Welfare Consequences of Sustainable Finance*

Harrison Hong[†] Neng Wang[‡] Jinqiang Yang[§]

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Abstract

Shareholders face restrictions to hold firms that have net-zero carbon emissions. These mandates address the global-warming externality by rewarding corporations that invest in decarbonization like direct air capture with a lower cost of capital. We quantify their welfare consequences by developing a model where a higher aggregate decarbonization-to-productive capital ratio delays a climate tipping point — an absorbing state with frequent weather disasters. We compare the market economy with a welfare-maximizing mandate to the planner's solution. Welfare increases along with endogenous mandate standards over time. A cost-of-capital wedge formula for sustainable versus unsustainable firms summarizes shareholders' decarbonization costs.

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[†]Columbia University and NBER. E-mail: hh2679@columbia.edu

[‡]Columbia University and NBER. E-mail: neng.wang@columbia.edu

[§]Shanghai University of Finance and Economics. E-mail: yang.jinqiang@mail.sufe.edu.cn

1 Introduction

In light of the failure of society to implement carbon emissions taxes, the financial sector is being pressured to help keep global temperatures within 1.5^o Celsius above pre-industrial levels. Governments and activists are pushing shareholders toward sustainable finance mandates, whereby a fraction of their portfolios are restricted to hold firms that can meet net-zero emissions targets by 2050. A prominent example is the Glasgow Financial Alliance for Net Zero, which has commitments from 450 financial firms across 45 countries with $130 trillion of assets (*Wall Street Journal* on November 7, 2021). Another is the Network for Greening the Financial System (NGFS), which has similar proposals for bank lending.

The main idea behind these mandates, which follow from socially responsible investing, is that these restrictions will influence costs of capital and incentivize firms to reform, thereby addressing the global-warming externality. The European Union (and likely the Security Exchange Commission) is remedying greenwashing by requiring investors to disclose the carbon emissions of firms in their portfolios. As a result, major corporations, including even energy producers, have started to announced plans to meet certain net-zero emissions targets. Part of these targets will be achieved with a switch to renewables. But a recent Intergovernmental Panel on Climate Change (IPCC) special report (Rogelj et al. (2018)) estimates that much will rely on the accumulation of decarbonization capital, including afforestation and reforestation, soil carbon sequestration, bioenergy with carbon capture and storage (BECCs), and direct air capture (DAC).[1]

While prior work on socially responsible investing indicates that the cost-of-capital channel can be a material incentive,[2] there remains challenging questions regarding the welfare

[1] One reason is that for heavy industrial sectors like cement and steel, which generate nearly 20% of global CO2 emissions, switching fuel sources is not a viable option for achieving net-zero (de Pee et al. (2018)).

[2] The first model of green mandates and the cost-of-capital channel in a static CARA setting is Heinkel, Kraus, and Zechner (2001). Hong and Kacperczyk (2009) show how ethical investing mandates affect the sin companies' cost of capital. Recent work model how non-pecuniary tastes of green investors influence cross-sectional asset prices in a CAPM setting (Pastor, Stambaugh, and Taylor (2019), Pedersen, Fitzgibbons, and Pomorski (2020)), in a financial constraints setting (Oehmke and Opp (2020)) and in an information aggregation setting (Goldstein et al. (2021)). While exits or screens are the predominant form of mandates, mandates need not only be passive but also active via voting for environmentally friendly policies (Gollier and Pouget (2014), Broccardo, Hart, and Zingales (2020)).

consequences of these mandates. To what extent can they achieve first-best outcomes when it comes to mitigating global warming? In particular, can mandates be effective when there is a climate tipping point—an irreversible process leading to an absorbing state characterized by more frequent weather disasters such as heatwaves, droughts, hurricanes, and wildfires (Lenton et al. (2008), (Collins et al. (2019), National Academy of Sciences (2016))—that significantly increases the social cost of carbon (Cai et al. (2015), Cai and Lontzek (2019)).[3] What should be the qualification standards for a firm to be labeled sustainable? What is the impact on economic growth, particularly during the transition? And what about the consequences for firms' costs of capital and investors' portfolio performance?

To answer these questions, we introduce decarbonization capital into a dynamic stochastic general equilibrium model with standard capital stock as the sole input for producing a homogeneous good and the sole source of carbon emissions (Nordhaus (2017), Jensen and Traeger (2014)). Decarbonization capital only offsets carbon emissions, has no productive role, comes at the expense of forgone investment or dividend payouts and faces adjustment costs like productive capital. To effectively manage climate transition risk, more decarbonization capital stock is needed for a larger economy. We thus expect that the ratio of decarbonization-to-productive capital plays a critical role in delaying the arrival of a climate tipping point.

We model climate transition risk as a rare disaster (Rietz (1988), Barro (2006), Pindyck and Wang (2013), and Martin and Pindyck (2015)) via a stochastic transition (a Poisson jump process) from a "Good" climate state with infrequent weather disasters to an absorbing "Bad" climate state with frequent weather disasters (Lontzek et al. (2015)). Weather disaster arrivals in both "Good" and "Bad" climate states, which also follow Poisson jump processes, destroy both productive and decarbonization capital and lead to significant welfare loss for households with Epstein and Zin (1989) utility.[4] Importantly, both the climate state

[3] Higher temperatures lead to increased frequency and damage from hurricanes that make landfall (Grinsted, Ditlevsen, and Christensen (2019), Kossin et.al. (2020)). Similarly, the wildfires in the Western US states are also linked to climate change (Abatzoglou and Williams (2016)).

[4] Models with time-varying disaster arrival rates (Colin-Dufresne, Johannes, and Lochstoer (2016)), Gabaix (2012), Gourio (2012), and Wachter (2013)) have been shown to be quantitatively important to simultaneously explain business cycles and asset price fluctuations.

transition rate and the weather disaster arrival rates in both "Good" and "Bad" climate states are endogenous depending on the ratio of decarbonization-to-productive capital.

Since the mitigation costs are borne by the firm but its benefits are enjoyed by society in the form of lower aggregate risk, there is an externality that can be addressed by sustainable finance mandates to invest in firms that decarbonize. In contrast to the literature on mandates which often feature heterogeneous agents and incomplete markets, we generate a cost-of-capital channel by restricting a fixed fraction of the representative agent's portfolio (or total wealth in the economy) to hold firms that meet sustainability guidelines. The representative investor faces a short-selling constraint[5] but otherwise has access to a complete set of financial securities (e.g., all contingencies including idiosyncratic shocks are dynamically spanned).

To be included in the representative investor's sustainable portfolio, otherwise ex-ante identical firms have to invest a minimally required amount on decarbonization. The value of productive capital, i.e., Tobin's q, for sustainable and unsustainable firms, are endogenously determined by markets so as to leave value-maximizing firms indifferent between being sustainable or not — the Tobin's q or stock price is the same for all firms in equilibrium.[6] The risk-free rate, stock-market risk premium, Tobin's q for aggregate productive capital, and growth rates are jointly determined in equilibrium.

In this paper, we focus on the welfare-maximizing mandate with markets: the minimum decarbonization spending for firms to qualify with the goal of maximizing the welfare of the representative agent given competitive markets.[7] The qualification standards for a firm to be sustainable is endogenous, depending on the fraction of wealth that is restricted, the climate state, and the ratio of the aggregate decarbonization-to-productive capital. When the fraction of wealth that is indexed to sustainable finance mandates is larger, all else

[5]Institutions often face shorting constraints for a variety of business or legal reasons (see, e.g., Almazan et al. (2004).

[6]The decarbonization capital, which is unproductive and does not contribute to output, sits in the firm's assets but is not priced by markets other than through the mandate qualification mechanism.

[7]This is analogous to the solution concept of Lucas and Stokey (1983) for the Ramsey taxation problem in dynamic public finance in that the the government announces its mandate policy at time 0 and commits to it, and the private sectors optimize over time accordingly.

equal, each sustainable firm needs to make smaller contributions to decarbonization capital accumulation (i.e. qualifying standards are lower for being labeled sustainable).[8]

To address the welfare questions we posed, we compare outcomes of market economy with the welfare-maximizing mandate to the first-best solution in the planner's economy. For both economies, the ratio of decarbonization-to-productive capital rises gradually over time due to adjustment costs—from a low initial condition for the decarbonization-to-productive capital ratio until reaching the steady state when the ratio of decarbonization-to-productive capital then remains constant over time. The higher decarbonization-to-capital ratio reduces aggregate risk in the mandate-market economy just as in the planner's setting.

But these two solutions need not be the same. The reason is that whereas the planner jointly chooses mitigation and productive investments, firms in the mandate-market economy choose productive investments taking as given the required mitigation spending given by the welfare-maximizing mandate. One of our contributions is evaluating when these two solutions differ. Even if they are not identical, the mandate-market solution can in principle come quantitatively close to the decarbonization and welfare levels attained in the first-best economy. Hence, for our quantitative analysis, we are particularly interested in parameters when the planner wants to act now, i.e. make significant flow contributions to mitigation so as to smoothly ramp up to a high steady-state decarbonization-to-productive capital ratio.

A cost-of-capital wedge formula for sustainable versus unsustainable firms summarizes the costs of decarbonization for shareholders. In equilibrium, a firm that qualifies receive a lower cost of capital equal to its decarbonization investments divided by its Tobin's q. Since firms have the same Tobin's q in equilibrium, the growth paths of both sustainable and unsustainable firms are identical (path by path) over time. Sustainable firms have lower cashflows to pay out due to mitigation spending but have lower costs of capital (the expected returns required by the representative investor). That is, in equilibrium the cash-flow effect and the discount-rate effect exactly offset each other so as to leave all firms indifferent between

[8] The fraction of wealth that needs to be restricted to sustainable finance companies to implement the welfare-maximizing mandate can in principle be small since sustainable firms only have to satisfy a non-zero dividend constraint and hence can dedicate all their investments toward decarbonization.

being a sustainable and an unsustainable firm. The lower cost of capital for sustainable firms subsidizes their decarbonization, which they would have otherwise invested in productive capital or distributed to shareholders. The benefits of this mitigation accrue to the entire economy.

The cost-of-capital wedge tracks endogenous mandate standards required of sustainable firm decarbonization spending. Since these standards scale with the amount of decarbonization capital in the economy, the cost of capital wedge will also vary during the transition, tending to increase as the economy moves towards the steady state. Since there is a perfectly competitive and homogeneous goods market and capital is the only input, the limiting case of a mandate that restricts all wealth is a sustainable-finance tax on sales that funds a higher decarbonization-to-productive capital ratio.[9]

For our quantitative analysis, we calibrate the parameters of our model using key macro-finance moments from the asset pricing literature and empirical estimates of climate mitigation pathways. To generate a large act now effect, we consider as a baseline a tipping point arrival rate of once every 10 years (consistent with recent studies such as Lenton et al. (2019) who find that tipping points can occur even at current levels of warming), conditional damage estimates of weather shocks from panel regression estimates (see, e.g., Dell, Jones and Olken (2012)), and cost estimates of decarbonization based on direct air capture (Gates (2021), de Pee et al. (2018)).

We find that the welfare-maximizing mandate well approximates the planner's mitigation and welfare levels. The steady-state decarbonization-to-productive capital ratio in the market economy with the optimal mandate is 4.54% compared to 4.76% for the first-best outcome in the planner's economy. Given that the global capital stock is around 600 trillion dollars, a roughly 4.5% decarbonization-to-productive capital ratio implies 27 trillion dollars of decarbonization capital (i.e., book value) at the steady state.[10] The transition time to the steady state (conditional on not reaching the tipping point yet) is 16 years starting from no

[9]This sustainable-finance tax is in lieu of a tax on emissions that is analyzed integrated assessment models featuring an emissions sector (see, e.g. Golosov, Hassler, Krusell, and Tsyviski (2014)).

[10]Gadzinki, Schuller, and Vacchino (2018) estimate global capital stock (including both traded and non-traded assets) in 2016 to be between 500 and 600 trillion dollars.

decarbonization capital stock.

Aggregate contributions to decarbonization capital stock each year under the welfare-maximizing mandate is around 0.24% of physical capital stock in the steady state, compared to 0.25% for the planner's solution. This means spending of 1.44 trillion dollars per year towards decarbonization. The welfare-maximizing mandate is large enough to contribute toward aggregate net-zero emissions targets by 2050. The welfare gains from the mandate solution are substantial and close to the gains obtained by the planner's solution — almost 25% higher measured in the certainty equivalent wealth than in a purely competitive market setting with no mandates.

With 20% of wealth restricted to sustainable,[11] the social cost of decarbonization captured by the cost-of-capital wedge between sustainable and unsustainable firms is 0.50% per annum at the steady state.[12] In practice, even though there are industry differences in carbon emissions, the mandates are implemented within an industry using a best-in-class approach. Hence our estimates can also be taken as applying to a typical industry. A mandate that restricts all wealth to sustainable firms yields a welfare-maximizing sales tax rate of 0.24% to fund aggregate annual investments in decarbonization capital. The benefit of decarbonization is aggregate risk mitigation.

The two capital stocks approach in our paper differs from the two sector model of Eberly and Wang (2009), where their two capital stocks both produce goods and investors' preferences for portfolio diversification is the key force. Decarbonization capital in our model is unproductive and exists only to offset emissions. It only arises as a result of a mandate or a sustainable-finance tax. Our paper builds on Hong, Wang and Yang (2020), who model the regional-level mitigation of weather disasters, and the optimal capital tax to stimulate first-best level of flow spending for preparedness.

Our paper contributes to the emerging climate finance literature on the role of the finan-

[11] The Glasgow Initiative accounts for 21% of aggregate capital. More generally, according to US SIF Foundation in January 2019, around 38% of assets under management already undergo some type of sustainability screening (though not all of it is regarding decarbonization) and over 80% of these screens as implemented as passive portfolios.

[12] This cost-of-capital wedge is consistent with some preliminary estimates of expected returns based on Scope 1+2 emissions from Bolton and Kacperczyk (2020).

cial system in addressing global warming (see Hong, Karolyi, and Scheinkman (2020) for an overview). Bansal, Ochoa, and Kiku (2017) use a long-run risk model to evaluate the impact of higher temperature on growth stocks. Barnett, Brock, and Hansen (2020) provide an asset-pricing framework to confront climate model uncertainty. Engle et al. (2020) develop a method to hedge climate risks through trading of stock portfolios.

2 Model

2.1 Climate State

Consider the following model of the climate transition risk. Let $\mathcal{S}_t$ denote the climate state at time t. The economy starts from the good climate state, which we refer to as the $\mathcal{G}$ state and transitions to the bad state, which we refer to as the $\mathcal{B}$ state, at a time-varying endogenously determined probability per unit of time, which we denote by $\zeta_t > 0$. Let $\widetilde{\mathcal{J}}_t$ denote this jump process. Moreover, we assume that this transition is permanent, which means that the $\mathcal{B}$ state is absorbing. To capture the idea that weather disasters are more frequent in the $\mathcal{B}$ state than in the $\mathcal{G}$ state, we let the disaster arrival rate in the $\mathcal{G}$ state, $\lambda_t^{\mathcal{G}}$ to be smaller than that in the $\mathcal{B}$ state, $\lambda_t^{\mathcal{B}}$, i.e., $\lambda_t^{\mathcal{G}} < \lambda_t^{\mathcal{B}}$. We later discuss the details for these arrival states, $\lambda_t^{\mathcal{G}}$ and $\lambda_t^{\mathcal{B}}$, which are also endogenously determined. Next, we introduce the production side of the economy.

2.2 Firm Production and K Capital Accumulation

The firm's output at t, Y_t, is proportional to its capital stock, K_t, which we refer to as productive capital and is the only factor of production:

$$Y_t = AK_t\,, \tag{1}$$

where $A > 0$ is a constant that defines productivity for all firms. This is a version of widely-used AK models in macroeconomics and finance. All firms start with the same level of initial capital stock K_0 and have the same production and capital accumulation technology. Additionally, they are subject to the same shocks (path by path). That is, there is no idiosyncratic shock in our model. This simplifying assumption makes our model tractable and allows us to focus on the impact of the investment mandate on equilibrium asset pricing and resource allocation.

Investment. Let I_t denote the firm's investment. The firm's productive capital stock, K_t, evolves as:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-} d\mathcal{W}_t - (1 - Z)K_{t-} d\mathcal{J}_t \ . \tag{2}$$

As in Lucas and Prescott (1971) and Jermann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in I and K, and thus can be written as

$$\Phi(I, K) = \phi(i)K \ , \tag{3}$$

where $i = I/K$ is the firm's investment-capital ratio and $\phi(\cdot)$ is increasing and concave. This specification captures the idea that changing capital stock rapidly is more costly than changing it slowly. As a result, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement cost of capital exceeds one.

The second term captures continuous shocks to capital, where $\mathcal{W}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility (for the capital stock growth). This $\mathcal{W}_t$ is the common shock to all firms as in standard AK models in macro and finance.

Jump shocks. In both climate states ($\mathcal{S}_t = \{\mathcal{G}, \mathcal{B}\}$), the firm's capital stock K is also subject to an aggregate jump due to weather disasters such droughts, wildfires or hurricanes that destroy capital. We capture the effect of this jump shock via the third term, where $\{\mathcal{J}_t\}$ is a (pure) jump process with a climate-state-$\mathcal{S}_t$-dependent arrival rate $\lambda_t^{\mathcal{S}_t} > 0$. For brevity, we use λ_t to refer to $\lambda_t^{\mathcal{S}_t}$ when doing so causes no confusion.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1 - Z)$ of the firm's capital stock K_{t-}, as $Z \in (0, 1)$ is the recovery fraction. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit to the number of these jump shocks.[13] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears. We assume that the cumulative distribution function (cdf) and probability density function (pdf) for the recovery fraction, Z, conditional on a jump arrival at any time t, are time invariant. Let $\Xi(Z)$ and $\xi(Z)$ denote the cdf and pdf of Z, respectively.

Firm investment, dividends, and mitigation spending (contribution). At any time t, the firm uses its output AK_t to finance investment I_t, pay cash flows (dividends) CF_t to shareholders, and make contributions X_t towards aggregate mitigation spending to be described in detail soon. As a result, we have

$$AK_t = I_t + CF_t + X_t \,. \tag{4}$$

We use **boldfaced** notations for aggregate variables. Before discussing the endogenous jump arrival rate λ_{t-}, we first introduce emissions, emission removals, and the dynamics of decarbonization capital stock $\mathbf{N}$.

