \qquad \forall j, k \quad (12)$$

$$Q^{I}_{jk} = Q^{P}_j - Q^{S}_{jk} \qquad \forall j, k \quad (13)$$

$$Q^{U}_{jk} = \theta_{jk} - Q^{S}_{jk} \qquad \forall j, k \quad (14)$$

As was previously stated, the model accounts for the maximization of the expected profit value, eq 1, taking into account first-stage costs related to variable production costs and costs for product changeovers and an expected second-stage performance embedding the sales of each product, inventory costs, and a penalty for underproduction. Therefore, the maximization of the objective function establishes the most appropriate production policy that balances benefits with inventory costs (which control the overproduction) and the cost for production shortfalls (which measures the loss of profit due to the unavailability of a product and is modeled with a factor, c^{U}, of this profit value. For $c^{U} = 1$, the underproduction cost equals the profit lost due to the unsatisfied demand; higher or lower values of this parameter impose a stricter or more relaxed safeguard against underproduction (a product demand satisfaction level is not explicitly imposed). The last term on the right-hand side of the objective function is a timing term, which was incorporated to reduce degeneracy and to ensure that the operations will start as soon as possible when some float time exists. α is a parameter with a very small value that does not modify the optimality related to the rest of the terms in the objective function (e.g., α is on the order of 10^{-5} in the proposed case study).

The constraints in eqs 2−11 represent the first-stage constraints, which define the sequence and precedence constraints, the timing, and the number of batches to be produced. Equation 2 expresses the requirement that all tasks end within the time horizon H. Equation 3 is incorporated to assign to each batch the operation times of the product produced in that batch. The binary variable X_{bj} takes the value of 1 if product j is produced in batch b and 0 otherwise. The connections between the start and end times of each stage i within a batch b are provided by eq 4. To express the requirement that the initial time of every stage from one batch is after the same operation in the previous batch, the precedence constraint in eq 5 is used. In the same way, the sequence constraint in eq 6 ensures the zero-wait transfer policy within one batch. According to eq 7, at most one product j can be assigned to each batch b. In eq 8, the number of batches assigned to each product j is constrained to be the number of batches n_j produced of that product.

When a change from product j to product j' occurs, both X_{bj} and $X_{b'j'}$ take the value of 1, and the penalty term is active. However, to avoid the introduction of nonlinearities into the model, the aggregated variable $\text{XM}_{bjj'}$ is defined, which takes the value of 1 if a changeover from product j to product j' occurs and 0 otherwise (eq 9). Through eq 10, the number of batches processed is limited to the maximum number of batches defined, B. Finally, eq 11 defines the amount of each product produced.

Second-stage constraints, which evaluate, for each product in each scenario, the quantity sold, the final inventory requirements, and the production shortfalls at the end of the time horizon, are defined in eqs 12−14. Because an amount of product higher than the amount produced cannot be delivered, the quantity sold of each product is the minimum between the demand and the amount produced. (Equation 12 is handled internally within the modeling environment by two inequality constraints.)

Scheduling Model with Option Contracts

With the aim of implicitly facilitating the management of risk from an operational perspective, the possibility of selling some amount of products by exercising an option contract is considered and introduced into the model. Options are contracts that give the holder the possibility of purchasing a certain quantity of product at a specified price. Because the contracts are signed beforehand, the total amount of product that can be sold by exercising the respective put option, Q^{OC}_j, has to be considered independently of the scenario. However, the amount of products sold coming from option contracts, Q^{SOC}_{jk}, varies under the different scenarios.

Therefore, in addition to the number of batches and the scheduling decisions, the first-stage variables in this model (SCHEDOC) include the amount of options purchased. In the second stage, the quantity of product j sold in scenario k coming from a contract is assessed.

The new stochastic model formulation is as follows

SCHEDOC

$$\max \text{EPV} = \sum_k (\omega_k \{\sum_j [\gamma_j Q^{S}_{jk} + \gamma^{OC}_j\, Q^{SOC}_{jk} - c^{I}_j\, Q^{I}_{jk} - c^{U}(\gamma_j - c^{P}_j) Q^{U}_{jk}]\}) - \sum_j (c^{OC}_j\, Q^{OC}_j + c^{P}_j\, Q^{P}_j) - \sum_{b,j,j'} c^{c}_{jj'} \text{XM}_{bjj'} - \alpha \sum_{i,b} \text{Tin}_{ib} \quad (15)$$

subject to

eqs 2−11

$$Q^{S}_{jk} + Q^{SOC}_{jk} = \min(\theta_{jk}, Q^{P}_{j} + Q^{OC}_{j}) \qquad \forall j, k \quad (16)$$

$$Q^{I}_{jk} = Q^{P}_{j} + Q^{OC}_{j} - Q^{S}_{jk} - Q^{SOC}_{jk} \qquad \forall j, k \quad (17)$$

$$Q^{U}_{jk} = \theta_{jk} - Q^{S}_{jk} - Q^{SOC}_{jk} \qquad \forall j, k \quad (18)$$

$$Q^{S}_{jk} \leq Q^{P}_{j} \qquad \forall j, k \quad (19)$$

$$Q^{SOC}_{jk} \leq Q^{OC}_{j} \qquad \forall j, k \quad (20)$$

Equation 16 expresses the sales of each product in each scenario as the minimum between the demand θ_{jk} and the available quantity, which is the product produced plus the product coming from an option contract, Q^{OC}_{j}. (As for eq 11, this constraint is translated within the modeling environment into two inequality constraints.) Inventory requirements and production shortfalls are computed in constraints 17 and 18, respectively. Sales coming from the production of a given product for a given scenario, Q^{S}_{jk}, cannot be higher than the amount produced. This is expressed by constraint 19. On the other hand, the amount of product sold due to an option contract, Q^{SOC}_{jk}, must be lower than the total amount contracted, as stated by inequality 20.

