Astro Qualifier Jan. 2016

- There are 6 problems. Attempt them all as partial credits will be given.
- Write on only one side of the provided paper for your solutions.
- Write your alias (NOT YOUR REAL NAME) on the top of every page of your solutions.
- Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2 is the second page for the solution to problem 3.)
- Do not staple your exam when done.
- You must show your work to receive full credit.

Constants:

$G = 6.67259 \times 10^{-8}$ dyne cm^2 g^{-2}
$c = 2.99792458 \times 10^{10}$ cm s^{-1}
$h = 6.6260755 \times 10^{-27}$ erg s
$k = 1.380658 \times 10^{-16}$ erg K^{-1}
$\sigma = 5.67051 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
$m_p = 1.6726231 \times 10^{-24}$ g
$m_n = 1.674929 \times 10^{-24}$ g
$m_e = 9.1093897 \times 10^{-28}$ g
$m_H = 1.673534 \times 10^{-24}$ g
$e = 4.803206 \times 10^{-10}$ esu
1 eV $= 1.60217733 \times 10^{-12}$ erg
1 M$_\odot = 1.989 \times 10^{33}$ g
1 L$_\odot = 3.826 \times 10^{38}$ erg s^{-1}
1 pc $= 3.0857 \times 10^{18}$ cm
1 AU $= 1.4960 \times 10^{13}$ cm

1. Briefly define and discuss the relevance of the following terms to modern astronomy.
   (a) (1 point) Cepheid variable star
   (b) (1 point) Initial mass function
   (c) (1 point) tunneling in the context of the PPI chain reaction
   (d) (1 point) age-metallicity relation
   (e) (1 point) damped Lyα system (DLA)
   (f) (1 point) s-process
   (g) (1 point) G dwarf problem
   (h) (1 point) Tully-Fisher relation
   (i) (1 point) Thin disk
   (j) (1 point) isophotal radius

2. The specific intensity at the surface of stars is given by

$$I_\nu(u) = \int_0^\infty S_\nu(t)\frac{dt}{u}e^{-t/u}, \tag{1}$$

   where S_ν is the source function, t is the optical depth, and $u = \cos\theta$. In addition, the moments of order n of the radiative field $M_\nu(n)$ are

$$M_\nu(n) = \frac{1}{2}\int_{-1}^{1} I_\nu(u)u^n du, \tag{2}$$

   where $M_\nu(0) = J_\nu$ and $M_\nu(1) = H_\nu$.
   (a) If the source function inside the star is $S(\tau) = a + b\tau$, where a and b are functions of ν but not τ, calculate the specific intensity I_ν at the surface, for outgoing directions ($u \geq 0$).
   (b) the average intensity J_ν.
   (c) the Eddington flux H_ν.

3. Assume that as a pulsar slows down, the quantity

$$\frac{d\ln P}{dt} = b,$$

   where b is positive constant and P is the rotation period.
   (a) (5 points) If at time $t = 0$, $P = P_0$, find an expression for $P(t)$, the period as a function of time.
   (b) (3 points) If the initial rotation energy is E_0, find an expression for $E(t)$, the energy as a function of time
   (c) (2 points) If $P_0 = 10^{-3}$ s, at what time is the period 3 s?

4. (a) (3 points) Use the Virial Theorem to derive expressions for the quantized radius AND energy of a Bohr hydrogen atom.

   (b) (1 point) Calculate the energy AND wavelength of light emitted by a Brackett gamma ($n = 7$ to $n = 4$) emission photon.

   (c) (3 points) Assume you observe a massive, hot star that exhibits a Brackett gamma emission line that has a P-Cygni line profile. What is the physical interpretation of this line profile? Discuss how such a profile arises; include a picture that describes where each region of the line profile arises from.

   (d) (3 points) Describe the process by which winds are driven in massive stars. Also describe how the Doppler effect aids wind driving in these stars.

5. (a) (2 points) Calculate the force due to radiation pressure experienced by an object of radius, r, and density, ρ, in a circular orbit with semimajor axis, a, around the Sun. Assume that the object absorbs all radiation and re-emits it isotropically in its rest-frame.

   (b) (2 points) If the object were stationary, this force would act only in the radial direction away from the Sun. However, because of our object's orbital velocity, the direction of the incoming photons has a small non-radial component in the object's rest-frame, and the radiation pressure exerted by the Sun in part a) has a small non-radial component. Expressing the object's orbital velocity in terms of the Sun's mass and a, solve for the non-radial component of the radiation pressure. (This non-radial component can be thought of as a photon headwind known as Poynting-Robertson drag.)

   (c) (2 points) This headwind causes the orbital semimajor axis to decay over time. Write the time derivative of the semimajor axis due to Poynting-Robertson drag for the object in part a. Assume that the radial component of the radiation pressure force is very small compared to the Sun's gravitational pull, so we only need to consider the Poynting-Robertson component.

   (d) (2 points) If the object is orbiting at 1 AU, its radius is 100 μm and its density is 1 g/cm^3, then calculate the amount of time it takes to spiral into the Sun.

   (e) (2 points) The object in part (d) is typical of zodiacal dust (dust particles in the Solar system, primarily located between the Sun and Jupiter). What does the above calculation say about the theory that zodiacal dust was formed at the beginning of the Solar system?

6. The image below is derived from data collected by the *Extreme Ultraviolet Explorer* satellite (EUVE). It shows photons near 0.1 keV collected from a luminous Seyfert (active) galaxy 1H 0419−577 (upper left) that was the target of the observation, and a serendipitously discovered Am Herculis star, an accreting magnetic white dwarf star (lower right). The three-lobed structure of the image is an artifact of the telescope.

   (a) (5 points) This observation had a total exposure time of 171,841 seconds. During the observation, 5529 photons total were collected in the region 60 arcseconds in radius around the Seyfert, and 4733 photons were collected from the region 60 arcseconds in radius around the AM Her. In the background region, which is 230 arcseconds in radius, and excludes the regions around the Seyfert and around the AM Her that are each 105 arc seconds in radius, 6291 photons were collected. What are the average net count rates and uncertainties from the Seyfert and from the AM Her? Are these values significantly different? (Hint: you may assume that Poisson statistics apply.)

(b) (5 points) During the first 10,000 seconds of the observation, only 308, 288, and 394 photons were collected from the Seyfert, the AM Her and the background region, respectively. What were the net count rates and uncertainties from the Seyfert and from the AM Her during this time period. Are these values significantly different?