2.3 Aggregate Emissions, Emission Removals, and Decarbonization Capital Stock N

We assume that the aggregate emissions $\mathbf{E}$ is proportional to aggregate productive capital:

$$\mathbf{E}_{t-} = \mathbf{e}\mathbf{K}_{t-} \,, \tag{5}$$

where $\mathbf{e} > 0$ is a constant and $\mathbf{K}_t$ is the sum (integral) of K_t by all firms: $\mathbf{K}_t = \int K_t^{\nu} d\nu$.[14] That is, aggregate emissions increases linearly with the size of the production sector of the economy, which is measured by the aggregate capital stock $\mathbf{K}$ or equivalently GDP ($A\mathbf{K}$).

Similarly, we assume that the aggregate emission removals $\mathbf{R}$ is proportional to the aggregate decarbonization capital stock $\mathbf{N}$:

$$\mathbf{R}_{t-} = \tau \mathbf{N}_{t-} \,, \tag{6}$$

[13]Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

[14]We integrate over a continuum of firms with unit measure.

where $\tau > 0$ is a constant. Equations (5) and (6) state that both aggregate emissions and carbon removals are given by an "AK"-type of technology.

Let $\mathbf{X}_t$ denote the aggregate investment in decarbonization capital stock, e.g., aggregate mitigation spending. In equilibrium, $\mathbf{X}_t$ is the sum of mitigation spending by all firms: $\mathbf{X}_t = \int X_t^\nu d\nu$. The aggregate decarbonization capital stock $\mathbf{N}$ evolves as follows:

$$\frac{d\mathbf{N}_t}{\mathbf{N_{t-}}} = \omega(\mathbf{X}_{t-}/\mathbf{N_{t-}})dt + \sigma d\mathcal{W}_t - (1-Z)d\mathcal{J}_t \,. \tag{7}$$

The control $\mathbf{X}_{t-}/\mathbf{N_{t-}}$ in (7) for $\mathbf{N}$ accumulation at the aggregate level is analogous to the investment-capital ratio I_{t-}/K_{t-} in (2) for K accumulation at the firm level. That is, absent jumps, $\omega(\mathbf{X}_{t-}/\mathbf{N_{t-}})$, the drift of $d\mathbf{N}_t/\mathbf{N}_{t-}$, is analogous to $\phi(I_{t-}/K_{t-})$, the drift of dK_t/K_{t-}. We assume that $\omega(\,\cdot\,)$ is increasing and concave as we do for $\phi(\,\cdot\,)$. This specification captures the idea that changing $\mathbf{N}$ rapidly is more costly than changing it slowly.

Equation (7) implies that the growth rate for the decarbonization capital stock $\mathbf{N}$, $d\mathbf{N}_t/\mathbf{N}_{t-}$, is subject to the same diffusion and jump shocks as the growth rate of capital stock K, dK_t/K_{t-}, path by path (e.g., for each realized jump and recovery fraction Z). This explains why the last two terms in (7) take the same form as those in (2).

Let $\mathbf{n}_{t-}$ denote the aggregate decarbonization stock $\mathbf{N}_{t-}$ scaled by $\mathbf{K}_{t-}$:

$$\mathbf{n}_{t-} = \frac{\mathbf{N}_{t-}}{\mathbf{K}_{t-}} \,. \tag{8}$$

Using Ito's lemma, we obtain the following dynamics for $\mathbf{n}_t$:

$$\frac{d\mathbf{n}_t}{\mathbf{n}_{t-}} = \left[\omega(\mathbf{x}_{t-}/\mathbf{n}_{t-}) - \phi(\mathbf{i}_{t-})\right] dt \,. \tag{9}$$

Note that there is no uncertainty for the dynamics of $\mathbf{n}_t$. This is because the growth rates for the two types of capital stock are subject to the same jump-diffusion shocks.[15] Next, we describe the distribution for the recovery fraction Z.

2.4 Mitigation and Externality

Global warming increases the arrival rate of the bad state (ζ_t) and the frequencies of disaster arrivals in both climate states ($\lambda_{t-}^{\mathcal{G}}$ and $\lambda_{t-}^{\mathcal{B}}$). Therefore, ζ_t, $\lambda_{t-}^{\mathcal{G}}$, and $\lambda_{t-}^{\mathcal{B}}$ are all increasing

[15] Note that $\mathbf{n}_t$ remains constant even when climate state transitions.

in the aggregate emissions $\mathbf{E}_{t-}$ and decreasing in the aggregate emissions removals $\mathbf{R}_{t-}$. As $\mathbf{E}_{t-} = \mathbf{e}\mathbf{K}_{t-}$ and $\mathbf{R}_{t-} = \tau\mathbf{N}_{t-}$ (see equations (5) and (6)), we may write ζ_t, $\lambda_{t-}^{\mathcal{G}}$, and $\lambda_{t-}^{\mathcal{B}}$ as functions that are increasing in $\mathbf{K}_{t-}$ and decreasing in $\mathbf{N}_{t-}$. We assume that the effects of $\mathbf{K}_{t-}$ and $\mathbf{N}_{t-}$ on ζ_{t-} can be summarized via $\mathbf{n}_{t-}$, i.e., the homogeneity property holds. Hence, we write $\zeta(\mathbf{n}_{t-})$, where $\zeta'(\mathbf{n}_{t-}) < 0$. That is, decarbonization delays the tipping point. Additionally, we assume that $\zeta''(\mathbf{n}_{t-}) > 0$. The rate at which $\zeta(\mathbf{n}_{t-})$ decreases with $\mathbf{n}_{t-}$, $|\zeta'(\mathbf{n}_{t-})|$, decreases with $\mathbf{n}_{t-}$.

Similarly, we assume that both $\lambda_{t-}^{\mathcal{G}}$, and $\lambda_{t-}^{\mathcal{B}}$ are homogeneous of degree zero in $\mathbf{K}_{t-}$ and $\mathbf{N}_{t-}$, which means they are functions of the pre-jump scaled *aggregate* decarbonization stock $\mathbf{n}_{t-} = \mathbf{N}_{t-}/\mathbf{K}_{t-}$ and the climate state $\mathcal{S}$.[16] To highlight the dependence of λ_{t-} on $\mathbf{n}_{t-}$ and $\mathcal{S}_{t-}$ explicit, we write $\lambda_{t-} = \lambda_{t-}^{\mathcal{S}_{t-}} = \lambda(\mathbf{n}_{t-}; \mathcal{S}_{t-})$. Intuitively, increasing $\mathbf{n}$ lowers the jump arrival rate, $\lambda'(\mathbf{n}_{t-}; \mathcal{S}_{t-}) < 0$. Additionally, the marginal impact of $\mathbf{N}$ on the change of λ decreases as $\mathbf{N}$ increases, i.e., $\lambda''(\mathbf{n}_{t-}; \mathcal{S}_{t-}) > 0$.

As disaster shocks are aggregate and disaster damages are only curtailed by *aggregate* decarbonization stock $\mathbf{N}$, absent mandates or other incentive programs, firms have no incentives to mitigate on their own in a competitive economy and how much each individual firm spends on mitigation has no impact on its own payoff.

2.5 Sustainable Finance Mandate: α in Type-S and $(1-\alpha)$ in Type-U Firms

The agent has to invest an α fraction of the entire aggregate wealth in a sustainable type-S firm. The risk-averse representative agent is required to meet the sustainable investment mandate at all times when allocating assets. In other words, on the demand side for financial assets, the representative agent holds and invests the entire wealth of the economy between sustainable (S) firms, unsustainable (U) firms, and the risk-free bonds.

On the supply side, a portfolio of S firms and a portfolio of U firms will arise endogenously in equilibrium, which we refer to as S-portfolio and U-portfolio, respectively. For a firm to

[16] This assumption implies that the expected damage over a small dt period, which is $\lambda_{t-}\mathbb{E}(1-Z)\mathbf{K}_{t-}dt$, doubles if we simultaneously double both the size of the productive sector ($\mathbf{K}_{t-}$) and the size of the decarbonization capital stock ($\mathbf{N}_{t-}$). This property is consistent with sustainable long-term growth.

qualify to be type-S, it has to spend at least $M_t = m_t K_t$, i.e., a fraction m_t of its capital on mitigation via a portfolio of decarbonization technologies so as to reduce disaster risk. That is, $\mathbf{1}_t^S = 1$ if and only if the firm's mitigation spending X_t satisfies:

$$X_t \geq M_t \,. \tag{10}$$

Otherwise, it is labeled a type-U for unsustainable and $\mathbf{1}_t^S = 0$.

The S and U portfolios include all the S and U firms, respectively. Let $\mathbf{Q}_t^S$ and $\mathbf{Q}_t^U$ denote the aggregate market value of the S portfolio and of the U portfolio at t, respectively. The total market capitalization of the economy, $\mathbf{Q}_t$, is given by

$$\mathbf{Q}_t = \mathbf{Q}_t^S + \mathbf{Q}_t^U \,. \tag{11}$$

In equilibrium, the investment mandate requires that the total capital investment in the S portfolio, $\mathbf{Q}_t^S$, has to be at least an α fraction of the total stock market capitalization $\mathbf{Q}_t$:

$$\mathbf{Q}_t^S \geq \alpha \mathbf{Q}_t \,. \tag{12}$$

2.6 Dynamic Consumption and Asset Allocation

The representative agent makes consumption, asset allocation, and risk management decisions. We use individual and aggregate variables for the agent interchangeably as we have a continuum of identical agents (with unit measure). For example, the aggregate wealth, $\mathbf{W}_t$, is equal to the representative agent's wealth, W_t. Similarly, the aggregate consumption, $\mathbf{C}_t$, is equal to the representative agent's consumption, C_t.

The representative agent has the following investment opportunities: (a) the S-portfolio which includes all the sustainable firms; (b) the U-portfolio which includes all other firms that are unsustainable; (c) the risk-free asset that pays interest at a risk-free interest rate r process determined in equilibrium.[17]

[17]To be precise, as markets are dynamically complete, the economy also has actuarially fair insurance claims for weather disasters (with every possible recovery fraction Z) and the insurance contracts contingent on climate transition as well as diffusion shocks. But we suppress these zero-net-supply claims since they do not change the allocations in the economy as shown in Pindyck and Wang (2013) and Hong, Wang and Yang (2020).

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that the representative agent has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right] , \tag{13}$$

where $f(C,V)$ is known as the normalized aggregator given by

$$f(C,V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^\chi}{((1-\gamma)V)^{\chi-1}} . \tag{14}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\chi = (1-\psi^{-1})/(1-\gamma)$. Unlike expected utility, recursive preferences as defined by (13) and (14) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\chi-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature.

If $\gamma = \psi^{-1}$ so that $\chi = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C,V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho V. \tag{15}$$

Comment. In our model, the representative agent represents investors in the whole economy including both the private and public sectors. We may also interpret our representative-agent model as one with heterogeneous agents where an α fraction of them are sustainable investors, who have investment mandates (e.g., large asset managers and sovereign wealth funds), and the remaining $1-\alpha$ fraction do not. The sustainable investors group has inelastic demand for sustainable firms and moreover they do not lend their shares out for other investors to short sustainable firms.

2.7 Competitive Equilibrium with Mandates

Let $\mathbf{Y}_t$, $\mathbf{C}_t$, $\mathbf{I}_t$, and $\mathbf{X}_t$ denote the aggregate output, consumption, investment, and mitigation spending, respectively. Using an individual firm's resource constraint (4) and adding across all type-S and U firms, we obtain the aggregate resource constraint:

$$\mathbf{Y}_t = \mathbf{C}_t + \mathbf{I}_t + \mathbf{X}_t \,. \tag{16}$$

We define the competitive equilibrium subject to the investment mandate defined by (10) as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the S portfolio, the U portfolio, and the risk-free asset subject to the investment mandate; (2) each firm chooses its status (S or U), and investment policy I to maximize its market value; (3) all firms that choose sustainable investment policies are included in the S portfolio and all remaining firms are included in the U portfolio; and (4) all markets clear.

The market-clearing conditions include (i) the net supply of the risk-free asset is zero; (ii) the representative agent's demand for the S portfolio is equal to the total supply by firms choosing to be sustainable; (iii) the representative agent's demand for the U portfolio is equal to the total supply by firms choosing to be unsustainable; (iv) the goods market clears, i.e., the aggregate resource constraint given in (16) holds.

3 Equilibrium Solution under Optimal Mandate

In this section, we solve for the equilibrium solution with the sustainable finance mandate. For a firm to be sustainable at t, it has to spend the minimal required m_t fraction of its productive capital stock K_t. We work within the set of m_t specifications where we can write m_t as a function of $\mathbf{n}_t$ and climate state $\mathcal{S}_t$: $m_t = m(\mathbf{n}_t; \mathcal{S}_t)$. We assume that a firm's mitigation is observable and contractible. While spending on aggregate risk mitigation yields no monetary payoff for the firm, doing so allows it to be included in the S-portfolio.

3.1 Firm Optimization

A value-maximizing firm chooses whether to be sustainable or unsustainable depending on which strategy yields a higher value. By exploiting our model's homogeneity property, we conjecture and verify that the equilibrium value of a type-j firm, where $j = \{S, U\}$, Q_t^j, at time t must satisfy:

$$Q_t^j = q^j(\mathbf{n}_t; \mathcal{S}_t) K_t^j \,, \tag{17}$$

where $q^j(\mathbf{n}_t; \mathcal{S}_t)$ is Tobin's q for a type j-firm as a function of $\mathbf{n}_t$ and climate state $\mathcal{S}_t$.

A type-j firm maximizes its present value:

$$\max_{I^j, X^j} \mathbb{E}\left(\int_0^\infty e^{-\int_0^t r^j(\mathbf{n}_v; \mathcal{S}_v)dv} CF^j(\mathbf{n}_t; \mathcal{S}_t) dt\right) \,, \tag{18}$$

where $r^j(\mathbf{n}_t; \mathcal{S}_t)$ is the expected cum-dividend return for a type-j firm in equilibrium.[18] In equation (18), $CF^j(\mathbf{n}_t; \mathcal{S}_t)$ is the firm's cash flow at t, which is given by

$$CF^S(\mathbf{n}_t; \mathcal{S}_t) = AK_t^S - I_t^S(\mathbf{n}_t; \mathcal{S}_t) - X_t^S(\mathbf{n}_t; \mathcal{S}_t) \text{ and } CF^U(\mathbf{n}_t; \mathcal{S}_t) = AK_t^U - I_t^U(\mathbf{n}_t; \mathcal{S}_t) \,, \tag{19}$$

as an unsustainable firm spends nothing on mitigation.

In equilibrium, as mitigation spending has no direct benefit for the firm, if the firm chooses to be U, i.e., $\mathbf{1}_t^S = 0$, it will set $X_t^U = 0$. Moreover, even if a firm chooses to be an S firm, it has no incentive to spend more than M_t, i.e., (10) always binds for a type-S firm.

That is, it is optimal for a sustainable firm to set x_t^S as:

$$x_t^S = \frac{X_t^S}{K_t^S} = m(\mathbf{n}_t; \mathcal{S}_t) \,. \tag{20}$$

All other firms spend nothing on mitigation and hence are unsustainable, i.e., $x_t^U = 0$. Since the fraction of total wealth allocated to meet the sustainability investment mandate is $\alpha \in (0, 1]$, there is a continuum of firms (with unit measure), and all sustainable firms are identical, the scaled aggregate mitigation spending, $\mathbf{x}_t$, is given by

$$\mathbf{x}_t = \frac{\mathbf{X}_t}{\mathbf{K}_t} = \frac{\alpha X_t^S}{K_t^S} = \alpha x_t^S = \alpha m(\mathbf{n}_t; \mathcal{S}_t) \,. \tag{21}$$

[18]Additionally, we impose the standard transversality condition for (18).

Next, we consider the firm's investment problem when it takes the sustainability mandate $\{m_t = m(\mathbf{n}_t; \mathcal{S}_t) : t \geq 0\}$ as given. We solve for optimal investment policies for both types of firms. A type-j firm's objective (18) implies that $\int_0^u e^{-\int_0^t r^j(\mathbf{n}_v;\mathcal{S}_v)dv} CF^j(\mathbf{n}_t; \mathcal{S}_t)dt + e^{-\int_0^u r^j(\mathbf{n}_v;\mathcal{S}_v)dv} Q_u^j$ is a martingale under the physical measure, where $r^j(\mathbf{n}; \mathcal{S})$ is the cost of capital that the firm takes as given. The firm takes the scaled aggregate decarbonization capital stock $\mathbf{n}$, aggregate mitigation spending $\mathbf{x}(\mathbf{n}; \mathcal{S})$, and aggregate investment $\mathbf{i}(\mathbf{n}; \mathcal{S})$ as given. The following Hamilton-Jacobi-Bellman (HJB) equation characterizes the firm's value function in climate state $\mathcal{G}$:

$$
\begin{aligned}
r^j(\mathbf{n}; \mathcal{G}) Q^j(K^j, \mathbf{n}; \mathcal{G}) = \max_{I^j} \; & CF^j(\mathbf{n}; \mathcal{G}) + \Phi(I^j, K^j) Q_K(K^j, \mathbf{n}; \mathcal{G}) + \frac{1}{2}(\sigma K^j)^2 Q_{KK}(K^j, \mathbf{n}; \mathcal{G}) \\
& + \left[\omega(\mathbf{x}(\mathbf{n}; \mathcal{G})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}; \mathcal{G}))\right] \mathbf{n} Q_{\mathbf{n}}^j(K^j, \mathbf{n}; \mathcal{G}) \\
& + \lambda(\mathbf{n}; \mathcal{G}) \mathbb{E}\left[Q^j(ZK^j, \mathbf{n}; \mathcal{G}) - Q^j(K^j, \mathbf{n}; \mathcal{G})\right] \\
& + \zeta(\mathbf{n}) (Q^j(K^j, \mathbf{n}; \mathcal{B}) - Q^j(K^j, \mathbf{n}; \mathcal{G})) \, ,
\end{aligned} \tag{22}
$$

where $CF^j(\mathbf{n}; \mathcal{G})$ is the cash flow for a type-j firm in climate state $\mathcal{G}$ given by (19). In (22), $\mathbb{E}[\,\cdot\,]$ is the conditional expectation operator with respect to the distribution of recovery fraction Z. We have a similar equation without the last transition term for the firm's value function $Q^j(K^j, \mathbf{n}; \mathcal{B})$ in climate state $\mathcal{B}$. Let $cf^j(\mathbf{n}; \mathcal{S}) = CF^j(\mathbf{n}; \mathcal{S})/K^j$ denote the scaled cash flow for a type-j firm.