Risk Management

The proposed stochastic optimization approach attempts to account for uncertainty by optimizing the expected profit without reflecting and controlling the variability of performances associated with each specific scenario. Although the schedules obtained could be considered more robust than the deterministic ones based on nominal parameter values, by taking a purely expected profit maximization perspective, the model assumes that the decision maker is risk-neutral or indifferent to profit variability. Therefore, there is no guarantee that the process will perform at a certain level over all of the uncertain parameters space. The only guarantee is that the average is optimized.[19,20]

The idea underlying risk management is the incorporation of the tradeoff between risk and profit within the decision-making process, thus leading to a multiobjective optimization problem in which the expected performance and the risk measure are the two objectives. Different criteria for assessing the variability of performance have been proposed in the literature.[19,21] Recent applications are also reported in the area of process design and long-term planning under the concept of both risk management approaches[15,22] and robust optimization models.[20] These works pursue the same objective of finding a set of solutions reflecting different individual criteria when making decisions under uncertainty. The applicability of some of these metrics in scheduling under uncertainty is assessed next.

The variance in performance for a given portfolio is one of the metrics commonly used for quantifying variability. However, two significant drawbacks of this approach to risk management have been identified. On one hand, the variance is a symmetric measure of dispersion around the expected value; therefore, in an attempt to reduce the dispersion of the values around the mean, some decisions leading to favorable results are discarded. On the other hand, this approach introduces nonlinearities into the model, thus increasing the computational requirements of these models.

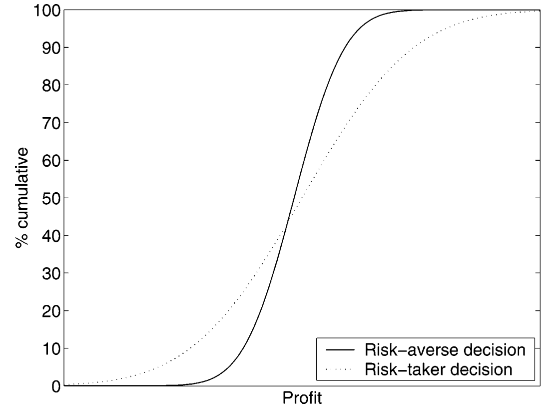

Figure 1. Examples of financial risk curves.

In view of these limitations, three alternative approaches for integrating risk management have been considered and appended as a second criterion to the objective function: the financial risk definition analyzed by Barbaro and Bagajewicz,[15] the downside risk definition proposed by Eppen et al.,[17] and the worst-case performance.

To understand and assess more easily the tradeoffs between risk and profit, the risk curve is used, which is the cumulative curve that indicates the level of incurred risk at each profit. Here, the idea is to obtain schedules that guarantee some expected profit at reduced risk exposures. Depending on the decision maker's attitude toward risk, low risk for some conservative profit aspiration levels or low risk at higher profit aspiration levels (even if risk at lower profit values increases) would be desired. Hypothetical examples of these extremes are depicted in Figure 1.

Financial Risk. Financial risk[15] is a probabilistic approach to risk management defined as the probability of not meeting a certain target profit Ω and is mathematically expressed as

$$\mathrm{FR}_{\Omega} = \sum_{k} \omega_k Z_{k\Omega} \quad (21)$$

where

$$Z_{k\Omega} = \begin{cases} 1 & \text{if } \mathrm{PV}_k < \Omega \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \quad (22)$$

The risk term is included in the objective function as stated by

$$\rho \mathrm{FR}_{\Omega} = \sum_{k} \omega_k \rho Z_{k\Omega} \quad (23)$$

where the goal programming weight ρ is incorporated to manage the tradeoff between the two criteria.

To force the new integer variable $Z_{k\Omega}$ to take the value of 1 if the profit is less than the corresponding target value Ω, constraint 24 is required. Otherwise, a value of 0 in the optimal solution is assured by the risk term in the objective function

$$\mathrm{PV}_k \geq \Omega - U Z_{k\Omega} \qquad \forall k \quad (24)$$

The PV_k term includes all sales and cost terms defined in the objective function (eqs 1 and 15), except for the timing term.

Downside Risk. Downside risk[17] is an alternative risk measure defined as the expected value of the

positive deviation from the target, δ_Ω

$$\mathrm{DR}_\Omega = E(\delta_{k\Omega}) \tag{25}$$

where

$$\delta_{k\Omega} = \begin{cases} \Omega - \mathrm{PV}_k & \text{if } \mathrm{PV}_k < \Omega \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \tag{26}$$

It is worthwhile to note that, whereas financial risk is defined as a probability, downside risk is an expected value.

