ASTRONOMY QUALIFYING EXAM
August 2020

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173 $\times 10^{-5}$ eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

a) [4 pts] Consider a spherical dust grain in orbit about the Sun that is subjected to the Sun's gravity and radiation pressure. Assuming the object is perfectly absorptive, derive β, the force felt due to radiation pressure divided by that felt due to the Sun's gravity. Your answer should be in terms of the Sun's mass (M) and luminosity (L) as well as the grain's density (ρ) and radius (r).

b) [2 pts] Your answer in part a) should show that β depends on r^{-1}. In reality, β does not go to infinity as r gets smaller and smaller. In fact, β drops sharply for grain radii below $\sim$1000 Angstroms. Why?

c) [4 pts] What is the maximum value of β that a dust grain lost by a comet on a circular orbit can have and still remain bound to the Sun? Since the comet's escape velocity is small, assume the grain's orbital velocity still matches that of the comet when it is initially lost from the comet.

PROBLEM 2

a) [3 pts] If the Sun subtends a solid angle Ω on the sky, and the bolometric flux from the Sun on the Earth is f, show that the flux at the solar photosphere is $\pi f/\Omega$.

b) [2 pts] The angular diameter of the Sun is 0.57 degrees. Calculate the solid angle subtended by the Sun, in steradians.

c) [2 pts] The solar flux at the Earth is $f = 1.4 \times 10^6$ erg s^{-1} cm^{-2} = 1.4 kW m^{-2}. Use *Part a* and the Stefan-Boltzmann law to derive the effective surface temperature of the Sun.

d) [3 pts] Derive an expression for the surface temperature of the Sun, in terms of only its solid angle, its flux per unit wavelength, $f_\lambda(\lambda_1)$ at the Earth at one particular wavelength λ_1 and fundamental constants.

PROBLEM 3

The light from a star is measured in the V-band. The star is observed to have a parallax angle of 0.04 arcsec. You can assume that m_v is small so the star appears very bright (lots of photon intensity).

a) [3 pts] Estimate the distance to the star (in meters).

b) [4 pts] What is the minimum diameter telescope needed (in meters) to observe the parallax angle? Chose a reasonable observation wavelength in the V-band.

c) [3 pts] What focal length, f, is needed (in meters) for this telescope if you were to measure the star's parallax angle with a CCD camera which has 1 micron pixels?

PROBLEM 4

a) [2 pts] What is the luminosity function of a class of astronomical objects?

b) [3 pts] The Schechter luminosity function is commonly used to model the luminosity function of galaxies, which has the following functional form:

$$\phi(\mathsf{L}) = \frac{\phi^*}{\mathsf{L}^*}\left(\frac{\mathsf{L}}{\mathsf{L}^*}\right)^{-\alpha} \exp\frac{-\mathsf{L}}{\mathsf{L}^*}, \tag{1}$$

with three parameters, ϕ^*, α, and L^*. What is the total luminosity of this class of objects? Simplify the formula using the Γ function,

$$\Gamma(\mathsf{x}) = \int_0^\infty \mathsf{y}^{(\mathsf{x}-1)}\mathsf{e}^{-\mathsf{y}}\mathsf{dy}. \tag{2}$$

c) [2 pts] For a class of objects with $\phi^* = 0.016$ Mpc^{-3}, $\alpha = 0.9$, and $\mathrm{L}^* = 10^{10}\mathrm{L}_\odot$, what is the total number density of this class of objects ($\Gamma(0.1) = 9.5$)?

d) [3 pts] A class of objects, located at a fixed distance, has a Schechter luminosity function with $\alpha = 0.9$. It is composed of two populations, one obscured and one normal. The normal population contributes to 70% of the total population intrinsically. The obscured population is dimmed by a factor of two by intervening obscuration. A survey has a limit to detect the L^* objects of the normal population. Considering all the two sub-classes of objects detected in this survey, what is the observed fraction of obscured objects to the total number of detections? Express the answers using the incomplete Γ function,

$$\Gamma(\mathsf{a}, \mathsf{x}) = \int_\mathsf{a}^\infty \mathsf{y}^{(\mathsf{x}-1)}\mathsf{e}^{-\mathsf{y}}\mathsf{dy}. \tag{3}$$

PROBLEM 5

a) [3 pts] A quasar can be assumed to emit a power law spectrum in the far UV. The functional form of the power law (observed on earth) is $\nu F(\nu) = N(\nu/\nu_0)^\alpha$ where $N = 6 \times 10^{-11}\,\mathrm{erg\,s^{-1}\,cm^{-2}}$, $\nu_0 = 3.29 \times 10^{15}\,\mathrm{Hz}$ and $\alpha = -0.7$. The quasar is located at a distance of 938.2 Mpc. What is the rate of photoionizing photons Q_0 emitted by the quasar?

b) [3 pts] The Strömgren sphere is the region around a source of ionizing photons in which all the hydrogen will be ionized. Another way to look at it is that in the ionized region, all of the ionizing photons Q_0 will be used up. Assuming a pure hydrogen gas with a density of $n = 10^4\,\mathrm{cm^{-3}}$, and an effective recombination coefficient of $\alpha_A = 4.18 \times 10^{-13}\,\mathrm{cm^3\,s^{-1}}$, determine the size of the ionized region in parsecs.

c) [2 pts] The recombination coefficient given above is the one for "Case A", whereby all photons created by recombination escape the plasma, including photons produced by free electrons recombining to the ground state. Such photons, however, will have high enough energy to ionize hydrogen. "Case B" therefore assumes that the photons recombining to ground will not escape, but instead will ionize another hydrogen. In that case, will α_B be larger than α_A, or smaller?

d) [2 pts] The warm ionized medium that comprises a quasar narrow-line region cools chiefly by collisionally-excited line emission from abundant ions. An example is the forbidden lines [O III]$\lambda\lambda 4959, 5007$. Use this fact to estimate the temperature of the gas in the narrow-line region.

PROBLEM 6

Consider a spherically symmetric main sequence (MS) star.

a) [3 pts] Consider a spherical shell at radial position $(r, r + dr)$. Draw all the forces on it, and use them to derive the equation of hydrostatic equilibrium (HSE).

b) [1 pt] Assuming you wrote the HSE using r as independent variable, rewrite it using the enclosed mass $m(r)$ as independent variable.

c) [2 pts] Now finish off the equations of stellar structure by remembering we need two conservation laws and a way to relate pressure to other quantities. You may use either r or m as independent variable and you are given that ϵ is the energy generation rate per gram.

d) [2 pts] How is energy transported in MS stars?

e) [2 pts] What energy transport process is *not* important in MS stars? What type of stars is this additional process important in and why?