By using our model's homogeneity property, $Q_t^j = q^j(\mathbf{n}_t; \mathcal{S}_t) K_t^j$ for $\mathcal{S}_t = \mathcal{G}, \mathcal{B}$, we obtain the following ODE for $q^j(\mathbf{n}; \mathcal{G})$, the Tobin's q in the climate state $\mathcal{G}$:

$$
\begin{aligned}
r^j(\mathbf{n}; \mathcal{G}) q^j(\mathbf{n}; \mathcal{G}) = & \max_{i^j} \; cf^j(\mathbf{n}; \mathcal{G}) + (\phi(i^j) - \lambda(\mathbf{n}; \mathcal{G})(1 - \mathbb{E}(Z))) q^j(\mathbf{n}; \mathcal{G}) \qquad (23) \\
& + \left[\omega(\mathbf{x}(\mathbf{n}; \mathcal{G})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}; \mathcal{G}))\right] \mathbf{n} q_{\mathbf{n}}^j(\mathbf{n}; \mathcal{G}) + \zeta(\mathbf{n})(q^j(\mathbf{n}; \mathcal{B}) - q^j(\mathbf{n}; \mathcal{G})) \, .
\end{aligned}
$$

Similarly, we have the following HJB equation for $q^j(\mathbf{n}; \mathcal{B})$, the Tobin's q in the $\mathcal{B}$ state:

$$
\begin{aligned}
r^j(\mathbf{n}; \mathcal{B}) q^j(\mathbf{n}; \mathcal{B}) = \max_{i^j} \; & cf^j(\mathbf{n}; \mathcal{B}) + (\phi(i^j) - \lambda(\mathbf{n}; \mathcal{B})(1 - \mathbb{E}(Z))) q^j(\mathbf{n}; \mathcal{B}) \\
& + \left[\omega(\mathbf{x}(\mathbf{n}; \mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}; \mathcal{B}))\right] \mathbf{n} q_{\mathbf{n}}^j(\mathbf{n}; \mathcal{B}) \, .
\end{aligned} \tag{24}
$$

Unlike (23), there is no transition term involving $\zeta(\mathbf{n})$ from $\mathcal{B}$ to $\mathcal{G}$ as $\mathcal{B}$ is an absorbing state.

The investment FOCs for both types of firms implied by (24) and (23) in both $\mathcal{S}$ states are the following well known conditions in the q-theory literature:

$$q^j(\mathbf{n};\mathcal{S}) = \frac{1}{\phi'(i^j(\mathbf{n};\mathcal{S}))}\,. \tag{25}$$

A type-j firm's marginal benefit of investing is equal to its marginal q, $q^j(\mathbf{n};\mathcal{S})$, multiplied by $\phi'(i^j(\mathbf{n};\mathcal{S}))$. Equation (25) states that this marginal benefit, $q^j(\mathbf{n};\mathcal{S})\phi'(i^j(\mathbf{n};\mathcal{S}))$, is equal to one, the marginal cost of investing at optimality. The homogeneity property implies that a firm's marginal q is equal to its average q (Hayashi, 1982).

Let $g(\mathbf{n};\mathcal{S})$ denote a type-j firm's expected growth rate including the effect of jumps. In the $\mathcal{B}$ state, the expected growth rate is

$$g^j(\mathbf{n};\mathcal{B}) = \phi(i^j(\mathbf{n};\mathcal{B})) - \lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))\,, \tag{26}$$

and in the $\mathcal{G}$ state, the expected growth rate is

$$g^j(\mathbf{n};\mathcal{G}) = \phi(i^j(\mathbf{n};\mathcal{G})) - \lambda(\mathbf{n};\mathcal{G})(1-\mathbb{E}(Z)) - \zeta(\mathbf{n})\frac{q^j(\mathbf{n};\mathcal{G}) - q^j(\mathbf{n};\mathcal{B})}{q^j(\mathbf{n};\mathcal{G})}\,. \tag{27}$$

As $x^S(\mathbf{n};\mathcal{S}) = m(\mathbf{n};\mathcal{S})$ and $x^U(\mathbf{n};\mathcal{S}) = 0$, we have $cf^S(\mathbf{n};\mathcal{S}) = A - i^S(\mathbf{n};\mathcal{S}) - m(\mathbf{n};\mathcal{S})$ for a type-S firm and $cf^U(\mathbf{n};\mathcal{S}) = A - i^U(\mathbf{n};\mathcal{S})$ for a type-U firm.

3.2 Representative Agent's Optimization

Return dynamics of type-j portfolios. Let $\mathbf{Q}_t^j = \mathbf{q}(\mathbf{n}_t;\mathcal{S}_t)\mathbf{K}_t^j$ denote the market value of a type-j portfolio at t in state $\mathcal{S}_t$, where $j = \{S, U\}$. Similarly, let $\mathbf{D}_t^S$ and $\mathbf{D}_t^U$ denote the aggregate dividends of the S portfolio and of the U portfolio at t, respectively.

First, consider the $\mathcal{B}$ state. We later show that the equilibrium cum-dividend return for the type-j portfolio in this state is:

$$\frac{d\mathbf{Q}_t^j + \mathbf{D}_{t-}^j dt}{\mathbf{Q}_{t-}^j} = r^j(\mathbf{n}_{t-};\mathcal{B})dt + \sigma d\mathcal{W}_t - (1-Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{B})dt\right). \tag{28}$$

Note that the last two terms in equation (28) are martingales. The diffusion volatility is equal to σ as in equation (2). The third term on the right side of equation (28) is a jump term capturing the effect of disasters on return dynamics. Both the diffusion volatility and jump terms are martingales (and this is why $r^j(\mathbf{n}_{t-};\mathcal{B})$ is the expected return.) Note that the only difference between the S- and U-portfolio is the expected return $r^j(\mathbf{n}_{t-};\mathcal{B})$. The diffusion and jump terms are the same as those in the capital dynamics given in equation (2). We verify these equilibrium results in Appendix B.

For the $\mathcal{G}$ state, we later show that the equilibrium cum-dividend return for the type-j portfolio is:

$$\frac{d\mathbf{Q}_t^j + \mathbf{D}_{t-}^j dt}{\mathbf{Q}_{t-}^j} = r^j(\mathbf{n}_{t-};\mathcal{G})dt + \sigma d\mathcal{W}_t - (1-Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{G})dt\right) + \frac{\mathbf{q}(\mathbf{n}_{t-};\mathcal{B}) - \mathbf{q}(\mathbf{n}_{t-};\mathcal{G})}{\mathbf{q}(\mathbf{n}_{t-};\mathcal{G})}\left(d\widetilde{\mathcal{J}}_t - \zeta(\mathbf{n}_{t-})dt\right). \tag{29}$$

The first three terms for the $\mathcal{G}$ state on the right side of (29) are the counterparts of the three terms on the right side of (28) for the $\mathcal{B}$ state. The key difference between (29) and (28) is that in the $\mathcal{G}$ state we have the $(\widetilde{\mathcal{J}}_t)$ jump term, which captures the effect of climate transition from the $\mathcal{G}$ state to the absorbing $\mathcal{B}$ state. Note that the last term describes the percentage change of the portfolio value upon the arrival of the tipping point, which also equals the percentage change of Tobin's q associated with the climate state transition. This is because unlike the weather disaster shock $\mathcal{J}_t$, the climate state transition shock $\widetilde{\mathcal{J}}_t$ does not change $\mathbf{K}$. This last term is also a martingale as $r^j(\mathbf{n}_{t-};\mathcal{G})$ is defined as the expected return for the type-j portfolio.

Wealth dynamics. Let W_t denote the representative agent's wealth. Let H_t^S and H_t^U denote the dollar amount invested in the S and U portfolio, respectively. Let H_t denote the agent's wealth allocated to the market portfolio at t. That is, $H_t = H_t^S + H_t^U$. The dollar amount invested in the risk-free asset is , $(W_t - H_t)$.

In state $\mathcal{B}$, the agent accumulates wealth as:

$$dW_t = \left[r(\mathbf{n}_{t-};\mathcal{B})\left(W_{t-} - H_{t-}\right) - C_{t-}\right]dt + \left(r^S(\mathbf{n}_{t-};\mathcal{B})H_{t-}^S + r^U(\mathbf{n}_{t-};\mathcal{B})H_{t-}^U\right)dt + \sigma H_{t-}d\mathcal{W}_t - (1-Z)\,H_{t-}(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{B})dt). \tag{30}$$

The first term in (30) is the interest income from savings in the risk-free asset minus consumption. The second term is the expected return from investing in the S and U portfolios.

Note that the expected returns are different: $r^S(\mathbf{n};\mathcal{B})$ and $r^U(\mathbf{n};\mathcal{B})$ for the S and U portfolios, respectively. The third and fourth terms are the diffusion and jump martingale terms for the stock market portfolio. Note that the stochastic (shock) components of the returns (diffusion and jumps) for the two portfolios are identical path by path. In state $\mathcal{G}$, the agent accumulates wealth as:

$$
\begin{aligned}
dW_t =& \left[r(\mathbf{n}_{t-};\mathcal{G})\left(W_{t-}-H_{t-}\right)-C_{t-}\right]dt+\left(r^S(\mathbf{n}_{t-};\mathcal{G})H^S_{t-}+r^U(\mathbf{n}_{t-};\mathcal{G})H^U_{t-}\right)dt+\sigma H_{t-}d\mathcal{W}_t \\
&-\left[(1-Z)\left(d\mathcal{J}_t-\lambda(\mathbf{n}_{t-};\mathcal{G})dt\right)-\frac{\mathbf{q}(\mathbf{n}_{t-};\mathcal{B})-\mathbf{q}(\mathbf{n}_{t-};\mathcal{G})}{\mathbf{q}(\mathbf{n}_{t-};\mathcal{G})}\left(d\widetilde{\mathcal{J}}_t-\zeta(\mathbf{n}_{t-})dt\right)\right]H_{t-}\,. \qquad (31)
\end{aligned}
$$

The last term captures the effect of climate state transition to $\mathcal{B}$ on the agent's wealth accumulation.

In equilibrium, the fraction of total wealth allocated to the S-portfolio, which we denote by $\pi^S = H^S/(H^S+H^U) = H^S/W$, is equal to the fraction of wealth mandated to invest in the S portfolio: $\pi^S=\alpha$. The remaining $1-\pi^S$ fraction of total wealth is allocated to the U-portfolio. That is, we have $H^S_t=\alpha W_t=\mathbf{Q}^S_t=\alpha\mathbf{Q}_t$, $H^U_t=(1-\alpha)W_t=\mathbf{Q}^U_t=(1-\alpha)\mathbf{Q}_t$, and $W_t=\mathbf{Q}_t=\mathbf{Q}^S_t+\mathbf{Q}^U_t$.

Let $V_t=V(W_t,\mathbf{n}_t;\mathcal{S}_t)$ denote the household's value function. The HJB equation for the household's value function in state $\mathcal{B}$, $V(W,\mathbf{n};\mathcal{B})$, satisfies (see Appendix A for details):

$$
\begin{aligned}
0=\max_C\ & f(C,V;\mathcal{B})+\left[\left(r^S(\mathbf{n};\mathcal{B})\alpha+r^U(\mathbf{n};\mathcal{B})(1-\alpha)\right)W-C+\lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))W\right]V_W \\
&+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\mathbf{n}V_{\mathbf{n}}+\frac{\sigma^2W^2V_{WW}}{2} \\
&+\lambda(\mathbf{n};\mathcal{B})\mathbb{E}\left[V\left(ZW,\mathbf{n};\mathcal{B}\right)-V(W,\mathbf{n};\mathcal{B})\right]\,. \qquad (32)
\end{aligned}
$$

Similarly, we have the following HJB equation for $V(W,\mathbf{n};\mathcal{G})$ in the $\mathcal{G}$ state:

$$
\begin{aligned}
0=\max_C\ & f(C,V;\mathcal{G})+\left[\left(r^S(\mathbf{n};\mathcal{G})\alpha+r^U(\mathbf{n};\mathcal{G})(1-\alpha)\right)W-C+\lambda(\mathbf{n};\mathcal{G})(1-\mathbb{E}(Z))W\right]V_W \\
&+\zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n};\mathcal{G})-\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}WV_W+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right]\mathbf{n}V_{\mathbf{n}}+\frac{\sigma^2W^2V_{WW}}{2} \\
&+\lambda(\mathbf{n};\mathcal{G})\mathbb{E}\left[V\left(ZW,\mathbf{n};\mathcal{G}\right)-V(W,\mathbf{n};\mathcal{G})\right] \\
&+\zeta(\mathbf{n})\left[V\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}W,\mathbf{n};\mathcal{B}\right)-V(W,\mathbf{n};\mathcal{G})\right]\,. \qquad (33)
\end{aligned}
$$

Compared with the HJB equation (32) for $V(W, \mathbf{n}; \mathcal{B})$, we have two additional terms in the HJB equation (33) for $V(W, \mathbf{n}; \mathcal{G})$ in state $\mathcal{G}$ due to the transition of the climate state from $\mathcal{G}$ to $\mathcal{B}$. The last term captures the direct effect of change of wealth (Tobin's q) on the expected value function change. The other effect is via the change of wealth accumulation rate (the V_W drift term).

The FOC for consumption C in both climate states is given by the following equation:

$$f_C(C, V; \mathcal{S}) = V_W(W, \mathbf{n}; \mathcal{S}) \,. \tag{34}$$

We show that $V_t = V(W_t, \mathbf{n}_t; \mathcal{S}_t)$ is homogeneous with degree $1-\gamma$ in W:

$$V(W, \mathbf{n}; \mathcal{S}) = \frac{1}{1-\gamma}(u(\mathbf{n}; \mathcal{S})W)^{1-\gamma} \,, \tag{35}$$

where $u(\mathbf{n}; \mathcal{S})$ is a welfare measure proportional to the representative household's certainty equivalent wealth to be determined endogenously. Substituting (35) into the FOC (34) yields the following linear consumption rule with a time-varying MPC that depends on $\mathbf{n}$ and $\mathcal{S}$:[19]

$$C(W, \mathbf{n}; \mathcal{S}) = \rho^{\psi} u(\mathbf{n}; \mathcal{S})^{1-\psi} W \,. \tag{36}$$

Substituting (36) and (35) into the HJB equation (32), we obtain the following ODE for $u(\mathbf{n}; \mathcal{B})$ in state $\mathcal{B}$:

$$\begin{aligned} 0 \;=\; & \frac{\rho^{\psi} u(\mathbf{n}; \mathcal{B})^{1-\psi} - \rho}{1-\psi^{-1}} + \alpha r^S(\mathbf{n}; \mathcal{B}) + (1-\alpha) r^U(\mathbf{n}; \mathcal{B}) - \rho^{\psi} u(\mathbf{n}; \mathcal{B})^{1-\psi} + \lambda(\mathbf{n}; \mathcal{B})(1-\mathbb{E}(Z)) \\ & + \left[\omega(\mathbf{x}(\mathbf{n}; \mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}; \mathcal{B}))\right] \frac{\mathbf{n} u'(\mathbf{n}; \mathcal{B})}{u(\mathbf{n}; \mathcal{B})} - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n}; \mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \,. \end{aligned} \tag{37}$$

Similarly, we obtain the following ODE for $u(\mathbf{n}; \mathcal{G})$ in state $\mathcal{G}$:

$$\begin{aligned} 0 \;=\; & \frac{\rho^{\psi} u(\mathbf{n}; \mathcal{G})^{1-\psi} - \rho}{1-\psi^{-1}} + \alpha r^S(\mathbf{n}; \mathcal{G}) + (1-\alpha) r^U(\mathbf{n}; \mathcal{G}) - \rho^{\psi} u(\mathbf{n}; \mathcal{G})^{1-\psi} + \lambda(\mathbf{n}; \mathcal{G})(1-\mathbb{E}(Z)) \\ & + \left[\omega(\mathbf{x}(\mathbf{n}; \mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n}; \mathcal{B}))\right] \frac{\mathbf{n} u'(\mathbf{n}; \mathcal{G})}{u(\mathbf{n}; \mathcal{G})} - \frac{\gamma \sigma^2}{2} + \frac{\lambda(\mathbf{n}; \mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\ & + \zeta(\mathbf{n}) \frac{\mathbf{q}(\mathbf{n}; \mathcal{G}) - \mathbf{q}(\mathbf{n}; \mathcal{B})}{\mathbf{q}(\mathbf{n}; \mathcal{G})} + \frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{u(\mathbf{n}; \mathcal{B}) q(\mathbf{n}; \mathcal{B})}{u(\mathbf{n}; \mathcal{G}) q(\mathbf{n}; \mathcal{G})}\right)^{1-\gamma} - 1\right] \,. \end{aligned} \tag{38}$$

As we discussed earlier, the last two terms are only present in state $\mathcal{G}$ not $\mathcal{B}$ as stochastic transition only occurs from $\mathcal{G}$ to the absorbing $\mathcal{B}$ state.

[19]Since our model is a representative-agent framework, the aggregate financial wealth, $\mathbf{W}_t$, is equal to W_t for all t. We thus simply use these two interchangeably.

3.3 Market Equilibrium

The equilibrium risk-free rate (r_t), the expected returns for the S and U portfolios (r_t^S and r_t^U), and Tobin's average q (q_t) for all firms are functions of $\mathbf{n}_t$ given the climate state $\mathcal{S}_t$. For brevity, whenever causing no confusion, we suppress the dependence on the climate state $\mathcal{S}$.

As a firm can choose being either sustainable or not, it must be indifferent between the two options at all time. That is, in equilibrium, all firms have the same Tobin's q, which in equilibrium is also Tobin's average q for the aggregate economy ($\mathbf{q}$):

$$q^S(\mathbf{n};\mathcal{S}) = q^U(\mathbf{n};\mathcal{S}) = \mathbf{q}(\mathbf{n};\mathcal{S})\,. \tag{39}$$

Equations (25) and (39) imply that the investment-capital ratio for all firms is the same and equal to the aggregate $\mathbf{i}(\mathbf{n};\mathcal{S})$ for a given scaled decarbonization capital stock $\mathbf{n}$ and the climate state $\mathcal{S}$:

$$i^S(\mathbf{n};\mathcal{S}) = i^U(\mathbf{n};\mathcal{S}) = \mathbf{i}(\mathbf{n};\mathcal{S})\,. \tag{40}$$

As a result, the cash flows difference between a U and an S firm is exactly the firm's mitigation spending $m(\mathbf{n};\mathcal{S})$:

$$cf^U(\mathbf{n};\mathcal{S}) - cf^S(\mathbf{n};\mathcal{S}) = m(\mathbf{n};\mathcal{S})\,, \tag{41}$$

where $cf^U(\mathbf{n};\mathcal{S}) = A - i(\mathbf{n};\mathcal{S})$ is the scaled cash flow for a U firm.