The incorporation of this metric within the modeling framework requires the addition of the following term in the objective function and the additional set of constraints defined by eqs 28 and 29

$$\rho \mathrm{DR}_\Omega = \sum_k \omega_k \rho \delta_k \tag{27}$$

$$\delta_k \geq \Omega - \mathrm{PV}_k \qquad \forall k \tag{28}$$

$$\delta_k \geq 0 \qquad \forall k \tag{29}$$

Barbaro and Bagajewicz[15] showed the following quantitative relationship between the financial and downside risk measures

$$\mathrm{DR}_\Omega = \int_{-\infty}^{\Omega} \mathrm{FR}_\xi \, \mathrm{d}\xi \tag{30}$$

Therefore, downside risk is defined as the area under the risk curve from profit $\xi = -\infty$ to $\xi = \Omega$, where Ω is the profit target. Using downside risk in the framework of two-stage stochastic models results in multiobjective problems and solution stages similar to those obtained in the case of financial risk. The advantage of using downside risk is that one can obtain a spectrum of solutions with different risks without the need of introducing binary variables. The only known problem is that downside risk is not monotone with risk, that is, one solution having a smaller downside risk than another does not necessarily present a smaller financial risk. This was illustrated by Barbaro and Bagajewicz.[15]

Worst-Case Scenario. The worst-case profit is also adopted as an alternative metric to control or reduce the probability of meeting unfavorable scenarios. A major difference with respect to the other approaches is that the probability information of the problem cannot be used. Moreover, both the expected profit and the profit in the worst-case scenario are to be maximized.

The term defined in eq 31 has to be incorporated into the objective function, and the additional constraint defined in eq 32 is also required

$$\rho \mathrm{WC} = \rho \mathrm{WPV} \tag{31}$$

$$\mathrm{WPV} \leq \mathrm{PV}_k \qquad \forall k \tag{32}$$

To illustrate and assess the effectiveness of these approaches to risk management, each of these approaches is applied to a scheduling case study as described in the following section.

Case Study

The methodology for incorporating risk management into the framework of two-stage stochastic programming for scheduling under uncertainty is tested using a modified description of the example presented by Petkov and Maranas.[3]

The multiproduct batch plant considered consists of one defined production line involving four production stages. Five different products are to be produced within a time horizon H of 1 week (168 h). To represent the demand uncertainty, 100 independent scenarios were simulated through Monte Carlo sampling from the given probability distributions.

The problem data related to batch sizes, processing times, sales prices, costs, and expected demand values for each product is given in Tables 1–5. A value for the production shortfall cost, c^{U}, of 2 is used throughout the example. The standard deviations of demands are assumed to be 50% of their mean values. Although this deviation is high, it makes sense in such a short time horizon where the required amounts can vary from null orders to some considerable quantities.

To solve all of the models, the formulations were implemented in GAMS[23] and solved using the MIP solver of CPLEX (7.0) on an AMD Athlon 2000 computer.

First, the deterministic problem with nominal demand values was solved, and the schedule thus obtained was evaluated for the different scenarios, i.e., the profit values that would be obtained for each particular scenario were computed. The deterministic formulations

Table 1. Batch Sizes, BS_j

product	batch size
j1	60
j2	80
j3	100
j4	60
j5	60

Table 2. Processing Times, Top_{ij}

product	stage 1	stage 2	stage 3	stage 4
j1	10	4	10	1
j2	3	10	6	12
j3	4	12	6	10
j4	16	3	8	4
j5	7	2	5	3

Table 3. Sale Prices, γ_j, γ_j^{OC}; Production Costs, c_j^{P}; Costs for Option Contracts, c_j^{OC}; and Inventory Costs, c_j^{I}

product	γ_j	γ_j^{OC}	c_j^{P}	c_j^{OC}	c_j^{I}
j1	9	9	4.5	5	0.8
j2	9	9	4.5	5	0.8
j3	12	12	6	6.5	1
j4	12	12	6	6.5	1
j5	8	8	4	4.5	0.6

Table 4. Expected Product Demands

product	demand
j1	180
j2	160
j3	300
j4	120
j5	150

Table 5. Costs for Product Changeovers, $c_{jj'}^{\mathrm{C}}$

	j1	j2	j3	j4	j5
j1	0	1	5	3	2
j2	2	0	4	5	1
j3	1	1	0	1	2
j4	1	2	2	0	3
j5	3	2	5	4	0

Table 6. Deterministic and Stochastic Results from the Scheduling Model SCHED

	deterministic					stochastic				
	j1	j2	j3	j4	j5	j1	j2	j3	j4	j5
n_j	3	2	3	2	3	4	2	4	3	3
Q_j^{P}	180	160	300	120	180	240	160	400	180	180
$E(Q_{jk}^{\mathrm{S}})$	153	132	256	100	139	177	132	295	119	139
$E(Q_{jk}^{\mathrm{I}})$	27	28	44	20	41	63	28	105	61	41
$E(Q_{jk}^{\mathrm{U}})$	31	21	49	19	9	7	21	10	0	9
$\mathrm{PV}_{\mathrm{nom}}$	4508					3059				
EPV	1686					2140				

