ASTRONOMY QUALIFYING EXAM

August 2022

Notes and Instructions

• There are 6 problems. Attempt them all as partial credit will be given.
• Write on only one side of the paper for your solutions.
• Write your alias on the top of every page of your solutions.
• Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
• You must show your work to receive full credit.

Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173×10^{-5} eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

Before the advent of relativity, physicists tried to figure out the source of energy that powers stars. One suggestion was that the stellar energy was gravitational energy that is being radiated away as the cloud of gas contracts. A star starts out as a huge cloud of gas which starts to collapse due to its own self-gravity. Gas falls toward the center and releases energy and this could possibly explain the energy production in stars. In this problem, we will check to see if this is a plausible explanation. Assume a spherically symmetric cloud with mass M and radius R. Use the virial theorem to:

(a) (6 points) Solve for the total energy being radiated away. Hint: you can assume an average cloud density.

(b) (4 points) Estimate the age of the sun, assuming it has always radiated at its current luminosity. Is this age reasonable? Justify your answer.

PROBLEM 2

The hot gas component of galaxy clusters emits in the X-ray band and is more massive than the stellar component in the clusters. Assume the gas has a uniform temperature and decreasing density profile from the center of the cluster.

(a) (2 points) Galaxy clusters are massive objects. Why does the gas halo not collapse to the center of the cluster because of the gravitational pull? Write a relevant equation to explain this.

(b) (2 points) What is the thermal energy density of the gas as a function of temperature and electron number density?

(c) (2 points) The gas is emitting thermal bremsstrahlung emission with an emissivity of

$$\epsilon \simeq 3.0 \times 10^{-27} \sqrt{\frac{T}{1\mathrm{K}}} \left(\frac{n_e}{1\ \mathrm{cm}^{-3}}\right)^2 \mathrm{erg\ cm^{-3}\ s^{-1}}, \tag{1}$$

where T is the temperature of the gas and n_e is the electron number density. What is the cooling time for this gas to exhaust all of its internal energy by radiation?

(d) (2 points) What is the Hubble time, and what is its value?

(e) (1 point) Where in a cluster do you expect that the cooling time can be shorter than the Hubble time and why?

(f) (1 point) What will happen to the gas that is cooled and why?

PROBLEM 3

In the "Schuster-Schwarzschild Model" line formation is assumed to occur in a finite layer (the "reversing layer") of thickness τ_ν, above a sharp photosphere. The intensity from the photosphere is I_0. The incoming intensity at the surface is $I^- = 0$. In the reversing layer the continuum opacity is zero and the lines are purely scattering. Using the 2-Stream approximation,

$$\mathsf{I} = \begin{cases} I^+ & : \quad \mu \geq 0 \\ I^- & : \quad \mu < 0 \end{cases}$$

where $\mu = \cos\theta$, show that:

(a) (2 points)

$$\begin{aligned} H_\nu &= \frac{1}{4}(I^+ - I^-) \\ J_\nu &= \frac{1}{2}(I^+ + I^-) \end{aligned}$$

(b) (2 points) What is the general expression for the source function? What is the physical meaning of ϵ and what is its value?

(c) (2 points) Now we consider the transfer equation at only two angles with $\mu = \pm 1/2$ (this is a form of the discrete ordinates method).
Using the transfer equation at the two angles, show that $H_\nu =$ constant.

(d) (2 points) Using the transfer equation at the two angles, and the upper boundary condition show that $J_\nu = 2H_\nu(2t_\nu + 1)$.

(e) (2 points) Using the lower boundary condition show that $H_\nu = \frac{I_0}{4(1+\tau_\nu)}$.

PROBLEM 4

You are planning to conduct high resolution optical spectroscopy toward Barnard's Star, whose current coordinates are α = 17:57:48.5 and δ=+04:41:36.2. It is 1.8 pc away from the Sun and it has a V-band magnitude of 9.51 (Vega system).

(a) (1 point) What is constantly changing about Barnard's Star that needs to be considered when planning observations? What time of year is be to observed this star from the ground and why?

(b) (1 point) What is the flux density of the star in V-band if the flux zero-point is 3.636×10^{-20} erg cm^{-2} s^{-1} Hz^{-1}?

(c) (1 point) This observation will be source noise limited, what distribution describes the uncertainty of these measurements and what is the simplest equation for the uncertainty σ in this case (1 point).

(d) (4 points) Derive an expression for the number of photons observed in a given Δt and calculate the number in a single resolution element for the ARCES spectrograph on the APO 3.5m at a wavelength of 5175 Angstroms. The ARCES spectrograph has a resolution of R$\sim$31,500 in the optical band. Assume that the V-band flux density is the flux density at 5175 Angstroms.

(e) (1 point) What is the maximum exposure time to avoid detector non-linearity? The detector goes non-linear at 35,000 ADU and the gain of the detector is 3.8 e$^-$ ADU^{-1}.

(f) (2 points) Demonstrate mathematically that multiple short exposures are equivalent to a single long exposure. Why is a single long exposure a bad idea in the first place and why do we typically take multiple exposures during observations?

PROBLEM 5

(a) (3 points) Describe the burning process on the main sequence. Explain the difference of the sun on the main sequence and a $1.5M_\odot$ star.

(b) (3 points) Describe He burning in the lower mass stars and intermediate mass stars. What is the mass range for each approximately? Compare the timescale of helium burning (lifetime on the helium main sequence) to that of hydrogen burning (lifetime on the main sequence).

(c) (4 points) Describe the following burning stages in stars: C, Ne, O, Si burning (give the main reactants and products, note if any free neutrons are produced).

PROBLEM 6

(a) (2 points) Draw a typical velocity rotation curve for a spiral galaxy. What does the observed rotation curve tell us about the matter distribution in spiral galaxies?

(b) (3 points) Describe the Tully-Fisher relationship for spiral galaxies and why it is important.

(c) (5 points) Assume a spiral galaxy has a mass to light ratio γ. Use the virial theorem to derive an expression for the galaxy's dynamical mass in terms of its γ, L (luminosity), v_c, and R (radius).