Since each S firm spends $m(\mathbf{n}_t;\mathcal{S}_t)K_t^S$ units on mitigation and all firms are of the same size, we have the following relation between the scaled mitigation $m(\mathbf{n};\mathcal{S})$ at the firm level and scaled mitigation at the aggregate level $\mathbf{x}(\mathbf{n};\mathcal{S}) = \mathbf{X}(\mathbf{n};\mathcal{S})/\mathbf{K}$:

$$m(\mathbf{n};\mathcal{S}) = \frac{\mathbf{x}(\mathbf{n};\mathcal{S})}{\alpha} \geq \mathbf{x}(\mathbf{n};\mathcal{S})\,. \tag{42}$$

The mitigation spending mandate for a firm, $m(\mathbf{n};\mathcal{S})$, is $1/\alpha$ times the aggregate scaled mitigation, $\mathbf{x}(\mathbf{n};\mathcal{S})$, as only an α fraction of firms commit to being sustainable. In equilibrium, the aggregate consumption $\mathbf{c}(\mathbf{n};\mathcal{S})$ is equal to the aggregate dividend $\mathbf{cf}(\mathbf{n};\mathcal{S})$:

$$\mathbf{c}(\mathbf{n};\mathcal{S}) = \mathbf{cf}(\mathbf{n};\mathcal{S}) = A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})\,. \tag{43}$$

Cost-of-capital wedge. It is helpful to use $\theta^j(\mathbf{n};\mathcal{S})$ to denote the wedge between the expected return for a type-j firm, $r^j(\mathbf{n};\mathcal{S})$, and the aggregate stock-market return, $r^M(\mathbf{n};\mathcal{S})$, and write for $j = \{S, U\}$,

$$r^j(\mathbf{n};\mathcal{S}) = r^M(\mathbf{n};\mathcal{S}) + \theta^j(\mathbf{n};\mathcal{S})\,. \tag{44}$$

As an α fraction of the total stock market is the S portfolio and the remaining $1-\alpha$ fraction is the U portfolio, we have

$$r^M(\mathbf{n};\mathcal{S}) = \alpha \cdot r^S(\mathbf{n};\mathcal{S}) + (1-\alpha)\cdot r^U(\mathbf{n};\mathcal{S})\,. \tag{45}$$

Using (24) for both S- and U-portfolios, we obtain

$$\theta^U(\mathbf{n};\mathcal{S}) \quad = \quad \frac{\mathbf{X}}{\mathbf{Q}} = \frac{\mathbf{x}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} = \frac{\alpha m(\mathbf{n};\mathcal{S})}{q(\mathbf{n};\mathcal{S})} > 0\,. \tag{46}$$

Equation (46) states that investors demand a higher rate of return to invest in U firms than in the aggregate stock market. The expected return wedge between the U-portfolio and the market portfolio is equal to $\theta^U(\mathbf{n};\mathcal{S})$, which is equal to the aggregate mitigation spending $\mathbf{X}$ divided by aggregate stock market value $\mathbf{Q}$. This ratio $\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{q}(\mathbf{n};\mathcal{S})$ can be viewed as a "tax" on the unsustainable firms by investors in equilibrium.

Substituting (44) into (45) and using (46), we obtain:

$$\theta^S(\mathbf{n};\mathcal{S}) = -\frac{1-\alpha}{\alpha}\,\theta^U(\mathbf{n};\mathcal{S}) = -\frac{1-\alpha}{\alpha}\frac{\mathbf{x}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} = -\,(1-\alpha)\,\frac{m(\mathbf{n};\mathcal{S})}{q(\mathbf{n};\mathcal{S})} < 0\,. \tag{47}$$

The cost-of-capital difference between U and S firms is given by

$$r^U(\mathbf{n};\mathcal{S}) - r^S(\mathbf{n};\mathcal{S}) = \theta^U(\mathbf{n};\mathcal{S}) - \theta^S(\mathbf{n};\mathcal{S}) = \frac{1}{\alpha}\frac{\mathbf{x}(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n})} = \frac{m(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\,. \tag{48}$$

By being sustainable, a firm lowers its cost of capital from $r^U(\mathbf{n};\mathcal{S})$ to $r^S(\mathbf{n};\mathcal{S})$ by $r^U(\mathbf{n};\mathcal{S}) - r^S(\mathbf{n};\mathcal{S})$. To enjoy this benefit, the firm spends $m(\mathbf{n};\mathcal{S})$ on mitigation. To make it indifferent between being sustainable and not, the cost-of-capital wedge is given by $r^U(\mathbf{n};\mathcal{S}) - r^S(\mathbf{n};\mathcal{S}) = m(\mathbf{n};\mathcal{S})/\mathbf{q}(\mathbf{n};\mathcal{S})$, the ratio between the firm's mitigation spending, $m(\mathbf{n};\mathcal{S})K$, and its market value, $\mathbf{q}(\mathbf{n};\mathcal{S})K$.

Equilibrium risk-free rate $r(\mathbf{n};\mathcal{S})$ and expected market return $r^M(\mathbf{n};\mathcal{S})$. Building on Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we calculate the aggregate stock-market risk premium in the absorbing $\mathcal{B}$ state, $r^M(\mathbf{n};\mathcal{B}) - r(\mathbf{n};\mathcal{B})$, by using

$$r^M(\mathbf{n};\mathcal{B}) - r(\mathbf{n};\mathcal{B}) = \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{B})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right] . \tag{49}$$

The first term in (49) is the standard diffusion shock contribution to the equity risk premium. The second term is the jump shock contribution to the equity risk premium. The equilibrium risk-free rate in state $\mathcal{B}$, $r(\mathbf{n};\mathcal{B})$, is

$$\begin{aligned} r(\mathbf{n};\mathcal{B}) = & \frac{c(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} + \phi(\mathbf{i}(\mathbf{n};\mathcal{B})) - \gamma\sigma^2 + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} \\ & - \lambda(\mathbf{n};\mathcal{B})\mathbb{E}\left[(1-Z)Z^{-\gamma}\right] . \end{aligned} \tag{50}$$

The aggregate stock-market risk premium in the $\mathcal{G}$ state, $r^M(\mathbf{n};\mathcal{G}) - r(\mathbf{n};\mathcal{G})$ is given by

$$\begin{aligned} r^M(\mathbf{n};\mathcal{G}) - r(\mathbf{n};\mathcal{G}) = & \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{G})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right] \\ & + \zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\left[\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\right)^{-\gamma} - 1\right] . \end{aligned} \tag{51}$$

The equilibrium interest rate in state $\mathcal{G}$, $r(\mathbf{n};\mathcal{G})$, is given by

$$\begin{aligned} r(\mathbf{n};\mathcal{G}) = & \frac{c(\mathbf{n};\mathcal{G})}{\mathbf{q}(\mathbf{n};\mathcal{G})} + \phi(\mathbf{i}(\mathbf{n};\mathcal{G})) - \gamma\sigma^2 + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right]\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{G})}{\mathbf{q}(\mathbf{n};\mathcal{G})} \\ & - \lambda(\mathbf{n};\mathcal{G})\mathbb{E}\left[(1-Z)Z^{-\gamma}\right] - \zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\right)^{-\gamma} . \end{aligned} \tag{52}$$

The effects of the climate transition risk in the $\mathcal{G}$ state on the equilibrium risk-free rate and market risk premium are captured by the last terms in (52) and (51), respectively.

Aggregate $\mathbf{i}(\mathbf{n};\mathcal{S})$, $\mathbf{q}(\mathbf{n};\mathcal{S})$, and $\mathbf{c}(\mathbf{n};\mathcal{S})$ for a given $\mathbf{x}(\mathbf{n};\mathcal{S})$ process. For a given $\mathbf{x}(\mathbf{n};\mathcal{B})$ process, we obtain the aggregate scaled investment $\mathbf{i}(\mathbf{n};\mathcal{B})$ for state $\mathcal{B}$ by solving

$$\begin{aligned} 0 \;=\; & \frac{(A - \mathbf{i}(\mathbf{n};\mathcal{B}) - \mathbf{x}(\mathbf{n};\mathcal{B}))\,\phi'(\mathbf{i}(\mathbf{n};\mathcal{B})) - \rho}{1-\psi^{-1}} + \phi(\mathbf{i}(\mathbf{n};\mathcal{B})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\ & + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} - \frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n};\mathcal{B}) + \mathbf{n}\mathbf{x}'(\mathbf{n};\mathcal{B})}{A - \mathbf{i}(\mathbf{n};\mathcal{B}) - \mathbf{x}(\mathbf{n};\mathcal{B})}\right) . \end{aligned} \tag{53}$$

For state $\mathcal{G}$, we obtain the aggregate scaled investment $\mathbf{i}(\mathbf{n};\mathcal{G})$ by solving

$$\begin{aligned}
0 \;=\; & \frac{(A-\mathbf{i}(\mathbf{n};\mathcal{G})-\mathbf{x}(\mathbf{n};\mathcal{G}))\,\phi'(\mathbf{i}(\mathbf{n};\mathcal{G}))-\rho}{1-\psi^{-1}}+\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \\
& +\left[\omega(\mathbf{x}/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{G})}{\mathbf{q}(\mathbf{n};\mathcal{G})}-\frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n};\mathcal{G})+\mathbf{n}\mathbf{x}'(\mathbf{n};\mathcal{G})}{A-\mathbf{i}(\mathbf{n};\mathcal{G})-\mathbf{x}(\mathbf{n};\mathcal{G})}\right)\,, \\
& +\frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{(A-\mathbf{i}(\mathbf{n};\mathcal{B})-\mathbf{x}(\mathbf{n};\mathcal{B}))\mathbf{q}(\mathbf{n};\mathcal{G})^{\psi}}{(A-\mathbf{i}(\mathbf{n};\mathcal{G})-\mathbf{x}(\mathbf{n};\mathcal{G}))\mathbf{q}(\mathbf{n};\mathcal{B})^{\psi}}\right)^{\frac{1-\gamma}{1-\psi}}-1\right]. \qquad (54)
\end{aligned}$$

The last term in (54) captures the effect of climate state transition. For both states, Tobin's q, $\mathbf{q}(\mathbf{n};\mathcal{S})$, is given by the standard FOC as in the q theory literature:

$$\mathbf{q}(\mathbf{n};\mathcal{S})=\frac{1}{\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))}\,. \qquad (55)$$

3.4 Market Economy with Welfare-Maximizing Mandate

For a given level of α, we endogenize the criterion at the firm level characterized by the mitigation threshold $M_t = m(\mathbf{n_t};\mathcal{S}_t)K_t$ for a firm to qualify as a sustainable firm. Specifically, at time 0, the planner announces $\{M_t;\, t\geq 0\}$ and commits to the announcement with the goal of maximizing the representative agent's utility given in equation (13) taking into account that the representative agent and firms take the mandate as given and optimize in competitive equilibrium.[20]

For a given mitigation spending process $\mathbf{x}$, we may write the agent's value function as follows:

$$J(\mathbf{K},\mathbf{N};\mathcal{S})=V(W,\mathbf{n};\mathcal{S})=\frac{1}{1-\gamma}(b(\mathbf{n};\mathcal{S})\mathbf{K})^{1-\gamma} \qquad (56)$$

where

$$b(\mathbf{n};\mathcal{S})=u(\mathbf{n};\mathcal{S})\times q(\mathbf{n};\mathcal{S})\,. \qquad (57)$$

For brevity, we suppress $\mathcal{S}$ whenever causing no confusion. Equation (57) follows from the equilibrium result that $W=\mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$ as all household's wealth is in the stock market, which is valued at $\mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$. Substituting $W=\mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$ into the agent's value function $V(W,\mathbf{n};\mathcal{S})$ given in (35) for the market economy yields $J(\mathbf{K},\mathbf{N};\mathcal{S})$ given in (56) and (57). Note that

[20] Broadly speaking, our mandate choice is related to the optimal fiscal and monetary policy literature (e.g., Lucas and Stokey, 1983) in macroeconomics. See Ljungqvist and Sargent (2018) for a textbook treatment.

$b(\mathbf{n};\mathcal{S})$ equals the product of $u(\mathbf{n};\mathcal{S})$ defined in (35), the objective for the agent, and the equilibrium Tobin's q, $q(\mathbf{n};\mathcal{S})$, the firm's objective. That is, $b(\mathbf{n};\mathcal{S})$ captures information from both the firm's and the household's optimization problems. The function $b(\mathbf{n};\mathcal{S})$ can be naturally interpreted as a welfare measure proportional to certainty equivalent wealth (scaled by the size of the economy $\mathbf{K}$).

Using the optimal consumption rule (36), the investment FOC (55), and the market clearing condition $c(\mathbf{n};\mathcal{S}) = A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})$, we obtain the following equilibrium condition:

$$\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))b(\mathbf{n};\mathcal{S})\,. \tag{58}$$

That is, (58) captures information about both the firm's and the household's optimization decisions. In Appendix B, we show that $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S}) \times q(\mathbf{n};\mathcal{S})$ satisfies the following ODE:

$$\begin{aligned} 0 \;=\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{B})-\mathbf{x}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{1-\psi^{-1}}-1\right] + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})} \\ &+\phi(\mathbf{i}(\mathbf{n};\mathcal{B})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\,, \end{aligned} \tag{59}$$

and

$$\begin{aligned} 0 \;=\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{G})-\mathbf{x}(\mathbf{n};\mathcal{G})}{b(\mathbf{n};\mathcal{G})}\right)^{1-\psi^{-1}}-1\right] + \left[\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{G})}{b(\mathbf{n};\mathcal{G})} \\ &+\phi(\mathbf{i}(\mathbf{n};\mathcal{G})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] + \frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{b(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{G})}\right)^{1-\gamma}-1\right]. \end{aligned} \tag{60}$$

This ODE for $b(\mathbf{n};\mathcal{S})$ summarizes information about both $u(\mathbf{n};\mathcal{S})$ and $q(\mathbf{n};\mathcal{S})$. Finally, the planner chooses $\mathbf{x}$ to maximize $J(\mathbf{K},\mathbf{N};\mathcal{S})$ (and equivalently $b(\mathbf{n};\mathcal{S})$), which yields:

$$\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \omega'(\mathbf{x}/\mathbf{n})b'(\mathbf{n};\mathcal{S})\,. \tag{61}$$

At the steady state where $\mathbf{n} = \mathbf{n}^*(\mathcal{S})$ for both $\mathcal{G}$ and $\mathcal{B}$, the drift of $\mathbf{n}$ is zero. The steady-state investment-capital ratio $\mathbf{i}^*(\mathcal{S})$ and mitigation spending $\mathbf{x}^*(\mathcal{S})$ thus satisfy

$$\omega(\mathbf{x}^*(\mathcal{S})/\mathbf{n}^*(\mathcal{S})) - \phi(\mathbf{i}^*(\mathcal{S})) = 0\,. \tag{62}$$

Additionally, we have

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^*(\mathcal{B})-\mathbf{x}^*(\mathcal{B})}{b(\mathbf{n}^*(\mathcal{B});\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}^*(\mathcal{B}))-\frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n}^*(\mathcal{B});\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right] \tag{63}$$

and

$$0 = \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^*(\mathcal{G})-\mathbf{x}^*(\mathcal{G})}{b(\mathbf{n}^*(\mathcal{G});\mathcal{G})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}^*(\mathcal{G}))-\frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n}^*(\mathcal{G});\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]+\frac{\zeta(\mathbf{n}^*(\mathcal{G}))}{1-\gamma}\left[\left(\frac{b(\mathbf{n}^*(\mathcal{G});\mathcal{B})}{b(\mathbf{n}^*(\mathcal{G});\mathcal{G})}\right)^{1-\gamma}-1\right]. \tag{64}$$

Summary. At the steady state where $d\mathbf{n}_t = 0$, the scaled decarbonization capital stock $\mathbf{n}^*(\mathcal{S})$, $\mathbf{i}^*(\mathcal{S})$, $\mathbf{x}^*(\mathcal{S})$, and the welfare measure $b(\mathbf{n}^*(\mathcal{S});\mathcal{S})$ jointly solve the four equations: the FOC (61) for $\mathbf{x}^*(\mathcal{S})$, the FOC (58) for $\mathbf{i}^*(\mathcal{S})$, the zero-drift condition (62) for $\mathbf{n}^*(\mathcal{S})$, and (63) for $b(\mathbf{n}^*;\mathcal{B})$ or (64) for $b(\mathbf{n}^*;\mathcal{G})$.

For the transition dynamics, the scaled mitigation spending $\mathbf{x}_t$, the investment-capital ratio $\mathbf{i}_t$, and the welfare measure b_t are all functions of the scaled decarbonization capital stock $\mathbf{n}_t$ and the climate state $\mathcal{S}_t$. We fully characterize the solution for the transition dynamics as follows. The functions $\mathbf{x}(\mathbf{n};\mathcal{S})$, $\mathbf{i}(\mathbf{n};\mathcal{S})$, and $b(\mathbf{n};\mathcal{S})$ jointly solve the ODE system of the following three equations: the FOC (58) for $\mathbf{i} = \mathbf{i}(\mathbf{n};\mathcal{S})$, the FOC (61) for $\mathbf{x} = \mathbf{x}(\mathbf{n};\mathcal{S})$, and the ODE (59) for $b(\mathbf{n}(\mathcal{B});\mathcal{B})$ in state $\mathcal{B}$ or the ODE (60) for $b(\mathbf{n}(\mathcal{G});\mathcal{G})$ in state $\mathcal{G}$, subject to the boundary conditions (for $\mathbf{n}^*(\mathcal{B})$ and $\mathbf{n}^*(\mathcal{G})$) at the steady state summarized above.

Sustainable Finance Tax as Limiting Case of Mandate with $\alpha = 1$. We show that the special case of our welfare-maximizing economy with investment mandate ($\alpha = 1$) yields the same outcome path by path at both the micro and macro level as an economy with capital taxation. Let τ_t denote the rate at which the government levies a tax on each firm's sales.

We define the competitive equilibrium with sustainable-finance taxation as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the

U-portfolio, S-portfolio, and the risk-free asset; (2) each firm chooses its investment policy I to maximize its market value by solving (18) where the firm's cash flow at t, $CF(\mathbf{n}_t;\mathcal{S})$, is given by

$$CF(\mathbf{n}_t;\mathcal{S}) = AK_t - I_t(\mathbf{n}_t;\mathcal{S}) - \tau_t K_t\,; \tag{65}$$

(3) all markets clear. The government sets the tax rate as follows:

$$\tau_t = \tau(\mathbf{n}_t;\mathcal{S}_t) = \mathbf{x}(\mathbf{n}_t;\mathcal{S}_t)\,. \tag{66}$$

As a result, this economy attains the same resource allocation as the welfare-maximizing economy with investment mandate $\alpha = 1$. The intuition is as follows. Because taxes are mandatory for all firms, using taxation, the government effectively makes all firms "sustainable." Since the government is benevolent maximizing the representative agent's welfare, it simply sets the sustainable-finance tax rate $\tau(\mathbf{n};\mathcal{S})$ to the same aggregate mitigation spending $\mathbf{x}(\mathbf{n};\mathcal{S})$ as in the economy with optimal investment mandate.