Table 7. Deterministic and Stochastic Results from the Scheduling Model with Option Contracts

	deterministic					stochastic				
	j1	j2	j3	j4	j5	j1	j2	j3	j4	j5
n_j	3	2	3	2	2	4	2	4	2	3
Q_j^{P}	180	160	300	120	120	240	160	400	120	180
Q_j^{OC}	0	0	0	0	30	0	31	0	33	4
$E(Q_{jk}^{\mathrm{S}})$	153	132	256	100	109	177	113	295	80	136
$E(Q_{jk}^{\mathrm{SOC}})$	0	0	0	0	17	0	31	0	33	4
$E(Q_{jk}^{\mathrm{I}})$	27	28	44	20	24	63	47	105	40	44
$E(Q_{jk}^{\mathrm{U}})$	31	21	49	19	22	7	10	10	7	8
$\mathrm{PV}_{\mathrm{nom}}$	4631					3746				
EPV	1592					2192				

can be simply derived from the stochastic formulations presented above (SCHED and SCHEDOC) considering only one scenario with the nominal demand values. The two-stage stochastic problem was solved next. The results obtained for both scheduling models are reported in Tables 6 and 7. The cumulative distributions of profit values for all of the scenarios obtained with the SCHEDOC model are plotted in Figure 2. Equivalent curves were obtained with the SCHED model.

One important thing to notice is that the deterministic formulations with nominal demands predict a solution that poorly represents the uncertain environment, i.e., the schedule obtained with the nominal parameters can be critically inefficient when another demand is ordered. Indeed, although the profits of the deterministically generated schedules are higher than those of their stochastic counterparts in the nominal scenario ($\mathrm{PV}_{\mathrm{nom}}$ in Tables 6 and 7), when the former are used in the presence of uncertainty, the expected profit value drops by nearly 63% with the SCHED model and 66% with the SCHEDOC model. Hence, the stochastic schedules perform better over the uncertain space than the deterministic ones, as is also reflected in the shift to the right of the risk curve (Figure 2). The deterministic models do not generate inventory to hedge from adverse scenarios, as the stochastic ones do, and hence the predicted schedules with nominal demand values provide fewer batches than the stochastic ones. Despite the higher inventory requirements, the stochastic schedules ensure a much better overall expected profit. Although the same trend is observed in both models presented, the flexibility obtained by the introduction of option contracts is reflected in a slightly better expected profit due to the fewer inventory requirements and the somewhat higher level of customer satisfaction.

To assess the value of knowing and using distributions for future outcomes, i.e., to evaluate the possible gain of solving the stochastic model, the value of stochastic solution (VSS) can be computed.[18] This value is the difference between the solution obtained from the stochastic formulation and the expected

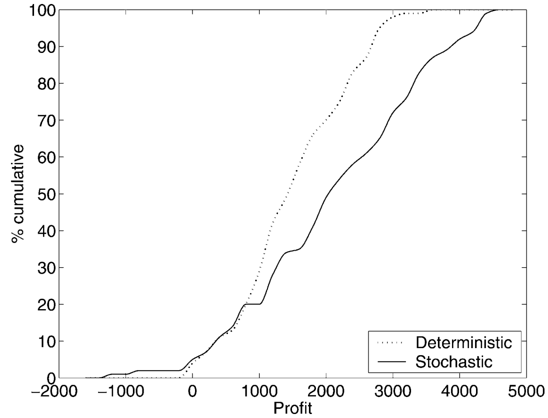

Figure 2. Risk curves obtained with the deterministic and stochastic SCHEDOC formulation.

Table 8. Profit Value in the Nominal Scenario ($\mathbf{PV_{nom}}$), Expected Profit (EPV), Worst Profit (WPV), Downside Risk (DR), and Financial Risk (FR) Values for the Different Approaches with the SCHED Model

	SCHED				
	$\mathrm{PV}_{\mathrm{nom}}$	EPV	WPV	$\mathrm{DR}_{\Omega=0}$	$\mathrm{FR}_{\Omega=0}$
deterministic	4508	1686	−62	1.23	0.02
stochastic	3059	2140	−1160	33.94	0.06
$\mathrm{FRisk}_{(\Omega=0,\rho=10000)}$	4190	1818	−29	0.29	0.01
$\mathrm{DRisk}_{(\Omega=0,\rho=1000)}$	4190	1818	−29	0.29	0.01
$\mathrm{worstcase}_{(\rho=1)}$	4190	1818	−29	0.29	0.01

value of the deterministic problem. Without option contracts, this value is about 27% (454 mu); with the introduction of option contracts, this value increases to 38% (600 mu).

Next, the impact of the proposed risk management procedures is investigated by solving the multiobjective models derived from the appendage of the alternate metrics to both SCHED and SCHEDOC models, with appropriate target profits and weight values. The results related to the profit value in the nominal scenario, the expected profit, the worst-case profit, and the downside and financial risk values obtained from the different approaches with both the SCHED and SCHEDOC models are summarized in Tables 8 and 9, respectively. Selected cumulative distributions of profit values over all of the scenarios are depicted in Figures 3 and 4, where the stochastic solution is included for comparison purposes.

