ASTRONOMY QUALIFYING EXAM
August 2024

Notes and Instructions

• There are 5 problems. 4 of the 5 questions count as your grade on the exam. You may choose to answer only 4 questions, or the question with the lowest grade will be dropped. The other 4 questions will be used to grade the exam.
• Write on only one side of the paper for your solutions.
• Write your alias on every page of your solutions.
• Number each page of your solutions with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
• You must show your work to receive full credit.

Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol,\odot} = 4.74$ mag
$R_\odot = 7 \times 10^{10}$ cm
$T_{\text{eff},\odot} = 5777$ K
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173 $\times 10^{-5}$ eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g
$m_H = 1.6735575 \times 10^{-24}$ g

PROBLEM 1

You discover a dense core in a giant molecular cloud (GMC) with $T = 20$ K, a density of 5×10^{-20} g cm^{-3}, and a distance of 700 pc from Earth.

(a) (3 points) Calculate the Jeans mass of this core.

(b) (1 point) If this core has a mass of 10 $M_\odot$, will it collapse?

(c) (2 points) You observe a star in this GMC cloud that has an absolute visual magnitude of $M_V = -1.1$ and an apparent magnitude of $m_V = 9.2$. How many magnitudes of extinction (A_V) is there along the line of sight to this star?

(d) (2 points) What is interstellar reddening, and how does it relate to extinction?

(e) (2 points) If it is common for GMCs with 5000 $M_\odot$ to collapse, why are high mass stars still rare? Refer to the Jeans mass in your answer.

PROBLEM 2

We learn about astronomical objects through the radiation they emit. Radiation is produced when an electron is accelerated. For the following two interactions given in (a) and (b), list and explain:

- The name of the type of radiation.
- How the acceleration happens physically.
- A type of astronomical object who's electromagnetic radiation is dominated by this type of radiation.
- The electromagnetic band that the radiation is observed in typically for the example given.

(a) (2 points) The interaction between a free electron and an ion.

(b) (2 points) The interaction between an electron and a magnetic field.

(c) (2 points) Stars emit approximately blackbody radiation, but their spectral shape is not exactly blackbody. What is the difference between a stellar spectrum and a blackbody, and what is the physical origin of the difference? Explain.

(d) (1 point) What astronomical phenomenon exhibits the most perfect blackbody spectrum in the universe?

(e) (3 points) Compton scattering occurs between a photon and an electron. It is distinguished from Thompson scattering because the scattering is not elastic, i.e., the electron recoils. Let the initial energy of the photon be ϵ_1, and the final energy be ϵ_2. Derive the equation for the final energy of the photon.

PROBLEM 3

(a) (2 points) An elliptical galaxy is measured at $z = 0.05$. What is the distance of the galaxy based on its redshift?

(b) (2 points) Calcium offers two strong absorption lines at 393.3 nm and at 396.9 nm, known as the K and H lines. What will be the observed wavelengths of the Ca H and K lines for this galaxy?

(c) (2 points) The K line from the entire galaxy has a Gaussian profile with an observed dispersion (σ) of 0.34 nm. What is the main factor causing this broadening? Quantify this factor.

(d) (1 point) In the nucleus region of the galaxy, the H_α emission line (centered at 6562.8 Å rest-frame) is observed with a dispersion of 250 Å. Calculate the dynamical state of the material emitting the line?

(e) (3 points) These materials are assumed to orbit a supermassive black hole at the center of the galaxy. Estimate the distance to the black hole in Schwarzschild radius unit.

PROBLEM 4

(a) (2 points) Write down the equation of radiative transfer for a plane-parallel atmosphere and define all the terms.

(b) (3 points) Assuming that there is no external irradiation at the surface, show that

$$\mathsf{I}_\lambda = \mathsf{S}_\lambda(1 - \mathsf{e}^{-\tau_\lambda})$$

(c) (5 points) What is I_λ in terms of S_λ for the optically thin and optically thick cases? For each case, do you expect to see emission or absorption lines at the wavelengths of large opacity, κ_λ?

PROBLEM 5

In this problem you are asked to discuss and compare two types of H fusion which occur in stars along with the chemical evolution of nitrogen.

(a) (2 points) Write down the three reaction steps in the PPI reaction. Show all isotopes and bi-products involved.

(b) (2 points) Write down the six reactions in the CN cycle. Show all isotopes and bi-products involved. Identify the two relatively fast reactions and the slowest reaction of the six. Why is carbon referred to as a catalyst?

(c) (2 points) Make a qualitative comparison of PP and the CN cycle in terms of the threshold temperature and temperature sensitivity of the energy generation coefficient ϵ, i.e., $d\epsilon/dT$. Discuss the relative amount that each cycle contributes to the total energy generation in the Sun's core.

(d) (2 points) Explain the relevance of the CN cycle to the evolution of the total nitrogen abundance in a galaxy. Explain what stellar types (mass ranges) are thought to produce significant amounts of N.

(e) (2 points) The figure below shows the universal behavior of the N/O abundance ratio as a function of metallicity, as measured by O/H. Note the flat behavior at low metallicities and the upward turn starting at around solar metallicity of about 8.7. Explain this change in slope.

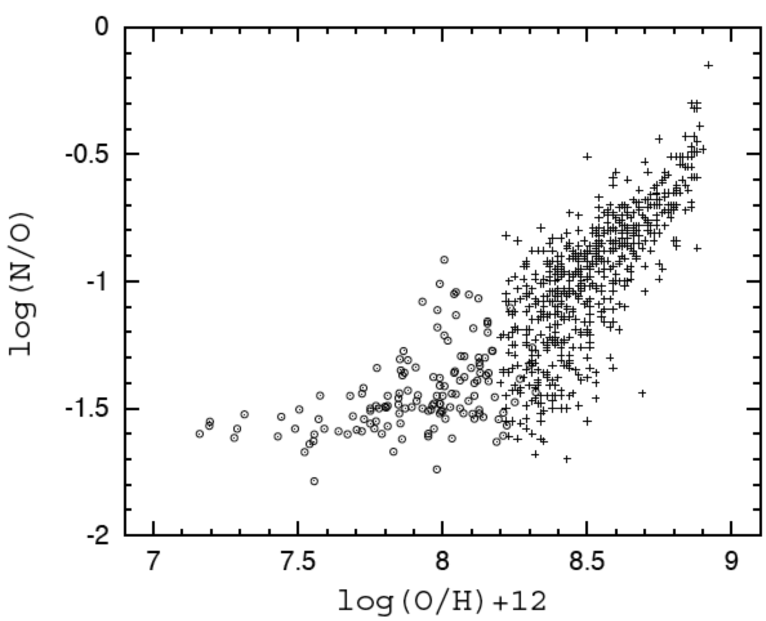

Fig. 4. The N/O–O/H for H II regions in spiral (pluses) and irregular (circles) galaxies.