While taxation typically distorts intertemporal decisions and hence is inefficient,[21] taxation proceeds in our model allow the government to fund the accumulation of decarbonization capital stock, substantially reducing the disaster risk arrival rates so that the equilibrium resource allocation with taxation is much closer to the planner's first-best solution.

4 Planner's Solution: First Best

We next report the first-best solution where the planner chooses aggregate $\mathbf{C}$, $\mathbf{I}$, and $\mathbf{X}$ to maximize the representative agent's utility defined in (13)-(14).

As our planner's model also features the homogeneity property, it is convenient to work with scaled variables at the aggregate level, $\mathbf{i}_t = \mathbf{I}_t/\mathbf{K}_t$, $\mathbf{x}_t = \mathbf{X}_t/\mathbf{K}_t$, $\mathbf{c_t} = \mathbf{C}_t/\mathbf{K}_t$. In Appendix C, we show that $\mathbf{x}(\mathbf{n};\mathcal{S})$ and $\mathbf{i}(\mathbf{n};\mathcal{S})$ for state $\mathcal{S} = \mathcal{G},\mathcal{B}$ satisfy the following

[21] See Chamley (1986) and Judd (1986) for seminal contributions.

equations:

$$\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \omega'(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n})b'(\mathbf{n};\mathcal{S})\,, \tag{67}$$

$$\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{S})-\mathbf{x}(\mathbf{n};\mathcal{S})}{b(\mathbf{n};\mathcal{S})}\right)^{-\psi^{-1}} = \phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))b(\mathbf{n};\mathcal{S})-\phi'(\mathbf{i}(\mathbf{n};\mathcal{S}))\mathbf{n}b'(\mathbf{n};\mathcal{S})\,. \tag{68}$$

The welfare measure proportional to certainty equivalent wealth in state $\mathcal{B}$, $b(\mathbf{n};\mathcal{B})$, solves the following ODE:

$$\begin{aligned}0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{B})-\mathbf{x}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\\ &\quad+\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\,,\end{aligned} \tag{69}$$

and the corresponding welfare measure in state $\mathcal{G}$, $b(\mathbf{n};\mathcal{G})$, solves the following ODE:

$$\begin{aligned}0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{G})-\mathbf{x}(\mathbf{n};\mathcal{G})}{b(\mathbf{n};\mathcal{G})}\right)^{1-\psi^{-1}}-1\right]+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{G})}{b(\mathbf{n};\mathcal{G})}\\ &\quad+\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]+\frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{b(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{G})}\right)^{1-\gamma}-1\right]\,.\end{aligned} \tag{70}$$

Compared with (69) for state $\mathcal{B}$, we have a new term (the last term) due to the climate state transition.

Because the $\mathcal{B}$ state is absorbing, we first solve the triple, $b(\mathbf{n};\mathcal{B})$, $\mathbf{i}(\mathbf{n};\mathcal{B})$, and $\mathbf{x}(\mathbf{n};\mathcal{B})$, for state $\mathcal{B}$ by using (67), (68), and (69). Then, we solve the triple, $b(\mathbf{n};\mathcal{G})$, $\mathbf{i}(\mathbf{n};\mathcal{G})$, and $\mathbf{x}(\mathbf{n};\mathcal{G})$, for state $\mathcal{G}$ by using (67), (68), and (70) using the $b(\mathbf{n};\mathcal{B})$ solution we have already obtained.

At the first-best steady state $\mathbf{n}^*(\mathcal{S})$ for both $\mathcal{G}$ and $\mathcal{B}$ states, we have

$$\omega(\mathbf{x}^*(\mathcal{S})/\mathbf{n}^*(\mathcal{S}))-\phi(\mathbf{i}^*(\mathcal{S}))=0\,. \tag{71}$$

Substituting (71) into (69) yields the following steady-state condition in $\mathcal{B}$:

$$\begin{aligned}0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^*(\mathcal{B})-\mathbf{x}^*(\mathcal{B})}{b(\mathbf{n}^*(\mathcal{B});\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}^*(\mathcal{B}))-\frac{\gamma\sigma^2}{2}\\ &\quad+\frac{\lambda(\mathbf{n}^*(\mathcal{B});\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]\,.\end{aligned} \tag{72}$$

Using the results for state $\mathcal{B}$, we obtain the following condition for the steady state for $\mathcal{G}$:

$$
\begin{aligned}
0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^*(\mathcal{G})-\mathbf{x}^*(\mathcal{G})}{b(\mathbf{n}^*(\mathcal{G});\mathcal{G})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}^*(\mathcal{G}))-\frac{\gamma\sigma^2}{2} \\
& +\frac{\lambda(\mathbf{n}^*(\mathcal{G});\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]+\frac{\zeta(\mathbf{n}^*(\mathcal{G}))}{1-\gamma}\left[\left(\frac{b(\mathbf{n}^*(\mathcal{G});\mathcal{B})}{b(\mathbf{n}^*(\mathcal{G});\mathcal{G})}\right)^{1-\gamma}-1\right]. \qquad (73)
\end{aligned}
$$

Summary. At the first-best steady state where $d\mathbf{n}_t = 0$, the scaled decarbonization capital stock $\mathbf{n}^*(\mathcal{S})$, $\mathbf{i}^*(\mathcal{S})$, $\mathbf{x}^*(\mathcal{S})$, and the welfare measure $b(\mathbf{n}^*(\mathcal{S});\mathcal{S})$ jointly solve the four equations: the FOC (67) for $\mathbf{x}^*(\mathcal{S})$, the FOC (68) for $\mathbf{i}^*(\mathcal{S})$, the zero-drift condition (71) for $\mathbf{n}^*(\mathcal{S})$, and (72) for $\mathbf{n}^*(\mathcal{S})$ or (73) for for $b(\mathbf{n}^*;\mathcal{G})$.

For the transition dynamics, the scaled mitigation spending $\mathbf{x}_t$, the investment-capital ratio $\mathbf{i}_t$, and the welfare measure b_t are all functions of the scaled decarbonization capital stock $\mathbf{n}_t$ and the climate state $\mathcal{S}_t$. We fully characterize the solution for the transition dynamics as follows. The functions $\mathbf{x}(\mathbf{n};\mathcal{S})$, $\mathbf{i}(\mathbf{n};\mathcal{S})$, and $b(\mathbf{n};\mathcal{S})$ jointly solve the ODE system of the following three equations: the FOC (68) for $\mathbf{i} = \mathbf{i}(\mathbf{n};\mathcal{S})$, the FOC (67) for $\mathbf{x} = \mathbf{x}(\mathbf{n};\mathcal{S})$, and the ODE (69) for $b(\mathbf{n}(\mathcal{B});\mathcal{B})$ in state $\mathcal{B}$ or the ODE (70) for $b(\mathbf{n}(\mathcal{G});\mathcal{G})$ in state $\mathcal{G}$, subject to the boundary conditions (for $\mathbf{n}^*(\mathcal{B})$ and $\mathbf{n}^*(\mathcal{G})$) at the steady state summarized above.

5 Comparing Welfare-Maximizing Mandate with Markets to First-Best

First, we consider the $\mathcal{B}$ state. By comparing the solutions under mandate-market equilibrium given in (59)-(58) with the solutions for the planner's economy given in (67)-(69), we note that the difference between the solutions for the two cases solely arises from the investment (I) FOCs. Specifically, the FOC for i in the mandate-market equilibrium setting given in (58) is different from the FOC for the planner's problem, given in (68).

Next, we explain why the two investment FOCs, (58) and (68), are different. First, for the planner's problem, $\mathbf{i}$ is chosen to maximize the welfare measure $b(\mathbf{n};\mathcal{B})$ given by the following ODE:

$$
\begin{aligned}
0 \;=\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}^{FB}(\mathbf{n};\mathcal{B})-\mathbf{x}^{FB}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}^{FB}(\mathbf{n};\mathcal{B}))-\frac{\gamma\sigma^2}{2} \\
&+\left[\omega(\mathbf{x}^{FB}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}^{FB}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}+\frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right], \qquad (74)
\end{aligned}
$$

which implies the following FOC for investment:

$$
\rho\left(\frac{A-\mathbf{i}^{FB}(\mathbf{n};\mathcal{B})-\mathbf{x}^{FB}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{-\psi^{-1}}=\phi'(\mathbf{i}^{FB}(\mathbf{n};\mathcal{B}))b(\mathbf{n};\mathcal{B})-\phi'(\mathbf{i}^{FB}(\mathbf{n};\mathcal{B}))\mathbf{n}b'(\mathbf{n};\mathcal{B})\,. \quad (75)
$$

In contrast, for the mandate-market equilibrium case, the firm chooses i to maximize market value, i.e., $q(\mathbf{n};\mathcal{B})$, taking the mitigation spending $\mathbf{x}$ and the evolution of the state variable $\mathbf{n}$ as well the household's $u(\mathbf{n};\mathcal{B})$ as given.

We show that the firm's optimization problem is equivalent to another optimization problem where the decision maker chooses i to maximize $b(\mathbf{n};\mathcal{B})$, where $b(\mathbf{n};\mathcal{B}) = u(\mathbf{n};\mathcal{B}) \times q(\mathbf{n};\mathcal{B})$ satisfies the following ODE:

$$
\begin{aligned}
0 \;=\;& \max_i \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-i-\mathbf{x}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\phi(i)-\frac{\gamma\sigma^2}{2} \\
&+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}b'(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}+\frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \qquad (76)
\end{aligned}
$$

The implied FOC in equilibrium at the aggregate level for $\mathbf{i}$ is then

$$
\rho\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{B})-\mathbf{x}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{-\psi^{-1}}=\phi'(\mathbf{i}(\mathbf{n};\mathcal{B}))b(\mathbf{n};\mathcal{B})\,. \qquad (77)
$$

Note that the two ODEs, (74) for the planner's problem and (76) for the mandate-market economy are the same at the aggregate level. However, the two FOCs, (75) for the planner, and (77) in the mandate-market economy, are different. When a firm optimizes, it takes the drift of $\mathbf{n}$ as given. In contrast, when choosing $\mathbf{i}$, the planner takes into account the impact of $\mathbf{i}$ on the future dynamics of $\mathbf{n}$ internalizing the impact of decarbonization capital stock accumulation on aggregate investment $\mathbf{i}$.

In other words, when accumulating physical capital stock, there are two consequences at the aggregate level: (1) reducing the resources for the representative household's consumption as $\mathbf{I}$ crowds out $\mathbf{C} = \mathbf{Y} - \mathbf{I} - \mathbf{X}$ and (2) decreasing the scaled aggregate decarbonization capital stock $\mathbf{n} = \mathbf{N}/\mathbf{K}$ in the future by increasing $\mathbf{K}$. The planner takes both costs into account when optimizing but firms do not in the mandate-market economy. This can be seen clearly by rewriting the planner's FOC, (75) as follows:

$$\rho \left(\frac{A - \mathbf{i}^{FB}(\mathbf{n};\mathcal{B}) - \mathbf{x}^{FB}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})} \right)^{-\psi^{-1}} + \phi'(\mathbf{i}^{FB}(\mathbf{n};\mathcal{B}))\mathbf{n}b'(\mathbf{n};\mathcal{B}) = \phi'(\mathbf{i}^{FB}(\mathbf{n};\mathcal{B}))b(\mathbf{n};\mathcal{B})\,, \quad (78)$$

The second term on the left side of (78) captures the increased cost on welfare due to a lower $\mathbf{n} = \mathbf{N}/\mathbf{K}$ in the future. The planner takes this indirect effect into account while the firms in the market economy with mandates do not.

Another way to see this difference between the two economies is to note that in the planner's economy, the aggregate mitigation $\mathbf{x}$ and investment $\mathbf{i}$ satisfy the following condition implied by the FOCs for $\mathbf{x}$ and investment $\mathbf{i}$, (shown in Appendix C):

$$\frac{\omega'(\mathbf{x}/\mathbf{n})}{\phi'(\mathbf{i})} = \frac{J_{\mathbf{K}}(\mathbf{K},\mathbf{N};\mathcal{S})}{J_{\mathbf{N}}(\mathbf{K},\mathbf{N};\mathcal{S})}\,. \quad (79)$$

That is, to enjoy the same aggregate risk mitigation benefit from the decarbonization capital stock accumulation, we need to target $\mathbf{n}$. Therefore, increasing capital stock $\mathbf{K}$ requires the planner to increase $\mathbf{N}$ according to (79) so that $\mathbf{n} = \mathbf{N}/\mathbf{K}$ remains the same. This force is missing in the firm's FOC in the market economy with mandates.

Finally, the same argument applies for the $\mathcal{G}$ state. For brevity, we leave the details out.

6 Quantitative Analysis

In this section, we calibrate our model to study how well mandates approximate the first-best solution. We focus on the parameter region where the social planner wants to act now to decarbonize—that is, where the planner makes significant annual contributions to mitigation so as to smoothly ramp up to a high steady-state decarbonization-to-productive capital ratio.

6.1 Functional Form Specifications for the Model

We begin by specifying various functional forms in our model.

Capital accumulation processes for firm-level capital K and aggregate $\mathbf{N}$. As in Pindyck and Wang (2013), we specify the investment-efficiency function $\phi(i)$ for a firm as

$$\phi(i) = i - \frac{\eta_K \, i^2}{2} \, , \tag{80}$$

where η_K measures the degree of adjustment costs.

At the aggregate level, we assume that the controlled drift function for the aggregate decarbonization stock $\mathbf{N}$ takes the same form as that for capital stock K at the firm level:

$$\omega(\mathbf{x}/\mathbf{n}) = (\mathbf{x}/\mathbf{n}) - \frac{\eta_{\mathbf{N}} \, (\mathbf{x}/\mathbf{n})^2}{2} \, , \tag{81}$$

but with a different capital adjustment cost parameter $\eta_{\mathbf{N}}$ for decarbonization capital. Note that $\mathbf{x}/\mathbf{n} = \mathbf{X}/\mathbf{N}$ is the aggregate investment $\mathbf{X}$ in the decarbonization capital stock scaled by $\mathbf{N}$, which is analogous to the firm's investment level scaled by capital stock: $i = I/K$.

Delaying the tipping point arrival. By accumulating decarbonization capital stock, the society decreases the tipping point arrival rate from $\zeta_0 > 0$ to

$$\zeta(\mathbf{n}) = \zeta_0 (1 - \mathbf{n}^{\zeta_1}) \, , \tag{82}$$

where $0 < \zeta_1 < 1$. The lower the value of ζ_1 the more efficient the decarbonization capital stock is at curtailing the tipping point arrival.

Conditional damage and weather disaster arrival rates. In a given climate state $\mathcal{S}_t$ (i.e., $\mathcal{B}$ or $\mathcal{G}$), we model as in Barro and Jin (2011) and Pindyck and Wang (2013) the stochastic damage upon the arrival of a weather disaster by assuming that the stochastic recovery fraction, $Z \in (0, 1)$, of capital stock is governed by the following cdf:

$$\Xi(Z) = Z^{\beta} \, , \tag{83}$$

where $\beta > 0$ is a constant. To ensure that our model is well defined (and economically relevant moments are finite), we require $\beta > \max\{\gamma - 1, 0\}$. That is, the damage caused by a disaster follows a fat-tailed power-law function (Gabaix (2009)).

In a given climate state, decarbonization capital can also ameliorate the damage to economic growth by reducing the frequencies of these high temperature events. Specifically, we use the following specification for the disaster arrival rate $\lambda(\mathbf{n}; \mathcal{S})$ in state $\mathcal{S}$:

$$\lambda(\mathbf{n}; \mathcal{S}) = \lambda_0^{\mathcal{S}}(1 - \mathbf{n}^{\lambda_1}), \tag{84}$$

where $\lambda_0^{\mathcal{S}} > 0$ is the arrival rate absent any decarbonization capital stock and $\lambda_1 \in (0, 1)$ measures how efficient decarbonization capital stock reduces the weather shock arrival. A lower value of λ_1 is associated with a more efficient decarbonization technology, *ceteris paribus*. The expected aggregate growth rate in state $\mathcal{B}$, $\mathbf{g}(\mathbf{n}; \mathcal{B})$, is

$$\mathbf{g}(\mathbf{n}; \mathcal{B}) = \phi(\mathbf{i}(\mathbf{n}; \mathcal{B})) - \lambda(\mathbf{n}; \mathcal{B})\ell, \tag{85}$$

where ℓ, the expected fractional capital loss conditional on a jump arrival, is given by

$$\ell = \mathbb{E}(1 - Z) = \frac{1}{\beta + 1}. \tag{86}$$

Note that a lower value of β is associated with a more damaging and also more fat tailed disaster. The first term in (85), $\phi(\mathbf{i})$, is the expected growth absent jumps and the second term adjusts for the effect of jumps. In state $\mathcal{G}$, the expected growth rate, $\mathbf{g}(\mathbf{n}; \mathcal{G})$, is

$$\mathbf{g}(\mathbf{n}; \mathcal{G}) = \phi(\mathbf{i}(\mathbf{n}; \mathcal{G})) - \lambda(\mathbf{n}; \mathcal{G})\ell - \zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n}; \mathcal{G}) - \mathbf{q}(\mathbf{n}; \mathcal{B})}{\mathbf{q}(\mathbf{n}; \mathcal{G})}. \tag{87}$$

The last term in (87) captures the effect of the climate state transition on the expected growth rate in state $\mathcal{G}$. The first two terms in (87) are similar to those in (85) for state $\mathcal{B}$.

6.2 Baseline Calibration

Our model has 13 parameters in total. We next choose parameter values based on well-known key macro-finance moments and empirical studies on climate mitigation pathways involving decarbonization. Our calibration exercise is intended to highlight the extent to which mandates can approximate the social planner's solution when the planner wants to act now to decarbonize.

We summarize the values of these parameters for our baseline analysis in Table 1.

Table 1: Parameter Values

Parameters	Symbol	Value
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	3.5%
coefficient of relative risk aversion	γ	3
productivity for K	A	14%
adjustment cost parameter for K	η_K	11
adjustment cost parameter for $\mathbf{N}$	$\eta_{\mathbf{N}}$	11
diffusion volatility for $\mathbf{N}$ and K	σ	14%
jump arrival rate from good state to bad state	ζ_0	0.1
jump arrival rate from good state to bad state	ζ_1	0.1
power-law exponent	β	39
jump arrival rate with $\mathbf{n} = 0$ under good state	$\lambda_0^{\mathcal{G}}$	0.05
jump arrival rate with $\mathbf{n} = 0$ under bad state	$\lambda_0^{\mathcal{B}}$	1
mitigation technology parameter	λ_1	0.3

All parameter values, whenever applicable, are continuously compounded and annualized.