From the results obtained, one can see how the risk management approaches try to restructure the profit distributions so that the defined risk and subsequent dispersion of profit values is limited while an acceptable expected revenue is maintained. Moreover, the risk curves obtained lie below the curve with maximum expected profit at low profit values. However, as expected, they intersect the maximum expected profit curve at some point.

With the SCHED model (see Table 8), the same solution is attained with the three alternative approaches, and a clear reduction of risk at a target profit of 0 is attained, although the realization of some of the scenarios would still show a negative return. The possibility of using option contracts (SCHEDOC model) translates into greater flexibility, as can be observed from the high number of configurations that are obtained with the different risk measures at different weight values (see Table 9). In this sense, one important thing to notice is that the last four cases reported in

Table 9. Profit Value in the Nominal Scenario (PV_{nom}), Expected Profit (EPV), Worst Profit (WPV), Downside Risk (DR), and Financial Risk (FR) Values for the Different Approaches with the SCHEDOC Model

	SCHEDOC						
	PV_{nom}	EPV	WPV	$DR_{\Omega=0}$	$DR_{\Omega=200}$	$FR_{\Omega=0}$	$FR_{\Omega=200}$
deterministic	4631	1592	−49	0.96	8.63	0.02	0.07
stochastic	3746	2192	−1179	21.04	31.23	0.05	0.06
$FRisk_{(\Omega=0,\rho=10000)}$	2997	2189	−1222	21	31.03	0.02	0.06
$FRisk_{(\Omega=200,\rho=50000)}$	4075	1949	−144	1.44	3.44	0.01	0.01
$DRisk_{(\Omega=0,\rho=1000)}$	4219	1883	0	0	4	0	0.02
$DRisk_{(\Omega=200,\rho=1000)}$	4391	1754	173	0	0.27	0	0.01
$worstcase_{(\rho=1)}$	4397	1750	181	0	0.57	0	0.03

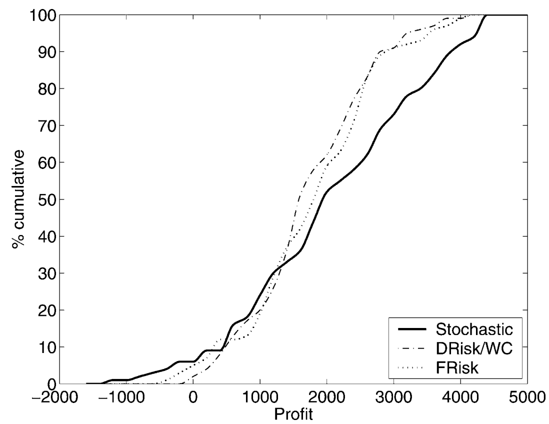

Figure 3. Risk curves obtained with the scheduling model.

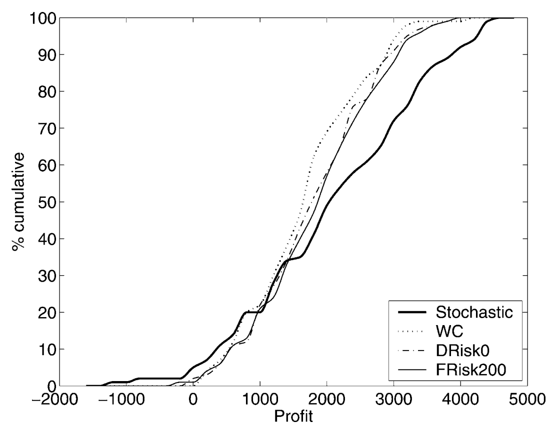

Figure 4. Risk curves obtained with the scheduling model with option contracts.

the table lead to the same schedule, and the different revenues come from the different contracts purchased. The inclusion of options leads also to more effective risk management. This conclusion is based on the observation that schedules are obtained with the entire risk curve above the target 0, i.e., schedules that ensure a positive return within the entire uncertain region (last three cases in Table 9). Despite the fact that the expected profit values of these more robust solutions are lower, they are higher than the expected profit of the deterministic schedule for all of the curves identified, and the revenue in the nominal scenario is only around 6% lower than the optimum found with the deterministic schedule.

The three alternative approaches implemented seem appropriate to face the uncertainty in the decision-making process and to illustrate different risk profiles. The computational time required to obtain robust solutions ranges from 200 to 20 000 s depending on the metric, the target profit, and the weight values.

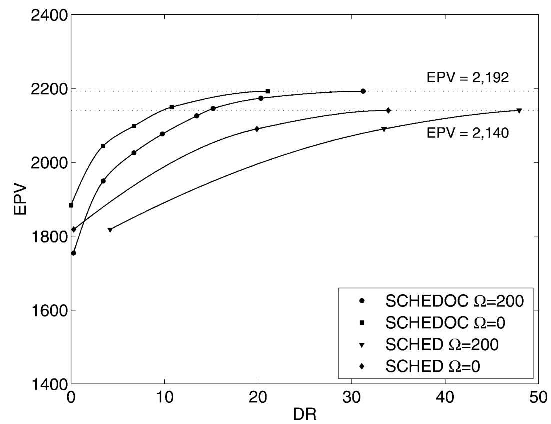

Figure 5. Tradeoff between the expected profit and the downside risk.

