ASTRONOMY QUALIFYING EXAM
August 2014

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

Use the Virial Theorem to:

a) (6 points) Derive the internal temperature of the Sun. How much hotter is this value compared to the Sun's effective surface temperature?

b) (4 points) Derive the Jeans Mass of a molecular cloud that is starting to collapse, thereby starting the star formation process.

PROBLEM 2

a) (2 points) Calculate the zeroth and first moments of the plane-parallel radiation transport equation and explain why we can't simply solve the moment equations rather than the original equation.

b) (3 points) Explain the physical content of the grey atmosphere approximation and derive the radiation transport equation, its the zeroth and first moments in that approximation.

c) (5 points) The Rosseland mean opacity is defined as the grey opacity that reproduces the total flux with the following approximations:

1. The pressure is isotropic
2. The radiation field is in LTE

Use parts (a) and (b) to derive the Rosseland mean opacity. What is the meaning of these 2 approximations and when are they valid?

PROBLEM 3

a) (2 points) What is the definition of the luminosity function of a class of astronomical objects?

b) (3 points) The Schechter luminosity function is commonly used to model the luminosity function of galaxies, which has the following functional form:

$$\phi(\mathsf{L}) = \frac{\phi^*}{\mathsf{L}^*}\left(\frac{\mathsf{L}}{\mathsf{L}^*}\right)^{-\alpha} \exp\frac{-\mathsf{L}}{\mathsf{L}^*},$$

with three parameters, ϕ^*, α, and L^*. What is the total luminosity of this class of objects? Simplify the formula using the Γ function,

$$\Gamma(x) = \int_0^\infty y^{(x-1)} e^{-y} dy.$$

c) (2 points) For a class of objects with $\phi^* = 0.016$ Mpc^{-3}, $\alpha = 0.9$, and $L^* = 10^{10} L_\odot$, what is the total number density of this class of objects ($\Gamma(0.1) = 9.5$)?

d) (3 points) A class of objects, located at a fixed distance, has a Schechter luminosity function with $\alpha = 0.9$. It is composed of two populations, one obscured and one normal. The normal population contributes to 70% of the total population intrinsically. The obscured population is dimmed by a factor of two by intervening obscuration.

A survey has a limit to detect the L^* objects of the normal population. Considering the two sub-classes of objects detected in this survey, what is the observed fraction of obscured objects to the total number of detections? Express the answers using the incomplete Γ function,

$$\Gamma(a, x) = \int_a^\infty y^{(x-1)} e^{-y} dy.$$

PROBLEM 4

Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric parallax is 0.37921 ± 0.00158 arcsec and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimajor axis of the reduced mass is 7.61 arcsec. The ratio of the distances of Sirius A and Sirius B from the center of mass is $a_A/a_B = 0.466$.

a) (3 points) Find the mass of each member of the system.

b) (3 points) The absolute bolometric magnitudes of Sirius A and Sirius B are 1.36 and 8.79, respectively. Determine their luminosities. Express your answers in terms of the luminosity of the Sun.

c) (2 points) The effective temperature of Sirius B is 24,790 K. Estimate its radius, and compare your answer to the radii of the Sun and Earth.

d) (2 points) Estimate the surface gravity of Sirius B in cgs units. Compare your answer to the surface gravity of the Sun and Earth.

PROBLEM 5

a) (6 points) Compute the Kelvin-Helmholtz timescale for the Sun. Assume the virial theorem and that the density of the star at any distance from its center is equal to the star's average density.

b) (2 points) Assuming that 10 eV could be released by every atom in the Sun through chemical reactions, estimate how long the Sun could shine at its current rate through chemical processes alone. For simplicity, assume that the Sun is composed entirely of hydrogen.

c) (2 points) Assuming that the Sun is 100% hydrogen, and that only the inner 10% of the Sun's mass becomes hot enough to burn hydrogen, estimate how long the Sun could shine at its current rate through nuclear reactions alone. Assume that 0.7% of the mass of hydrogen is converted to energy in forming a helium nucleus.

PROBLEM 6

The Universe contains different components that have different equations of state. These include matter, radiation, and dark energy.

a) (1 point) What is the equation of state for matter? Explain.

b) (1 point) What is the equation of state for radiation? Explain.

c) (2 points) What is the best current estimate for the equation of state for dark energy? Explain.

d) (6 points) Derive the cosmic scale factor as a function of time for a cosmic component with constant equation of state w, assuming a flat Universe.