Preferences parameters. We choose consensus values for the coefficient of relative risk aversion, $\gamma = 3$, and the time rate of preferences, $\rho = 3.5\%$ per annum. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero to values as high as two.[22] We choose $\psi = 1.5$ which is larger than one, as in Bansal and Yaron (2004) and the long-run risk literature for asset-pricing purposes.

Parameters for productive and decarbonization capital accumulation process. We set the productivity parameter to $A = 14\%$ per annum and the capital adjustment parameter $\eta_K = 11$ to target an average q of 2.26 and an average growth rate of $g = 3.13\%$ per annum in the pre-climate-change sample when the disaster arrival rate is very low (close to zero, i.e., $\mathbf{n} \approx 0$).[23] These values of A and η_K are in the range of empirical estimates reported in Eberly, Rebelo, and Vincent (2012). Decarbonization capital has no

[22]Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Guvenen (2006) reconciles the conflicting evidence on the elasticity of intertemporal substitution from a macro perspective.

[23]That is, we suppose the $\mathcal{G}$ state is permanent and absorbing for this calibration.

productivity but faces adjustment costs comparable to those for physical capital. We thus set the decarbonization capital adjustment cost parameter $\eta_{\mathbf{N}}$ to the calibrated value of η_K in the interest of parsimony and also under the premise that direct air capture and plants are themselves a form of physical capital. We set the annual diffusion volatility at $\sigma = 14\%$ (as in Pindyck and Wang, 2013) to target a historical stock market risk premium of about 6% per annum (Mehra and Prescott, 1985).

Parameters for delaying the tipping point. Recent studies indicate that tipping points in the climate system can occur even at current levels of warming (Lenton et al. (2019)). To generate a sizeable act-now effect, we intentionally set the arrival rate of a tipping point to be once in a decade: $\zeta_0 = 0.1$. We then build on estimates from Gates (2020) who proposes that spending roughly \$5 trillion dollars each year on carbon capture can forever eliminate the problem of global warming (this estimate is based on \$100 per ton cost of capture and there are 51 billion tons of carbon emissions per year). We consider a more modest scenario similar to de Pee et al. (2018) where spending a couple of trillion dollars per year on decarbonization of heavy industries can substantially delay a tipping point from once a decade to around once every 50 years—we do this by setting $\zeta_1 = 0.1$.[24]

Parameters for weather disasters and conditional damage functions. Since weather disasters like droughts are associated with high annual temperatures, we calibrate the parameter ($\lambda_0^{\mathcal{G}}$) describing the arrival rate of weather disasters in the good climate state $\mathcal{G}$ and the parameter (β) related to expected damages conditional on arrival ($\ell = (\beta+1)^{-1}$) using a set of panel regressions documenting the adverse effects of weather shocks in the form of extreme temperatures for economic growth (Dell, Jones, and Olken (2012)).[25]

[24]Reforestation also has the potential to contribute to keeping global temperatures from breaching the 1.5^o Celsius barrier. This adjustment process is also expensive like building direct air capture plants (Bastin et al. (2019), and Griscom et al. (2017)).

[25]This panel regression approach initially focused on how weather affects crop yields (Schenkler and Roberts (2009)) by using location and time fixed effects. But it is now applied to many other contexts including economic growth and productivity. The main idea is that abnormally high annual temperature fluctuations are plausibly exogenous shocks that causally trace out the impact of higher temperatures on output. Burke, Hsiang, and Miguel (2015) find that the effects of temperature on growth is nonlinear. But we stay with the linear specification from Dell, Jones and Olken (2012) in this paper.

First, we calibrate β as follows. For the median country, abnormal temperature over one year lower the GDP growth rate by 2.5% per annum. To match this moment, we set $\beta = 39$ as the implied expected fractional capital loss is $\ell = 1/(\beta + 1) = 1/40 = 2.5\%$ per annum. Second, using Dell, Jones, and Olken (2012), we infer that the weather disaster arrival rate in the good climate state is low and around $\lambda_0^{\mathcal{G}} = 0.05$ per annum in the pre-climate-change sample. In other words, such weather disaster events are still uncommon, occurring in a few percent of the country-year observations. Our analysis is most apt for the median country in their sample. But our model can be recalibrated for any subset of countries. For the $\mathcal{B}$ climate state, we set $\lambda_0^{\mathcal{B}} = 1$, a many times increase in weather disaster frequency, following studies of tipping points cited in the Introduction. Third, we set $\lambda_1 = 0.3$ for $\lambda(\mathbf{n}; \mathcal{S})$ so that the decarbonization-to-productive capital ratio $\mathbf{n}$, which reduces temperatures, not only delays a tipping point also reduces the frequency of weather disasters, as is often modeled in climate science and integrated assessment models.

6.3 Steady States: Markets with Welfare-Maximizing Mandate versus Planner's First-Best Solution

The column labeled "mandate" in Table 2 reports the steady-state mandate solution and the column labeled "planner" reports the planner's solution in the $\mathcal{G}$ state. Our focus is on understanding the annual contributions to decarbonization and the steady-state decarbonization-to-capital stock $\mathbf{n}^*$. Annual contributions are 0.24% under the mandate compared to 0.25% under the planner's solution. Since the global physical capital stock is about 600 trillion dollars, the aggregate contribution to decarbonization stock, $\mathbf{x}^*$, is roughly $1.44 = 0.24\% \times 600$ trillion dollars per year. The steady-state decarbonization-to-productive capital ratio, $\mathbf{n}^*$, is 4.54% under the mandate and 4.76% in the planner's economy. This implies that the aggregate decarbonization capital stock $\mathbf{N}$ is about 27 trillion dollars (the book value of decarbonization capital) at the steady state. The transition time conditional on being in the $\mathcal{G}$ state in the market economy with the mandate is 16.25 years compared to 15.33 years for the planner's economy.

In other words, the outcome in the market economy with mandates is quite close to the

Table 2: THE EFFECT OF MANDATE/PLANNER IN THE STEADY STATE FOR THE MARKET ECONOMY WITH MANDATES UNDER THE $\mathcal{G}$ STATE.

variable	notation	mandate	planner
scaled mitigation spending	$\mathbf{x}^*$	0.24%	0.25%
scaled decarbonization stock	$\mathbf{n}^*$	4.54%	4.76%
scaled aggregate investment	$\mathbf{i}^*$	5.32%	5.18%
Tobin's average q	$\mathbf{q}^*$	2.41	2.33
scaled aggregate consumption	$\mathbf{c}^*$	8.44%	8.57%
expected GDP growth rate	$\mathbf{g}^*$	3.54%	3.47%
(real) risk-free rate	r	1.12%	1.22%
stock market risk premium	$r^M - r$	5.92%	5.93%
time from $\mathbf{n} = 0$ to $0.99\mathbf{n}^*$ conditional in $\mathcal{G}$	$\mathbf{t}(0.99)$	16.25	15.33

$\mathbf{t}(0.99)$ is the transition time from $\mathbf{n}_0 = 0$ to $\mathbf{n} = 0.99\mathbf{n}^*$, where $\mathbf{n}^*$ is the steady-state value of $\mathbf{n}$ in the respective economy, conditional on no climate transition from $\mathcal{G}$ to $\mathcal{B}$ before $\mathbf{t}(0.99)$.

planner's first-best solution when it comes to the mitigation variables. The same is true for the other macroeconomic variables. Investment is slightly higher at 5.32% for mandates compared to 5.18% in the planner's solution, while consumption is slightly lower at 8.44% compared to 8.57% under first best. Tobin's q as a result is also slightly higher under mandates at 2.41 compared to 2.33 for the planner's solution. The growth rate is slightly higher under the mandate, while the interest and risk-premium are slightly lower.

6.4 Transition Dynamics and Comparison to Planner's Outcomes

In this subsection, we discuss the transition dynamics. We also highlight the extent to which welfare-maximizing mandates (Section 3.4) approximates the planner's first-best outcomes (Section 4) along the transition path.

Mitigation, consumption, investment and welfare gains under mandates versus planner's solution. In Figure 1, we examine the optimal mitigation $\mathbf{x}$, investment $\mathbf{i}$, consumption $\mathbf{c}$, and a welfare measure (proportional to the certainty-equivalent wealth) b as functions of $\mathbf{n}$ in the $\mathcal{G}$ state. As $\mathbf{n}_t$ is deterministically evolving over time t conditional on no climate state transition, Figure 1 also captures the corresponding conditional transition

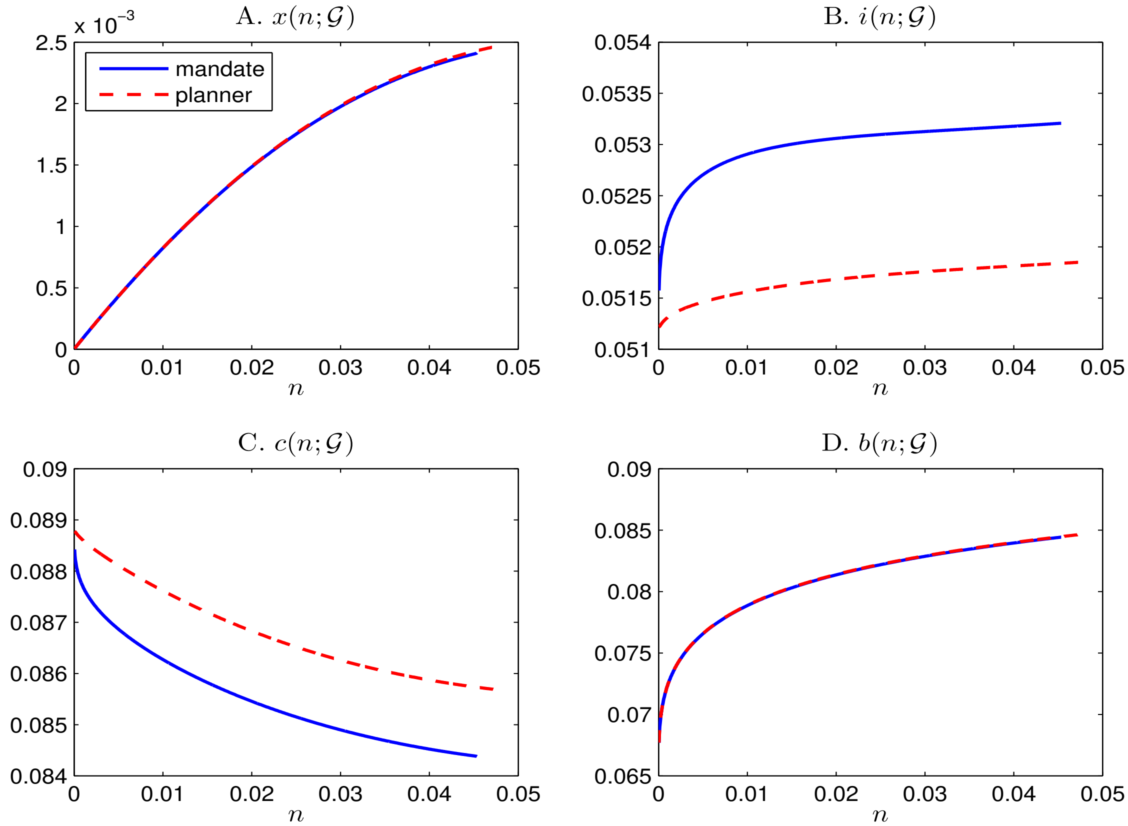

Figure 1: This figure plots the aggregate mitigation spending $\mathbf{x}$, aggregate investment $\mathbf{i}$, aggregate consumption $\mathbf{c}$ and the aggregate welfare measure b as functions of the scaled decarbonization capital stock $\mathbf{n}$ in state $\mathcal{G}$. The parameters values are reported in Table 1.

dynamics of the economy. All these aggregates are dependent on the ratio of decarbonization capital to physical capital, $\mathbf{n}$. For all four panels, the blue lines indicate the optimal solution under mandates and the red lines describe the planner's first-best solution.

Panel A shows that the solution under mandates tracks the planner's first-best solution well up to the steady-state $\mathbf{n}^* = 4.54\%$ under mandates in state $\mathcal{G}$. The planner's solution peaks at a higher value $\mathbf{n}^* = 4.76\%$ at the steady state. That is, the mitigation spending under the optimal mandate is only materially below the first-best when $\mathbf{n}$ is sufficiently close the planner's steady state $\mathbf{n}^* = 4.76\%$. This is intuitive as the marginal return of mitigation is quite high when $\mathbf{n}$ is not too high.

Panel B shows that investment $\mathbf{i}$ is higher under the mandate than in the planner's solution in state $\mathcal{G}$. As we discussed earlier, as firms in the competitive market economy (even with mandates) do not take into account the impact of their capital accumulation decisions on the aggregate variables, they over-invest compared with the planner. In contrast, the planner fully takes into account more decarbonization capital stock $\mathbf{N}$ is necessary to effectively protect a larger economy (with a larger $\mathbf{K}$).

Since the sum of $\mathbf{i}$, $\mathbf{c}$, and $\mathbf{x}$ equals the constant productivity A, consumption $\mathbf{c}$ is lower in the market economy under the mandate than in the planner's solution as we see from Panel C. As risk mitigation is a public good, $\mathbf{n} = 0$ is the laissez faire market economy outcome with no mandate. Therefore, mandates move all three of these policies in the market solution much closer towards the planner's solution.

Panel D shows that the welfare measure $b(\mathbf{n}; \mathcal{G})$ for the mandate solution is almost identical to that in the planner's economy for the levels of $\mathbf{n}$ up to the steady-state level of $\mathbf{n}^*$ under optimal mandate, i.e., for $\mathbf{n} \leq \mathbf{n}^* = 4.54\%$. This is good news as mandates are effectively incentivizing firms to reform and contribute to decarbonization. In fact, the welfare gains are large. Under the unmitigated competitive market solution, the welfare measure (proportional to the certainty equivalent wealth) $b(0; \mathcal{G})$ is 0.068. This measure at the steady state, $b(\mathbf{n}^*; \mathcal{G})$, is 0.085 under the mandate's solution and similarly under the planner's solution.

We thus obtain a 25% gain in terms of the society's willingness to pay (in units of consumption goods/dollars) for an optimal mandate. The magnitudes are large as the world (based on the estimates that we use from the literature) with a tipping point and no mitigation is dismal. However, the market economy with mandates still falls short of delivering the planner's first-best steady state level of $\mathbf{n}^*_{FB}$, which in our numerical example is about 5% higher than the $\mathbf{n}^* = 4.54\%$.

Weather disaster arrivals $\lambda(\mathbf{n}; \mathcal{G})$ and the aggregate growth rate $\mathbf{g}(\mathbf{n}; \mathcal{G})$. In Figure 2, we examine how the weather disaster arrival rate $\lambda(\mathbf{n}; \mathcal{G})$ and the expected aggregate growth rate $\mathbf{g}(\mathbf{n}; \mathcal{G})$ vary with $\mathbf{n}$. Panel A shows that the disaster arrival rate $\lambda(\mathbf{n}; \mathcal{G})$ falls with $\mathbf{n}$, as the society builds up the decarbonization capital, the climate tipping point arrival is delayed and the the disaster arrival rate also falls. Additionally, the arrival rate $\lambda(\mathbf{n}; \mathcal{G})$

in the two economies are identical up to the steady-state level of $\mathbf{n}^* = 4.54\%$ in the market economy with mandates. This is because $\lambda(\mathbf{n};\mathcal{S})$ solely depends on $\mathbf{n}$, which can be seen from $\lambda(\mathbf{n};\mathcal{S}) = \lambda_0^{\mathcal{S}}(1-\mathbf{n}^{\lambda_1})$ given in (84).

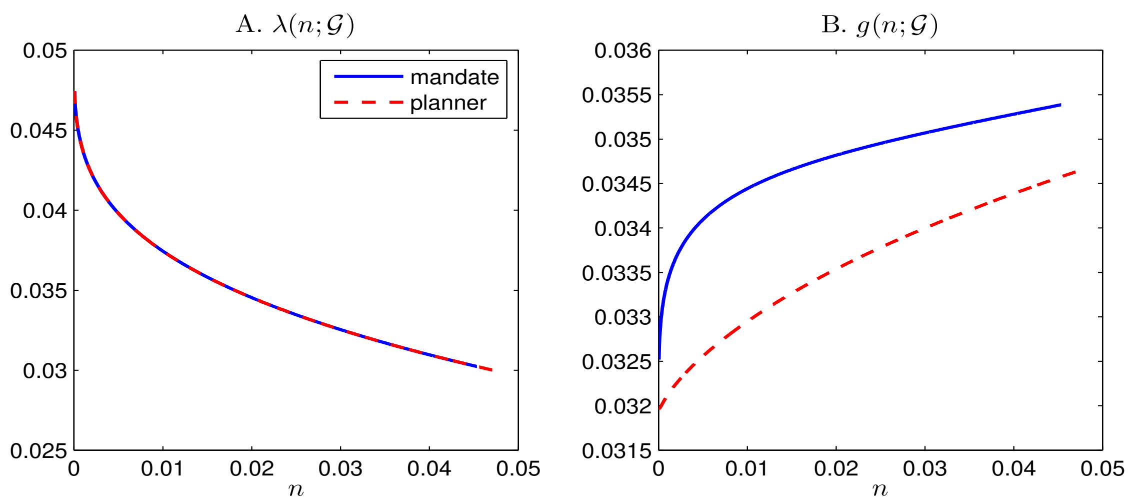

Figure 2: This figure plots the jump arrival rate $\lambda(\,\cdot\,;\mathcal{G})$ and the expected growth rate $g(\,\cdot\,;\mathcal{G})$ as functions of the scaled decarbonization capital stock $\mathbf{n}$ in the $\mathcal{G}$ state. The parameters values are reported in Table 1.

Panel B shows that the expected growth rate $\mathbf{g}(\mathbf{n};\mathcal{G})$ rises with decarbonization-to-productive capital ratio. There are three forces determining $\mathbf{g}(\mathbf{n};\mathcal{G})$: the investment channel $\mathbf{i}$, the expected loss given a disaster arrival and the expected value destruction due to the expected tipping point arrival, which can be seen from (87). Quantitatively, the investment channel $\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))$ dominates growth. Since the aggregate investment-capital ratio $\mathbf{i}$ is higher in the market economy with mandates than in the planner's economy due to externality, the growth rate in the mandate economy is thus also higher than in the planner's economy. The difference in the two growth rates are quantitatively significant.