(Note that the primary purpose of this work is to propose a risk management framework rather than to develop the most efficient solution algorithm.) It is worthwhile mentioning the increased combinatorial complexity associated with the financial risk procedure due to the additional binary variables. On the other hand, the simple worst-case procedure shows a remarkable efficiency in identifying a schedule with good performance over the uncertain region, and its computational requirements are significantly lower. Another interesting result to consider is that the worst-case profits obtained with the worst-case approach with a weight value $\rho = 1$ match the maximum worst profit that would be obtained from the maximization of the worst case as the unique optimization criterion, thus establishing an upper bound of the measure.

The tradeoff between risk and profit was further investigated by parametrically varying the weight of the corresponding risk term in the objective function. Pareto curves obtained by managing downside risk for both models with target profits of 0 and 200 mu are depicted in Figure 5. Pareto curves obtained with the worst-case approach are plotted in Figure 6.

Each of these pareto points corresponds to one risk curve, i.e., an alternative scheduling policy; the decision of which one to implement is up to the decision maker. As expected, a reduction of the downside risk or a better worst profit can be attained at the expense of a reduction in the expected profit value. As the weight value decreases, the expected profit value of the solutions converges to the maximum value obtained with the stochastic model. The latter is also attained by increasing the target profit at a fixed weight value. Furthermore, the effectiveness of introducing option contracts is also illustrated by the fact that the curves obtained with the SCHEDOC model lie above those obtained

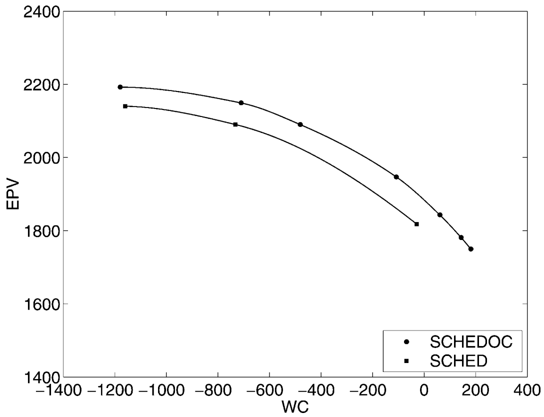

Figure 6. Tradeoff between the expected profit and the worst profit.

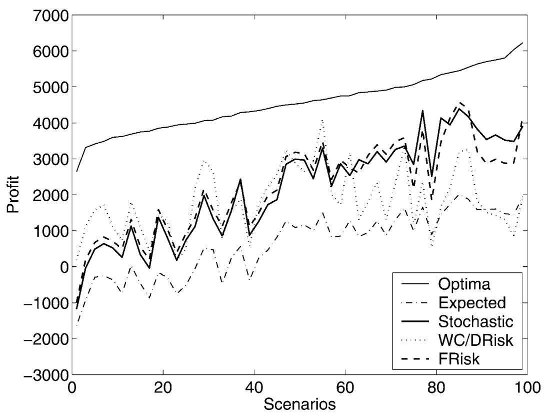

Figure 7. Profit values of the alternative approaches in the different scenarios.

without considering option contracts. Therefore, for a given level of risk, higher benefits are obtained by holding option contracts.

With the aim of comparing the different approaches directly within one specific scenario, the profit value that would be attained depending on the schedule implemented is depicted for all of the scenarios in Figure 7. The maximum profit value that would be attained if the optimum schedule for each scenario were implemented is also included, along with the expected profit of each optimal schedule over all realizations (optima and expected values, respectively, in Figure 7). These results were obtained by solving the deterministic scheduling model for each particular realization of demand uncertainty and by evaluating the resulting schedules over all of the scenarios. As stated above, the poor representation of the uncertain environment by the deterministic formulation is also observed in this graphic from the significant difference between the optimum profit value of each scenario and its expectation when the uncertainty is faced. From this figure, the higher variability of the stochastic solution in comparison with the solutions attained by controlling risk can be also observed. On the other hand, it is worthwhile to note that all of the schedules obtained under the different approaches show a coherent performance over the scenarios, which can be considered as an indication of the representability of the scenarios sampled. Moreover, despite the lower expected profit value, the negative returns of these scenarios are avoided through the managing of risk.

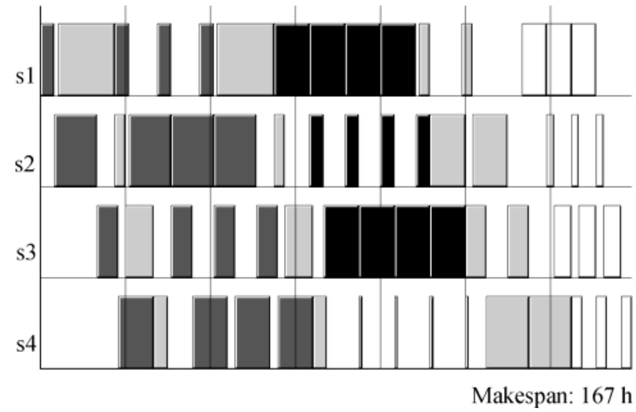

Figure 8. Schedule with the maximum expected profit.

