

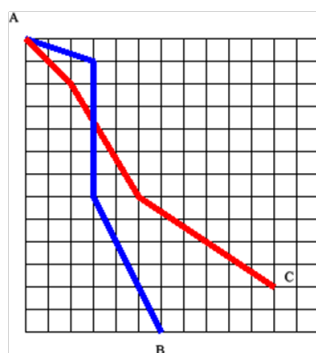


# Sign Determination of Algebraic Numbers

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## Objectives

- Compare lengths of different polygonal path.
- The length of a path is an algebraic number, not an integer nor a rational number.



## Background

- It is the main barrier to accurately classify some of the most fundamental computational problems in Euclidean space, such as the shortest path problem, the minimum spanning tree problem and the traveling salesman problem.
- It is the simplest among the problems of the sign determination of algebraic numbers of high degree, which are related to polynomial identity testing problem.
- It has been used to show hardness of problems in other area such as approximation of 3-player Nash equilibrium.

## Preliminary Results

- We have proved better bounds based on pigeon hole principle and prime number theorem.
- We design a space efficient algorithm to find extreme close gap.
- We discover

$$\xi_7 + \xi_{14} + \xi_{39} + \xi_{70} + \xi_{72} + \xi_{76} + \xi_{85} \\ = 47.42163068019049036900034846$$

$$\xi_{13} + \xi_{16} + \xi_{46} + \xi_{55} + \xi_{67} + \xi_{73} + \xi_{79} \\ = 47.42163068019049036881196876$$

## Relevant References

- "The open problems project".  
<http://maven.smith.edu/~orourke/TOPP/>.
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- Qi Cheng and Yu-Hsin Li. "Finding the smallest gap between sums of square roots". The 9th Latin American Theoretical Informatics Symposium, 2010.