Transition Time. Next, in Figure 3, we plot the transition path of $\mathbf{n}_t$ over time t conditional on no climate transition from $\mathcal{G}$ to $\mathcal{B}$ before reaching the steady state $\mathbf{n}^*(\mathcal{G})$ in state $\mathcal{G}$. Again, we see that the society reaches a higher steady state $\mathbf{n}^*_{FB} = 4.76\%$ under the planner's solution than $\mathbf{n}^* = 4.54\%$ under the mandate but the differences are modest as the

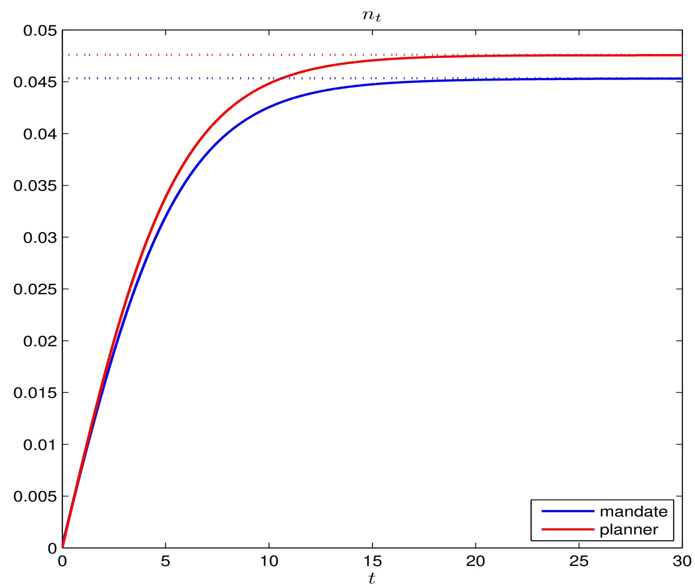

Figure 3: This figure plots the transition path of $\mathbf{n}_t$ over time conditional on the economy is in state $\mathcal{G}$ until reaching its steady state $\mathbf{n}^*(\mathcal{G})$ for both the market economy with mandates and the planner's economy. The first-best steady state is $\mathbf{n}^*_{FB}(\mathcal{G}) = 4.76\%$ for the planner's economy and the steady state is $\mathbf{n}^*(\mathcal{G}) = 4.54\%$ for the market economy with mandates in state $\mathcal{G}$. The parameters values are reported in Table 1.

welfare-maximizing mandate approximates the first-best solution.

6.5 Cost-of-capital Wedge

In Figure 4, we analyze the costs of accumulating decarbonization capital to firms and investors. We consider three investment mandate levels: $\alpha = 10\%, 20\%, 30\%$. For these three levels of α, our mandate solution can all be implemented. Naturally when α is lower, each firm needs to spend more to qualify for the sustainable portfolio but it also gets compensated with a larger cost-of-capital wedge in equilibrium.

The blue solid lines depict the solution when 10% of wealth is indexed to sustainable mandates ($\alpha = 10\%$). The qualifying standard $m(\mathbf{n}; \mathcal{G})$ increases with $\mathbf{n}$, peaking at around 2.4% (which equals $\mathbf{x}^* = 0.24\%/10\%$) per annum at the steady state. That is, a firm would need to spend 2.4% of its capital on decarbonization to qualify for the sustainable portfolio at the steady state. The sustainable firms get compensated for their contributions with a significant cost-of-capital wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$, which equals $\mathbf{x}^*(\mathcal{G})/(\alpha\mathbf{q}^*(\mathcal{G}))$ at the steady state. Quantitatively, in our analysis, this wedge is 1% per annum in the market economy with mandates, which is about 10% lower than the wedge of 1.1% in the planner's economy. This difference is primarily due to the fact that Tobin's q in our mandated market economy is higher than in the planner's economy.[26]

As we increase α from 10% to 20% (the black dotted line) and 30% (the red dashed line), the qualification standard falls and so do the cost-of-capital wedges. Current estimates have sustainable finance mandates α at around 20% (the black dotted line). Firms have to spend around 1.2% (which equals $\mathbf{x}^*/\alpha = 0.24\%/20\%$) at the steady state to qualify as sustainable and get compensated around 0.5% (which equals $\mathbf{x}^*/(\alpha\mathbf{q}^*) = 0.5\%$) via the cost-of-capital wedge in the market economy with mandates. Note that tthe optimal ramp-up schedules of both $m(\mathbf{n};\mathcal{G})$ and cost-of-capital wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ are non-linear. Moreover, our model clearly predicts that as decarbonization $\mathbf{n}$ ramps up, both qualification standards and the cost to investors rise.

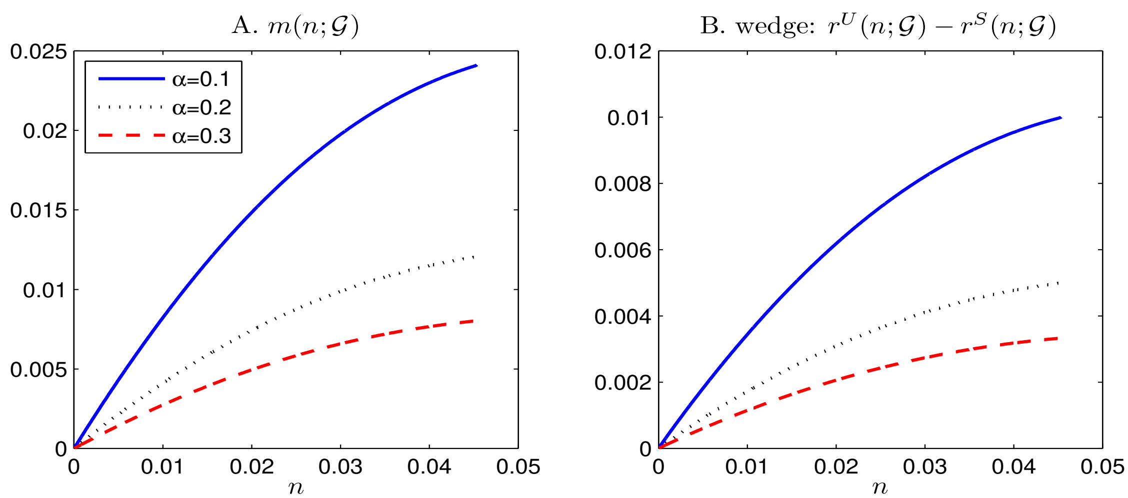

Figure 4: This figure plots the mitigation spending mandate $m(\mathbf{n};\mathcal{G})$ and the cost-of-capital wedge $r^U(\mathbf{n};\mathcal{G}) - r^S(\mathbf{n};\mathcal{G})$ as functions of scaled decarbonization capital stock $\mathbf{n}$ in state $\mathcal{G}$. The parameters values are reported in Table 1.

[26]This follows from $\mathbf{q} = 1/\phi'(\mathbf{i})$ and $\mathbf{i}$ is higher in the market economy with mandates than in the planner's economy as firms do not internalize the negative externality of investing on increasing the risk of climate transition, as we discussed earlier.

6.6 Comparative Statics

Table 3: THE EFFECT OF ζ_0 AND ζ_1 IN THE STEADY STATE FOR THE MARKET ECONOMY WITH MANDATES UNDER THE $\mathcal{G}$ STATE.

variable	notation	baseline	$\zeta_0 = 0.05$	$\zeta_1 = 0.05$
scaled mitigation spending	$\mathbf{x}^*$	0.24%	0.19%	0.21%
scaled decarbonization stock	$\mathbf{n}^*$	4.54%	3.56%	3.84%
scaled aggregate investment	$\mathbf{i}^*$	5.32%	5.39%	5.38%
Tobin's q	$\mathbf{q}^*$	2.41	2.45	2.45
scaled aggregate consumption	$\mathbf{c}^*$	8.44%	8.42%	8.41%
expected GDP growth rate	$\mathbf{g}^*$	3.54%	3.61%	3.60%
(real) risk-free rate	r	1.12%	1.12%	1.12%
stock market risk premium	$r^M - r$	5.92%	5.92%	5.92%
time from $\mathbf{n} = 0$ to $0.99\mathbf{n}^*$ conditional in $\mathcal{G}$	$\mathbf{t}(0.99)$	16.25	12.53	13.21

$\mathbf{t}(0.99)$ is the transition time from $\mathbf{n}_0 = 0$ to $\mathbf{n} = 0.99\mathbf{n}^*$, where $\mathbf{n}^*$ is the steady-state value of $\mathbf{n}$ in the respective economy, conditional on no climate transition from $\mathcal{G}$ to $\mathcal{B}$ before $\mathbf{t}(0.99)$.

Table 3 reports how the steady-state market mandate equilibrium outcomes change as we vary our key parameters involving the climate tipping point. The column labeled "baseline" repeats the baseline results from Table 2 to ease our comparisons. The column labeled "$\zeta_0 = 0.05$" reports the results as we increase the expected tipping point arrival time from 10 years to 20 years absent mitigation (i.e., reducing ζ_0 by half from 0.1 to 0.05). Mitigation spending $\mathbf{x}^*$ falls from 0.24% to 0.19% per annum and there is still a large steady state $\mathbf{n}^*$ of 3.56% even with reduced risk of a tipping point arrival. The other variables do not differ much with the baseline. The column labeled "$\zeta_1 = 0.05$" reports the results when we have a more effective mitigation technology reducing the tipping point risk. as the mitigation technology is more effective, mitigation spending $\mathbf{x}^*$ falls from 0.24% in our baseline to 0.21% per annum and the steady state $\mathbf{n}^*$ increases to 3.84%. The effects of reducing ζ_1 on other variables are much less significant, similar to what we found for the effect of changing ζ_0.

In sum, our findings here are consistent with the view that anticipating the tipping point arrival generates an act-now effect. This is because it takes time and it is costly to accumulate decarbonization capital, there is a desire for preparedness or acting now even if there is only a modest probability of the climate entering into an absorbing bad state. Because in this $\mathcal{B}$

state, weather disasters are so frequent, starting the accumulation of decarbonization capital once we are in the bad state would be too late and take many years, which causes a significant welfare loss.

7 Conclusion

Sustainable finance mandates have grown significantly in the last decade in lieu of government failures to address climate disaster externalities. Firms that spend enough resources on mitigation of these externalities qualify for sustainable finance mandates. These mandates incentivize otherwise ex-ante identical unsustainable firms to become sustainable for a lower cost of capital. We present and solve a dynamic stochastic general equilibrium model featuring the gradual accumulation of nonproductive but protective decarbonization capital to study the welfare consequences. The model is tractable, including a simple formula that characterizes the cost-of-capital wedge between sustainable and unsustainable firms that captures the costs of decarbonization for shareholders. We find that the welfare-maximizing mandate well approximates the planner's mitigation and welfare levels. There are a number of testable implications that can be taken to the data.

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Appendices

A Household's Optimization Problem

First, we consider the problem in the $\mathcal{B}$ state.

The $\mathcal{B}$ state. Using the same procedure as in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we can show that the jump hedging demand is zero (for all levels of Z) in equilibrium. Therefore, we may rewrite the household's wealth dynamics given by (30) in the $\mathcal{B}$ state as follows

$$\begin{aligned} dW_t = & \left[r(\mathbf{n}_{t-};\mathcal{B})W_{t-} - C_{t-}\right]dt + \left[r^S(\mathbf{n}_{t-};\mathcal{B})\pi^S_{t-} + r^U(\mathbf{n}_{t-};\mathcal{B})(1-\pi^S_{t-}) - r(\mathbf{n}_{t-};\mathcal{B})\right]H_{t-}\,dt \\ & + \sigma H_{t-} d\mathcal{W}_t - (1-Z)\,H_{t-}(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{B})dt)\,, \end{aligned} \tag{A.88}$$

where $\pi^S = H^S/(H^S+H^U) = H^S/H$.

The following HJB equation characterizes the value function $V(W,\mathbf{n};\mathcal{B})$:

$$\begin{aligned} 0 = \max_{C,\pi^S,H} & \left[r(\mathbf{n};\mathcal{B})W - C + \left(r^S(\mathbf{n};\mathcal{B})\pi^S + (r^U(\mathbf{n};\mathcal{B})(1-\pi^S) - r(\mathbf{n};\mathcal{B})\right)H + \lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))H\right]V_W \\ & + f(C,V;\mathcal{B}) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right]\mathbf{n}V_{\mathbf{n}} + \frac{\sigma^2H^2V_{WW}}{2} \\ & + \lambda(\mathbf{n};\mathcal{B})\mathbb{E}\left[V\left(W-(1-Z)H,\mathbf{n};\mathcal{B}\right) - V(W,\mathbf{n};\mathcal{B})\right]\,, \end{aligned} \tag{A.89}$$

subject to $\pi^S \geq \alpha$. Because the S- and the U-portfolio returns have the same (diffusion and jump) risk exposures with probability one, if $r^S > r^U$ were true, the optimality condition for π^S would imply counterfactually $\pi^S \to \infty$, as (A.89) is linear in π^S. Since $\pi^S \to \infty$ cannot be an equilibrium, it is necessary $r^S \leq r^U$. Indeed, we show that $r^S < r^U$ holds with strict inequality in equilibrium and the constraint $\pi^S = \alpha$ binds for households. We thus conclude that the household's value function satisfies the simplified HJB equation (32).

The FOC for consumption C is the standard condition given by (34). The FOC for the portfolio allocation to the risky asset, H, is given by

$$\begin{aligned} 0 = & [\pi^S r^S(\mathbf{n};\mathcal{B}) + (1-\pi^S)r^U(\mathbf{n};\mathcal{B}) - r(\mathbf{n};\mathcal{B}) + \lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))]V_W + \sigma^2 H V_{WW} \\ & - \lambda(\mathbf{n};\mathcal{B})\mathbb{E}\left[(1-Z)V_W\left(W-(1-Z)H,\mathbf{n};\mathcal{B}\right)\right]\,. \end{aligned} \tag{A.90}$$

Next, we consider the $\mathcal{G}$ state.

The $\mathcal{G}$ state. The household's wealth process in the $\mathcal{G}$ state evolves as follows:

$$\begin{aligned} dW_t = & \left[r(\mathbf{n}_{t-};\mathcal{G})W_{t-} - C_{t-}\right]dt \\ & + \left[(r^S(\mathbf{n}_{t-};\mathcal{G}) - r(\mathbf{n}_{t-};\mathcal{G}))\pi^S_{t-} + (r^U(\mathbf{n}_{t-};\mathcal{G}) - r(\mathbf{n}_{t-};\mathcal{G}))(1-\pi^S_{t-})\right]H_{t-}dt \\ & + \left[d\mathcal{W}_t - (1-Z)\left(d\mathcal{J}_t - \lambda(\mathbf{n}_{t-};\mathcal{G})dt\right) + \frac{\mathbf{q}(\mathbf{n}_{t-};\mathcal{B}) - \mathbf{q}(\mathbf{n}_{t-};\mathcal{G})}{\mathbf{q}(\mathbf{n}_{t-};\mathcal{G})}\left(d\widetilde{\mathcal{J}}_t - \zeta(\mathbf{n}_{t-})dt\right)\right]H_{t-}\,. \end{aligned} \tag{A.91}$$

Using essentially the same reasoning as for the $\mathcal{B}$ state, we obtain $\pi^S = \alpha$ in equilibrium and the following HJB equation for the value function, $V(W, \mathbf{n}; \mathcal{G})$:

$$
\begin{aligned}
0 = \max_{C,H} \; & \left[r(\mathbf{n};\mathcal{G})W - C + \left(r^S(\mathbf{n};\mathcal{G})\alpha + r^U(\mathbf{n};\mathcal{G})(1-\alpha) - r(\mathbf{n};\mathcal{G}) + \lambda(\mathbf{n};\mathcal{G})(1-\mathbb{E}(Z))\right) H\right] V_W \\
& + \zeta(\mathbf{n}) \frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})} H V_W + f(C,V) + \left[\omega(\mathbf{x}/\mathbf{n}) - \phi(\mathbf{i})\right] \mathbf{n} V_{\mathbf{n}} + \frac{\sigma^2 H^2 V_{WW}}{2} \\
& + \lambda(\mathbf{n};\mathcal{G}) \mathbb{E}\left[V\left(W - (1-Z)H, \mathbf{n};\mathcal{G}\right) - V(W,\mathbf{n};\mathcal{G})\right] \\
& + \zeta(\mathbf{n}) \left[V\left(W - \frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})} H, \mathbf{n};\mathcal{B}\right) - V(W,\mathbf{n};\mathcal{G})\right]. \qquad \text{(A.92)}
\end{aligned}
$$

And the FOC for the portfolio allocation to the risky asset portfolio, H, is given by

$$
\begin{aligned}
0 = & \left[\alpha r^S(\mathbf{n};\mathcal{G}) + (1-\alpha) r^U(\mathbf{n};\mathcal{G}) - r(\mathbf{n};\mathcal{G}) + \lambda(\mathbf{n};\mathcal{G})(1-\mathbb{E}(Z)) + \zeta(\mathbf{n}) \frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\right] V_W \\
& + \sigma^2 H V_{WW} - \lambda(\mathbf{n};\mathcal{G}) \mathbb{E}\left[(1-Z) V_W\left(W - (1-Z)H, \mathbf{n};\mathcal{G}\right)\right] \\
& + \zeta(\mathbf{n}) \frac{\mathbf{q}(\mathbf{n};\mathcal{B}) - \mathbf{q}(\mathbf{n};\mathcal{G})}{\mathbf{q}(\mathbf{n};\mathcal{G})} V_W\left(W - \frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})} H, \mathbf{n};\mathcal{B}\right). \qquad \text{(A.93)}
\end{aligned}
$$

Compared with (A.90), the FOC in the $\mathcal{B}$ state, (A.93) has an additional term capturing the effect of transition risk.

B Market Equilibrium

B.1 Market Equilibrium for a Given Mandate

First, a sustainable firm spends minimally on mitigation: $x^S = \frac{X^S}{K^S}$. Second, in equilibrium, the representative household invests all wealth in the stock market and holds no risk-free asset, $H = W$ and $W = \mathbf{Q}^S + \mathbf{Q}^U$. Third, the representative agent's (dollar amount) investment in the S portfolio is equal to the total market value of sustainable firms, $\pi^S = \alpha$ and (dollar amount) investment for the U portfolio is equal to the total market value of unsustainable firms, $\pi^U = 1 - \alpha$. Finally, goods market clears.