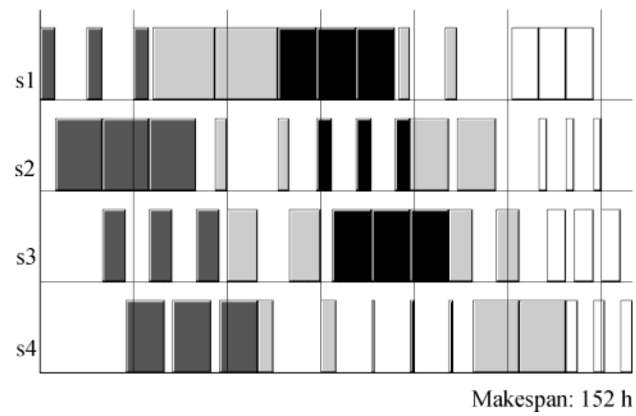

Figure 9. Schedule with the best worst-case scenario.

Finally, the schedule that maximizes the expected profit value and the schedule with the best worst profit identified with the SCHEDOC model are depicted in Figures 8 and 9, respectively.

Conclusions

The incorporation of demand uncertainties within the decision-making process of the short-term scheduling problem in multiproduct batch plants has been addressed in this work. A two-stage stochastic optimization model accounting for the maximization of the expected profit was presented and further extended to incorporate option contracts with the purpose of implicitly facing uncertainty. To incorporate the tradeoff between risk and profit within the decision process and, hence, to explicitly control the variability of performance, three alternative control measures were appended next as a second criterion to the objective functions: the financial risk, the downside risk, and the worst-case revenue. To illustrate the effectiveness of the proposed models, each one was applied to a scheduling case study.

From a comparison between the stochastic model and its deterministic counterpart with nominal demand values, the importance of the stochastic approach was shown. The expected profit value of the deterministically generated schedules dropped nearly 65% from the optimum value when the deterministic models were implemented in the presence of uncertainty. Stochastic modeling, on the other hand, led to a significant improvement in the expected performance over all particular realizations of demand uncertainty. Management of risk was attained with the three alter-

nate measures, and schedules with limited dispersion and acceptable expected profits were identified. The worst-case approach was shown to be very effective in identifying robust schedules with the best performance for the worst scenarios. Moreover, the availability of option contracts not only introduced flexibility to face the uncertainty but also led to schedules with higher expected revenues for a fixed level of risk.

Therefore, alternative scheduling policies can be easily identified by appropriately managing risk within the scheduling-making process. The schedule to be implemented is subject to the decision makers' attitude toward risk.

Acknowledgment

Financial support received from the Spanish Ministerio de Educación, Cultura y Deporte (FPU research grant to A.B.), and Ministerio de Ciencia y Tecnología (Project DPI2002-00856); from the Generalitat de Cataluny a (Project I0353); and from the European Community (Projects GIRD-CT-2000-00318 and GRD1-2000-25172) is fully appreciated. The support of the Ministerio de Educación, Cultura y Deporte, for the sabbatical stay of Dr. Bagajewicz is also acknowledged.

Nomenclature

b = index for batches
B = upper bound on the number of batches
BS_j = batch size of product j
$c^C_{jj'}$ = cost for product changeover between j and j'
c^I_j = unitary inventory cost of product j
c^{OC}_j = unitary cost for option contracts of product j
c^P_j = unitary production cost of product j
c^U = unitary cost for production shortfall
DR_Ω = downside risk at profit target Ω
$E(\cdot)$ = expected measure
EPV = expected profit value
FR_Ω = financial risk at profit target Ω
H = time horizon
i = index for stages
j = index for products
k = index for scenarios
n_j = number of batches produced of product j
PV_k = profit value of scenario k
Q^I_{jk} = inventory for product j in scenario k
Q^{OC}_j = amount of product j contracted
Q^P_j = amount of product j produced
Q^S_{jk} = quantity sold of product j in scenario k
Q^{SOC}_{jk} = amount of product j allocated in option contracts in scenario k
Q^U_{jk} = quantity not satisfied of product j in scenario k
T_{ib} = processing time of stage i in batch b
Tfn_{ib} = final processing time of stage i in batch b
Tin_{ib} = initial processing time of stage i in batch b
Top_{ij} = processing time of product j in stage i
U = high value
WPV = worst profit value
X_{bj} = binary variable indicating the assignment of product j in batch b
$XM_{bjj'}$ = variable indicating the change from product j to j' in batch b
$Z_{k\Omega}$ = binary variable indicating that the value of PV_k is lower than the target profit Ω

Greek Characters

α = low value
$\delta_{k\Omega}$ = positive deviation of the profit value from the target Ω in scenario k
γ^{OC}_j = sales price for option contracts
γ_j = sales price of product j
Ω = target profit
ω_k = probability of scenario k
ρ = weight value
θ_{jk} = demand for product j in scenario k