As in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), the risk-free asset holding is zero and $W^{\mathcal{J}} = ZW$, and $H = W = \mathbf{Q}^S + \mathbf{Q}^U = q^S(\mathbf{n};\mathcal{S})\mathbf{K}^S + q^U(\mathbf{n};\mathcal{S})\mathbf{K}^U = q(\mathbf{n};\mathcal{S})(\mathbf{K}^S + \mathbf{K}^U) = q(\mathbf{n};\mathcal{S})\mathbf{K}$. Additionally, using $\pi^S = \alpha$ and (A.90), in state $\mathcal{B}$ we obtain

$$
\alpha r^S(\mathbf{n};\mathcal{B}) + (1-\alpha) r^U(\mathbf{n};\mathcal{B}) = r(\mathbf{n};\mathcal{B}) + \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{B}) \mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right] = r^M(\mathbf{n};\mathcal{B}), \qquad \text{(B.94)}
$$

which implies (49). Similarly, in state $\mathcal{G}$ we obtain

$$
\begin{aligned}
r^M(\mathbf{n};\mathcal{G}) &= r(\mathbf{n};\mathcal{G}) + \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{G}) \mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right] + \zeta(\mathbf{n}) \frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})} \left[\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\right)^{-\gamma} - 1\right] \\
&= \alpha r^S(\mathbf{n};\mathcal{G}) + (1-\alpha) r^U(\mathbf{n};\mathcal{G}), \qquad \text{(B.95)}
\end{aligned}
$$

which implies (51).

Notice that as all firms have the same Tobin's q in equilibrium, using (24) we have $i^S(\mathbf{n};\mathcal{S}) = i^U(\mathbf{n};\mathcal{S}) = \mathbf{i}(\mathbf{n};\mathcal{S})$ and

$$\begin{aligned} \mathbf{q}(\mathbf{n};\mathcal{S}) &= \frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - m(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S})}{r^S(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})} \\ &= \frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S})}{r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})}\,. \end{aligned} \tag{B.96}$$

Using $\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha) r^U(\mathbf{n};\mathcal{S}) = r^M(\mathbf{n};\mathcal{S})$, $\mathbf{x} = \alpha m(\mathbf{n};\mathcal{S})$, and (B.96), we obtain

$$\begin{aligned} &\frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S})}{r^M(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})} \\ = &\frac{\alpha(A - \mathbf{i}(\mathbf{n};\mathcal{S}) - m(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S}))}{\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha) r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})} \\ &+\frac{(1-\alpha)(A - \mathbf{i}(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S}))}{\alpha r^S(\mathbf{n};\mathcal{S}) + (1-\alpha) r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})} \\ = &\frac{\alpha \mathbf{q}(\mathbf{n};\mathcal{S})(r^S(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})) + (1-\alpha)\mathbf{q}(\mathbf{n};\mathcal{S})(r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S}))}{\alpha(r^S(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})) + (1-\alpha)(r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S}))} = \mathbf{q}(\mathbf{n};\mathcal{S})\,. \end{aligned} \tag{B.97}$$

And then using

$$\begin{aligned} \mathbf{q}(\mathbf{n};\mathcal{S}) &= \frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S})}{r^M(\mathbf{n};\mathcal{S}) + \theta^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})} \\ &= \frac{A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x} + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S})}{r^M(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S})}\,, \end{aligned} \tag{B.98}$$

we obtain $(A - \mathbf{i}(\mathbf{n};\mathcal{S}) + [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\mathbf{nq}'(\mathbf{n};\mathcal{S}))\theta^U(\mathbf{n};\mathcal{S}) = \mathbf{x}(r^U(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S}))$ and $\theta^U(\mathbf{n};\mathcal{S}) = \mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{q}(\mathbf{n};\mathcal{S}) = \alpha m(\mathbf{n};\mathcal{S})/q(\mathbf{n};\mathcal{S})$ as shown in (46).

The optimal consumption rule given in (36) implies

$$c(\mathbf{n};\mathcal{S}) = \frac{C}{\mathbf{K}} = \frac{C}{W} q(\mathbf{n};\mathcal{S}) = \rho^{\psi} u(\mathbf{n};\mathcal{S})^{1-\psi} \mathbf{q}(\mathbf{n};\mathcal{S})\,. \tag{B.99}$$

And then substituting $c(\mathbf{n};\mathcal{S})$ given by (B.99) and the value function given in (35) into the HJB equation (32), in $\mathcal{B}$ state we obtain

$$\begin{aligned} 0 &= \frac{1}{1-\psi^{-1}}\left(\frac{c(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} - \rho\right) + \left(\alpha r^S(\mathbf{n};\mathcal{B}) + (1-\alpha) r^U(\mathbf{n};\mathcal{B}) - \frac{c(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} + \lambda(\mathbf{n};\mathcal{B})(1 - \mathbb{E}(Z))\right) \\ &\quad + [\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))]\,\frac{\mathbf{n}u'(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{B})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\ &= \frac{1}{1-\psi^{-1}}\left(\frac{c(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} - \rho\right) + \left(r^M(\mathbf{n};\mathcal{B}) - \frac{c(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} + \lambda(\mathbf{n};\mathcal{B})(1 - \mathbb{E}(Z))\right) \\ &\quad + [\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))]\,\frac{\mathbf{n}u'(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{B})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right]\,. \end{aligned} \tag{B.100}$$

By using (B.98) and the goods market clear condition, we obtain

$$\frac{c(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})} = r^M(\mathbf{n};\mathcal{S}) - g(\mathbf{n};\mathcal{S}) - [\omega(\mathbf{x}(\mathbf{n};\mathcal{S})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{S}))]\,\frac{\mathbf{n}\mathbf{q}'(\mathbf{n};\mathcal{S})}{\mathbf{q}(\mathbf{n};\mathcal{S})}\,. \tag{B.101}$$

Substituting (B.101) into (B.100) and using $c(\mathbf{n};\mathcal{S}) = A - \mathbf{i}(\mathbf{n};\mathcal{S}) - \mathbf{x}(\mathbf{n};\mathcal{S})$ and (B.99), we obtain

$$\begin{aligned}
0 \;=\;& \frac{1}{1-\psi^{-1}}\left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{B}) - \mathbf{x}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} - \rho\right) + \phi(\mathbf{i}(\mathbf{n};\mathcal{B})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\
&+ [\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))]\left(\frac{\mathbf{n}q'(\mathbf{n};\mathcal{B})}{q(\mathbf{n};\mathcal{B})} + \frac{\mathbf{n}u'(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{B})}\right) \\
0 \;=\;& \frac{1}{1-\psi^{-1}}\left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{B}) - \mathbf{x}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{B})} - \rho\right) + \phi(\mathbf{i}(\mathbf{n};\mathcal{B})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\
&+ [\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{B}))]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}q'(\mathbf{n};\mathcal{B})}{q(\mathbf{n};\mathcal{B})} - \frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n};\mathcal{B}) + \mathbf{n}\mathbf{x}'(\mathbf{n};\mathcal{B})}{A - \mathbf{i}(\mathbf{n};\mathcal{B}) - \mathbf{x}(\mathbf{n};\mathcal{B})}\right),
\end{aligned} \tag{B.102}$$

which implies (53).

Similarly, in the $\mathcal{G}$ state we obtain

$$\begin{aligned}
0 \;=\;& \frac{1}{1-\psi^{-1}}\left(\psi^{-1}\frac{c(\mathbf{n};\mathcal{G})}{\mathbf{q}(\mathbf{n};\mathcal{G})} - \rho\right) + \left(r^M(\mathbf{n};\mathcal{G}) + \lambda(\mathbf{n};\mathcal{G})(1-\mathbb{E}(Z)) + \zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\right) \\
&+ [\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{G}))]\,\frac{\mathbf{n}u'(\mathbf{n};\mathcal{G})}{u(\mathbf{n};\mathcal{G})} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\
&+ \frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{B})q(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{G})q(\mathbf{n};\mathcal{G})}\right)^{1-\gamma} - 1\right] \\
=\;& \frac{1}{1-\psi^{-1}}\left(\frac{A - \mathbf{i}(\mathbf{n};\mathcal{G}) - \mathbf{x}(\mathbf{n};\mathcal{G})}{\mathbf{q}(\mathbf{n};\mathcal{G})} - \rho\right) + \phi(\mathbf{i}(\mathbf{n};\mathcal{G})) - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\
&+ \frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{(A - \mathbf{i}(\mathbf{n};\mathcal{B}) - \mathbf{x}(\mathbf{n};\mathcal{B}))\mathbf{q}(\mathbf{n};\mathcal{G})^{\psi}}{(A - \mathbf{i}(\mathbf{n};\mathcal{G}) - \mathbf{x}(\mathbf{n};\mathcal{G}))\mathbf{q}(\mathbf{n};\mathcal{B})^{\psi}}\right)^{\frac{1-\gamma}{1-\psi}} - 1\right] \\
&+ [\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n}) - \phi(\mathbf{i}(\mathbf{n};\mathcal{G}))]\left(\frac{\psi}{1-\psi}\frac{\mathbf{n}q'(\mathbf{n};\mathcal{G})}{q(\mathbf{n};\mathcal{G})} - \frac{1}{1-\psi}\frac{\mathbf{n}\mathbf{i}'(\mathbf{n};\mathcal{G}) + \mathbf{n}\mathbf{x}'(\mathbf{n};\mathcal{G})}{A - \mathbf{i}(\mathbf{n};\mathcal{G}) - \mathbf{x}(\mathbf{n};\mathcal{G})}\right),
\end{aligned} \tag{B.103}$$

which implies (54).

Finally, we obtain the equilibrium risk-free rate in the $\mathcal{B}$ state given by (50) by substituting $r^M(\mathbf{n};\mathcal{B}) = r(\mathbf{n};\mathcal{B}) + \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{B})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right]$ into (B.101). Similarly, we obtain the risk-free rate in the $\mathcal{G}$ state given by (52) by substituting

$$r^M(\mathbf{n};\mathcal{G}) = r(\mathbf{n};\mathcal{G}) + \gamma\sigma^2 + \lambda(\mathbf{n};\mathcal{G})\mathbb{E}\left[(1-Z)(Z^{-\gamma}-1)\right] + \zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n};\mathcal{G}) - \mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\left[\left(\frac{\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}\right)^{-\gamma} - 1\right]$$

into (B.101).

B.2 Welfare-maximizing mandate

Using (35) and $W = \mathbf{q}(\mathbf{n};\mathcal{S})\mathbf{K}$ in equilibrium, we may rewrite the ODE (37) for $u(\mathbf{n};\mathcal{B})$ in the $\mathcal{B}$ state as:

$$\begin{aligned}
0 \;=\;& \frac{\rho}{1-\psi^{-1}}\left[\frac{c(\mathbf{n};\mathcal{B})}{q(\mathbf{n};\mathcal{B})}-1\right] \\
&+\left(\alpha r^{S}(\mathbf{n};\mathcal{B})+(1-\alpha)r^{U}(\mathbf{n};\mathcal{B})+\lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))-\frac{c(\mathbf{n};\mathcal{B})}{q(\mathbf{n};\mathcal{B})}\right) \\
&+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{B})}-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \qquad \text{(B.104)}
\end{aligned}$$

Then using (58) and $\mathbf{q}(\mathbf{n};\mathcal{B}) = \frac{1}{\phi'(\mathbf{i}(\mathbf{n};\mathcal{B}))}$, we obtain

$$\begin{aligned}
0 \;=\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{B})-\mathbf{x}(\mathbf{n};\mathcal{B})}{b(\mathbf{n};\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\left(\alpha r^{S}(\mathbf{n};\mathcal{B})+(1-\alpha)r^{U}(\mathbf{n};\mathcal{B})+\lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))\right) \\
&-\frac{c(\mathbf{n};\mathcal{B})}{q(\mathbf{n};\mathcal{B})}+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{B})}-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \qquad \text{(B.105)}
\end{aligned}$$

Using (B.101) and $g(\mathbf{n};\mathcal{B}) = \phi(\mathbf{i}(\mathbf{n};\mathcal{B})) - \lambda(\mathbf{n};\mathcal{B})(1-\mathbb{E}(Z))$ to simplify (B.105), we obtain:

$$\begin{aligned}
0 =\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}-\mathbf{x}}{b(\mathbf{n};\mathcal{B})}\right)^{1-\psi^{-1}}-1\right]+\phi(\mathbf{i}(\mathbf{n};\mathcal{B})) \\
&+\left(\omega(\mathbf{x}(\mathbf{n};\mathcal{B})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{B}))\right)\left(\frac{\mathbf{n}u'(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{B})}+\frac{\mathbf{n}q'(\mathbf{n};\mathcal{B})}{q(\mathbf{n};\mathcal{B})}\right)-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{B})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]. \qquad \text{(B.106)}
\end{aligned}$$

And then, using $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S})\times q(\mathbf{n};\mathcal{S})$, we obtain (59).

Similarly, we may rewrite the ODE (38) for $u(\mathbf{n};\mathcal{G})$ in the $\mathcal{G}$ state as:

$$\begin{aligned}
0 =\;& \frac{\rho}{1-\psi^{-1}}\left[\frac{c(\mathbf{n};\mathcal{G})}{q(\mathbf{n};\mathcal{G})}-1\right]+\alpha r^{S}(\mathbf{n};\mathcal{G})+(1-\alpha)r^{U}(\mathbf{n};\mathcal{G})+\lambda(\mathbf{n};\mathcal{G})(1-\mathbb{E}(Z)) \\
&+\zeta(\mathbf{n})\frac{\mathbf{q}(\mathbf{n};\mathcal{G})-\mathbf{q}(\mathbf{n};\mathcal{B})}{\mathbf{q}(\mathbf{n};\mathcal{G})}-\frac{c(\mathbf{n};\mathcal{G})}{q(\mathbf{n};\mathcal{G})}+\left[\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right]\frac{\mathbf{n}u'(\mathbf{n};\mathcal{G})}{u(\mathbf{n};\mathcal{G})}-\frac{\gamma\sigma^2}{2} \\
&+\frac{\lambda(\mathbf{n};\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]+\frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{B})q(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{G})q(\mathbf{n};\mathcal{G})}\right)^{1-\gamma}-1\right] \\
=\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{A-\mathbf{i}(\mathbf{n};\mathcal{G})-\mathbf{x}(\mathbf{n};\mathcal{G})}{b(\mathbf{n};\mathcal{G})}\right)^{1-\psi^{-1}}-1\right]+\left(\omega(\mathbf{x}(\mathbf{n};\mathcal{G})/\mathbf{n})-\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))\right)\left(\frac{\mathbf{n}u'(\mathbf{n};\mathcal{G})}{u(\mathbf{n};\mathcal{G})}+\frac{\mathbf{n}q'(\mathbf{n};\mathcal{G})}{q(\mathbf{n};\mathcal{G})}\right) \\
&+\phi(\mathbf{i}(\mathbf{n};\mathcal{G}))-\frac{\gamma\sigma^2}{2}+\frac{\lambda(\mathbf{n};\mathcal{G})}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma})-1\right]+\frac{\zeta(\mathbf{n})}{1-\gamma}\left[\left(\frac{u(\mathbf{n};\mathcal{B})q(\mathbf{n};\mathcal{B})}{u(\mathbf{n};\mathcal{G})q(\mathbf{n};\mathcal{G})}\right)^{1-\gamma}-1\right]. \qquad \text{(B.107)}
\end{aligned}$$

Finally, using $b(\mathbf{n};\mathcal{S}) = u(\mathbf{n};\mathcal{S})\times q(\mathbf{n};\mathcal{S})$, we obtain (60).

C Planner's Problem

The following coupled HJB equations characterize the planner's optimization problem in states $\mathcal{G}$ and $\mathcal{B}$:

$$0 = \max_{\mathbf{C},\,\mathbf{i},\,\mathbf{x}} \; f(\mathbf{C}, J; \mathcal{B}) + \phi(\mathbf{i})\mathbf{K}J_{\mathbf{K}} + \omega(\mathbf{x}/\mathbf{n})\mathbf{N}J_{\mathbf{N}} + \frac{\mathbf{K}^2 J_{\mathbf{KK}} + 2\mathbf{NK}J_{\mathbf{KN}} + \mathbf{N}^2 J_{\mathbf{NN}}}{2}\sigma^2$$
$$+ \lambda(\mathbf{n}; \mathcal{B})\mathbb{E}\left[J\left(Z\mathbf{K}, Z\mathbf{N}; \mathcal{B}\right) - J(\mathbf{K}, \mathbf{N}; \mathcal{B})\right] , \qquad \text{(C.108)}$$

$$0 = \max_{\mathbf{C},\,\mathbf{i},\,\mathbf{x}} \; f(\mathbf{C}, J; \mathcal{G}) + \phi(\mathbf{i})\mathbf{K}J_{K} + \omega(\mathbf{x}/\mathbf{n})\mathbf{N}J_{\mathbf{N}} + \frac{\mathbf{K}^2 J_{\mathbf{KK}} + 2\mathbf{NK}J_{\mathbf{KN}} + \mathbf{N}^2 J_{\mathbf{NN}}}{2}\sigma^2$$
$$+ \lambda(\mathbf{n}; \mathcal{G})\mathbb{E}\left[J\left(Z\mathbf{K}, Z\mathbf{N}; \mathcal{G}\right) - J(\mathbf{K}, \mathbf{N}; \mathcal{G})\right] + \zeta(\mathbf{n})\left[J(\mathbf{K}, \mathbf{N}; \mathcal{B}) - J(\mathbf{K}, \mathbf{N}; \mathcal{G})\right] , \qquad \text{(C.109)}$$

subject to the aggregate resource constraint at all t:

$$A\mathbf{K_t} = \mathbf{C_t} + \mathbf{i}_t\mathbf{K_t} + \mathbf{x}_t\mathbf{K_t} \, . \qquad \text{(C.110)}$$

The FOC for the scaled investment $\mathbf{i}$ is

$$f_{\mathbf{C}}(\mathbf{C}, J; \mathcal{S}) = \phi'(\mathbf{i})J_{\mathbf{K}}(\mathbf{K}, \mathbf{N}; \mathcal{S}) \, . \qquad \text{(C.111)}$$

The FOC for the scaled aggregate mitigation spending $\mathbf{x}$ is

$$f_{\mathbf{C}}(\mathbf{C}, J; \mathcal{S}) = \omega'(\mathbf{x}/\mathbf{n})J_{\mathbf{N}}(\mathbf{K}, \mathbf{N}; \mathcal{S}) \, , \qquad \text{(C.112)}$$

if the solution is strictly positive, $\mathbf{x} > 0$. Otherwise, $\mathbf{x} = 0$ as mitigation cannot be negative. The FOCs (C.111) and (C.112) imply that

$$\frac{\omega'(\mathbf{x}/\mathbf{n})}{\phi'(\mathbf{i})} = \frac{J_{\mathbf{K}}(\mathbf{K}, \mathbf{N}; \mathcal{S})}{J_{\mathbf{N}}(\mathbf{K}, \mathbf{N}; \mathcal{S})} \, , \qquad \text{(C.113)}$$

Substituting the agent's value function (56) into the FOCs (C.111)-(C.112) and the HJB equation (C.108)-(C.109) and simplifying, we obtain (67), (68), and (69)-(70).