Literature Cited

(1) Shah, N. Single- and Multisite Planning and Scheduling: Current Status and Future Challenges. In *Foundations of Computer-Aided Process Operations*; AIChE Symposium Series; American Institute of Chemical Engineers (AIChE): New York, 1998; pp 75−90.
(2) Vin, J. P.; Ierapetritou, M. G. A New Appraoch for Efficient Rescheduling of Multiproduct Batch Plants. *Ind. Eng. Chem. Res.* **2000**, *39*, 4228.
(3) Petkov, S. B.; Maranas, C. G. Multiperiod Planning and Scheduling of Multiproduct Batch Plants under Demand Uncertainty. *Ind. Eng. Chem. Res.* **1997**, *36*, 4864−4881.
(4) Lee, Y. G.; Malone, M. F. A General Treatment of Uncertainties in Batch Process Planning. *Ind. Eng. Chem. Res.* **2001**, *40*, 1507−1515.
(5) Vin, J. P.; Ierapetritou, M. G. Robust Short-Term Scheduling of Multiproduct Batch Plants under Demand Uncertainty. *Ind. Eng. Chem. Res.* **2001**, *40*, 4543−4554.
(6) Engell, S.; Märkert, A.; Sand, G.; Schultz, R. *Scheduling of a Multiproduct Batch Plant by Two-Stage Stochastic Integer Programming*; Technical Report 59, Preprint 520; Institut für Mathematik, Gerhard-Mercator-Universität, Duisburg, Germany, 2002.
(7) Harjunkoski, I.; Grossmann, I. E. Decomposition Techniques for Multistage Scheduling Problems Using Mixed-Integer and Constraint Programming Methods. *Comput. Chem. Eng.* **2002**, 1533−1552.
(8) Balasubramanian, J.; Grossmann, I. E. Scheduling Optimization under Uncertainty—An Alternative Approach. *Comput. Chem. Eng.* **2003**, 469−490.
(9) Reklaitis, G. V. Overview of scheduling and planning of batch process operations. In *Batch Process Systems Engineering*; Reklaitis, G. V.; Sunol, A. K.; Rippin, D. W. T.; Hortacsu, O., Eds.; Springer: Berlin, 1996; pp 660−705.
(10) Kondili, E.; Pantelides, C. C.; Sargent, R. W. H. A General Algorithm for Short-Term Scheduling of Batch Operations. I. MILP Formulation. *Comput. Chem. Eng.* **1993**, *17*, 211−227.
(11) Mockus, L.; Reklaitis, G. V. Mathematical Programming Formulation for Scheduling of Batch Operations Based on Nonuniform Time Discretization. *Comput. Chem. Eng.* **1997**, *21*, 1147.
(12) Pinto, J. M.; Grossmann, I. E. A Continuous Time Mixed Integer Linear Programming Model for Short-Term Scheduling of Multistage Batch Plants. *Ind. Eng. Chem. Res.* **1995**, *34*, 3037.
(13) Ierapetritou, M. G.; Floudas, C. A. Effective Continuous-Time Formulation for Short-Term Scheduling. 1. Multipurpose Batch Processes. *Ind. Eng. Chem. Res.* **1998**, *37*, 4341−4359.
(14) Méndez, C. A.; Henning, G. P.; Cerdá, J. An MILP Continuous-Time Approach to Short-Term Scheduling of Resource-Constrained Multistage Flowshop Batch Facilities. *Comput. Chem. Eng.* **2001**, *25*, 701−711.
(15) Barbaro, A.; Bagajewicz, M. J. Managing Financial Risk in Planning Under Uncertainty. *AIChE J.* **2003**, manuscript submitted.
(16) Barbaro, A.; Bagajewicz, M. J. Effect of Inventory and Contract Options to Hedge Financial Risk in Planning under Uncertainty. *AIChE J.* **2003**, manuscript submitted.
(17) Eppen, G. D.; Martin, R. K.; Schrage, D. A. A Scenario Approach to Capacity Planning. *Oper. Res.* **1989**, *37*, 517−527.
(18) Birge, J. R.; Louveaux, F. *Introduction to Stochastic Programming*; Springer: New York, 1997.
(19) Samsatli, N. J.; Papageorgiou, L. G.; Shah, N. Robustness Metrics for Dynamic Optimization Models under Parameter Uncertainty. *AIChE J.* **1998**, *44*, 1993−2006.

(20) Suh, M.; Lee, T. Robust Optimization Method for the Economic Term in Chemical Process Design and Planning. *Ind. Eng. Chem. Res.* **2001**, *40*, 5950−5959.

(21) Ahmed, S.; Sahinidis, N. V. Process Planning under Uncertainty. *Ind. Eng. Chem. Res.* **1998**, *37*, 1883−1892.

(22) Gupta, A.; Maranas, C. D. Market-Based Pollution Abatement Strategies: Risk Management using Emission Option Contracts. *Ind. Eng. Chem. Res.* **2003**, *42*, 802−810.

(23) Brooke, A.; Kendricck, D.; Meeraus, A. *GAMS*: *A User's Guide*. Scientific Press: Berkeley, CA, 1988.

Received for review June 30, 2003
Revised manuscript received November 5, 2003
Accepted November 19, 2003

IE030529F